Evolution of higher moments of multiplicity distribution

Radka Sochorová[a], Boris Tomášik[a,b]

[a]Faculty of Nuclear Sciences and Physical Engineering, Czech Technical University in Prague, Czech Republic
[b]Univerzita Mateja Bela, Banská Bystrica, Slovakia
boris.tomasik@umb.sk

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1. Motivation

The overall observed multiplicity of different types of particles from ultrarelativistic nuclear collisions agrees with the statistical model at temperatures above 160 MeV.

The phase transition temperature can be also determined from multiplicity fluctuations. Higher-order susceptibilities determined by lattice QCD are then compared with data on higher-order moments of multiuplicity distribution. The extracted temperature is usually lower than 160 MeV.

The main aim of this work is to study the evolution of the multiplicity distribution in fireball that cools down after the chemical freeze-out. Can different moments of multiplicity distribution be influenced by the temperature decrease differently?

Multiplicity distribution is evolved with the help of a master equation. Particularly we focus on the higher factorial moments from which all other kinds of moments, eg. central moments or the coefficients of skewness and kurtosis, can be calculated. We first study relaxation of factorial moments when the temperature is fixed. Then, the cooling scenario is investigated.

2. Relaxation of factorial moments

We consider a binary process $a_1 a_2 \leftrightarrow b_1\, b_2$ with $a \neq b$

The master equation for $P_n(\tau)$, the probability of finding n pairs $b_1\, b_2$ in dimensionless time τ has the following form

$$\frac{dP_n(\tau)}{d\tau} = \varepsilon[P_{n-1}(\tau) - P_n(\tau)] - [n^2 P_n(\tau) - (n+1)^2 P_{n+1}(\tau)]$$

where $n = 0, 1, 2, 3\, \dots$, $\varepsilon = G\langle N_{a_1}\rangle\langle N_{a_2}\rangle / L$, $\tau = t/\tau_0^c$ is the dimensionless time variable, $\tau_0^c = V/L$ is the relaxation time, V is the proper volume of the reaction, G is the creation term, L is the annihilation term, they are obtained as thermally averaged cross section $\langle \sigma v_{rel} \rangle$

From the master equation we can calculate factorial moments:
$F_2 = \langle N(N-1)\rangle$, $F_3 = \langle N(N-1)(N-2)\rangle$, $F_4 = \langle N(N-1)(N-2)(N-3)\rangle$

We let the distribution of the multiplicity approach equilibrium value with the help of master equation.

For numerical calculations were used binomial initial conditions:

$$P_1 = 0.005, P_0 = 0.995$$

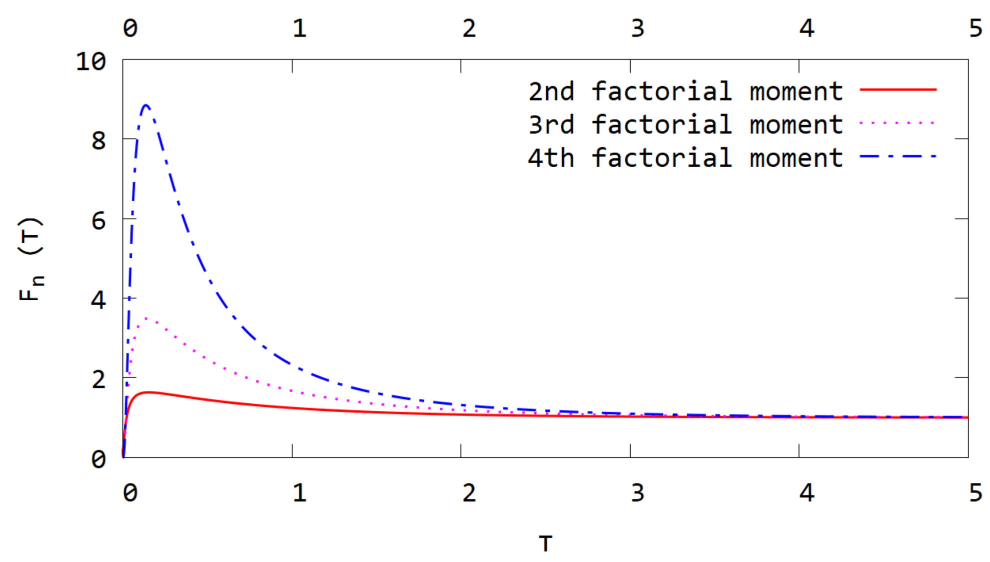

Figure: Time evolution of factorial moments divided by their equilibrium values for $\varepsilon = 0.1$

All curves converge to 1

Relaxation time for all moments is the same!

3. Higher moments in a cooling fireball

Realistic description of fireball evolution must include decreasing temperature.

If the temperature changes, also the relaxation time will change. Thus one cannot use dimensionless time because relaxation time was the typical scale in introducing dimensionless time. We need to go back to real time in the master equation and calculate creation and annihilation term for each temperature.

$$\frac{dP_n(t)}{dt} = \frac{G}{V}\langle N_{a_1}\rangle\langle N_{a_2}\rangle[P_{n-1}(t) - P_n(t)] - \frac{L}{V}[n^2 P_n(t) - (n+1)^2 P_{n+1}(t)]$$

We shall study, how higher moments evolve in a scenario with decreasing temperature.

Simple toy model: temperature and volume behave like in 1D longitudinally boost-invariant expansion (Bjorken scenario)

$$T^3 = T_0^3 \frac{t_0}{t}, \qquad V = V_0 \frac{t}{t_0}$$

Initial state for the evolution: $t_0 = 2.2$ fm/c, $T_0 = 165$ MeV, $V_0 = 125$ fm^3,
Final state for the evolution: $t_f = 10$ fm/c, $T_f = 100$ MeV,

As a model case we will study strangeness production via the reaction $\pi^+ + n \rightarrow K^+ + \Lambda$

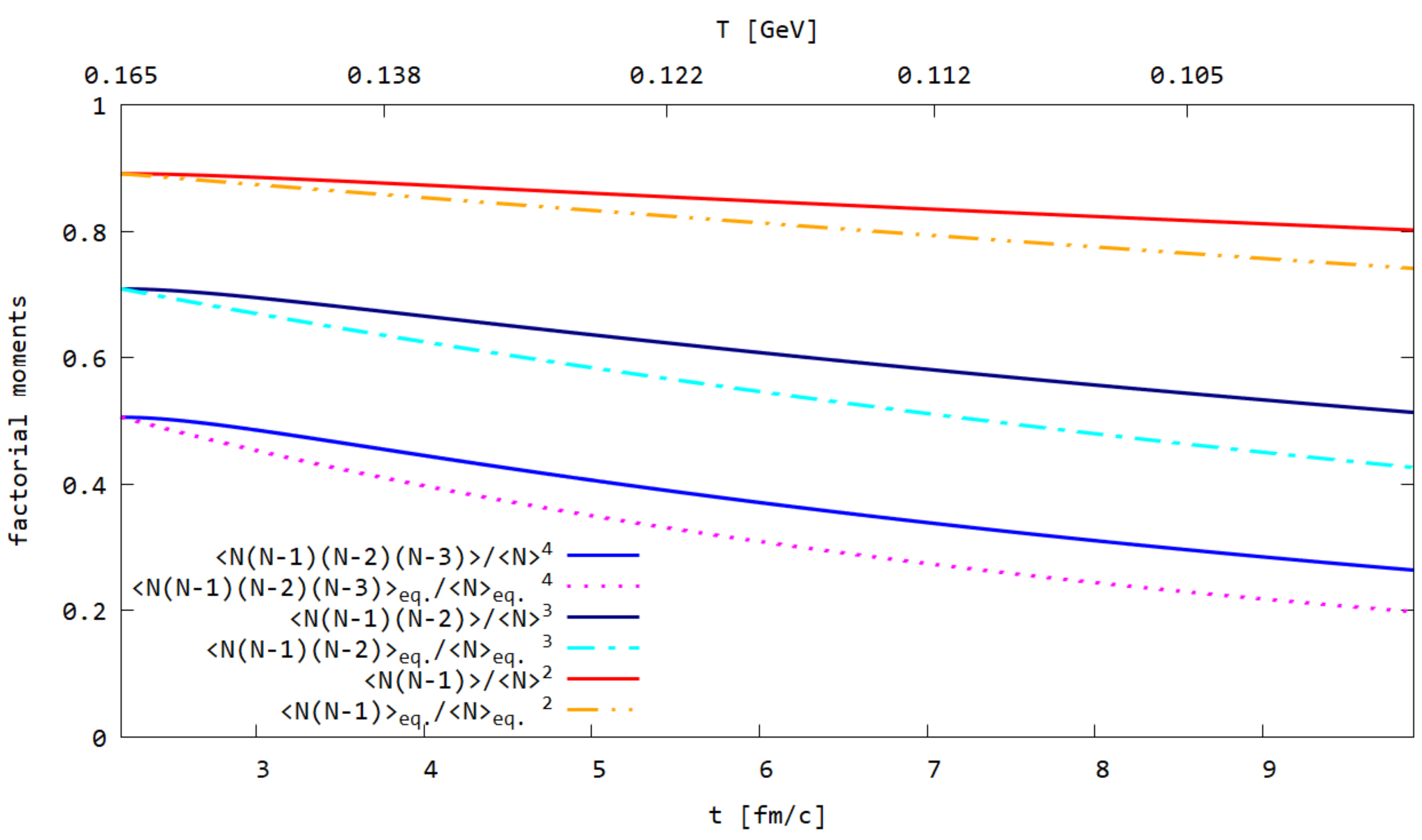

Figure: Scaled factorial moments for gradual change of temperature.
Solid lines: evolution of moments according to master equation. The value of $\langle v_{rel}\sigma\rangle$ scaled 200 times.
Dashed lines: equillibrium values at the given temperature.

4. The freeze-out temperature

At the hadronisation temperature we set the moments to equilibrium values, then we let them evolve. Their values are fixed when temperature is lowered to 100 MeV. Their values are off equilibrium there. If one would interpret the observed values of the moments as being established in equilibrium, what would be the extracted (apparent) freeze-out temperature?

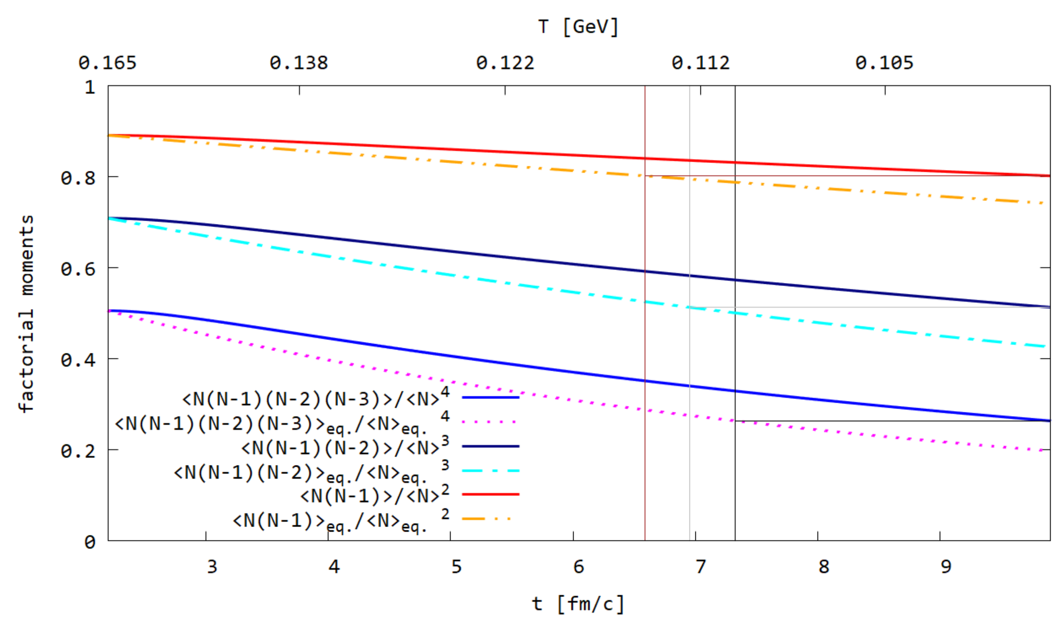

Figure: Apparent freeze-out temperature of factorial moments. Solid lines: evolved with master equation. $\langle v_{rel}\sigma\rangle$ scaled 200x. Dashed lines: equilibrium values.

In experimental data, more conveniently the central moments are used. Often, one uses their combinations like the coefficient of skewness or the coefficient of kurtosis.

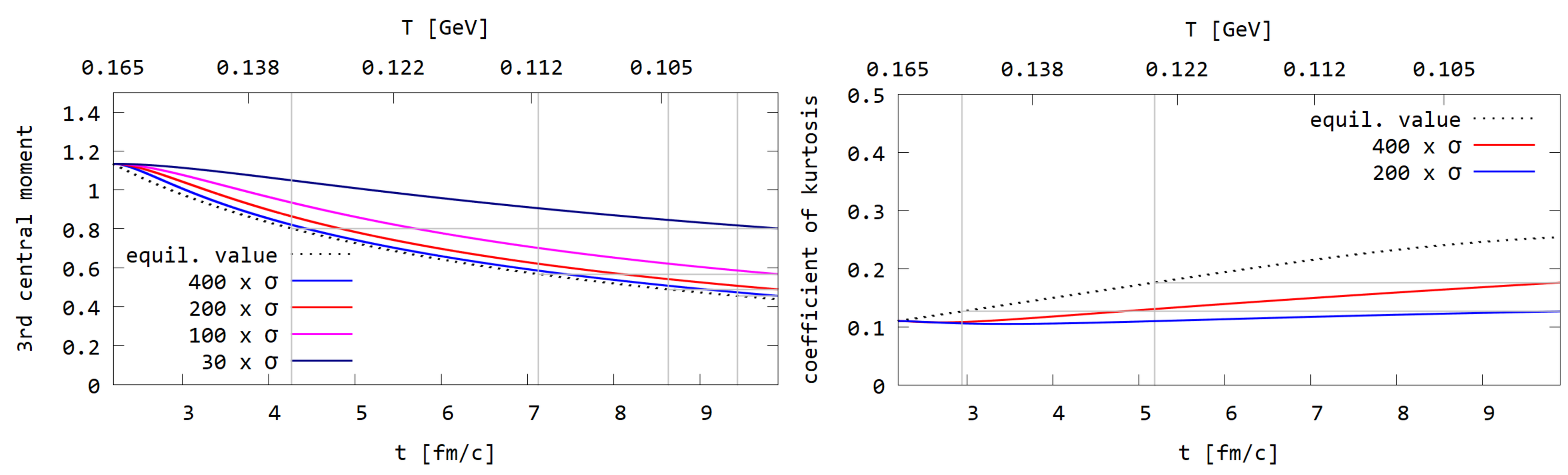

Figure Left: time evolution of the 3rd central moment. Right: time evolution of kurtosis. Different curves represent different values of $\langle \sigma v_{rel} \rangle$.

While the 3rd central moment is decreasing, the coefficient of kurtosis increases. The extracted apparent temperature strongly depends on the chosen observable. We have nonequilibrium evolution of the moments it is very difficult to determine the unique freeze-out temperature from them.

5. Conclusions

- If equilibrium is broken, higher factorial moments of multiplicity distribution differ more from their equilibrium values than the lower moments.
- Evolution of chemical reaction off equilibrium may mimmick different temperatures for different orders of the (factorial or central) moments (we demonstrated this on the example of $\pi^+ + n \rightarrow K^+ + \Lambda$)
- The behavior of the combination of the central moments depends on the combination of moments we choose.
- We should be very careful when we want to extract the freeze-out temperature from higher moments

References

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