$\eta/s(T)$ uncertainties from the equation of state

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ABSTRACT

We perform a global model-to-data comparison using a hydrodynamics model with the EKRT initial state [1]

- Compare analysis results based on four different equations of state
- Statistical analysis based on Bayesian statistics [2]
- Emulate the hydro model behavior for uninvestigated input values using Gaussian processes
- Calibrate the model to experimental data using Markov chain Monte Carlo sampling

Gaussian process

Model emulation using Gaussian processes: Set Y_a of values of observable y_a, corresponding to a set X of points in parameter space, has a multivariate normal distribution:

$$\mathcal{G} : X \to Y_a \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$$

where $\boldsymbol{\mu} = \mu(X) = \{\mu(\vec{x}_1), ..., \mu(\vec{x}_N)\}$ is the mean and $\boldsymbol{\Sigma} = \sigma(X, X)$ is the covariance matrix with covariance function $\sigma(\vec{x}, \vec{x}')$ (model-dependent choice; constant, linear, exponential, periodic, ...).

Choice: Radial-basis function (RBF) with a noise term

$$\sigma(\vec{x}, \vec{x}') = \theta_0 \exp\left(-\sum_{i=1}^{n} \frac{(x_i - x_i')^2}{2\theta_i^2}\right) + \theta_{\text{noise}}\delta_{\vec{x}\vec{x}'}$$

The *hyperparameters* $\vec{\theta} = (\theta_0, \theta_1, ...\theta_n, \theta_{\text{noise}})$ are not known a priori and must be estimated from the simulation output (emulator training). Emulator prediction for the model output y_0 at a point $\vec{x_0}$ can then be determined by writing a joint distribution

$$\begin{pmatrix} y_0 \\ Y_a \end{pmatrix} = \mathcal{N}\left(\begin{pmatrix} \mu(\vec{x_0}) \\ \mu(X) \end{pmatrix}, \begin{pmatrix} \Sigma_{0,0} & \Sigma_{0,X} \\ \Sigma_{X,0} & \Sigma_{X,X} \end{pmatrix}\right)$$

from which we can derive the conditional predictive mean and associated covariance as ($\mu(X) \equiv 0$)

$$y^*(\vec{x_0}) = \Sigma_{0,X}\Sigma_{X,X}^{-1}Y;\ \Sigma_{GP} = \Sigma_{0,0} - \Sigma_{0,X}\Sigma_{X,X}^{-1}\Sigma_{X,0}.$$

Bayes' theorem

Given a set $X = \{\vec{x}_k\}_{k=1}^N$ of points in parameter space and a corresponding set $Y = \{\vec{y}_k\}_{k=1}^N$ of points in observable space,

$$\boxed{P(\vec{x}^*|X, Y, \vec{y}^{\text{exp}}) \propto P(X, Y, \vec{y}^{\text{exp}}|\vec{x}^*)P(\vec{x}^*)}$$

- $P(\vec{x}^*|X, Y, \vec{y}^{\text{exp}})$ is the *posterior* probability distribution of $\vec{x}^*$ for given $(X, Y, \vec{y}^{\text{exp}})$
- $P(\vec{x}^*)$ is the *prior* probability distribution (simplest case: ranges of parameter values)
- $P(X, Y, \vec{y}^{\text{exp}}|\vec{x}^*)$ is the *likelihood* of $(X, Y, \vec{y}^{\text{exp}})$ for given $\vec{x}^*$

$$P(X, Y, \vec{y}^{\text{exp}}|\vec{x}^*) = \frac{1}{\sqrt{|2\pi\Sigma|}} \exp\left(-\frac{1}{2}(\vec{y}^* - \vec{y}^{\text{exp}})^T\Sigma^{-1}(\vec{y}^* - \vec{y}^{\text{exp}})\right),$$

where $\vec{y}^*$ is emulator estimate for the input parameter point $\vec{x}^*$ and Σ is the covariance matrix representing the uncertainties related to the model-to-data comparison. In this study

$$\Sigma = \text{diag}((\sigma_{\text{exp}})^2 + (\sigma_{\text{GP}})^2),$$

with σ_{exp} representing the experimental error and $(\sigma_{\text{GP}})^2$ is the GP emulator variance.

Markov chain Monte Carlo

The posterior distribution is sampled with Markov chain Monte Carlo (MCMC) method

- Random walk in parameter space, where each step is accepted or rejected based on a relative likelihood
- Converges to posterior distribution as number of steps $N \to \infty$
- Acceptance fraction of steps measures the quality of random walk; should be 0.2-0.5 D. Foreman-Mackey *et al.*, arXiv:1202.3665
- Autocorrelation time = Number of steps between independent samples; "Burn-in" takes a few autocorrelations, gathering enough samples $\sim \mathcal{O}(10)$ autocorrelations (with $\sim \mathcal{O}(100)$ walkers)

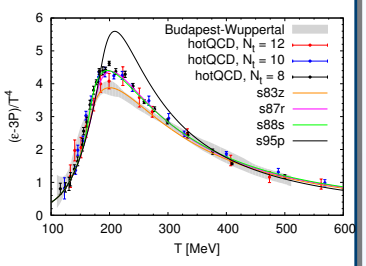
EKRT+hydrodynamics model and equations of state

- Initial energy density from the EKRT minijet saturation model [3]: $e(\vec{r}_T, \tau_s(\vec{r}_T)) = \frac{K_{sat}}{\pi}[p_{sat}(\vec{r}_T, K_{sat})]^4$; $\tau_s(\vec{r}_T) = 1/p_{sat}(\vec{r}_T, K_{sat})$
- 2+1D viscous hydrodynamics with linear temperature dependence on η/s

$$\eta/s(T) = S_{HG}(T_{min} - T) + (\eta/s)_{min},\ T < T_{min}$$
$$\eta/s(T) = S_{QGP}(T - T_{min}) + (\eta/s)_{min},\ T > T_{min}$$

The following equations of state were used in this study:

- s83z: Hadron gas particles from 2016 PDG summary tables, fit done to lattice data from [4]
- s87r: $M < 2$ GeV HG particles from 2005 PDG summary tables, fit to lattice data from [5]
- s88s: HG particles from 2016 PDG summary tables, fit to lattice data from [5]
- **s95p** [6]: $M < 2$ GeV HG particles from 2005 PDG summary tables, fit to lattice data from [7]



Results

The following observables were used for the data calibration:
Au+Au at $\sqrt{s_{NN}} = 200$ GeV:

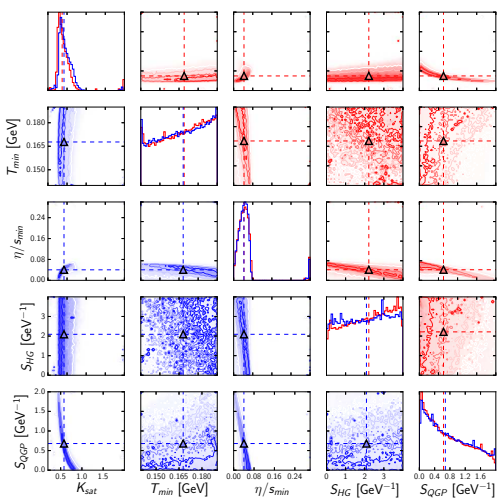
- N_{ch} in $|\eta| < 0.5$ in (0-5)%, (5-10)%, (10-20)%, (20-30)% and (20-30)% centrality [STAR, PRC **79**, 034909 (2009)]
- $v_2\{2\}$ in (0-5)%, (5-10)%, (10-20)%, (20-30)% and (20-30)% centrality [STAR, PRC **72**, 014904 (2005)]

Pb+Pb at $\sqrt{s_{NN}} = 2.76$ TeV:

- N_{ch} in $|\eta| < 0.5$ in (5-10)%, (10-20)%, (20-30)% and (20-30)% centrality [ALICE PRL **106**, 032301 (2011)]
- $v_2\{2\}$ in (5-10)%, (10-20)%, (20-30)% and (20-30)% centrality [ALICE PRL **116**, 132302 (2016)]

Pb+Pb at $\sqrt{s_{NN}} = 5.02$ TeV:

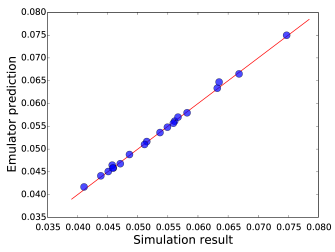
- N_{ch} in $|\eta| < 0.5$ in (10-20)%, (20-30)% and (20-30)% centrality [ALICE PRL **116**, 222302 (2016)]
- $v_2\{2\}$ in (10-20)%, (20-30)% and (20-30)% centrality [ALICE PRL **116**, 132302 (2016)]

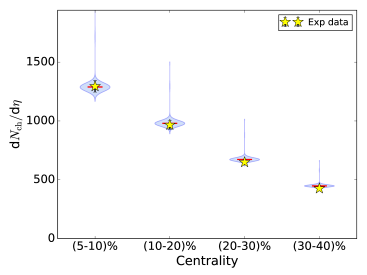


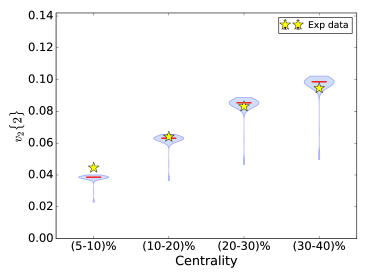
Left: Posterior probability distribution for EoS s87r, with kinetic decoupling T_{dec} at 150 MeV (blue) or 120 MeV (red).

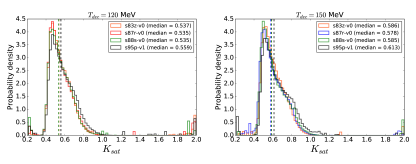
Below: 20-point emulator verification for s87r $v_2\{2\}$ at (10-20)% for Pb+Pb at $\sqrt{s_{NN}} = 2.76$ TeV with $T_{dec} = 150$ MeV

Right: Distribution of predicted model outputs for Pb+Pb at $\sqrt{s_{NN}} = 2.76$ TeV with $T_{dec} = 150$ MeV for N_{ch} (top) and $v_2\{2\}$ (bottom).



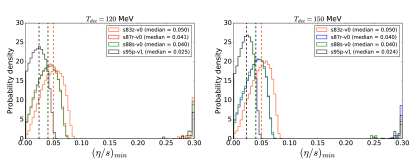


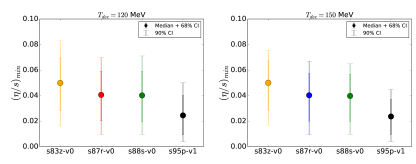


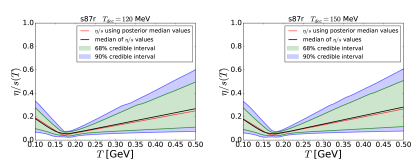


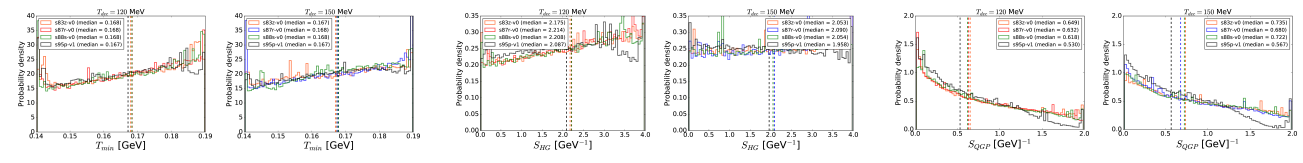
- Change of EoS has negligible effect on K_{sat}
- $\sim$ 60% increase in $(\eta/s)_{min}$ median when using s87r or s88s compared to **s95p**
- $\sim$ 25% increase in $(\eta/s)_{min}$ when using s83z compared to s87r and s88s
- Differences remain within the uncertainties, however
- T_{min} and the slope parameters remain unconstrained by the present analysis

  $\Rightarrow$ Introduce more observables to further limit the parameter space: Identified particle data ($N, \langle p_T \rangle$ of π, K, p...); higher flow harmonics (v_3, v_4); Xe+Xe data









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