Charmonium and bottomonium spectral functions from high precision lattice QCD computations

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in collaboration with

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Outline

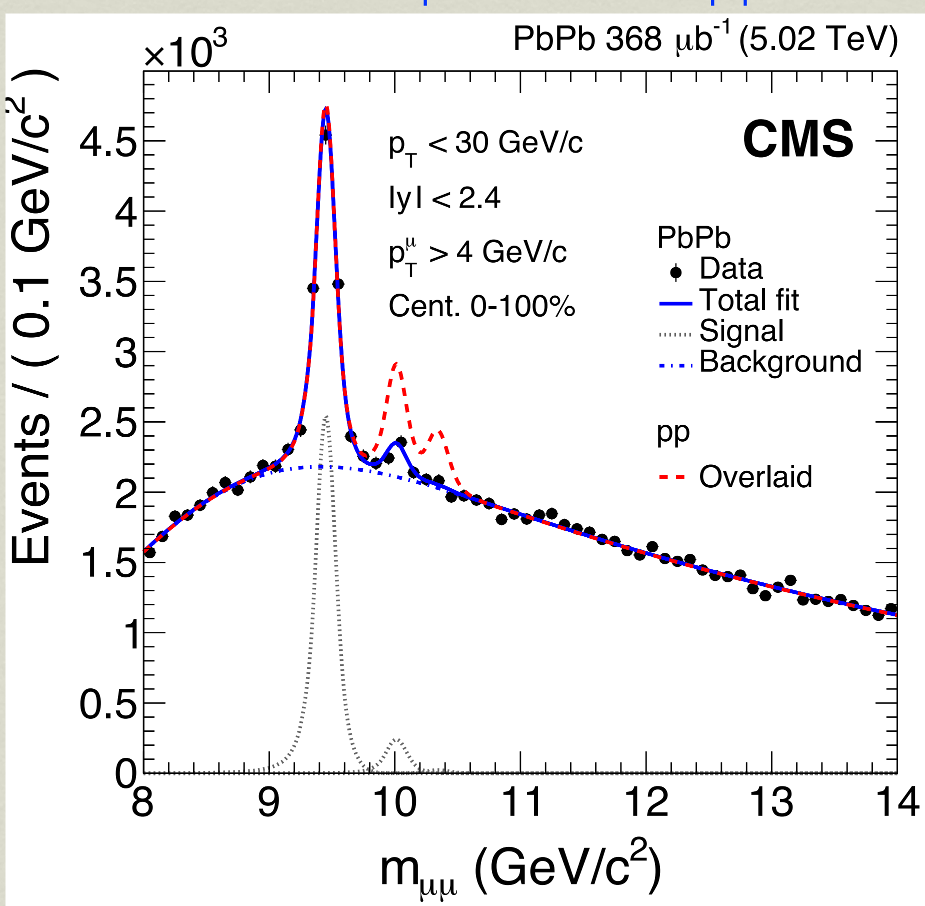
- Motivation
- Correlation functions & spectral functions of bottomonia
- Correlation functions & spectral functions of charmonia
- Summary & Outlook

Motivation

- Hadron spectral functions

—Carry all information about the in-medium properties of quarkonia

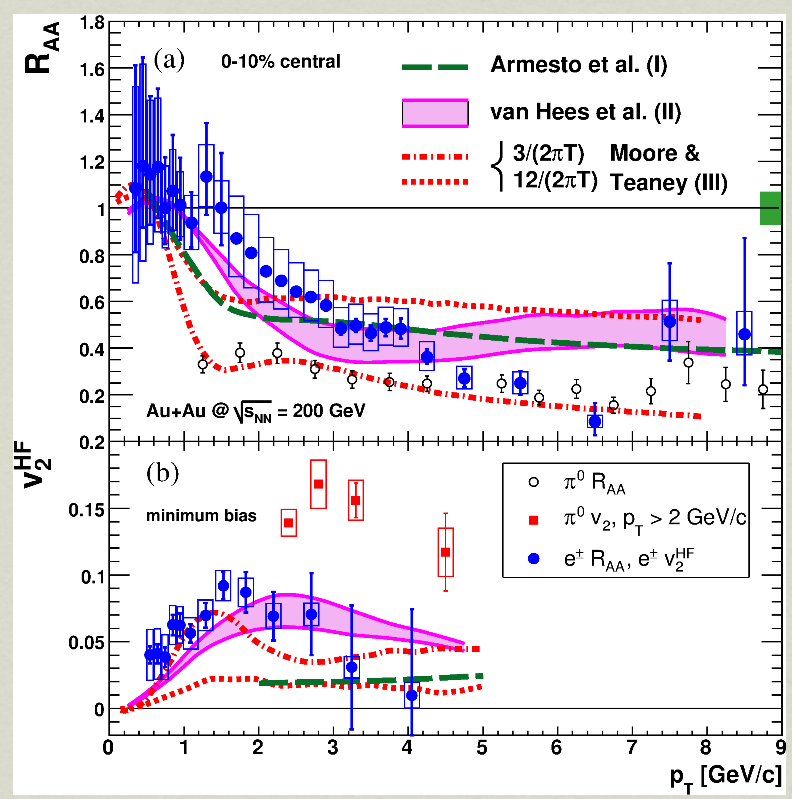
Quarkonia dissociation temperature
➛Understand the quarkonia suppression

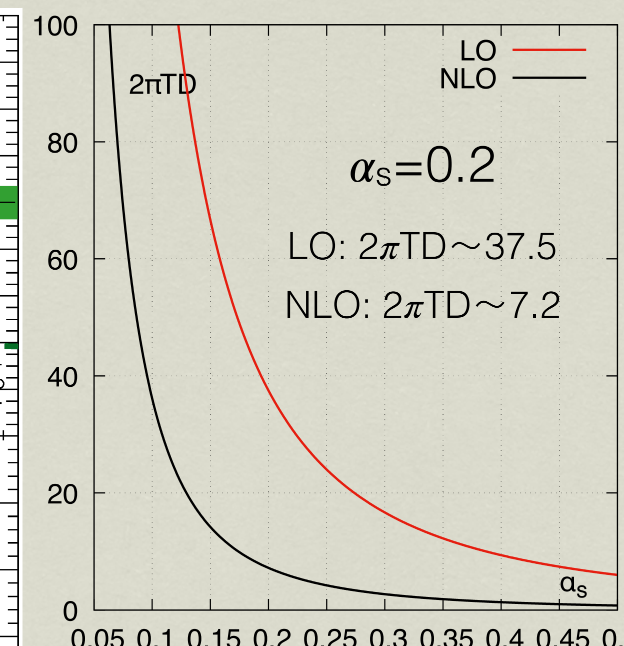


CMS, PRL120(2018) 142301

Heavy quark diffusion coefficient:

$$D = \frac{1}{6\chi_{00}} \lim_{\omega\to 0} \sum_{i=1}^{3} \frac{\rho^V_{ii}(\omega,\mathbf{0})}{\omega}$$





PHENIX, PRL98(2007)172301

Moore & Teaney, PRC71,064904

Caron-Huot & Moore, PRL100,052301

Spectral functions from LQCD

Temporal correlation function relates to spectral function:

discretized ~O(10)

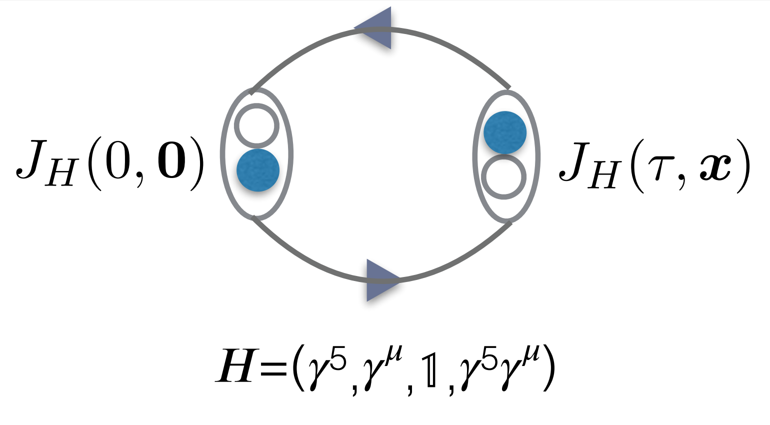
$$K(\omega,\tau,T) \equiv \frac{\cosh\left(\omega(\tau T - 1/2)\right)}{\sinh(\omega/2T)}$$

continuos ~O(1000)

$$G_H(\tau,\vec{p}) = \sum_{x,y,z} \exp(-i\,\vec{p}\cdot\vec{x})\langle J_H(0,\vec{0})J_H^\dagger(\tau,\vec{x})\rangle = \int \frac{d\omega}{2\pi} K(\omega,\tau,T)\rho_H(\omega,\vec{p},T)$$

ill-posed!

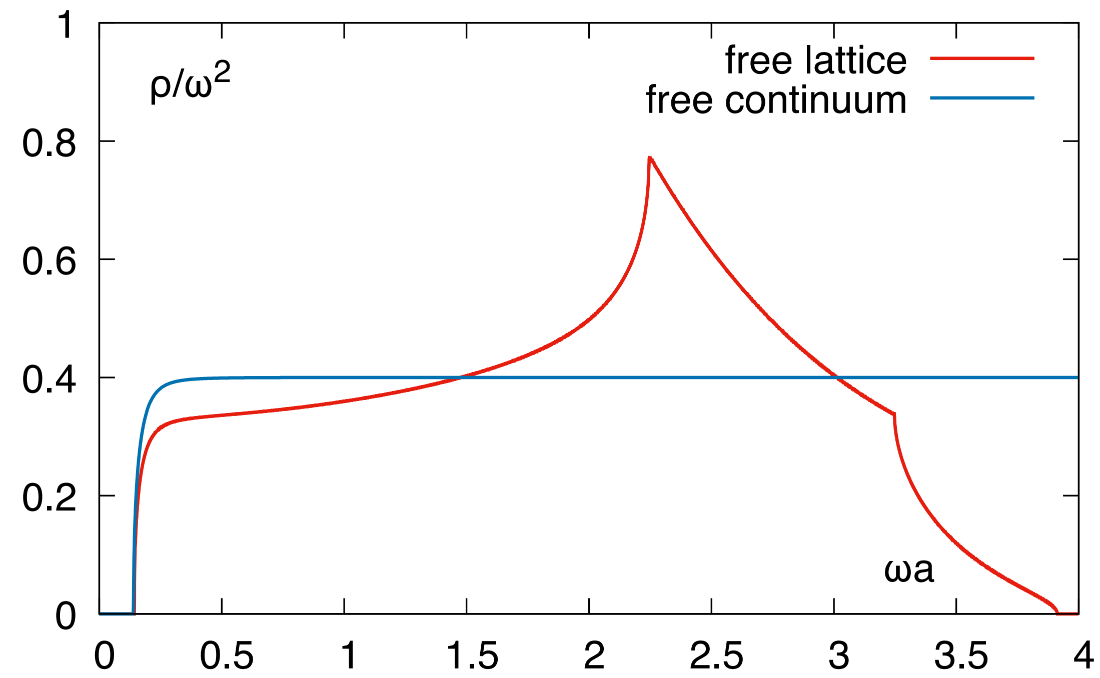
- ✳ New Bayesian Method Y. Burnier and A. Rothkopf, PRL 111,18,182003
- ✳ Backus-Gilbert Method B. B. Brandt, et al., PRD93, 054510(2016)
- ✳ Stochastic Approaches H.-T. Ding, et al., PRD97, 094503
- ✳ **Maximum Entropy Method** M. Asakawa, et al., PPNP. 46(2001) 445-508



Understanding of N_τ & default model (DM) dependence of output SPF is important

Prior information in the default model

- High frequency of the SPF :
  - ✳Free continuum SPF
  - ✳Free lattice SPF H.-T. Ding, et al, arXiv:0910.3098



- Low frequency of the SPF:
  - ✳Non-interacting: F. Karsch et al., PRD68, 014504; G. Aarts et al, NPB726, 93

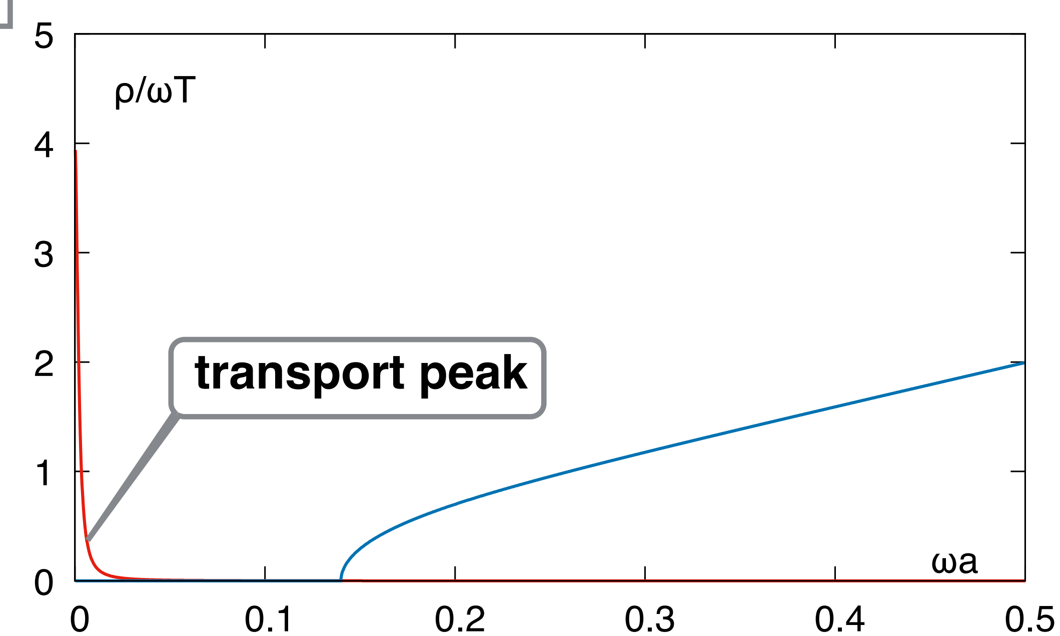
$$\rho_H(\omega) = N_c\left[(a_H^{(1)} + a_H^{(3)})I_1 + (a_H^{(2)} + a_H^{(3)})I_2\right]\omega\delta(\omega) \Longrightarrow D = \frac{1}{6\chi_{00}} \lim_{\omega\to 0} \sum_{i=1}^{3} \frac{\rho_{ii}^V(\omega,\mathbf{0})}{\omega} = \infty$$

$\omega\delta(\omega)$ gives infinite quark diffusion coefficient

  - ✳Interacting: P. Petreczky and D. Teaney, PRD73,014508

$$\delta(\omega) \to \frac{1}{\pi}\frac{\eta}{\omega^2+\eta^2} \Longrightarrow D \propto 1/\eta$$

$\delta(\omega)$ is smeared into Breit-Wigner form at $\omega \sim 0$



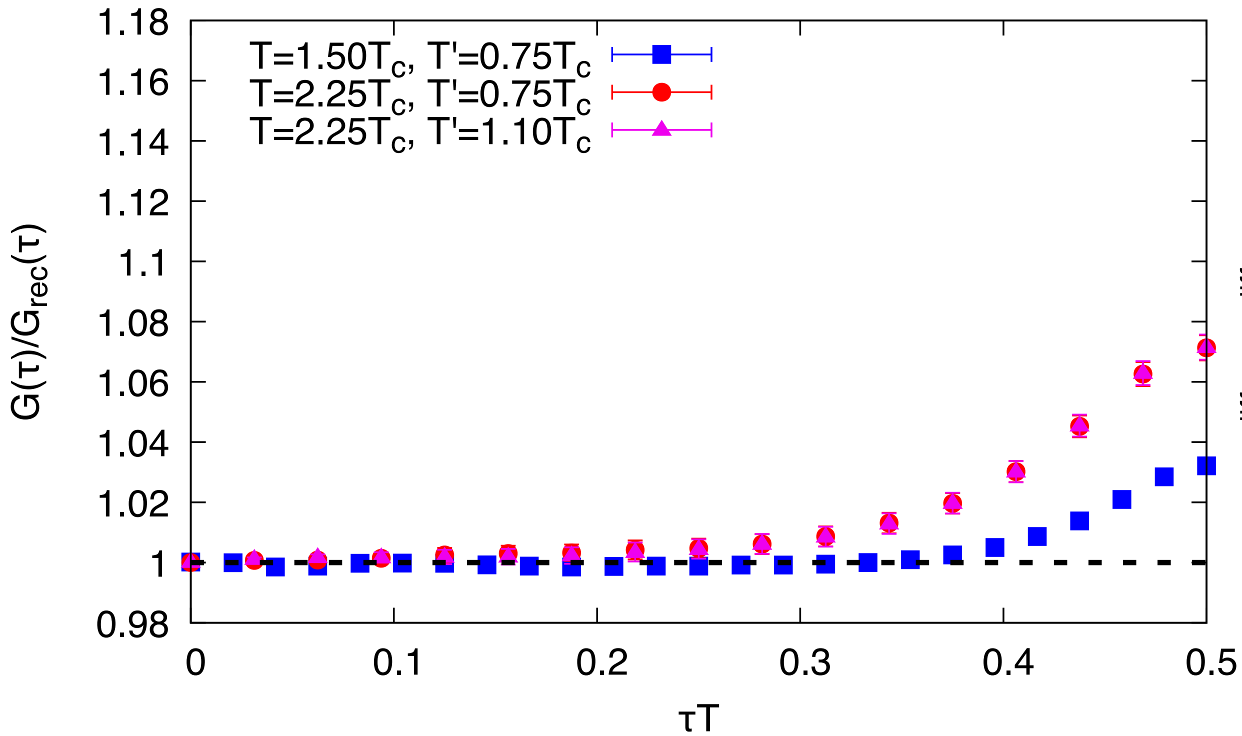
Bottomonia correlation functions in the VC channel

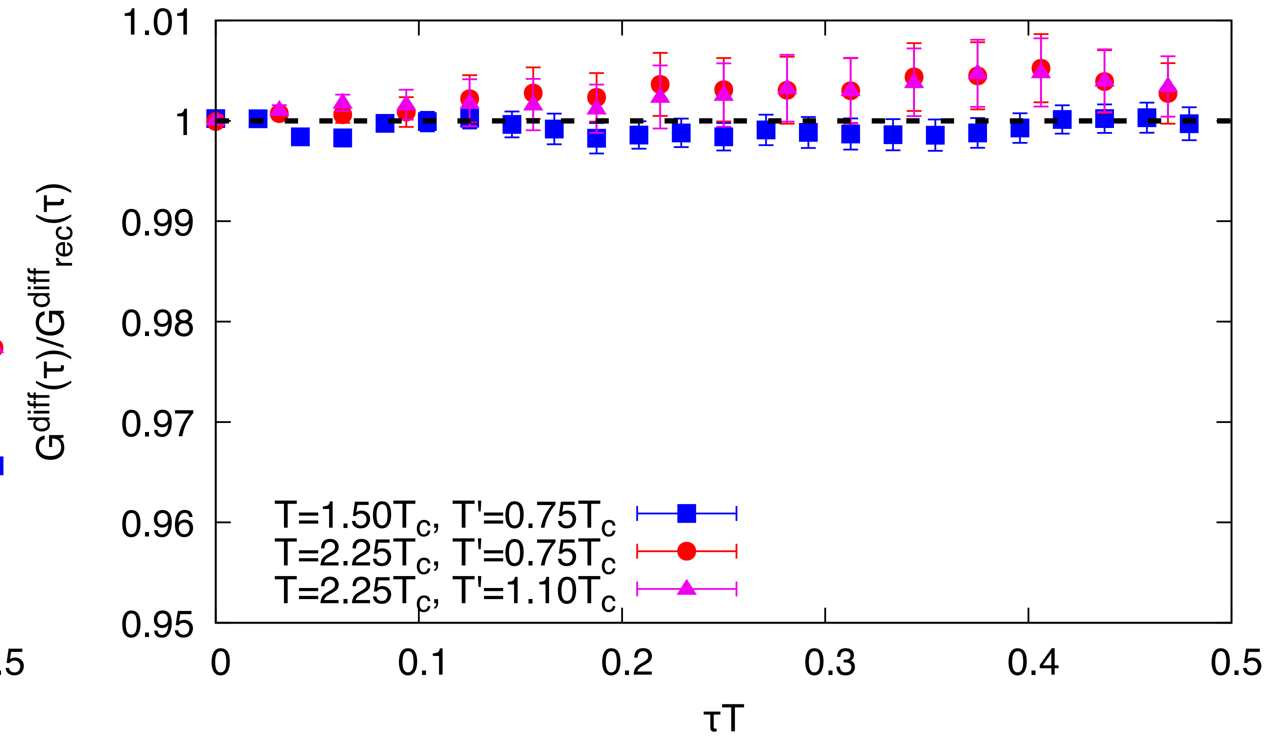
G/G_{rec} cancels out the trivial temperature dependence of $K(\omega,\tau,T)$:

$$\frac{G(\tau,T)}{G_{rec}(\tau,T;T')} = \frac{\int d\omega\ \rho(\omega,T)K(\omega,\tau,T)}{\int d\omega\ \rho(\omega,T')K(\omega,\tau,T)}$$

G_{diff} suppresses τ independent contributions, e.g. $\omega\delta(\omega)$ term in SPF:

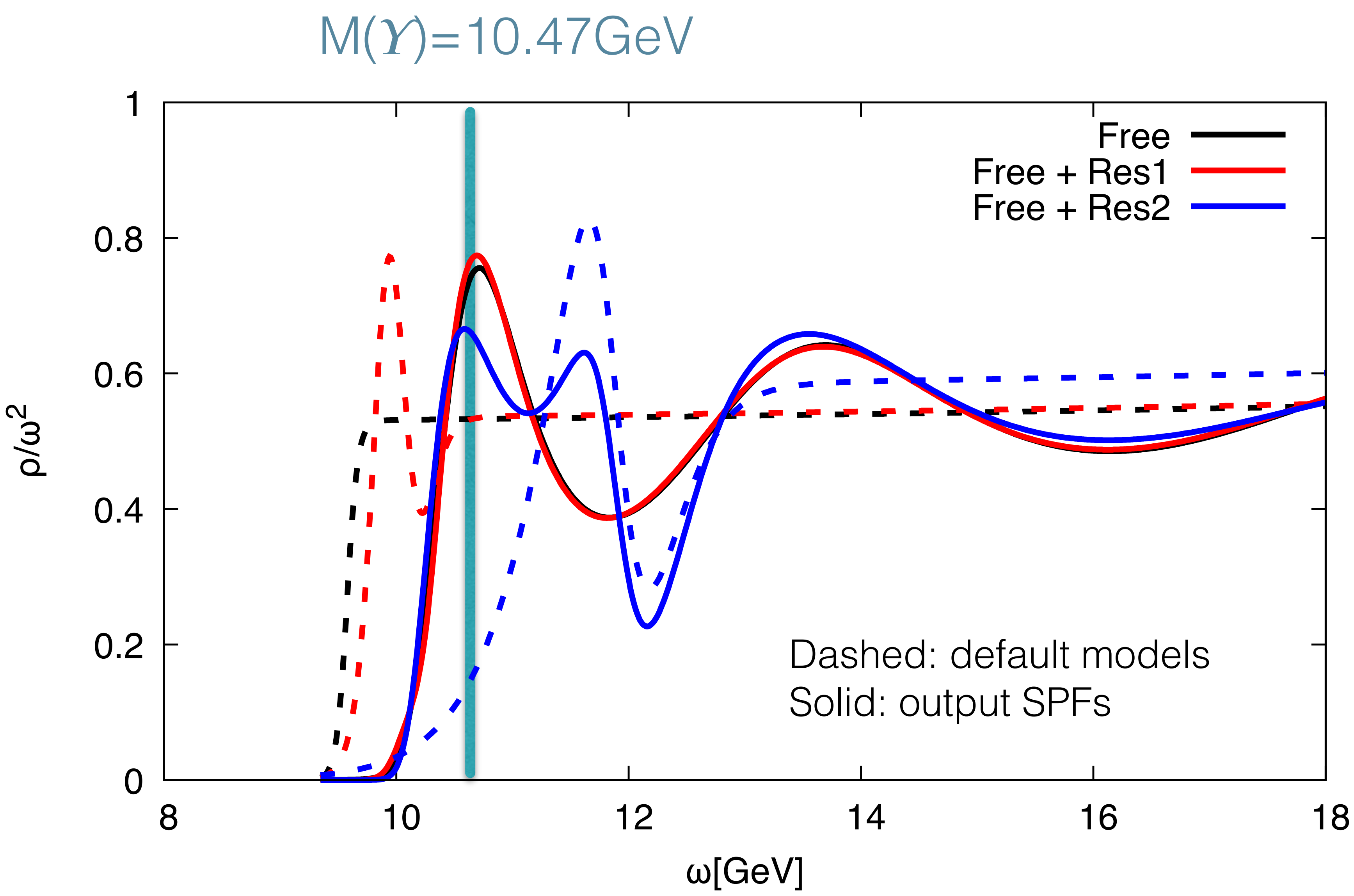
$$\frac{G^{\rm diff}(\tau/a)}{G^{\rm diff}_{rec}(\tau/a)} = \frac{G(\tau/a)-G(\tau/a+1)}{G_{rec}(\tau/a)-G_{rec}(\tau/a+1)}$$





- Thermal modification includes two parts: transport peak & resonance part
- Indication of no thermal modifications to Υ up to 2.25T_c

SPFs of bottomonia: VC channel at 0.75T_c



Default Models(DM):

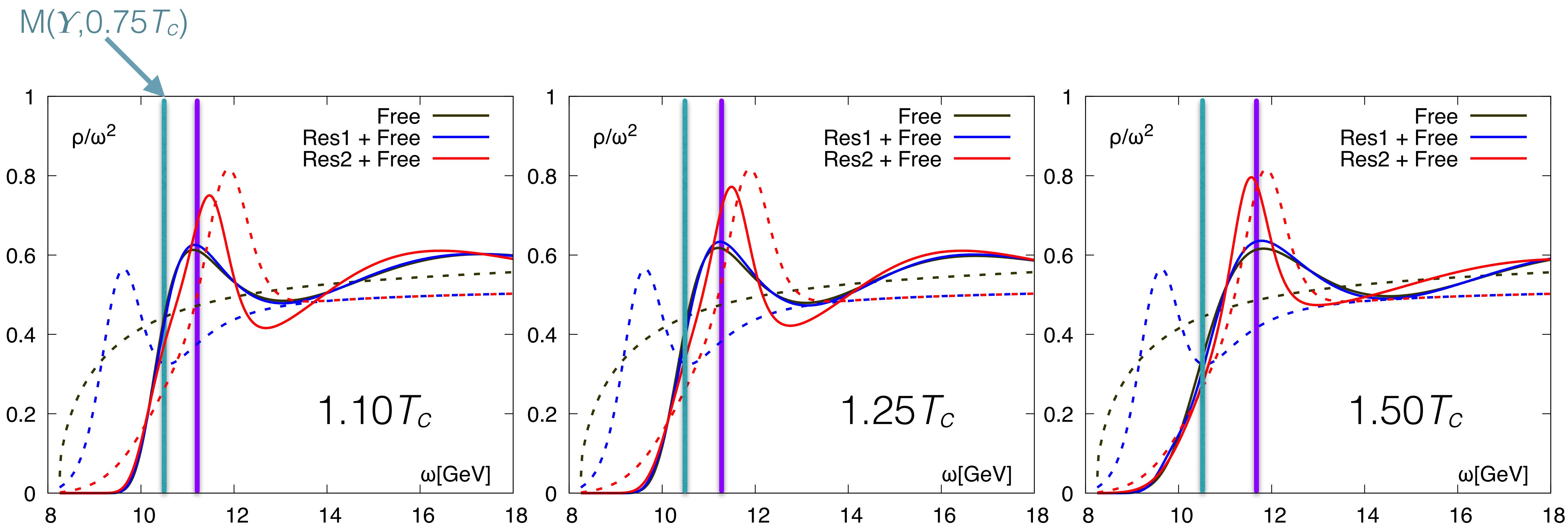
Free, Res1/Res2 + Free

Peak locations in DMs:

Res1 < M(ϒ), Res2 > M(ϒ)

- The first peaks in the output SPFs are independent of the DMs
- The peak location of the first peak is similar as the pole mass of ϒ

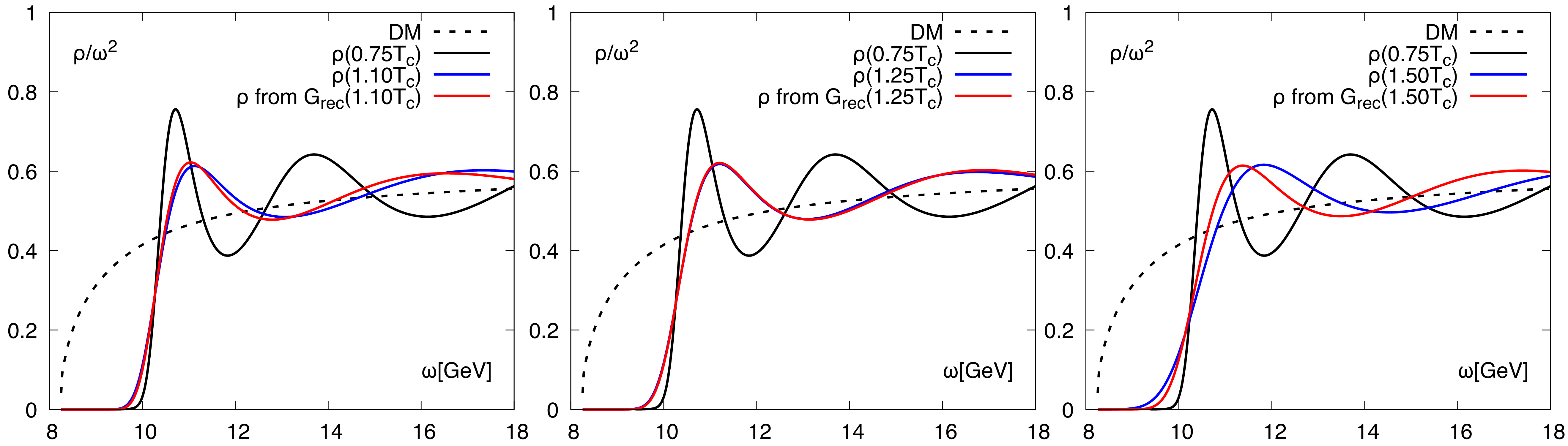
DM dep: bottomonia SPFs in the VC channel at $T>T_c$



Default Models = Free, Res1/Res2 + Free

- The peak location of the low-lying peak has minor DM dependence
- A small shift of first peak location (≥500MeV) is observed for $T>T_c$

N_τ dep: bottomonia SPFs in the VC channel at $T>T_c$



Default Model = Free

- After getting rid of N_τ dependence the shift of first peak location is almost gone

→ Υ seems to stay unmodified up to $1.50T_c$

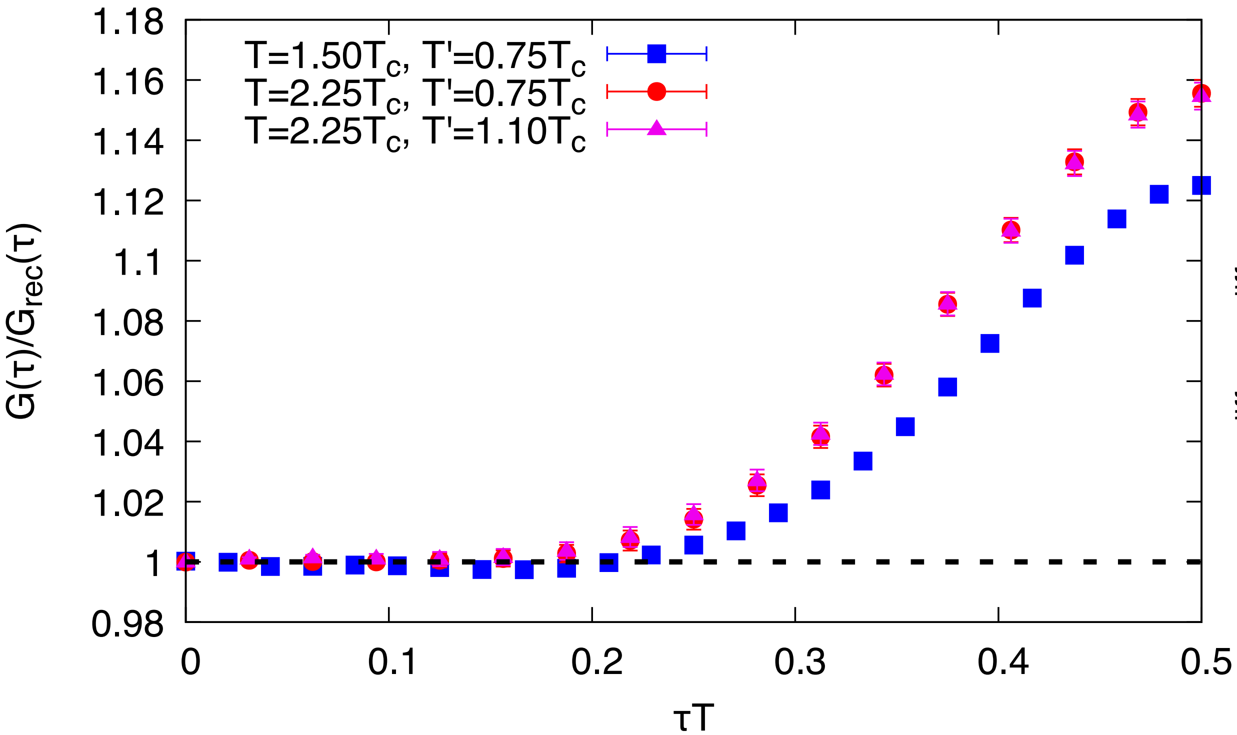
Charmonia correlation functions in the VC channel

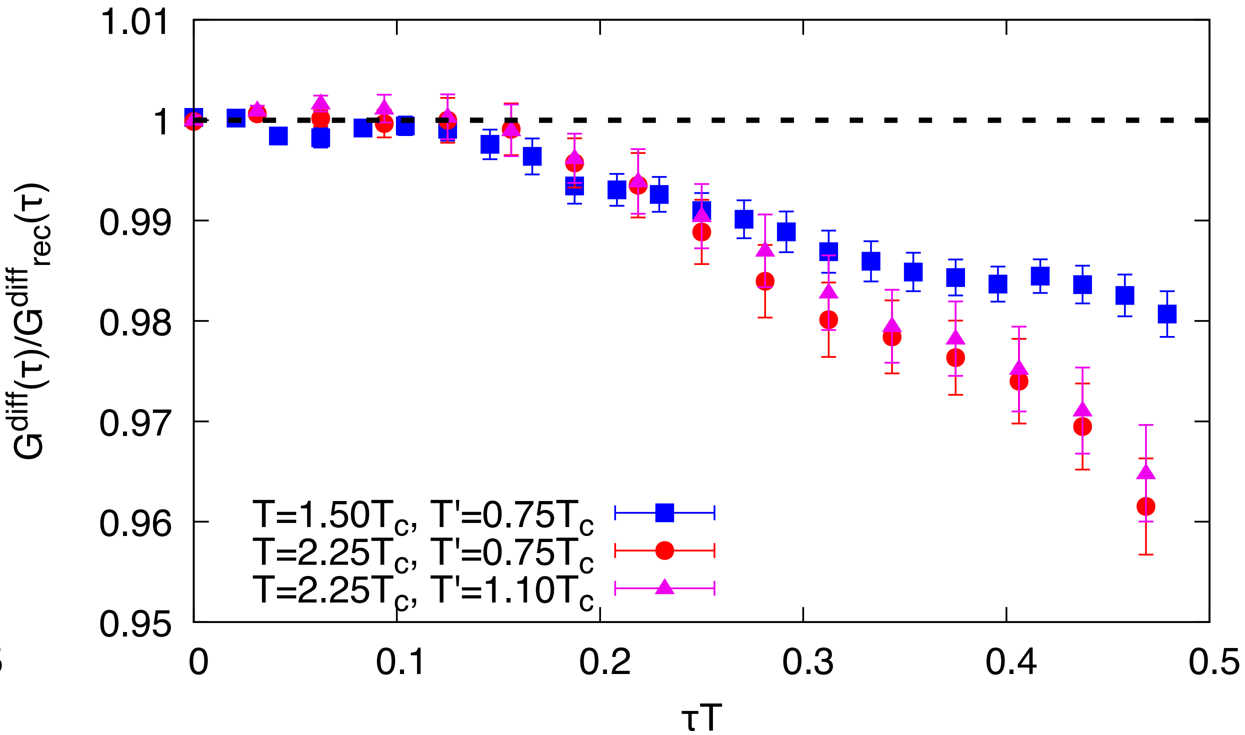
G/G_{rec} cancels out the trivial temperature dependence of $K(\omega,\tau,T)$:

$$\frac{G(\tau,T)}{G_{rec}(\tau,T;T')} = \frac{\int d\omega\ \rho(\omega,T) K(\omega,\tau,T)}{\int d\omega\ \rho(\omega,T') K(\omega,\tau,T)}$$

G_{diff} suppresses τ independent contributions, e.g. $\omega\delta(\omega)$ term in SPF:

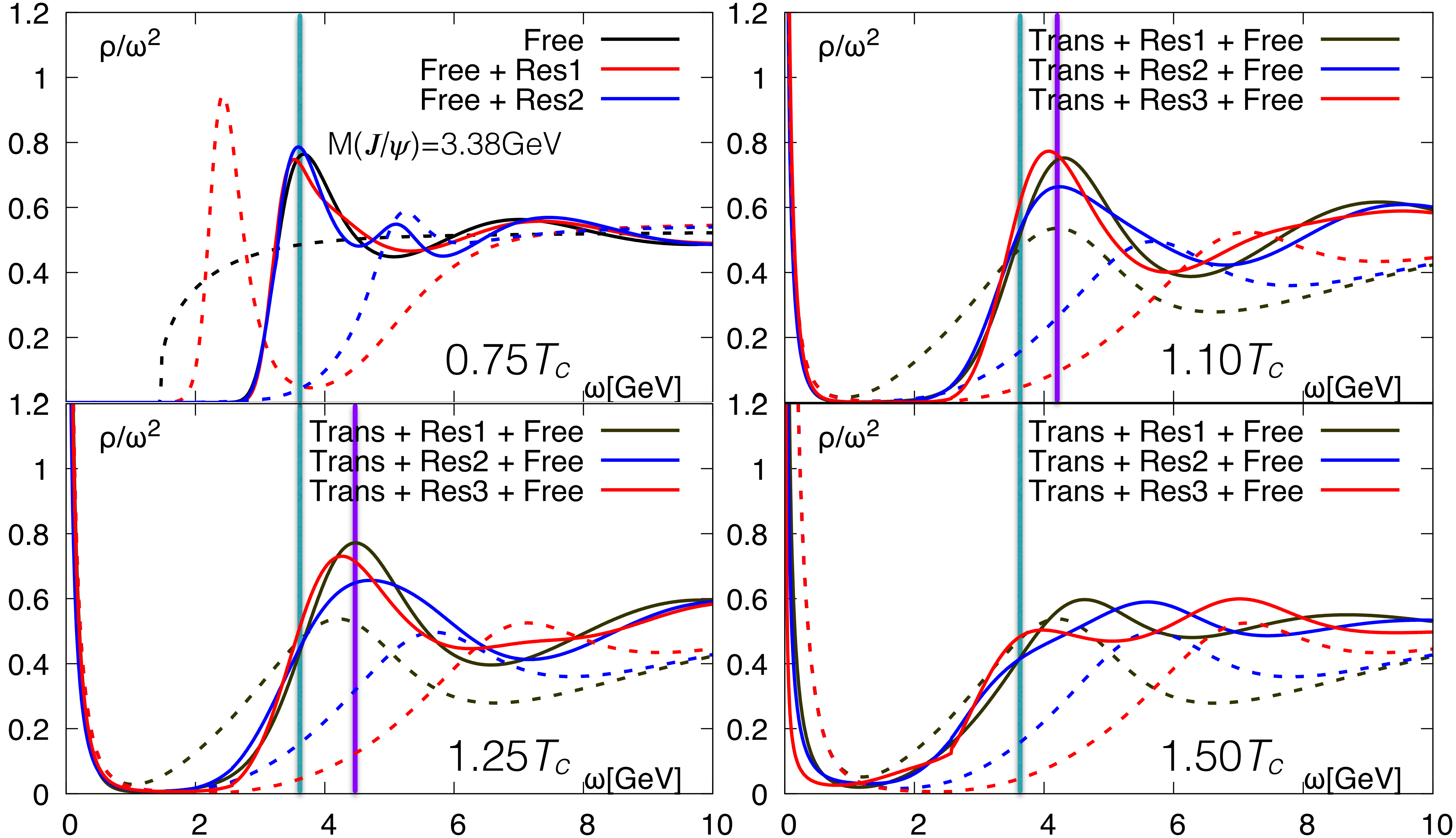
$$\frac{G^{\rm diff}(\tau/a)}{G^{\rm diff}_{rec}(\tau/a)} = \frac{G(\tau/a) - G(\tau/a+1)}{G_{rec}(\tau/a) - G_{rec}(\tau/a+1)}$$





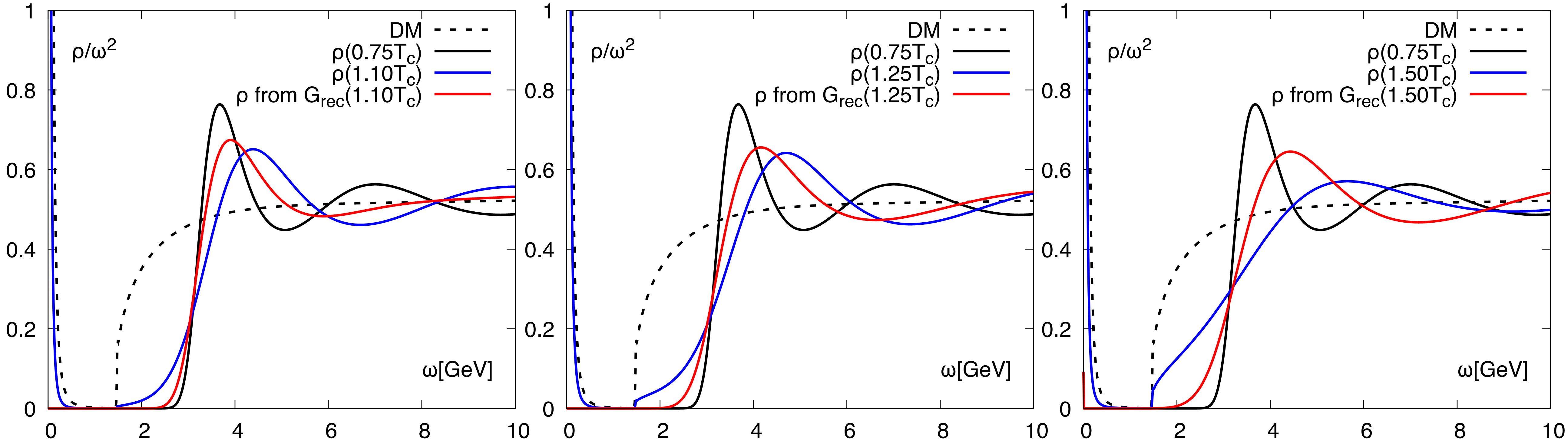
- Thermal modifications: more in charmonia than bottomonia correlators

DM dep: charmonia SPFs in VC channel at all T



- First low-lying peaks are stable at 0.75,1.10&1.25 T_c
- A shift of the first peak location(≥600MeV) is observed at 1.10 &1.25 T_c
- SPFs become flat at 1.50 T_c

N_τ dep: charmonia SPFs in the VC channel at $T > T_c$

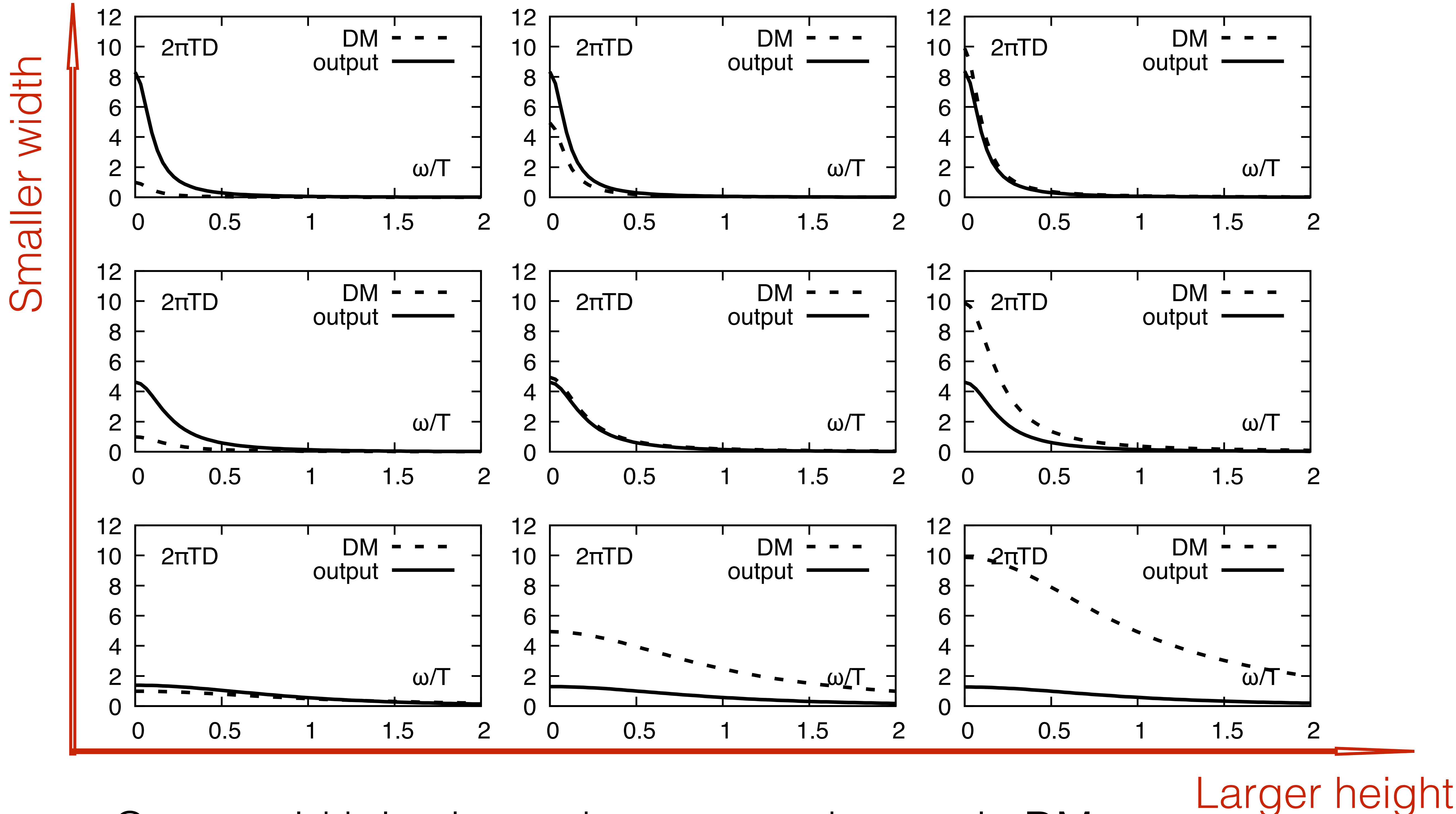


Default Model = Trans + Free

- After getting rid of N_τ dependence the shift of first peak location becomes smaller at $1.10T_c$ & $1.25T_c$

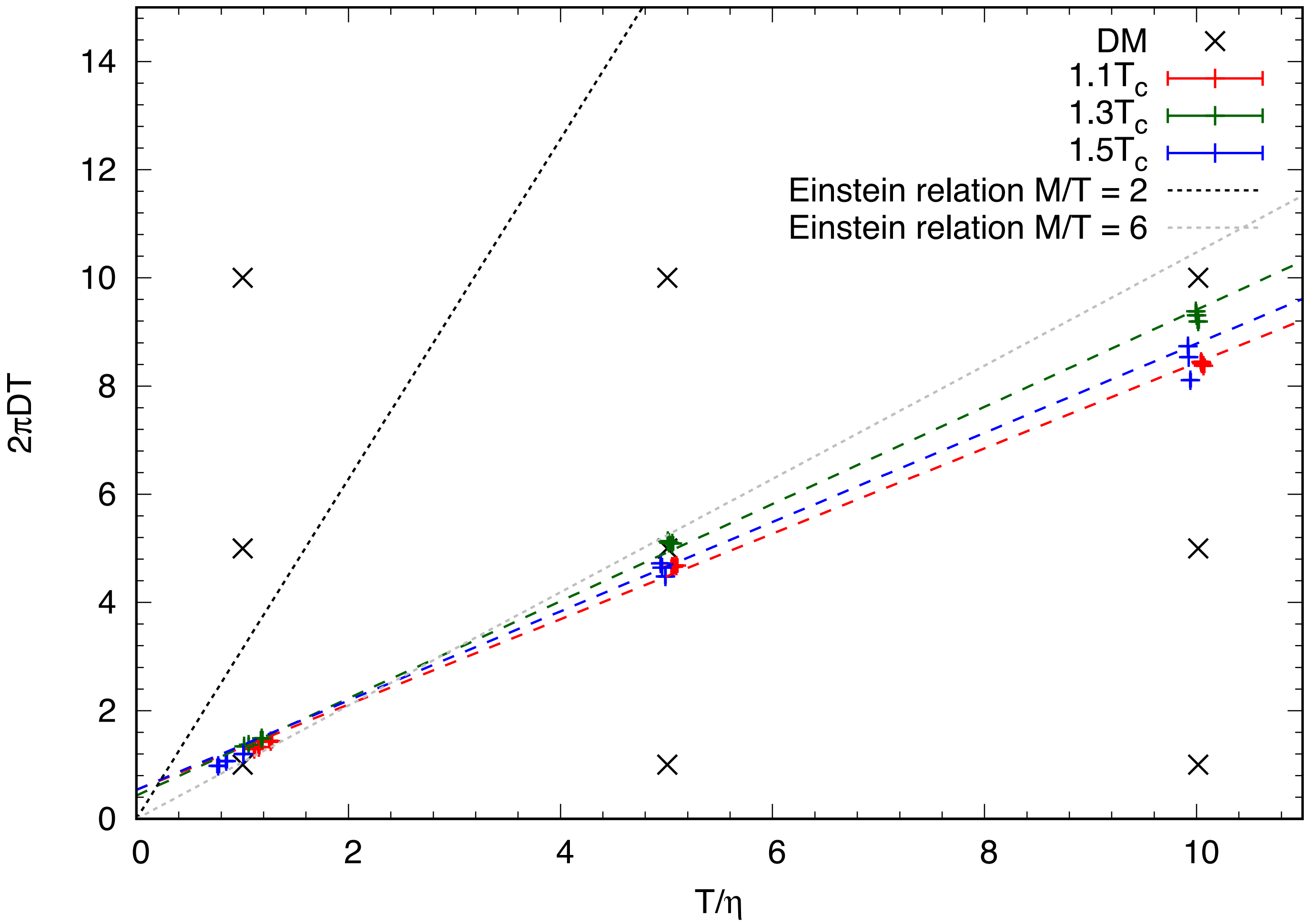
→ J/ψ suffers from more thermal modifications than Υ

Charm transport peak at 1.10T_c



- Output width is almost the same to the one in DM
- Output $2\pi TD$ varies as the width in DM changes

Charm quark diffusion coefficient at different T



Breit-Wigner ansatz for the transport peak in DM leads to fixed $2\pi TD \cdot \eta/T$:

$$D = \frac{1}{6\chi_{00}} \lim_{\omega\to 0} c \frac{\eta}{\omega^2+\eta^2} \Longrightarrow c \propto D\eta$$

$$G(N_\tau/2) = \int \frac{d\omega}{2\pi} c \frac{\omega\eta}{\omega^2+\eta^2} \frac{1}{\sinh(\omega/2T)} \approx \frac{c}{2N_\tau} (\omega \ll T)$$

MEM can only determine the coefficient c or $D\eta$

→ The diffusion coefficient can be determined once η/T is fixed

$2\pi TD$ = 0.789(11) T/η + 0.53(8) at 1.10T_c
$2\pi TD$ = 0.898(8) T/η + 0.43(2) at 1.25T_c
$2\pi TD$ = 0.825(9) T/η + 0.54(7) at 1.50T_c

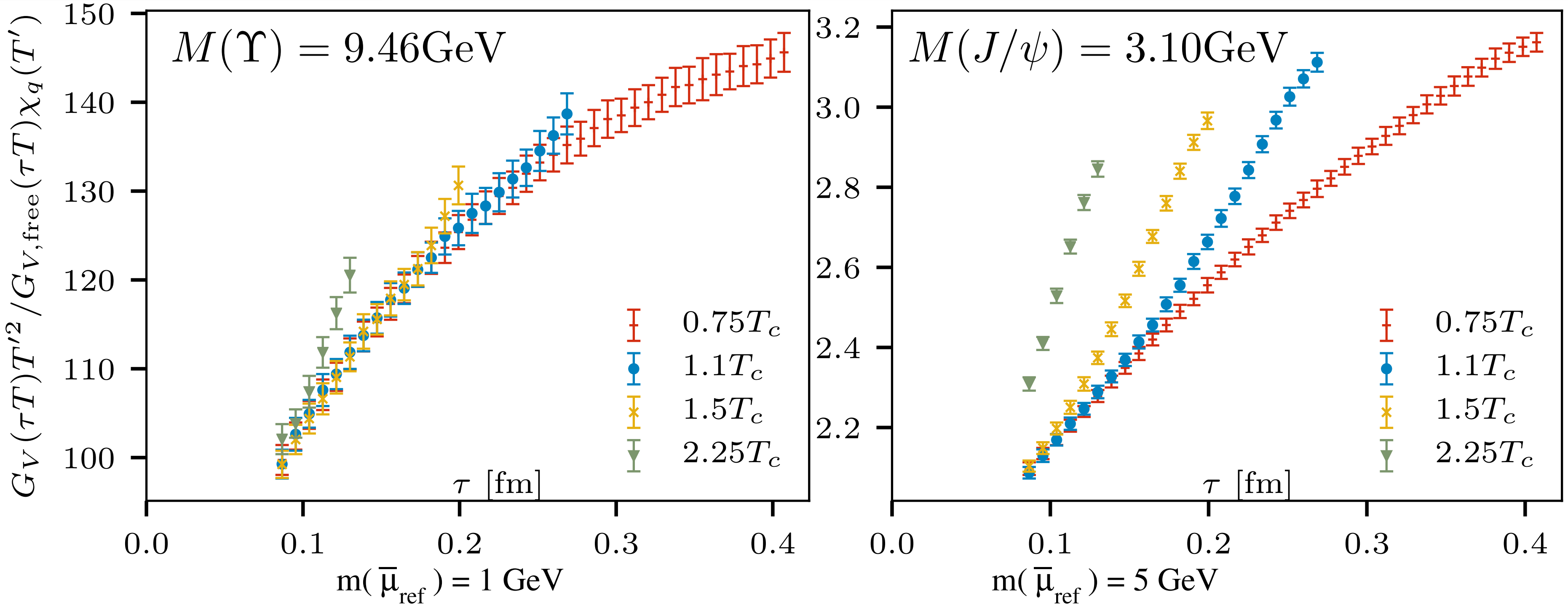
Summary & Outlook

We have performed simulations on large quenched isotropic lattices to calculate both the temporal correlation functions of charmonia and bottomonia

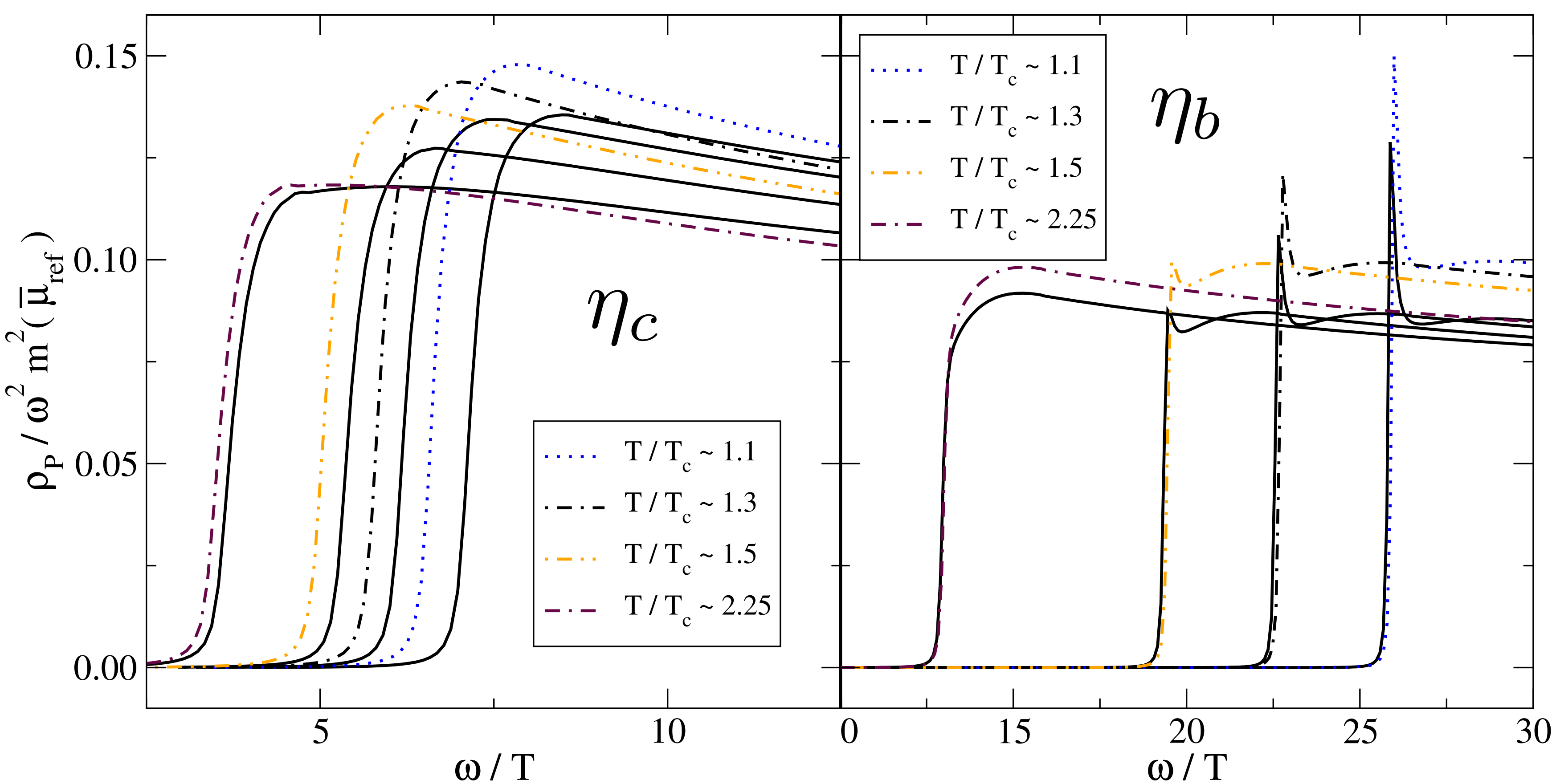
- ✳ Υ seems to stay unmodified up to $1.50T_c$ from spectral function analysis
- ✳ J/ψ suffers from more thermal modifications than Υ
- ✳ So far we observed a linear relation between $2\pi TD$ and T/η

◇ Outlook: . . .

Outlook



Continuum extrapolated vector correlators tuned to physical quarkonia masses.



[Y. Burnier, et al., JHEP11(2017)206]

Perturbative SPFs in the PS channel for charmonia and bottomonia.

[See **Anna-Lena Kruse**’s poster **No.766** on Tue. 17:00]

Thanks!

Backup: simulation details

- Large quenched QCD on isotropic lattices close to continuum
- Relativistic treatment of heavy quarks ($aM_Q<<1$)
- M_Q tuned to reproduce nearly physical J/ψ mass and Υ mass
- The relative errors of correlators: ~0.3%

β	a[fm]	M_V[GeV]	N_σ	N_τ	T/T_c	$\delta G/G[b\bar{b}/c\bar{c}]$
7.793	0.009fm	3.38($c\bar{c}$)	192	96	0.75	0.32%/0.37%
				64	1.10	0.21%/0.27%
				56	1.25	0.23%/0.26%
		10.47($b\bar{b}$)		48	1.50	0.17%/0.25%
				32	2.25	0.39%/0.39%

Backup: Inversion Methods: Maximum Entropy Method

A method based on Bayesian theorem to obtain the **most probable** solution

- Maximize the probability of having ρ given $\boldsymbol{D}$ & $\boldsymbol{G}$:

$$P[\rho|G,D,\alpha] = \frac{P[G|\rho,D,\alpha]\ P[\rho|D,\alpha]}{P[G|D,\alpha]}$$

ρ : spectral function
G : correlation function
D : default model

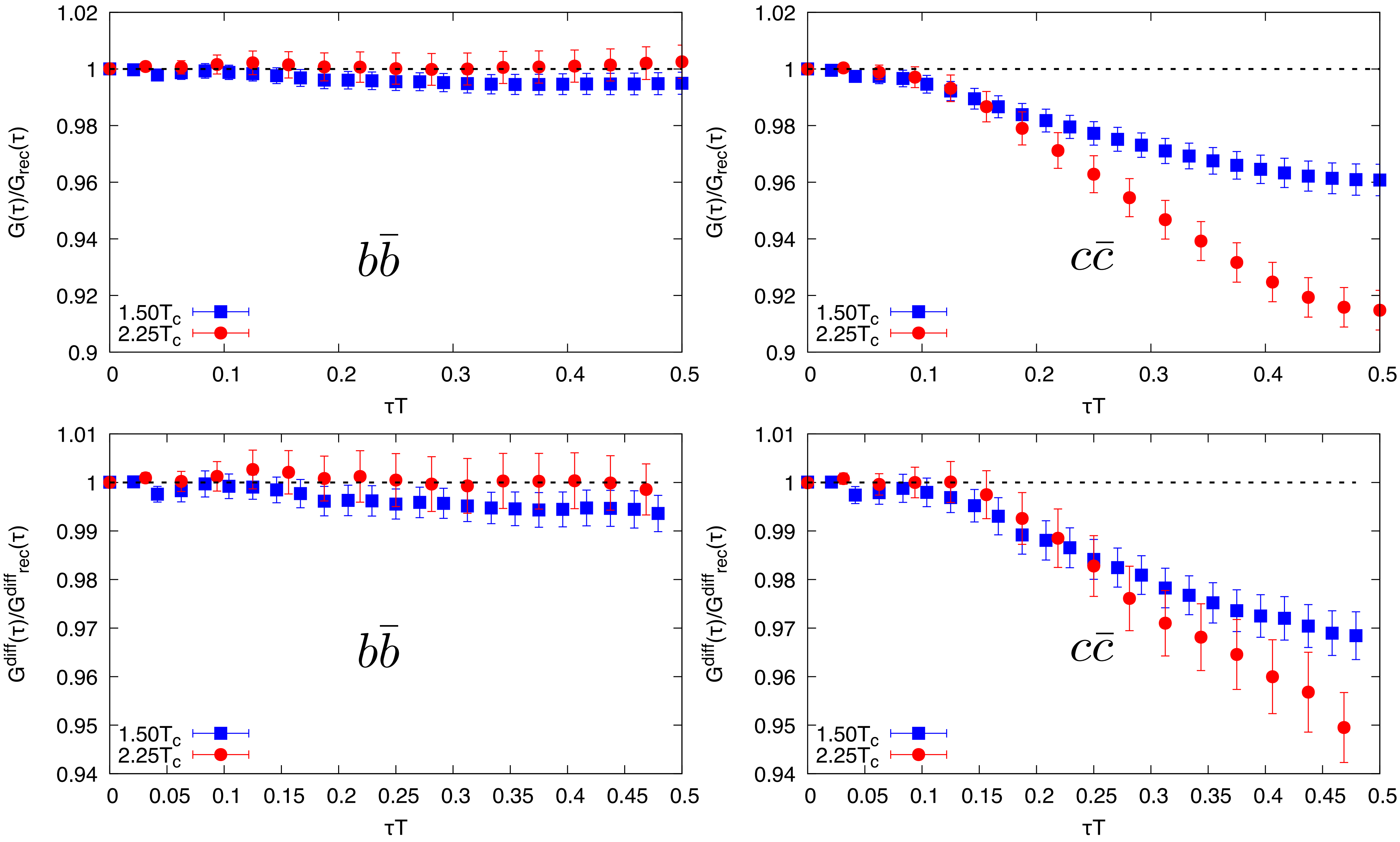
- Ingredients: $P[G|\rho,D,\alpha] \propto \exp(-\chi^2/2)$: likelihood function

$P[\rho|D,\alpha] \propto \exp(\alpha S)$: prior probability

Shannon-Jaynes entropy: $S[\rho] = \int \mathrm{d}\omega[\rho(\omega) - D(\omega) - \rho(\omega)\ln(\frac{\rho(\omega)}{D(\omega)})]$

- Check the dependence on DMs

Backup: Grec correlation function: PS channel



- Almost no transport contribution in PS channel.