The ideal relativistic fluid limit with polarization

David Montenegro[1], **Leonardo Tinti**[2], **Giorgio Torrieri**[1]

[1]Instituto de Física Gleb Wataghin (IFGW) - UNICAMP [2] ITP,JW Goethe Universitat, Frankfurt

Introduction and motivation

The purpose of our work is to define the ideal hydrodynamic limit if the fluid has polarization. This is necessary because

- obviously the experimental observation of Λ reaction plane polarization, and the reasonable expectation some polarization is present throghout the fluids evolution, before freezeout.

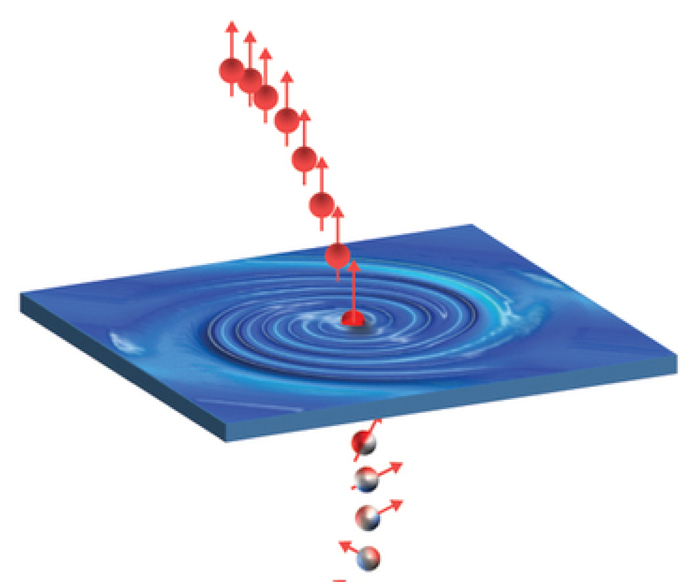

Figura 1

- Connection to anomalous transport, CME,CVE , magneto-hydrodynamics
  - Classical fields coupled to the fluid vs part of the fluid. (B^μ vs $\omega^{\mu\nu}$)
  - Anomalous vs symmetry respecting ($J^\mu = ... + \omega$ vs $T^{\mu\nu} + u_\alpha S^{\alpha\mu\nu}$)

  Very different physics but experimentally entangled . Eg, polarizability variation with baryo and isospin chemical potential can mimic the CME! all observables so far CP-respecting
- What is the role of Gauge symmetry? In a QGP most particles with spin are gluons, but "gluon polarization" and "angular momentum" dont separate (gauge changes go between one and another).

But this presents a conceptual difficulty! What is ideal hydro? Local isotropy, circulation conservation obviously broken. Continuum limit dubious (Is a "small vortex" indistinguisheable from a polarization spin state?) Only "instant thermalization" could work .

Development

This forces us to use lagrangian methods [1]

A definition of B, u_μ in terms of fields ϕ^I, lagrangian coordinates. $L = L(B)$ respects symmetries of fluid where

$$B = \det B^{IJ} \quad , \quad B^{IJ} = \partial_\mu \phi^J$$

$$u^\mu = \frac{1}{6\sqrt{B}} \epsilon^{\mu\alpha\beta\gamma} \epsilon_{IJK} \partial_\alpha \phi^I \partial_\beta \phi^J \partial_\gamma \phi^K$$

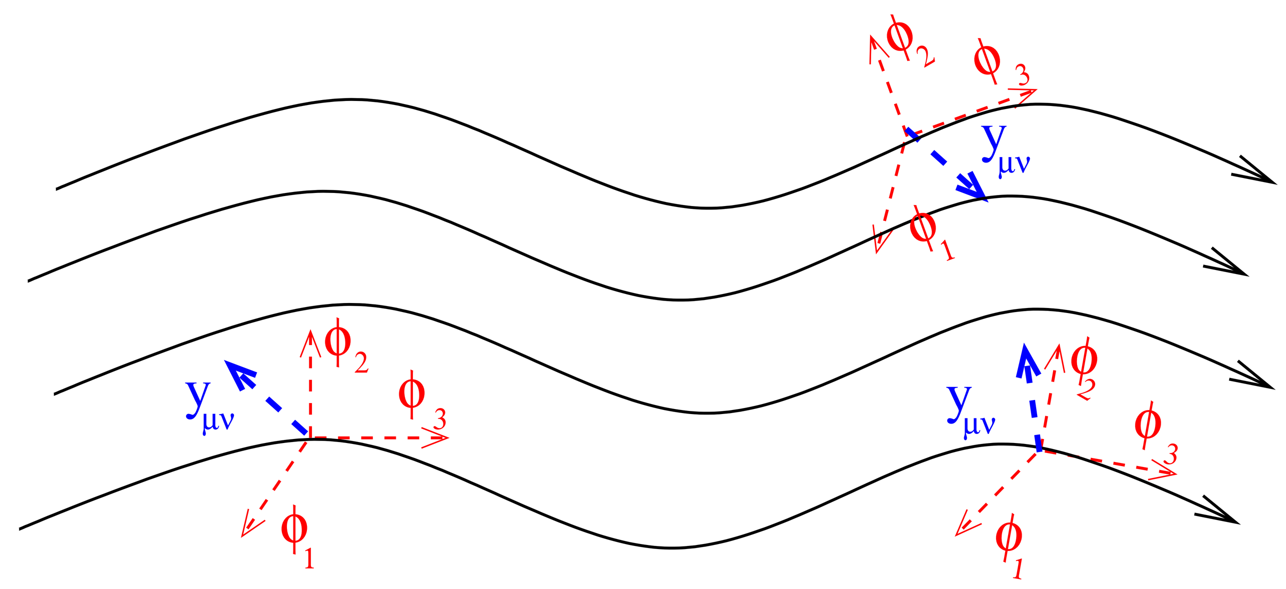

Figura 2

A definition of local polarization y from chemical shift symmetry [2]

$$\Psi_{\mu\nu}|_{comoving} = -\Psi_{\nu\mu}|_{comoving} = \exp\left[-\sum_{i=1,2,3} \alpha_i(\phi_I) \hat{T}_i^{\mu\nu}\right]$$

$$\alpha_i \to \alpha_i + \Delta\alpha_i(\phi_I) \Rightarrow L(b, y_{\alpha\beta} = u_\mu \partial^\mu \Psi_{\alpha\beta})$$

Ensures that polarization current (one index of the 3-tensor) always proportional to u_μ

We are now ready to combine polarization with the ideal hydrodynamic limit, defined as

(i) The dynamics within each cell is faster than macroscopic dynamics, and it is expressible only in term of local variables and with no explicit reference to four-velocity u^μ (gradients of flow are however permissible, in fact required to describe local vorticity).

(ii) Dynamics is dictated by local entropy maximization, within each cell, subject to constraints of that cell alone. Macroscopic quantities are assumed to be in local equilibrium inside each macroscopic cell

(iii) Only excitations around a hydrostatic medium are sound waves,vortices

Part (iii) forces polarization and vorticity to always be parallel to avoid a Goldstone mode.

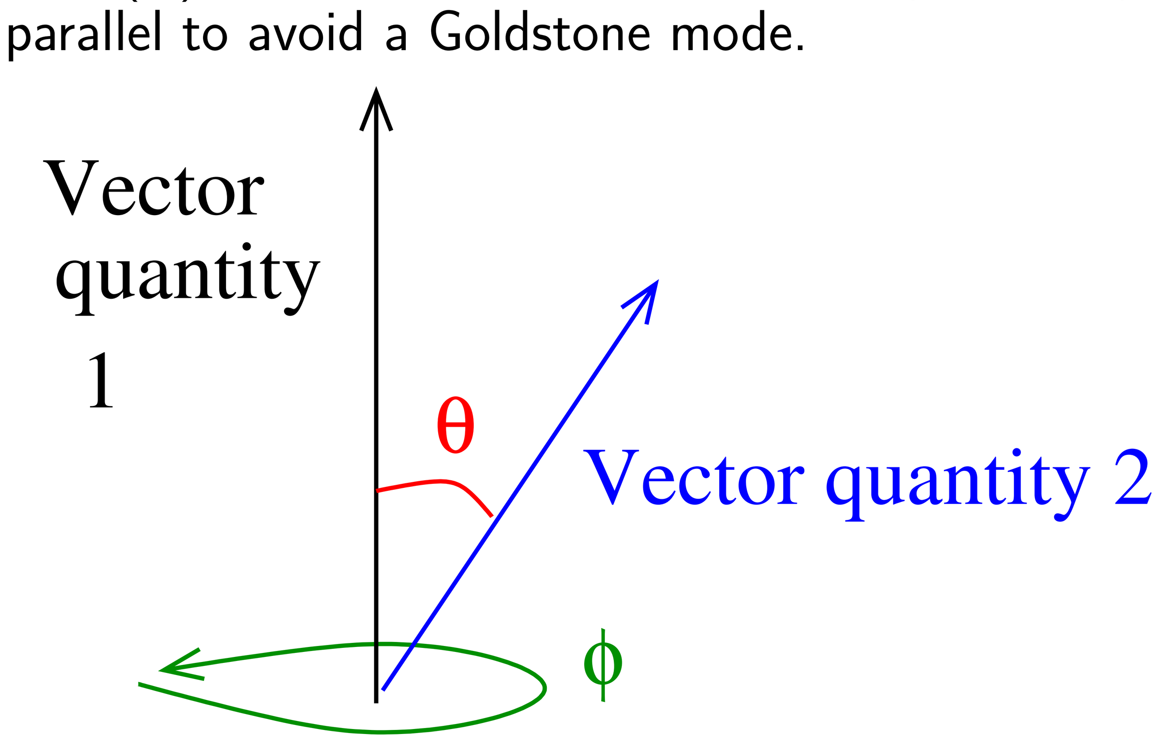

Figura 3

Equation of state is related to the Lagrangian in a way analogous to [3]. $L = \mathcal{F}$ where $\mathcal{F}$ is a free energy so

$$d\mathcal{F} = \frac{\partial \mathcal{F}}{\partial V} dV + \frac{\partial \mathcal{F}}{\partial e} de + \frac{\partial \mathcal{F}}{\partial [\omega_{\mu\nu}]} d[\omega_{\mu\nu}] = 0$$

This gives us a candidate for the Langrangian. The linear version wrt hydrostatics ,$\phi_I = X_I + \pi_L + \pi_T$, assuming $F(b, y) = F(b(1 - y^2))$, is [4]:

$$\mathcal{L} = (-F'(1)) \left\{ \frac{1}{2} (\dot{\pi})^2 - c_s^2 [\partial \pi]^2 \right\} +$$

$$+ f\zeta \left\{ \ddot{\pi}^i \partial_i \dot{\pi}_j + \ddot{\pi}_i \ddot{\pi}_j + \partial_j \dot{\pi}^i \partial_i \dot{\pi}_j + \partial_j \dot{\pi}_i \ddot{\pi}_j + \right.$$

$$\left. + (2\ddot{\pi}^i \partial_j \dot{\pi}_i - 2\ddot{\pi}_j \partial^i \dot{\pi}_j) + (\ddot{\pi}_i^2 - \ddot{\pi}_j^2) + (\partial_j \dot{\pi}_i^2 - \partial_i \dot{\pi}_j^2) \right\}$$

Note that sound-waves and vortices mix! Physically, the sound-wave compression changes the vortical susceptibility, which absorbs or emits angular momentum, i.e. vorticity. But this makes dispersion relation quartic

Causality and implications

the Free energy $\mathcal{F}$, and hence the local dynamics, is sensitive to an accelleration. As is well-known (Ostrogradski's theorem, Dirac runaway solutions) such Lagrangians are unstable and lead to causality violation. Note that one needs Lagrangians to see this! Indeed, looking at the phase and group velocity one will find superluminal propagation of sound-waves and vortices

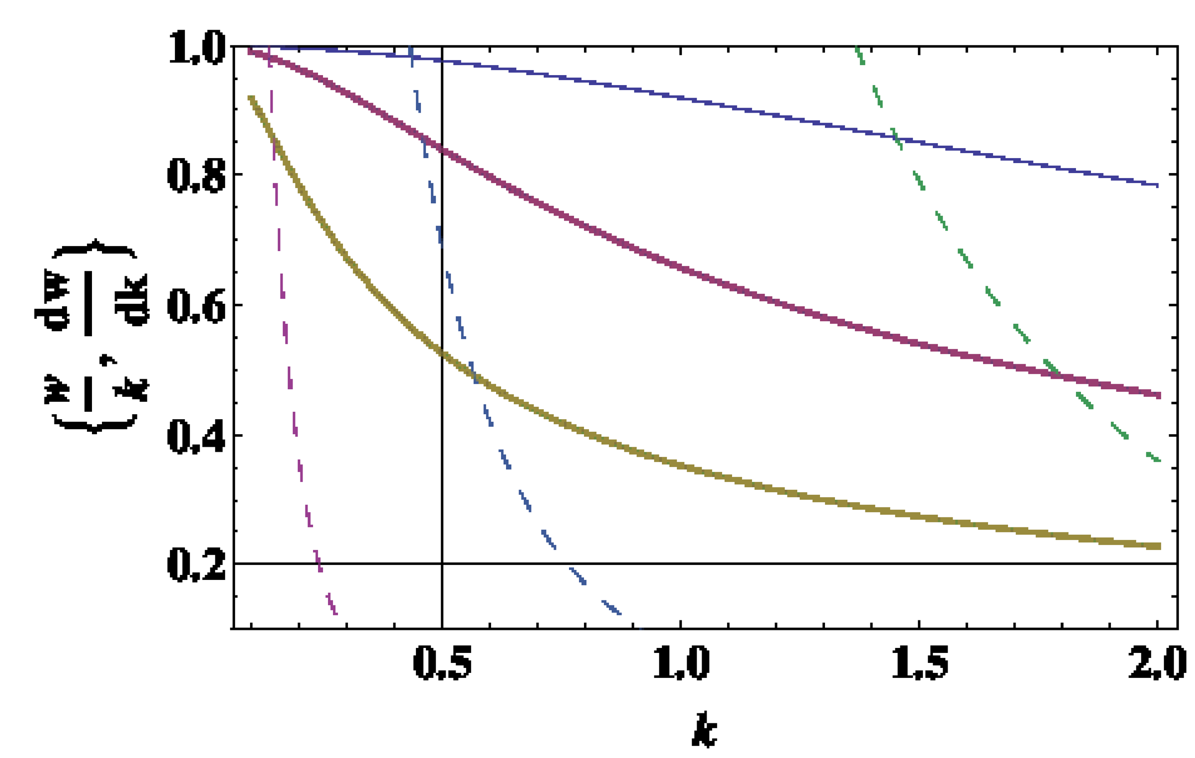

Figura 4

To fix this issue, one would need to update the proportionality of y on ω to an Israel-Stewart type equation.

$$\tau_\Omega u_\alpha \partial^\alpha \Omega_{\mu\nu} + \Omega_{\mu\nu} = \chi(b, \omega^2) \omega^{\mu\nu} = y^{\mu\nu}$$

But this means dissipative relaxation-type dynamics is present already at ideal fluid level A new lower limit for dissipation?

In conclusion, Lagrangian methods overcome the ambiguities of defining ideal hydrodynamics with polarization, but suggest a breakdown in causality already in the ideal limit, due to the "instant" alignement of polarization and vorticity. Fixing this issue might give rise to a new lower limit on dissipation.

References

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