Spectral function for overoccupied gluodynamics from real-time lattice simulations

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Physics motivation

Strong Yang-Mills fields appear in:

- weak coupling descriptions of QGP and heavy-ion collisions
- initial stages with $f \sim 1/g^2$ for $p \ll Q_s$
- infrared tail $f \sim T/\omega$ of thermal Bose distribution ($\omega \ll T$)

Their spectral properties? Controlled way to study initial stages?

Hard Thermal Loop (HTL) formalism [1] :

- description; scale separation between highly occupied soft modes $p \sim m$ and hard modes $p \sim \Lambda$ needed; expansion in $m/\Lambda \ll 1$.

Our objectives:

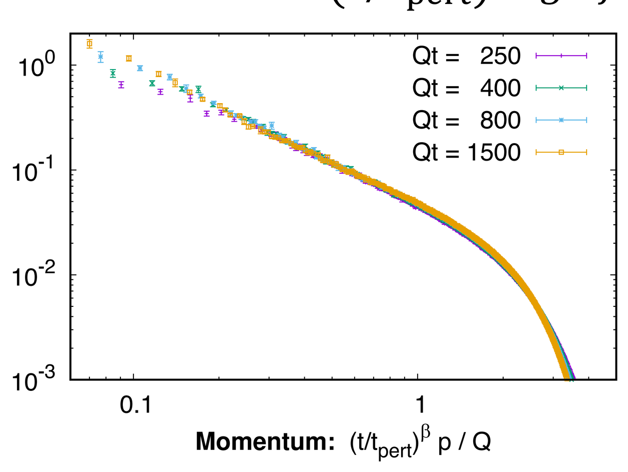
- develop non-perturbative approach not based on $m/\Lambda \ll 1$ to obtain spectral and statistical functions $\rho(t,\omega,p)$ and $F(t,\omega,p)$
- quantify to what extent HTL at LO is a good approx. of soft modes
- measure quantities exceeding LO HTL that are hard to compute diagrammatically. *Application*: damping rates $\gamma_{T/L}(p)$.

Studied system

Far-from-equilibrium, isotropic, overoccupied $f \sim 1/g^2$:

- undergoes a cascade of energy to the UV in a self-similar regime →
  $f(p,t) = t^\alpha f_S(t^\beta p)$ (see, e.g., [2])
- insensitive to details of the over-occupied initial condition
- scale separation grows with time as $m/\Lambda \sim (Qt)^{-2/7}$ (m: mass, Λ: hard scale, Q: constant scale)
- → HTL should be applicable.

Distribution: $(t/t_{\text{pert}})^{-\alpha} g^2 f$



Universal scaling exponents $\alpha = -4/7$ and $\beta = -1/7$

Theory

Classical-statistical lattice simulations:

- SU(N_c) gauge theory with $N_c = 2$ in temporal $A_0 = 0$ gauge
- large occupancies $f(p \sim \Lambda) \gg 1$ → dynamics approx. by class. EOM

Computation of spectral function ρ:

- perturb system at $t = t_{\text{pert}}$ with a source $j_{\boldsymbol{p}}(t,\boldsymbol{x}) \sim e^{i\boldsymbol{p}\cdot\boldsymbol{x}}\,\delta(t - t_{\text{pert}})$ (with restored Gauss law and in Coulomb gauge $\partial_j A_j = 0$ at t_{pert})
- employ *linear response theory*: split gauge field $A_j(t,\boldsymbol{x}) \mapsto A_j(t,\boldsymbol{x}) + a_j(t,\boldsymbol{x})$, solve newly developed linearized equations for $a_j(t,\boldsymbol{x})$ [3]
- extract retarded propagator from $a_j(t,\boldsymbol{p}) = \int dt' G_{R,jk}(t,t',\boldsymbol{p})\, j^k(t,\boldsymbol{p})$
- obtain spectral function from $G_{R,jk} = \theta(t-t')\,\rho_{jk}$
- use isotropy and homogeneity to improve the statistical uncertainty
- transverse polarization: $\rho_T(\boldsymbol{p}) = v_j^*(\boldsymbol{p})\rho^{jk}(\boldsymbol{p})v_k(\boldsymbol{p})$ with $\boldsymbol{v} \perp \boldsymbol{p}$
  longitudinal polarization: $\rho_L(\boldsymbol{p}) = p_j\, \rho^{jk}(\boldsymbol{p})\, p_k$

Further references

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Spectral function ρ

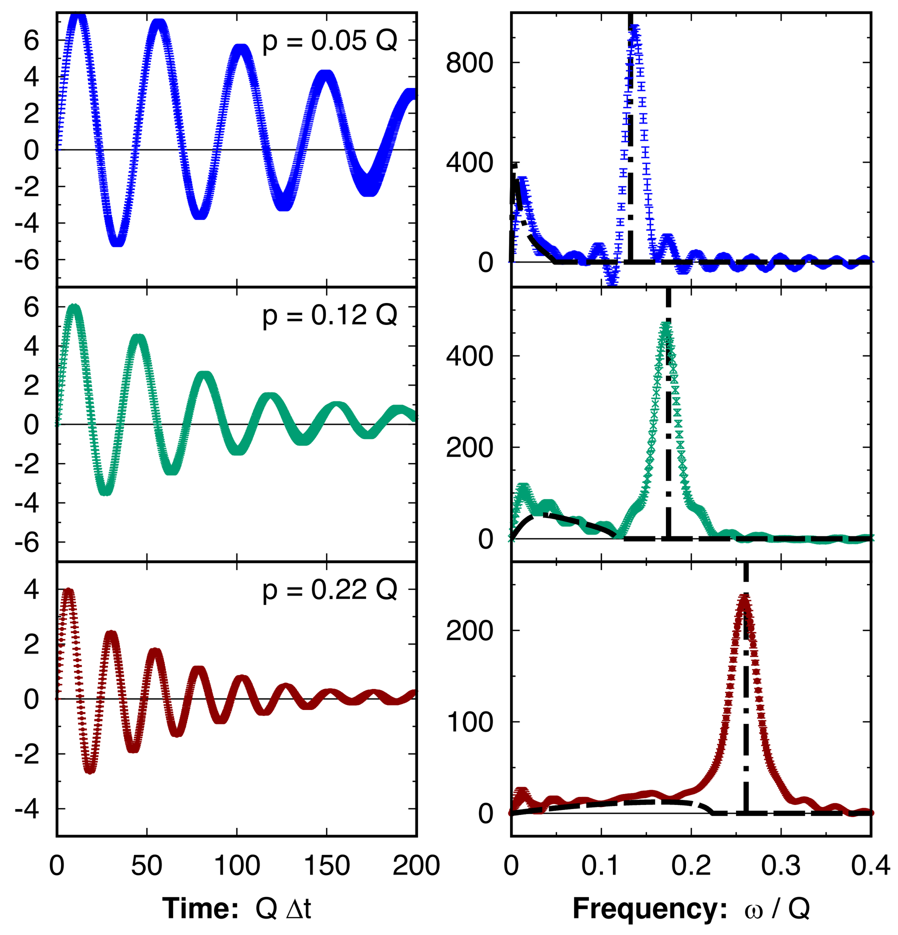
Transverse spectral function: ρ_T

ρ_T as function of $\Delta t = t - t'$ (left) or ω (right) at late time $t_{\text{pert}} \gg \Delta t$

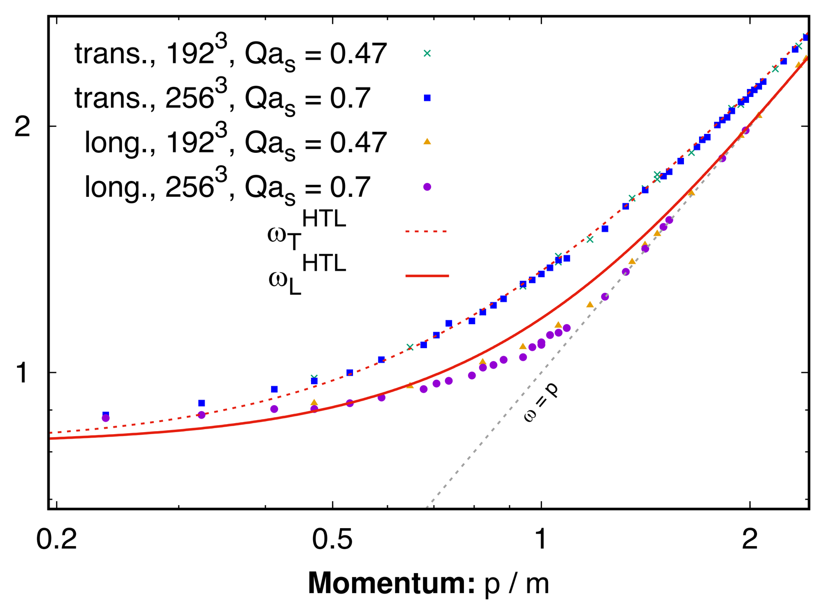
- black dashed lines: HTL at LO
- for $|\omega| \le p$: *Landau cut*
- damped oscillations, Lorentz peaks: *existence of quasi-particles* with dispersion $\omega_T(p)$ and damping rate $\gamma_T(p)$.

ρ_L is similar *(shown in paper)*

- but for $p \gtrsim m \approx 0.15\, Q$, quasiparticle peak suppressed
- Landau cut dominates then

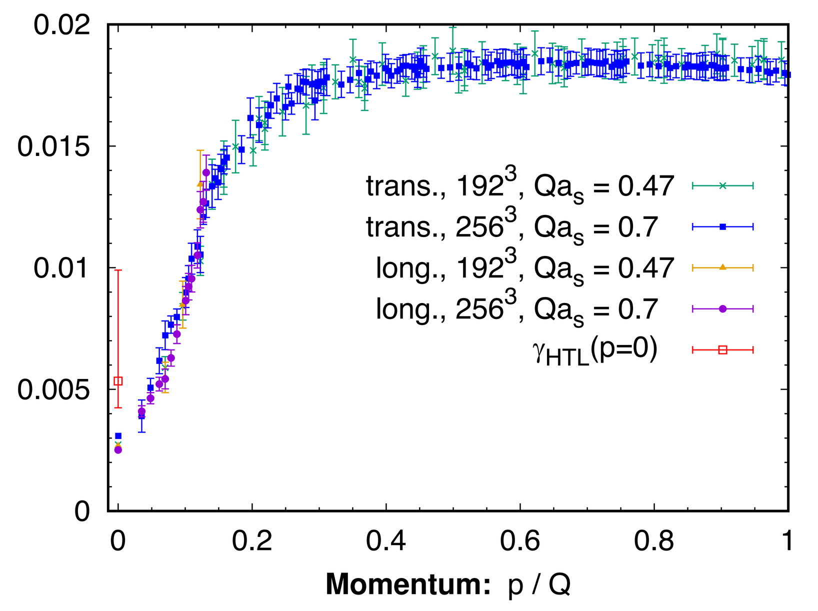


Extracted dispersion: $\omega(p)\,/\,m$



- extracted from peak position
- HTL predictions [1]: $\omega_{T,L}^{\text{HTL}}(p)$
- plasmon frequency $\omega_{\text{pl}} = \omega(p=0)$, asymptotic mass m_∞ (gap at $p \to \infty$), we get $m_\infty/\omega_{\text{pl}} = 0.96 \pm 0.03$
  (vs. HTL at LO: $\sqrt{2/3} \approx 0.82$).

Extracted damping rate: $\gamma(p)\,/\,Q$



- extracted by fitting to a damped oscillator
- HTL prediction [1]: $\gamma_{\text{HTL}}(p=0)$
- γ is beyond HTL at LO, it may contain non-perturbative contributions (magnetic scale)
- "isotropic" $\gamma_T \approx \gamma_L$ for $p \lesssim m$.

Statistical correlation function F

$$F_{jk}(t,t',\boldsymbol{p}) = \langle A_j(t,\boldsymbol{p})A_k^*(t',\boldsymbol{p})\rangle.$$

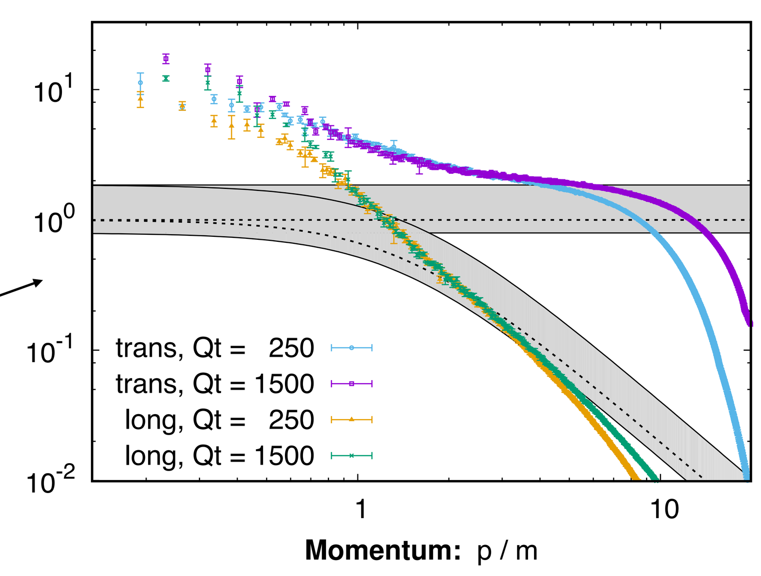
Observation: *(shown in paper)*

- $\partial_t\partial_{t'}F\,/\,\partial_t\rho = \partial_t\partial_{t'}F(t,\Delta t=0,p)$
  → ω-dependence of $\partial_t\partial_{t'}F$ as in $\partial_t\rho$

$\partial_t\partial_{t'}F(\Delta t = 0)$ **deviates from HTL:**

- HTL expectations as gray bands
  ($T_* \approx \int d^3p\, f^2 \,/\, 2\int d^3p\, f/p$.)
- enhanced $\partial_t\partial_{t'}F_{T,L}$ for $p \lesssim m$ visible

Ratio: $(\partial_t\partial_{t'}F\,/\,\partial_t\rho)\,/\,T_*$



Conclusion

a. We *developed a non-perturbative numerical approach* based on classical lattice simulations, linearized equations and linear response theory to extract the spectral function in over-occupied gauge systems.

b. *HTL at LO can describe* many observables but important *deviations* also found; for the first time, measurement of *damping rates* $\gamma_{T/L}(p)$

c. **Outlook:** We aim to use these techniques to study anisotropic, expanding systems for *initial stages* in heavy-ion collisions.