Bayesian estimation of heavy quark transport coefficients

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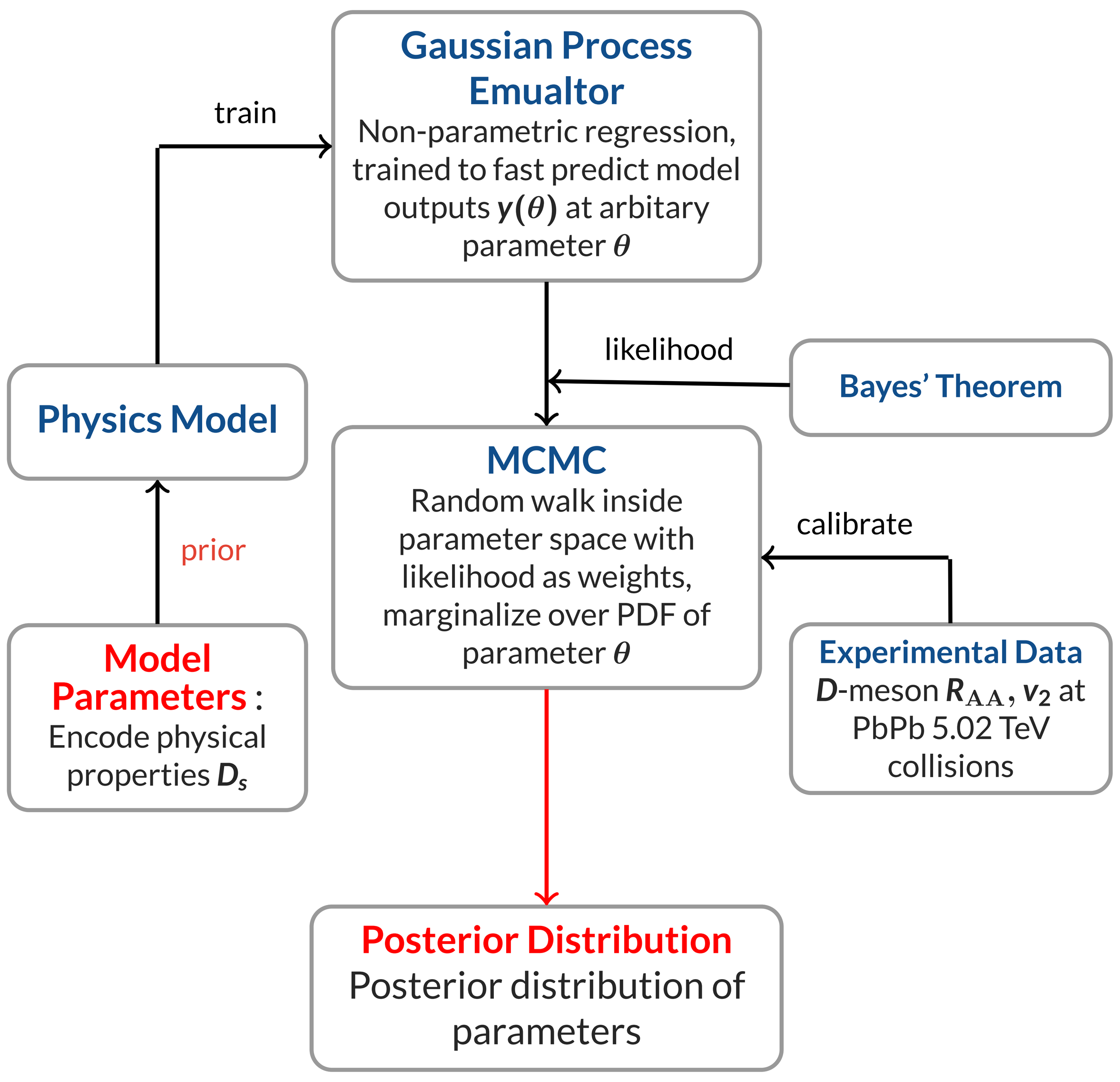
Model: improved Langevin dynamics with radiative energy loss

- Initial conditions: T$_R$ENTo+ FONLL
- In-medium propagation: $\frac{d\vec{p}}{dt} = -\eta_D(p)\vec{p} + \vec{\xi} + \vec{f_g}$
  - drag: $\eta_D(p) = \frac{2\pi T^2}{E} \cdot \frac{1}{[D_s 2\pi T]}$
  - thermal noise: $\langle \xi_i \xi_j \rangle = \frac{4\pi T^3}{[D_s 2\pi T]} \delta_{ij}$
  - gluon radiation: $\vec{f_g}\Delta t = -\sum_{i=1}^{\langle N_g \rangle} \vec{p}_{g,i}$,
    where $\langle N_g \rangle = \int dt dx dk_\perp^2 \frac{2\alpha_s P(x)}{\pi k_\perp^4} \sin^2\left(\frac{t-t_i}{2\tau_f}\right)\left(\frac{k_\perp^2}{k_\perp^2 + x^2 M^2}\right)^4 \cdot \frac{8\pi T^3}{[D_s 2\pi T]}$
- Hadronization: fragmentation + recombination; Hadronic stage: UrQMD
- Medium evolution: T$_R$ENTo+ iEbE-vishnu + UrQMD

Model Parameters: $D_s \to \theta$

$$D_s 2\pi T = \frac{1}{1+(\gamma^2 p)^2} \cdot \left[\alpha\left(1+\beta\left(\frac{T}{T_c}-1\right)\right)\right] + \frac{(\gamma^2 p)^2}{1+(\gamma^2 p)^2} \cdot [D_s 2\pi T]^{\mathrm{pQCD}}(\alpha_s)$$



Goal: model to data comparison

1. Simultaneously describe experimental data $y_{\rm exp}$
2. Quantitatively estimate diffusion coefficient D_s
3. Systematically compare between different models

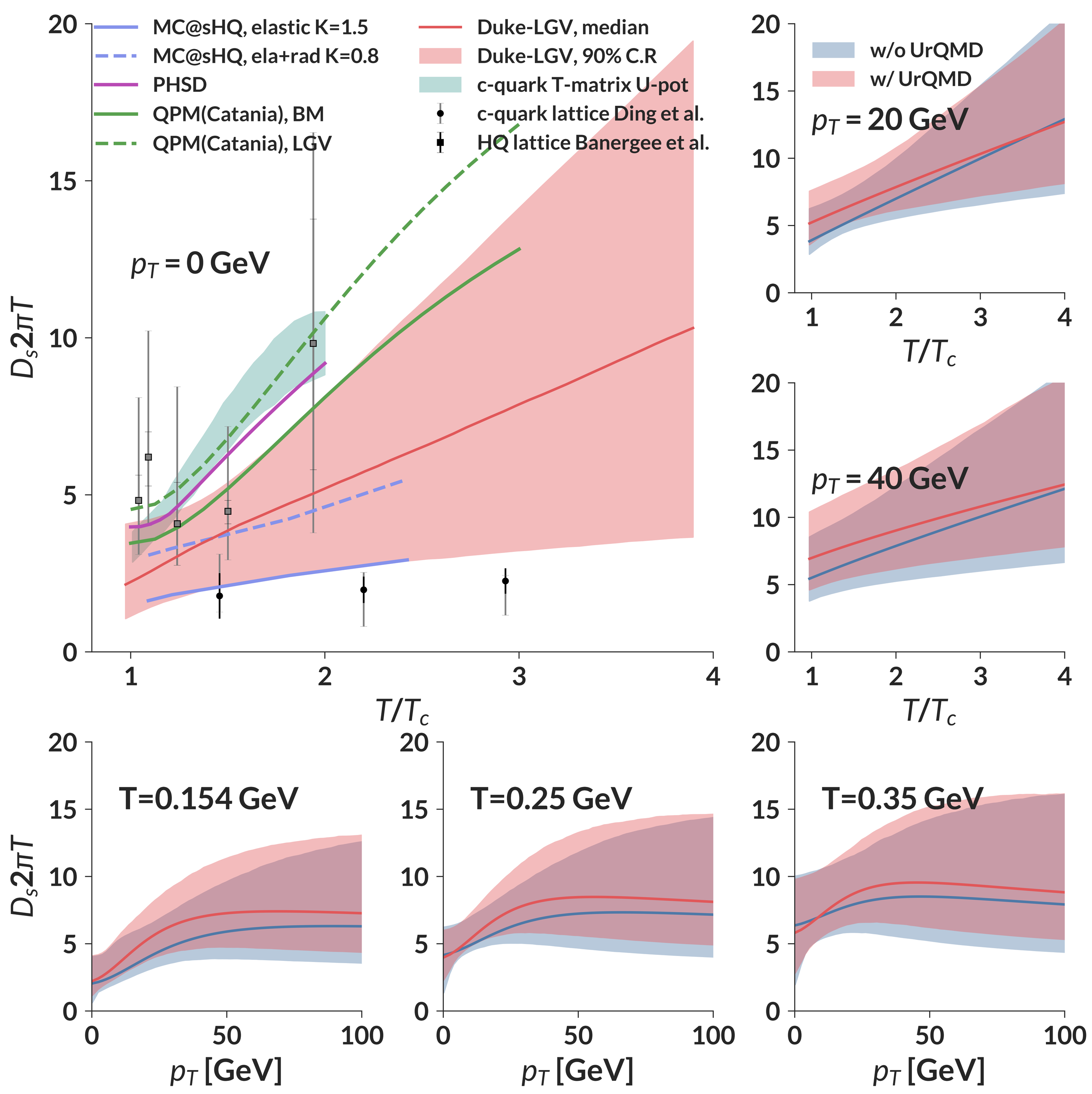
Methodology

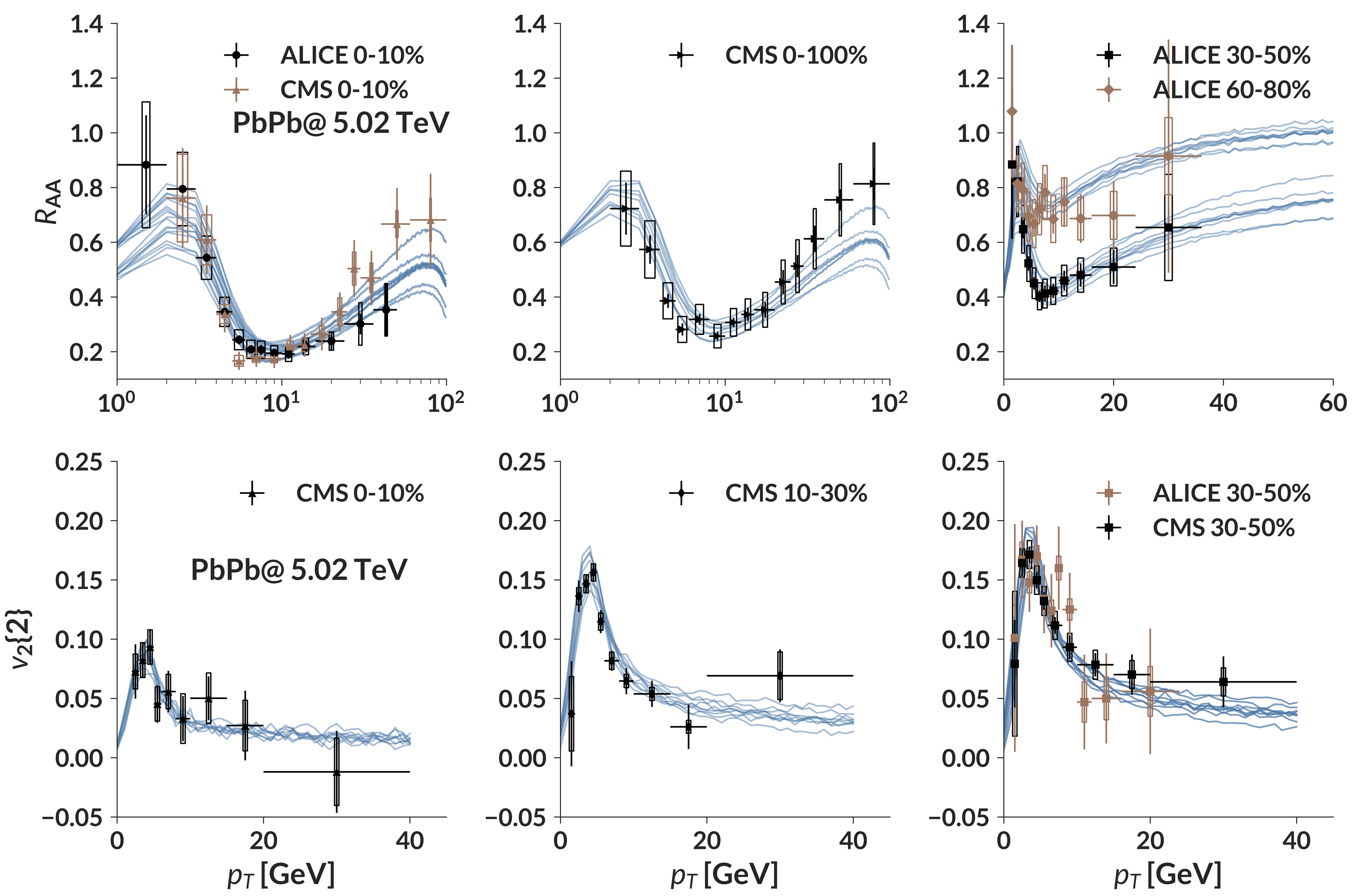
Bayes' Theorem:

$$p(\theta|y=y_{\rm exp}) \propto p(\theta) \times \mathcal{L}(y=y_{\rm exp}|\theta)$$

- Likelihood: $\mathcal{L}(y=y_{\rm exp}|\theta) \propto \exp[-(y(\theta)-y_{\rm exp})\Sigma^{-1}(y(\theta)-y_{\rm exp})^T]$
- Covariance matrix: $\Sigma = \Sigma_{\rm exp} + \Sigma_{\rm model} + \Sigma_{\rm GP}$

Calibration results

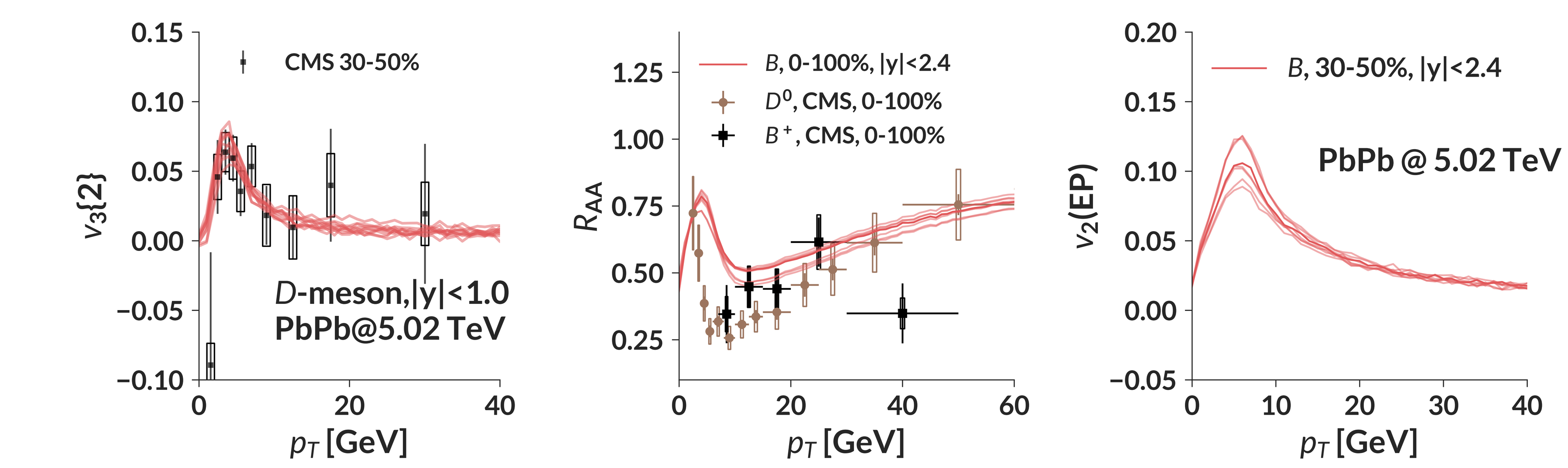


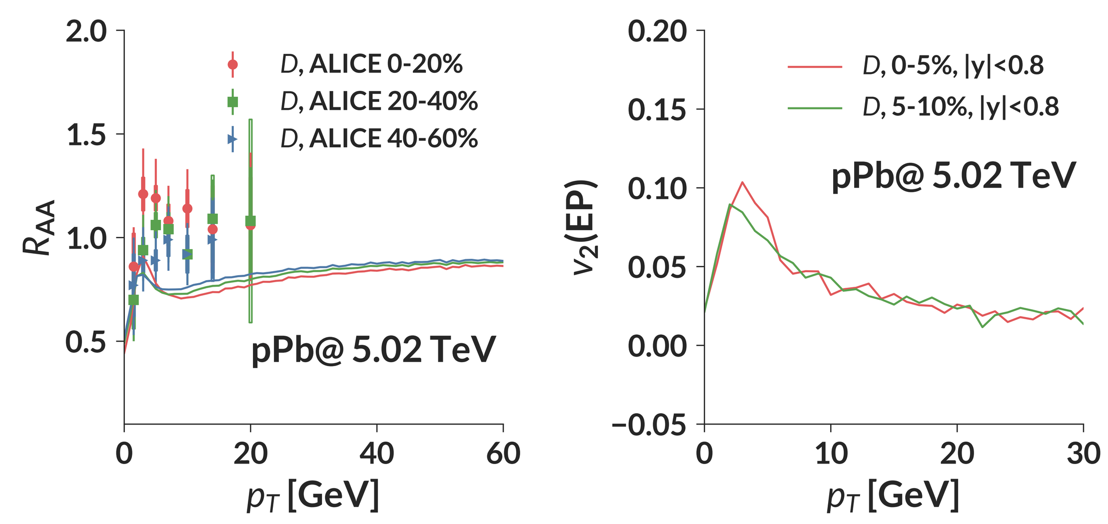


- $D_s 2\pi T$ increases with rising temperature
- Large non-perturbative contribution at low p_T
- Hadronic interaction → little impact on QGP phase D_s estimation

- After calibration, improved Langevin describes D-meson $R_{\rm AA}$ and v_2 data

Model Predictions

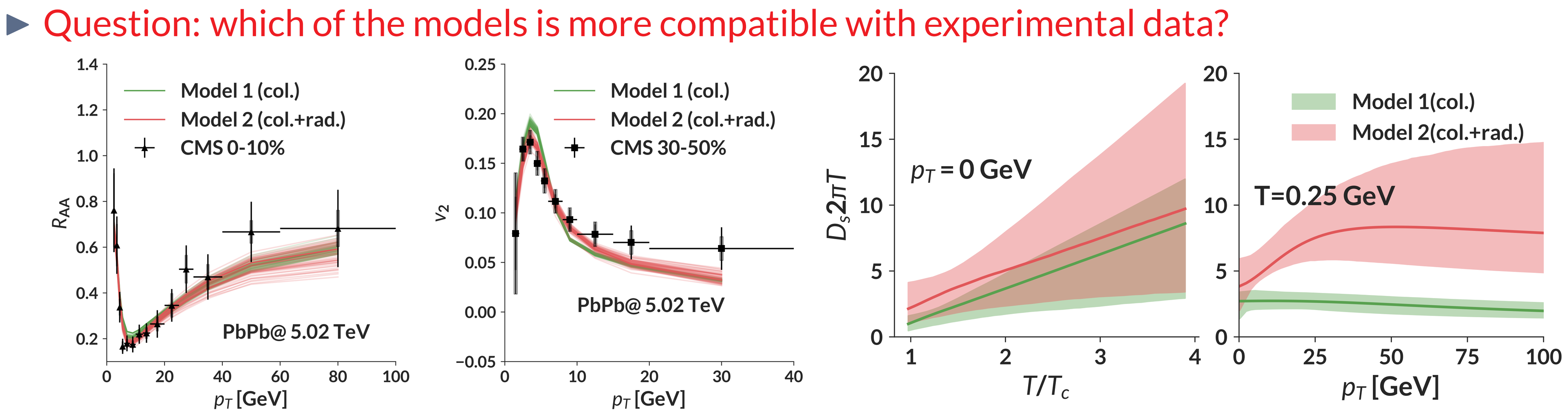




Predictions for :
- D-meson v_3
- B-meson $R_{\rm AA}$, v_2
- pPb 5.02 TeV

Bayes factor: differentiate between different models

- **Model 1**: collisional energy loss; **Model 2**: collisional + radiative energy loss
- After calibration, both models describe experimental $R_{\rm AA}$ and v_2 yet with different D_s: $M_1 \leftrightarrow D_{s1}$, $M_2 \leftrightarrow D_{s2}$
- Question: which of the models is more compatible with experimental data?



- Utilize other observables (e.g. two particle correlations)
- Bayes Factor K: provides comparative quality measurement between two models (posterior distributions)
  - ratio of (marginalized) likelihood of two competing hypothesis/models

$$K = \frac{p(y_{\rm exp}|M_1)}{p(y_{\rm exp}|M_2)} = \frac{\int p(\theta_1|M_1)\mathcal{L}(y_{\rm exp}|\theta_2, M_1)d\theta_1}{\int p(\theta_2|M_2)\mathcal{L}(y_{\rm exp}|\theta_2, M_2)d\theta_2}$$

  - $K < 1$: support M_2; $K > 100$: strongly support M_1
  - $K(\frac{\rm col.}{\rm col.+rad.}) < 0.01 \to$ Strongly favors col.+rad. energy loss model

Conclusion and outlook

- Extraction of $D_s 2\pi T$
- Prediction for D-meson v_3 in PbPb 5.02 TeV and $R_{\rm AA}$, v_2 in p-Pb collisions
- Hadronic rescattering has minor impact on QGP phase D_s extraction
- Bayes factor strongly favors col.+rad. model over pure col. model

Outlook

- Pre-equilibrium dynamics of heavy quarks
- Comparison between different transport models, such as an improved Langevin model and Linearized Boltzmann model

Reference

[1] Phys. Rev. C **97**, no. 1, 014907 (2018); Phys. Rev. C **94**, no. 2, 024907 (2016); Phys. Rev. C **92**, no. 1, 011901 (2015).
[2] ALICE: Phys. Rev. Lett. **120** (2018) no.10, 102301; EPJ Web Conf. **17**, 11800 (2018); CMS: Phys. Rev. Lett. **119**, no. 15, 152301 (2017); arXiv:1708.03497; arXiv: 1708.04962.
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