Transport coefficients of two-flavor quark matter from the Kubo formalism

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Abstract

Transport coefficients of quark plasma are the key inputs in the hydrodynamic description of heavy-ion collisions. We compute the four first-order transport coefficients of two-flavor quark plasma from the Kubo formalism at finite temperatures and densities. One of our key results is that the bulk viscosity exceeds the shear viscosity close to the chiral phase transition line. We find that the Wiedemann-Franz law for the ratio κ/σ does not hold, and we conjecture on the basis of the uncertainty principle a lower bound for the ratio $\kappa T/c_V \geq \hbar c^2/18k_B$, where c_V is the heat capacity per unit volume.

Kubo formulas

The first-order transport coefficients are given by the following Green-Kubo formulas

$$\eta = -\frac{1}{10}\frac{d}{d\omega}\mathrm{Im}\Pi^R_{\hat{\pi}_{\mu\nu}\hat{\pi}^{\mu\nu}}(\omega)\Big|_{\omega=0},$$

$$\zeta = -\frac{d}{d\omega}\mathrm{Im}\Pi^R_{\hat{p}^*\hat{p}^*}(\omega)\Big|_{\omega=0},$$

$$\kappa = \frac{1}{3T}\frac{d}{d\omega}\mathrm{Im}\Pi^R_{\hat{h}_\mu\hat{h}^\mu}(\omega)\Big|_{\omega=0},$$

$$\sigma = \frac{1}{3}\frac{d}{d\omega}\mathrm{Im}\Pi^R_{\hat{j}_\mu\hat{j}^\mu}(\omega)\Big|_{\omega=0},$$

where

$$\Pi^R_{\hat{X}\hat{Y}}(\omega) = -i\int_0^\infty dt e^{i\omega t}\int d\boldsymbol{x}\langle[\hat{X}(\boldsymbol{x},t),\hat{Y}(0)]\rangle_0$$

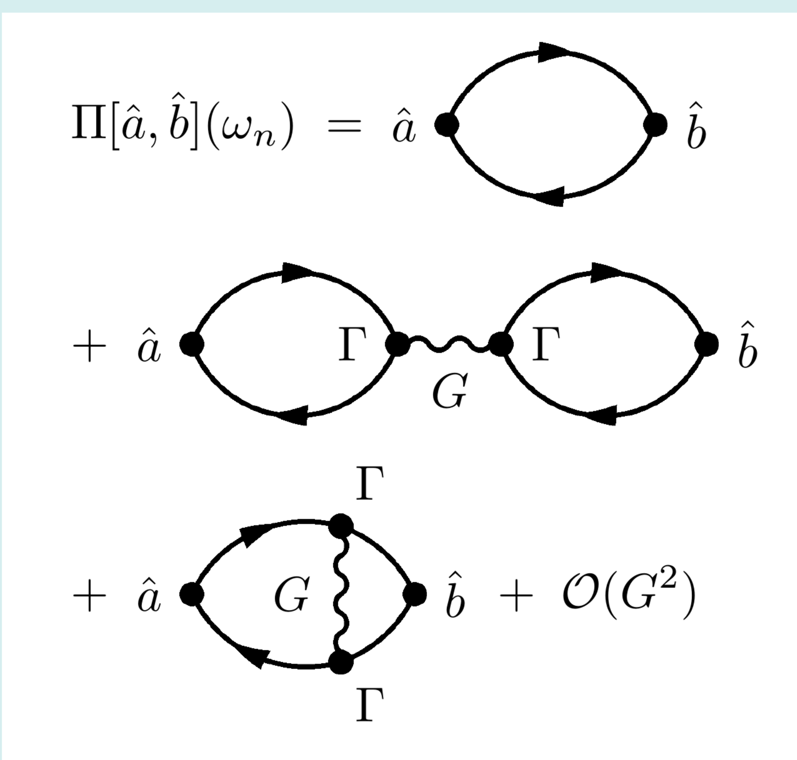
is the two-point equilibrium retarded Green's function; $\hat{h}_\mu$ and $\hat{j}_\mu$ are the heat current and electric current, respectively; $\hat{\pi}_{\mu\nu}$ is the shear stress tensor and $\hat{p}^*$ - the bulk viscous pressure.

Model and correlation functions

We use the two-flavor Nambu–Jona-Lasinio model with scalar and pseudoscalar interaction

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m_0)\psi + \frac{G}{2}\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\boldsymbol{\tau}\psi)^2\right],$$

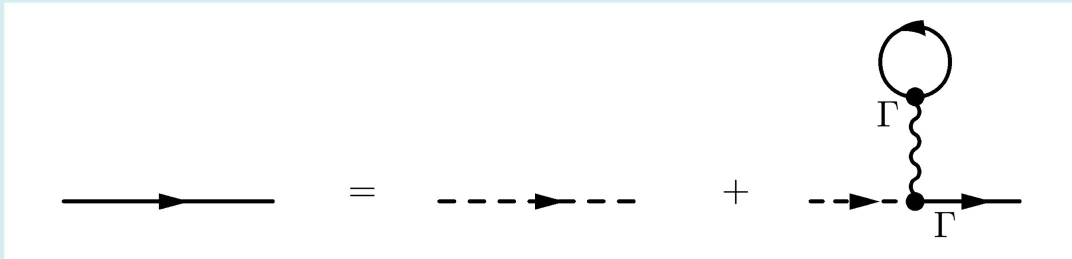
Two-point correlators are given by an infinite series of Feynman diagrams, where vertex corrections are suppressed by a factor of $1/N_c$



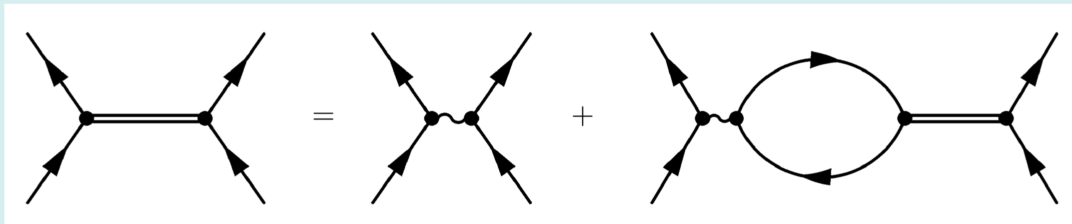
η, κ, σ are given by single-loop diagrams, whereas the bulk viscosity ζ includes an infinite series of multi-loop diagrams. All diagrams should be evaluated with *full quark propagators*.

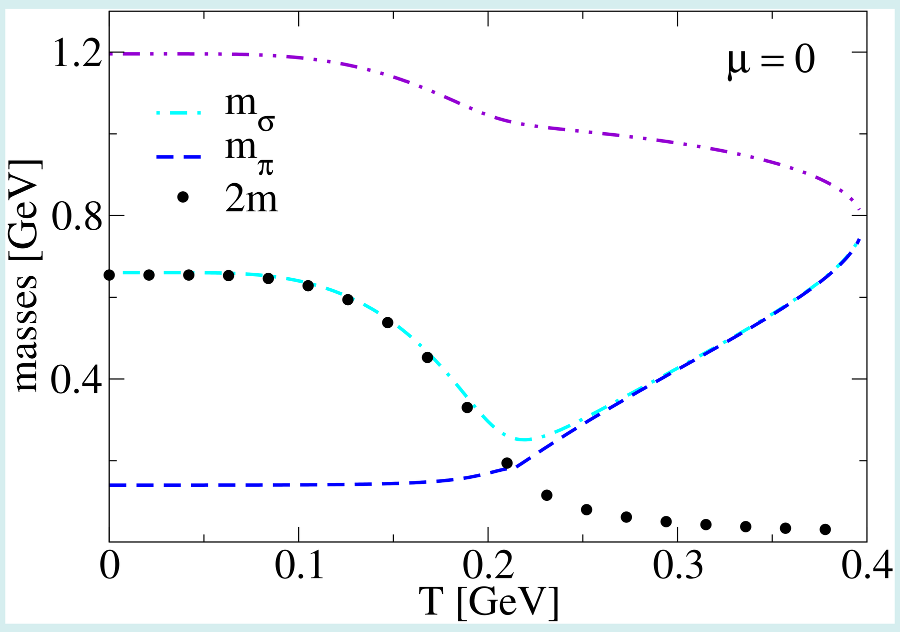
Quark and meson masses

The dynamically generated (constituent) quark mass is found from the gap equation



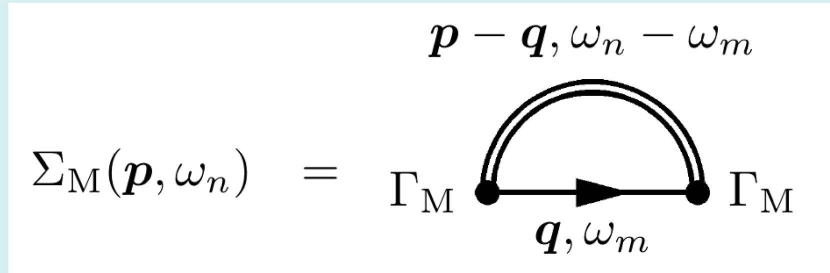
The meson propagators are obtained from the Bethe-Salpeter equation





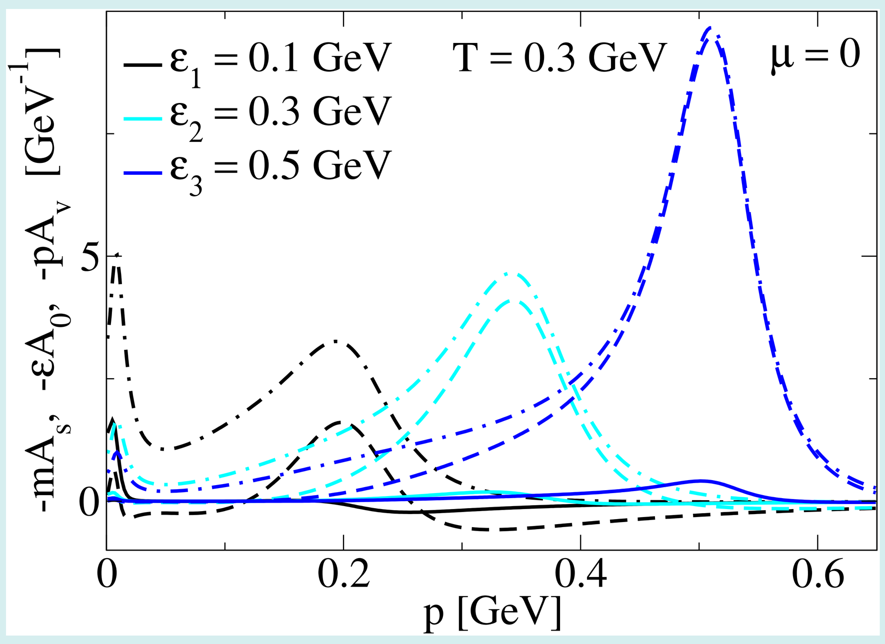
Quark spectral function

Dominant processes in the quark self-energy are the meson decay into quark-antiquark pair and its inverse process ($\Gamma_\pi = i\gamma_5\boldsymbol{\tau}$, $\Gamma_\sigma = 1$)

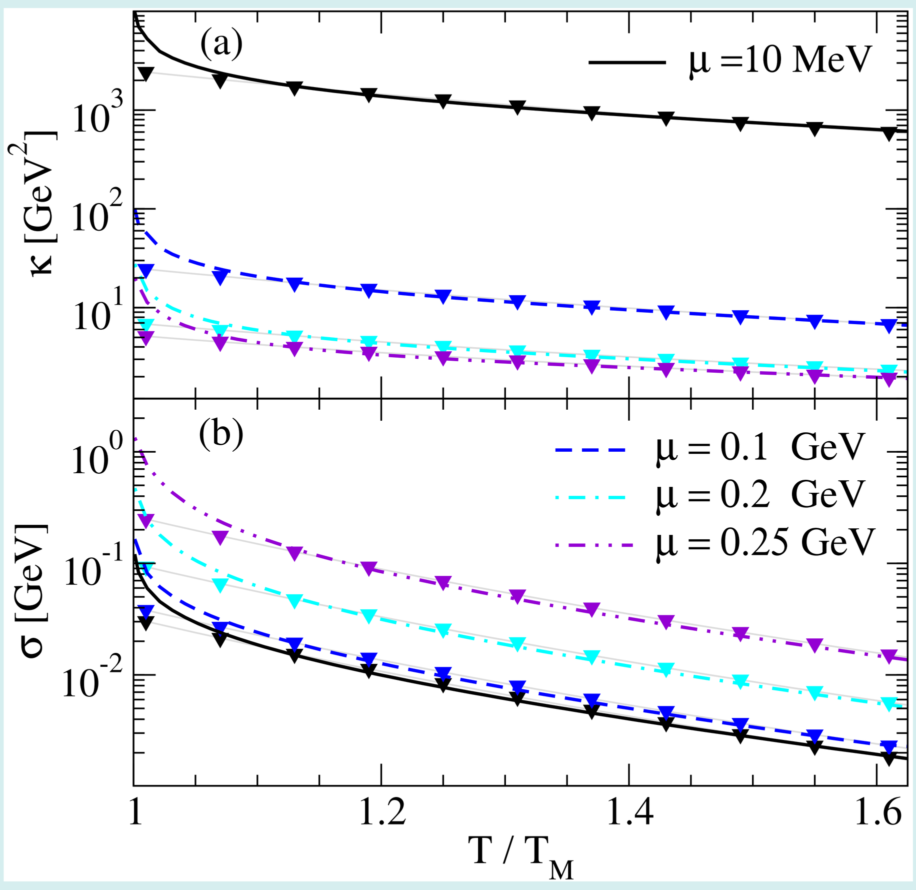


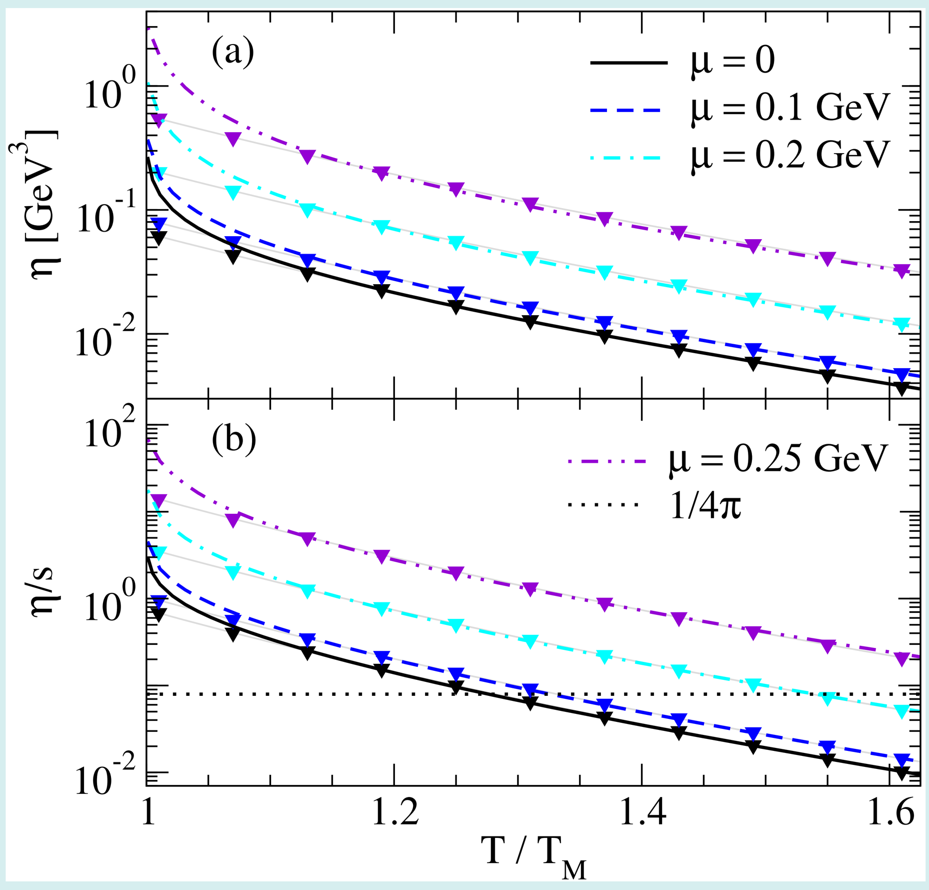
Quark self-energy and spectral function have three Lorentz components

$$\Sigma(p_0,\boldsymbol{p}) = m\Sigma_s + p_0\gamma_0\Sigma_0 - \boldsymbol{p\gamma}\Sigma_v,$$
$$A(p_0,\boldsymbol{p}) = -(mA_s + p_0\gamma_0 A_0 - \boldsymbol{p\gamma}A_v)/\pi.$$

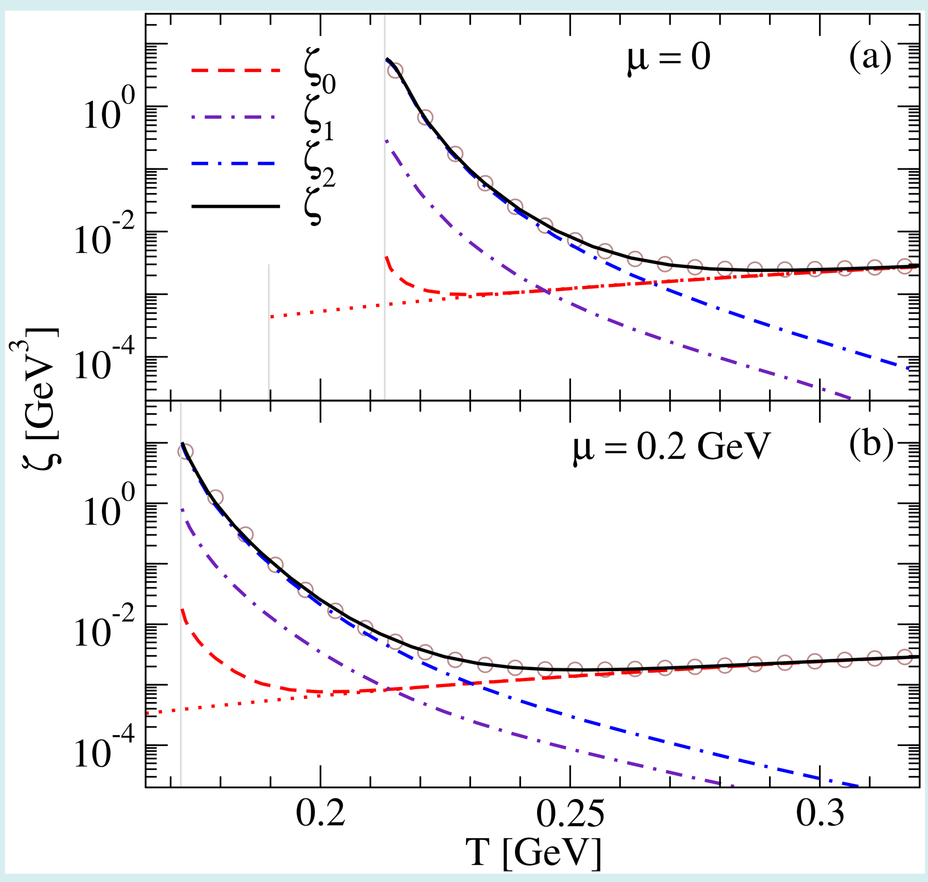


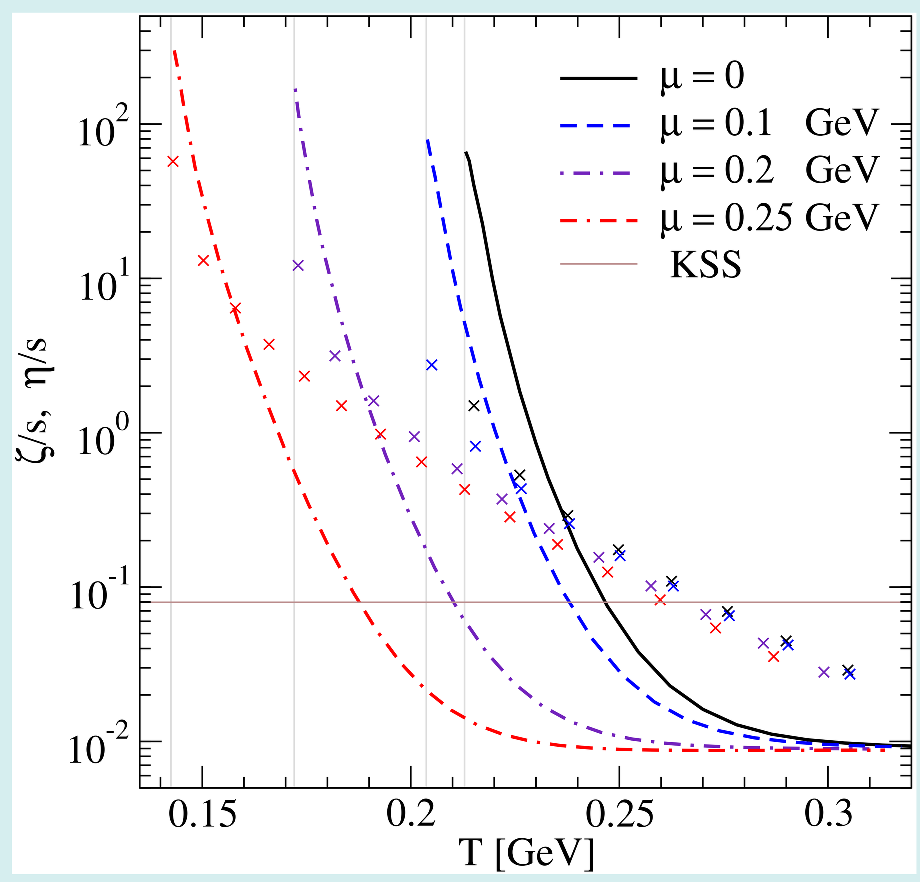
Numerical results for transport coefficients





- κ, σ and η show a universal dependence on the scaled temperature T/T_M.
- At high temperatures they scale as $\kappa \propto T^{-3}$, $\sigma \propto T^{-6}$, $\eta \propto T^{-6}$, and $\eta/s \propto T^{-9}$.
- At high temperatures the ratio η/s undershoots the KSS bound $1/4\pi$.
- We find the following relation between the conductivities $\kappa/\sigma = 9h^2/10\pi\alpha T \simeq \pi^3 T^3/5\alpha\mu^2$.
- From the uncertainty principle we conjecture a lower bound for the ratio $\kappa T/c_V \geq \hbar c^2/18k_B$.





- At low temperatures the bulk viscosity is dominated by multi-loop contributions ζ_1 and ζ_2.
- In this regime the bulk viscosity dominates the shear viscosity by factors $5 \div 20$.
- At high temperatures and densities one-loop contribution $\zeta_0 \propto T^3$ becomes dominant.

References

[1] A. Harutyunyan, D. H. Rischke and A. Sedrakian, *Transport coefficients of two-flavor quark matter from the Kubo formalism*, Phys. Rev. D **95** (2017) 114021, [arXiv:nucl-th/1702.04291].

[2] A. Harutyunyan and A. Sedrakian, *Bulk viscosity of two-flavor quark matter from the Kubo formalism*, Phys. Rev. D **96** (2017) 034006, [arXiv:hep-ph/1705.09825].