The ideal relativistic fluid limit with polarization

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Introduction and motivation

The purpose of our work is to define the ideal hydrodynamic limit if the fluid has polarization. This is necessary because

- obviously the experimental observation of $\Lambda$ reaction plane polarization, and the reasonable expectation some polarization is present throughout the fluids evolution, before freezeout.

This forces us to use lagrangian methods \cite{1}

\textbullet{} Classical fields coupled to the fluid vs part of the fluid. ($B^\mu \leftrightarrow u^{\mu\nu}$)

\textbullet{} Anomalous vs symmetry respecting ($J^\mu = ... + \omega \leftrightarrow T^{\mu\nu} + u_\mu S^{\mu\nu}$)

Very different physics but experimentally entangled. E.g. polarizability variation with baryo and isospin chemical potential can mimic the CME! all observables so far CP-respecting

- What is the role of Gauge symmetry? In a QGP most particles with spin are gluons, but "gluon polarization" and "angular momentum" dont separate (gauge changes go between one and another).

But this presents a conceptual difficulty! What is ideal hydro? Local isotropy, circulation conservation obviously broken (is a "small vortex" indistinguishable from a polarization spin state?) Only "instant thermalization" could work.

Development

This forces us to use lagrangian methods \cite{1}

A definition of $B$, $u_\mu$ in terms of fields $\phi^i$, lagrangian coordinates. $L = L(B)$ respects symmetries of fluid where

\[ B = \det \Omega_{ij} \hspace{1em}, \hspace{1em} \Omega_{ij} = \partial_i \phi^j \]

\[ u^\mu = \frac{1}{\sqrt{B}} e^{i\alpha\beta\gamma\epsilon^{ijk}} \partial_i \phi^j \partial_j \phi^K \]

Equation of state is related to the Lagrangian in a way analogous to \cite{3}. $L = \mathcal{F}$ where $\mathcal{F}$ is a free energy so

\[ d\mathcal{F} = \frac{\partial \mathcal{F}}{\partial \psi_v} d\psi_v + \frac{\partial \mathcal{F}}{\partial \psi_{\alpha\beta}} d\psi_{\alpha\beta} + \frac{\partial \mathcal{F}}{\partial \psi_{\mu\nu}} d\psi_{\mu\nu} = 0 \]

This gives us a candidate for the Lagrangian. The linear version wrt hydrostatics $\phi_1 = X_1 + \pi_t + \pi_L$, assuming $F(b, y) = F(b(1 - y^2))$, is \cite{4}:

\[ \mathcal{L} = \left\{-\mathcal{F}'(1) + \frac{1}{2} (\pi_t^2 - c_s^2 \partial \pi_t^2) + \right\} \]

\begin{align*}
+ & fc \left\{ \pi_1^2 \partial_\gamma \pi_1 + \pi_1 \partial_\gamma \pi_1 + \partial_\gamma \pi_1 \partial_\gamma \pi_1 + \partial_\gamma \pi_1 \partial_\gamma \pi_1 + \\
+ & (2 \pi_1 \partial_\gamma \pi_1 - 2 \pi_1 \partial_\gamma \pi_1) + (\partial_\gamma \pi_1 - \partial_\gamma \pi_1) + (\partial_\gamma \pi_1 - \partial_\gamma \pi_1) \right\}
\end{align*}

Note that sound-waves and vortices mix! Physically, the sound-wave compression changes the vortical susceptibility, which absorbs or emits angular momentum, i.e. vorticity. But this makes dispersion relation quartic

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\begin{tikzpicture}
\end{tikzpicture}
\end{center}

Causality and implications

the Free energy $\mathcal{F}$, and hence the local dynamics, is sensitive to an acceleration. As is well-known (Ostrogradski’s theorem, Dirac runaway solutions) such Lagrangians are unstable and lead to causality violation. Note that one needs Lagrangians to see this! Indeed, looking at the phase and group velocity one will find superluminal propagation of sound-waves and vortices

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To fix this issue, one would need to update the proportionality of $\gamma$ on $\omega$ to an Israel-Stewart type equation.

\[ \tau \Omega_{\mu\nu} \partial ^\alpha \Omega_{\mu\nu} + \Omega_{\mu\nu} = \chi(b, \omega^2) \omega_{\mu\nu} = \gamma_{\mu\nu} \]

But this means dissipative relaxation-type dynamics is present already at ideal fluid level! A new lower limit for dissipation?

In conclusion, Lagrangian methods overcome the ambiguities of defining ideal hydrodynamics with polarization, but suggest a breakdown in causality already in the ideal limit, due to the “instant” alignment of polarization and vorticity. Fixing this issue might give rise to a new lower limit on dissipation.

References


