Hydrodynamic tails and fluctuation bounds on the bulk viscosity

Mauricio Martinez, Thomas Schäffer

Physics Department, North Carolina State University, Raleigh, USA

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Abstract

We study the small frequency behavior of the bulk viscosity spectral function using stochastic fluid dynamics. We obtain a number of model independent results, including the long-time tail of the bulk stress correlation function, and the leading non-analyticity of the spectral function at small frequency. We also establish a lower bound on the bulk viscosity which is weakly dependent on assumptions regarding the range of applicability of fluid dynamics. The bound on the bulk viscosity ζ scales as

$$\zeta_{min} \sim (P - \frac{2}{3}\mathcal{E})^2 \sum_i D_i^2$$

where Di are the diffusion constants for energy and momentum, and P − 2/3 $\mathcal{E}$, where P is the pressure and $\mathcal{E}$ is the energy density, is a measure of scale breaking. Applied to the cold Fermi gas near unitarity this bound implies that the ratio of bulk viscosity to entropy density satisfies ζ/s $\gtrsim$ 0.1.

Hydrodnamic tails: formalism

We calculate the symmetrized correlation function

$$G_S^{\mathcal{OO}}(\omega,\mathbf{k}) = \int d^3x \int dt\, e^{i(\omega t - \mathbf{k}\cdot\mathbf{x})} \left\langle \frac{1}{2}\{\mathcal{O}(t,\mathbf{x}), \mathcal{O}(0,0)\} \right\rangle$$

where the operator $\mathcal{O}$ = P-2/3 $\mathcal{E}_0$ is the trace anomaly in any frame. This correlation function can be factorized in a set of two-point functions

$$\begin{aligned} G_S^{\mathcal{OO}}(\omega,0) \;=\; & \int \frac{d\omega'}{2\pi} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \left[2a_{\rho\rho}^2 \Delta_S^{\rho\rho}(\omega',\mathbf{k}) \Delta_S^{\rho\rho}(\omega-\omega',\mathbf{k}) \right. \\ & \left. + a_{\rho T}^2 \Delta_S^{\rho\rho}(\omega',\mathbf{k}) \Delta_S^{TT}(\omega-\omega',\mathbf{k}) + 2a_{TT}^2 \Delta_S^{TT}(\omega',\mathbf{k}) \Delta_S^{TT}(\omega-\omega',\mathbf{k}) \right] \end{aligned}$$

where the symmetrized density and temperature correlations functions are

$$\begin{aligned} \Delta_S^{\rho\rho}(\omega,\mathbf{k}) \;=\; 2\rho T \Bigg\{ & \frac{\Gamma k^4}{(\omega^2 - c_s^2 k^2)^2 + (\Gamma \omega k^2)^2} + \frac{\Delta c_P}{c_s^2} \frac{D_T \mathbf{k}^2}{\omega^2 + (D_T k^2)^2} \\ & - \frac{\Delta c_P}{c_s^2} \frac{(\omega^2 - c_s^2 k^2) D_T \mathbf{k}^2}{(\omega^2 - c_s^2 k^2)^2 + (\Gamma \omega k^2)^2} \Bigg\}, \end{aligned}$$

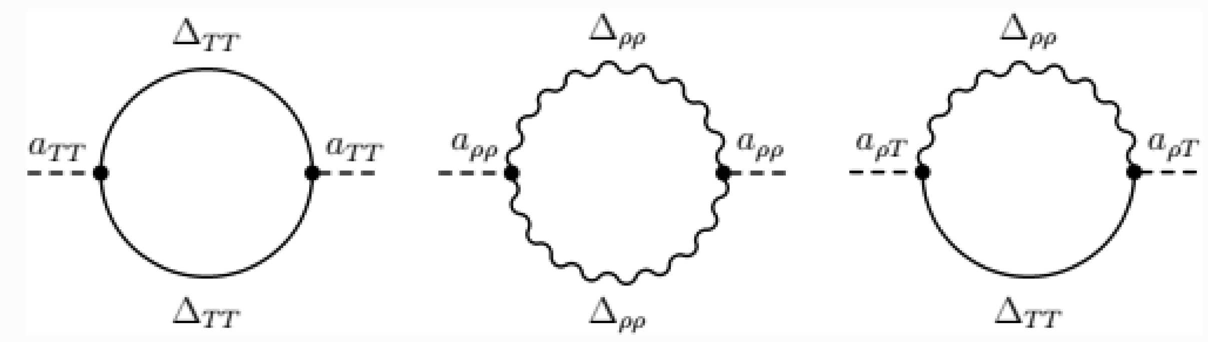
$$\Delta_S^{TT}(\omega,\mathbf{k}) \;=\; \frac{2T^2}{c_P} \frac{D_T \mathbf{k}^2}{\omega^2 + (D_T \mathbf{k}^2)^2}$$

Density correlation function receives contributions from both sound and diffusive modes while the temperature receives only contributions from the diffusive channel.

The fluctuation-dissipation theorem relates the symmetric and retarded response functions

$$G_S(\omega,\mathbf{k}) \simeq -\frac{2T}{\omega} \mathrm{Im}\, G_R(\omega,\mathbf{k})$$

Hydrodynamic tails: One loop diagrams



The leading infrared behavior of the retarded correlator

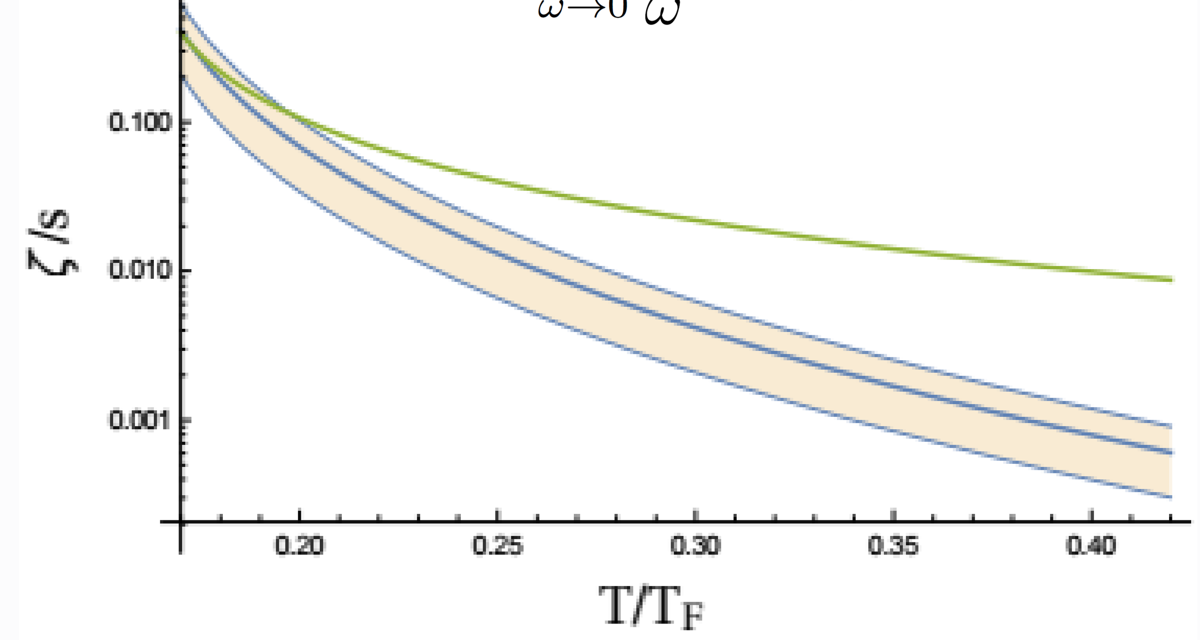
$$G_R^{\mathcal{OO}}(\omega,\mathbf{0}) = -A_T L(\omega,\Lambda,2D_T) - A_\Gamma L(\omega,\Lambda,\Gamma)$$

$$A_T = \frac{2a_{TT}^2 T^3}{c_P^2} + \frac{2a_{\rho\rho}^2 \rho^2 T (\Delta c_P)^2}{c_s^4} + \frac{a_{\rho T}^2 \rho T^2 \Delta c_P}{c_P c_s^2} \qquad A_\Gamma = \frac{a_{\rho\rho}^2 \rho^2 T}{c_s^4}$$

Fluctuation bound on $\zeta\,/\,s$

The Kubo formula is

$$\zeta = -\lim_{\omega\to 0} \frac{1}{\omega} \mathrm{Im}\, G_R^{\mathcal{OO}}(\omega,\mathbf{0})$$



The cut-off dependent term of the bulk viscosity

$$\zeta_\Lambda = \frac{1}{18\pi^2} \left(\frac{A_T \Lambda}{2D_T} + \frac{A_\Gamma \Lambda}{\Gamma} \right)$$

combines with the bare bulk viscosity to determine the physical bulk viscosity via RG procedure. This bound increases with *Λ*. The largest value of is determined by the breakdown scale of fluid dynamics

Lower bound of the bulk viscosity

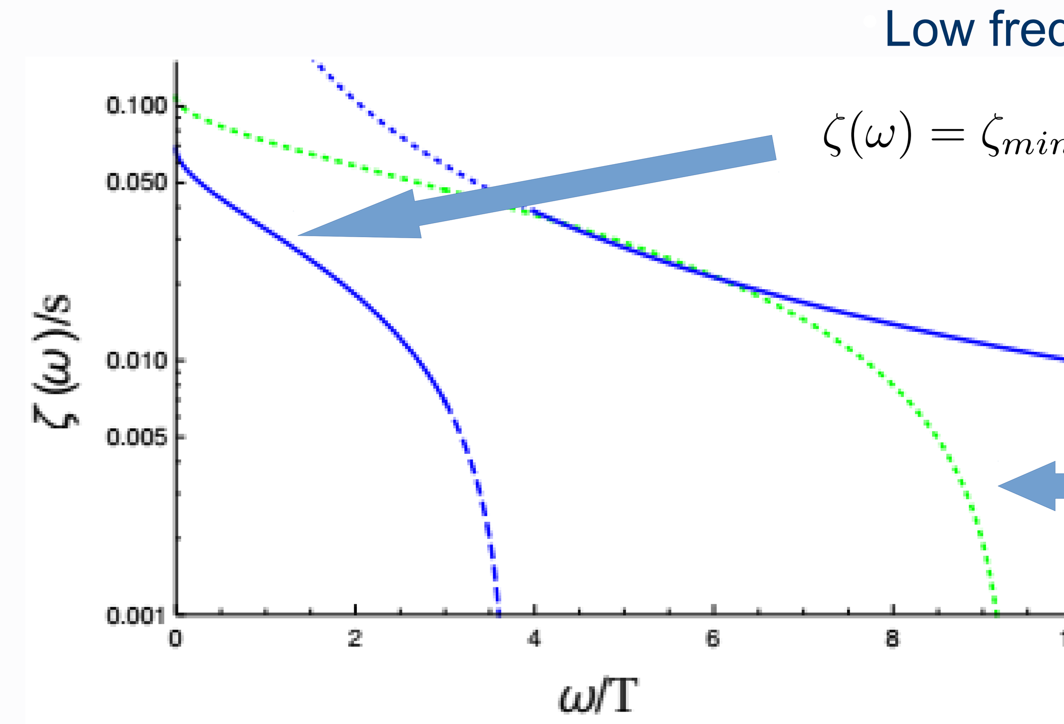
$$\zeta_{min} = \left(\frac{A_T}{2D_T^2} + \frac{\sqrt{5}A_\Gamma}{\sqrt{3}\Gamma^2} \right) \sqrt{\frac{T}{m}}$$

The spectral function is

$$\zeta(\omega) = -\frac{1}{9\omega} \mathrm{Im} G_R^{\mathcal{OO}}(\omega,\mathbf{0})$$

Low frequency tail

$$\zeta(\omega) = \zeta_{min} - \left(\frac{A_T}{(2D_T)^{3/2}} + \frac{A_\Gamma}{\Gamma^{3/2}} \right) \frac{\sqrt{\omega}}{36\sqrt{2}\pi}.$$



High frequency tail

$$\zeta(\omega) = \frac{\mathcal{C}}{36\pi\sqrt{m\omega}} \frac{1}{1 + a_s^2 m\omega}$$

Conclusions

We have shown that fluctuations provide a lower bound on the bulk viscosity that only depends on the thermal conductivity and shear viscosity as well as scale breaking in the equation of state. The physical mechanism for can be understood in terms of the rate of equilibration of thermal fluctuations. If the fluid is compressed then the equilibrium density and temperature change,and as result the mean square fluctuations in ρ, T, v have to change as well. However, the mechanism for fluctuations to adjust involves diffusion of heat and momentum, and does not take place instantaneously. As a consequence the fluid is slightly out of equilibrium, entropy increases, and the effective bulk viscosity is not zero. Extensions of this formalism to relativistic fluids is possible by using hydro-kinetics [2,3].

References

1. M. Martinez, T. Schäfer, Phys. Rev.A 96, 063607 (2017)

2. Y. Akamatsu et. al., Phys. Rev. C 95, 014909 (2016), Phys. Rev. C 97, 024902 (2017)

3. M. Martinez, T. Schäfer, Forthcoming