Equation of State for QCD with a critical point from the 3D Ising model

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I. Scaling EoS for 3D Ising model

The behavior of the magnetization $M$, the magnetic field $h$ and the reduced temperature $r = (T - T_c)/T_c$ close to the 3D Ising model critical point can be captured with the map [2, 3]:

$$M = M_0 R^\beta \theta,$$

$$h = h_0 R^\delta \tilde{h}(\theta),$$

$$r = R(1 - \theta^2),$$

where $\beta \approx 0.326, \delta \approx 4.8$ are the 3D Ising model critical exponents, $M_0, h_0$ are normalization constants and $\tilde{h}(\theta) = (\theta + a\theta^3 + b\theta^5)$ with $a = -0.76201, b = 0.00804$.

II. 3D Ising $\mapsto$ QCD mapping

A linear map from the 3D Ising model phase diagram onto the QCD one requires 6 parameters:

$$(r, h) \mapsto (T, \mu_B):$$

$$\frac{T - T_{c0}}{T_{c0}} = w(r \rho \sin \alpha_1 + h \sin \alpha_2),$$

$$\frac{\mu_B - \mu_{B0}}{\mu_{B0}} = w(-r \rho \cos \alpha_1 - h \cos \alpha_2).$$

We choose the critical point to be located on a parabola $\rightarrow$ 4 parameters:

$$\frac{T_c}{T_{c0}} = 1 + \kappa \left( \frac{\mu_B}{T_c} \right)^2 + O(\mu_B^4)$$

where $T_{c0} = 155$ MeV and $\kappa = -0.0149$ [4, 5] are the chiral transition temperature and the curvature of the transition line at $\mu_B = 0$.

III. Expansion coefficients

We assume that the Taylor coefficients from Lattice QCD [6, 4] can be written as the sum of an “Ising” contribution from the critical point and a “Non-Ising” one [7]:

$$T^4 C^\text{LAT}_{n}(T) = T^4 C^\text{Non-Ising}_{n}(T) + T^4 C^\text{Ising}_{n}(T)$$

For the parameters, we choose:

$$\mu_{B0} = 350 \text{MeV} \rightarrow T_C \approx 143.2 \text{MeV}, \quad \alpha_1 \approx 3.85^\circ$$

and

$$\alpha_2 - \alpha_1 = \frac{\pi}{2}, \quad W = 1, \quad J = 2$$

IV. Reconstruct the pressure

The full pressure can be reconstructed as the sum of an “Ising” critical contribution and “Non-Ising” one; the latter is expressed as a Taylor expansion [7]:

$$P(T, \mu_B) = T^4 \sum_n C^\text{Non-Ising}_{n}(T) \left( \frac{\mu_B}{T} \right)^n + T^4 C^\text{Ising}_{n}(T, \mu_B)$$

Results: The thermodynamics

From the pressure, we calculate entropy density, baryon density, energy density, specific heat and speed of sound [7]:

First order observables show a discontinuity for $\mu_B > \mu_{B0}$; second order observables show a peak (dip) at the critical point.

We performed an analysis on the possible values of $w, \rho$ while keeping the other parameters fixed, checking for thermodynamic stability and causality. In blue, the values corresponding to acceptable choices, in red the pathological ones.

Conclusions

I. This EoS matches Lattice QCD at low $\mu_B$, and contains a critical point in the 3D Ising model universality class.

II. It is ready for implementation in hydro simulations of HIC.

III. Requiring thermodynamic stability and causality sets constraints on the parameters.

IV. Using this family of EoS in hydro simulations and comparing with data can yield additional constraints.