Tachyonic instability of the scalar mode prior to the QCD critical point based on the functional renormalization-group method in the two-flavor case

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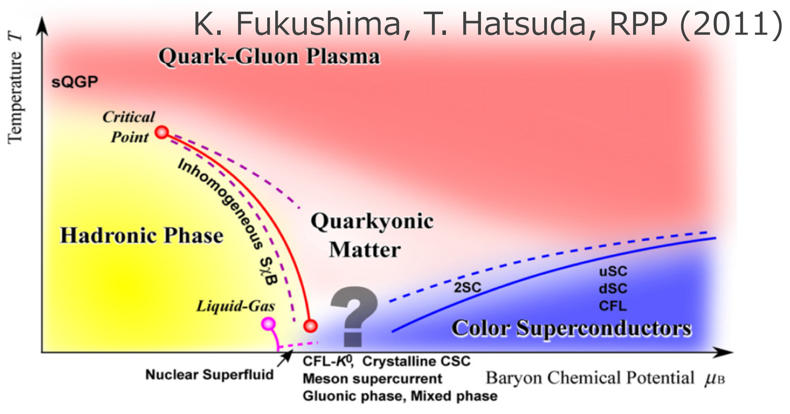
T. Y., Teiji Kunihiro, Kenji Morita, PTEP (2016) 073D01; T. Y., Teiji Kunihiro, Kenji Morita, Phys. Rev. D 96, 074028 (2017).

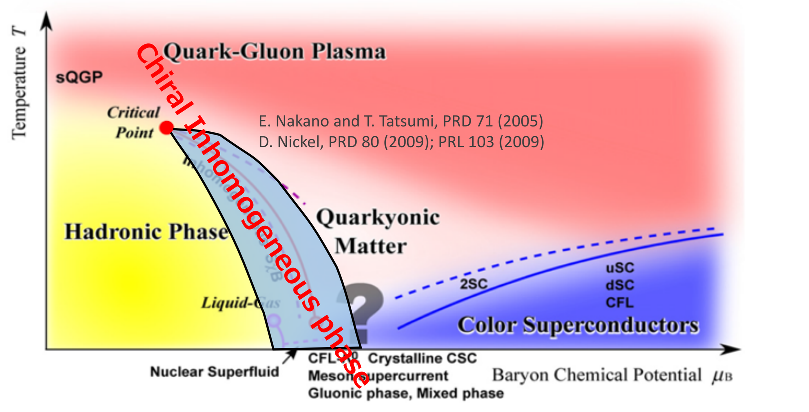
Introduction

What is the true nature of the phase transition between the hadronic phase and quark-gluon plasma phase?

First-order phase transition? QCD critical point (CP)?
Chiral inhomogeneous phase?

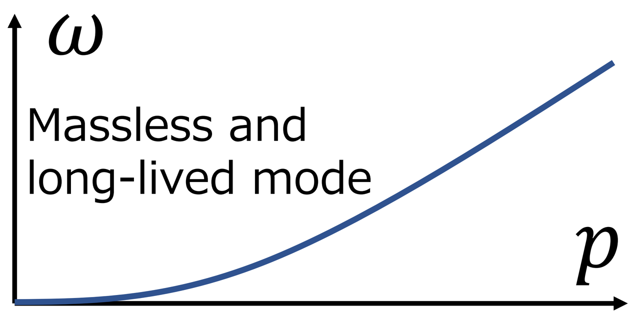
K. Fukushima, T. Hatsuda, RPP (2011)

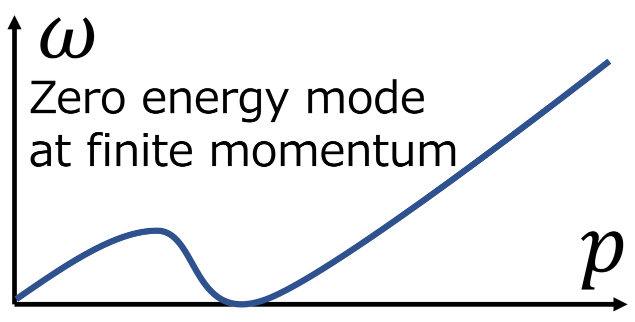




Dynamical quantities such as particle dispersion relations is useful to investigate the nature of the phase transition.

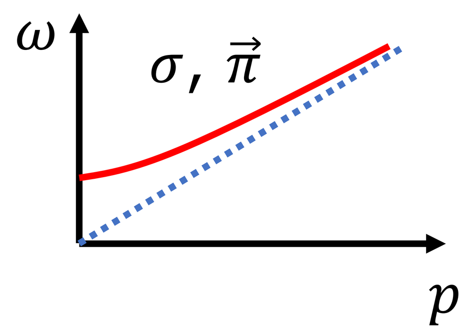
e.g.) Soft mode prior to a critical point — Unstable mode prior to an inhomogeneous phase



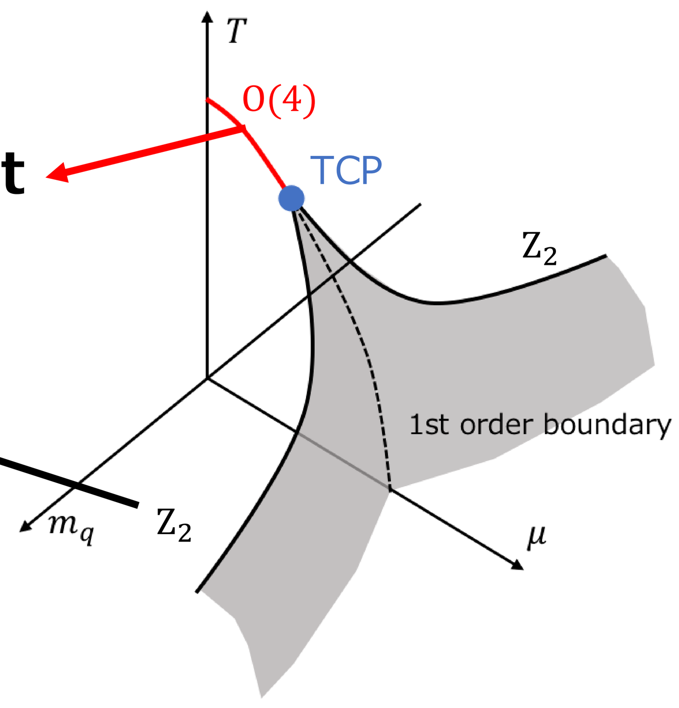


Even if the QCD CP exists, some aspects of the QCD CP such as the nature of the soft modes are unrevealed. The particle dispersion relations is also useful to investigate the nature of the QCD CP itself.

Different universality class and different soft modes depending on the current quark mass



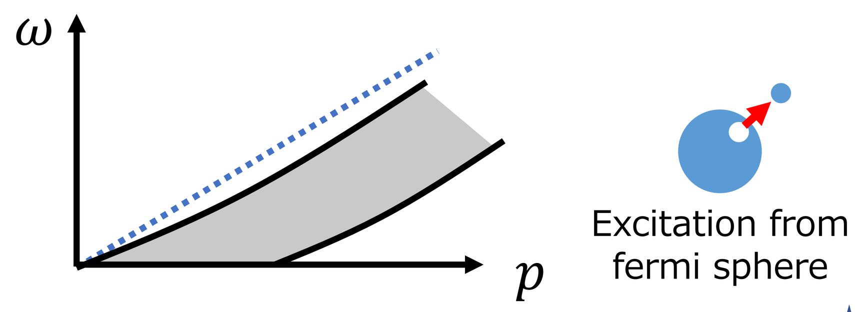
Soft meson quartet



Soft particle-hole modes

H. Fujii, M. Ohtani, PRD (2004)

D. T. Son, M. A. Stephanov, PRD (2004)



QCD CP(s) in $T-\mu-m_q$ space

Due to the breaking of the chiral and charge conjugation symmetries

$$G=\begin{pmatrix}\langle\bar\psi\psi\bar\psi\psi\rangle & \langle\bar\psi\gamma^0\psi\bar\psi\psi\rangle\\ \langle\bar\psi\psi\bar\psi\gamma^0\psi\rangle & \langle\bar\psi\gamma^0\psi\bar\psi\gamma^0\psi\rangle\end{pmatrix}$$

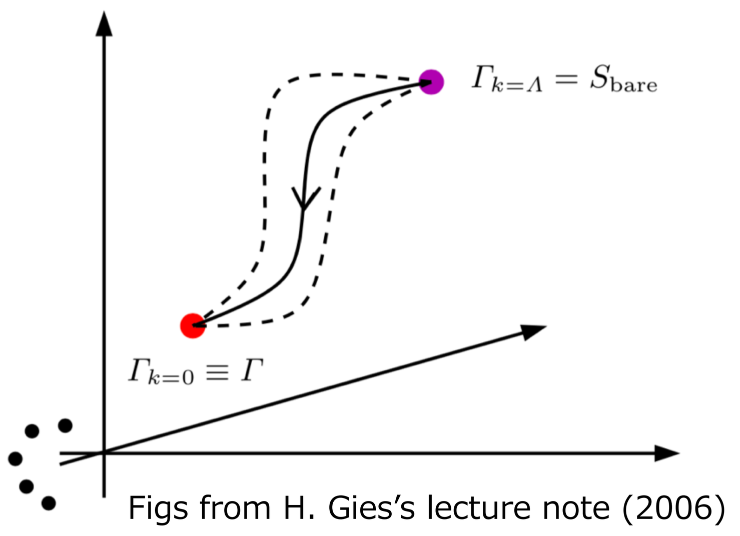
T. Kunihiro, PLB (1991)

Our Purpose and Method

We calculate the σ and π spectral functions around the QCD CP with the functional renormalization group (FRG).

FRG

C. Wetterich, PLB (1993)



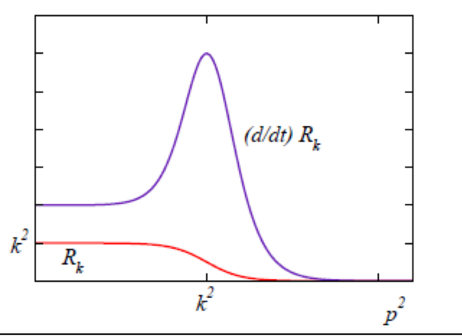
Figs from H. Gies's lecture note (2006)

$$\partial_k\Gamma_k[\Phi]=\frac{1}{2}\mathrm{STr}\left[\frac{\partial_k R_k}{\Gamma_k^{(2)}[\Phi]+R_k}\right]$$

Regulated action

$$S_k=S+\frac{1}{2}\Phi\cdot R_k\cdot\Phi$$

Regulator term
Suppress fluctuations with momentum lower than k

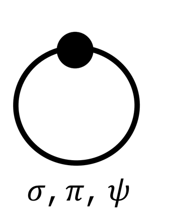


Quark-meson model with the local potential approx.

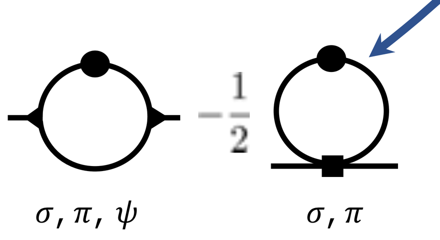
$$\Gamma_k[\bar\psi,\psi,\phi=(\sigma,\vec\pi)]=\int_0^{\frac{1}{T}}\int d^3\vec x\left\{\bar\psi(\partial\!\!\!/+g_s(\sigma+i\vec\tau\cdot\vec\pi\gamma_5)-\mu\gamma_0)\psi+\frac{1}{2}(\partial_\mu\phi)^2+U_k(\phi^2)-c\sigma\right\}$$

k-dependent effective potential

$$\partial_k U_k(\phi^2)=\frac{1}{2}$$



$$\partial_k G^{-1}_{\sigma(\pi),k}=$$



$$G^{(\text{in flow eq})}_{k,\sigma}(p)\approx\frac{1}{p^2+U''(\sigma_0)+R_k^B(p)}\quad\cdots$$

$$\Gamma^{(3)}_{\sigma\sigma\sigma}\approx U'''(\sigma_0)\quad\cdots$$

$$\sigma_0=\arg\min_\sigma(U_{k\to0}(\sigma^2)-c\sigma)$$

R. Tripolt, N. Strodthoff, L. Smekal, J. Wambach, PRD (2014); R. Tripolt, L. Smekal, J. Wambach, PRD (2014)

The analytic continuation of the Green's function can be performed in the flow equation by making use of 3D regulators.

K. Kamikado, N. Strodthoff, L. Smekal, J. Wambach, EPJ C (2014)
R. Tripolt, N. Strodthoff, L. Smekal, J. Wambach, PRD (2014); R. Tripolt, L. Smekal, J. Wambach, PRD (2014)

Temperature Green's fcn. $\mathcal{G}_{\sigma(\pi),k}(i\omega_n,\vec p)$ → Analytic continuation → Retarded Green's fcn. $G^R_{\sigma(\pi),k}(\omega,\vec p)$

Spectral fcn. $\rho_{\sigma(\pi)}(\omega,\vec p)=-\frac{1}{\pi}\mathrm{Im}G^R_{\sigma(\pi),0}(\omega,\vec p)$

Litim type 3D regulators [D. F. Litim, PRD (2001)]

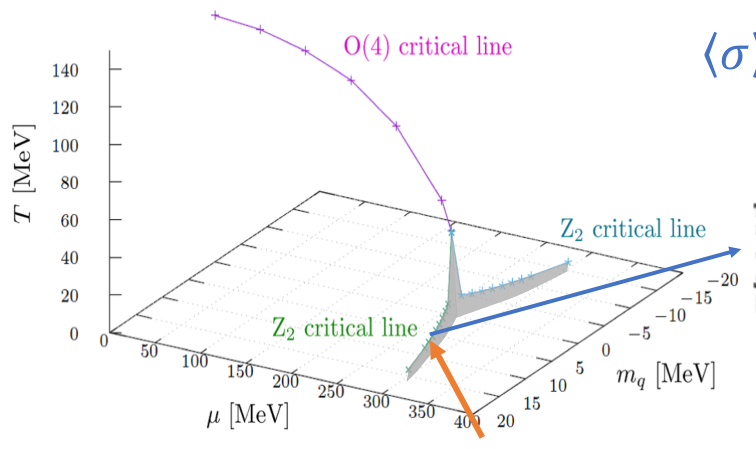
$$R_k^B(\vec q)=(k^2-\vec q^2)\theta(k^2-\vec q^2)\quad R_k^F(\vec q)=i\vec{q\!\!\!/}\left(\sqrt{\frac{k^2}{\vec q^2}}-1\right)\theta(k^2-\vec q^2)$$

Result

CP location in $T-\mu-m_q$ space

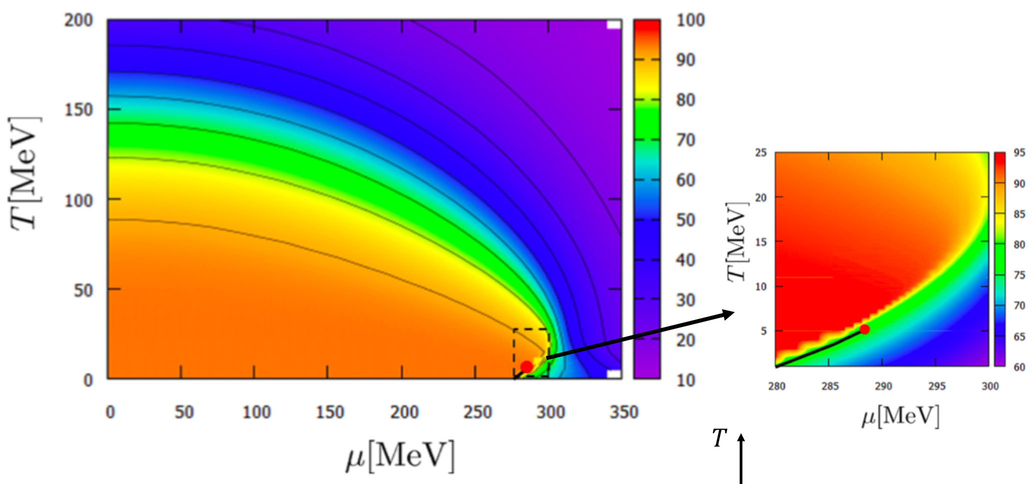
Λ	m_Λ/Λ	λ	g_s
1000 MeV	0.794	2.00	3.2

$$U_\Lambda(\phi^2)=\frac{1}{2}m_\Lambda^2\phi^2+\frac{1}{4}\lambda\phi^4$$

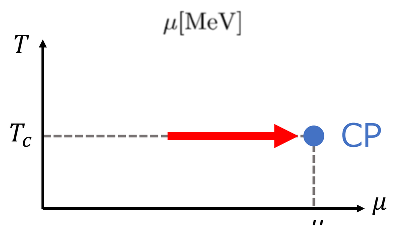


Physical current quark mass
$m_q=8.88$ MeV

$\langle\sigma\rangle$ at physical point

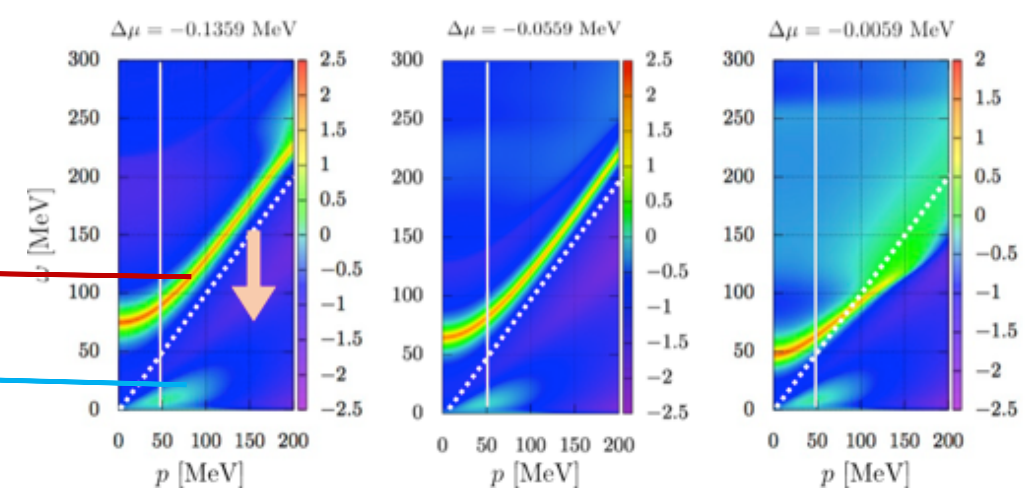


σ spectral function near CPs

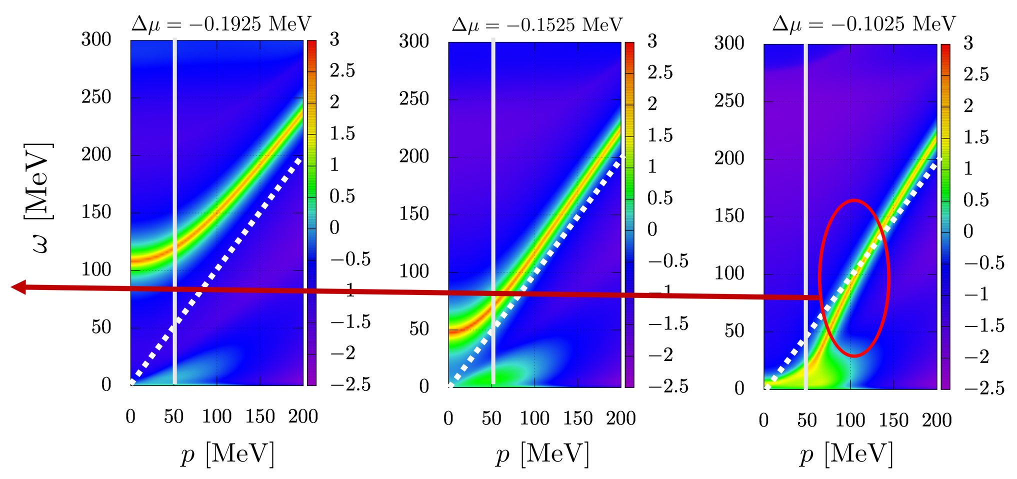


$\Delta\mu=\mu-\mu_c$

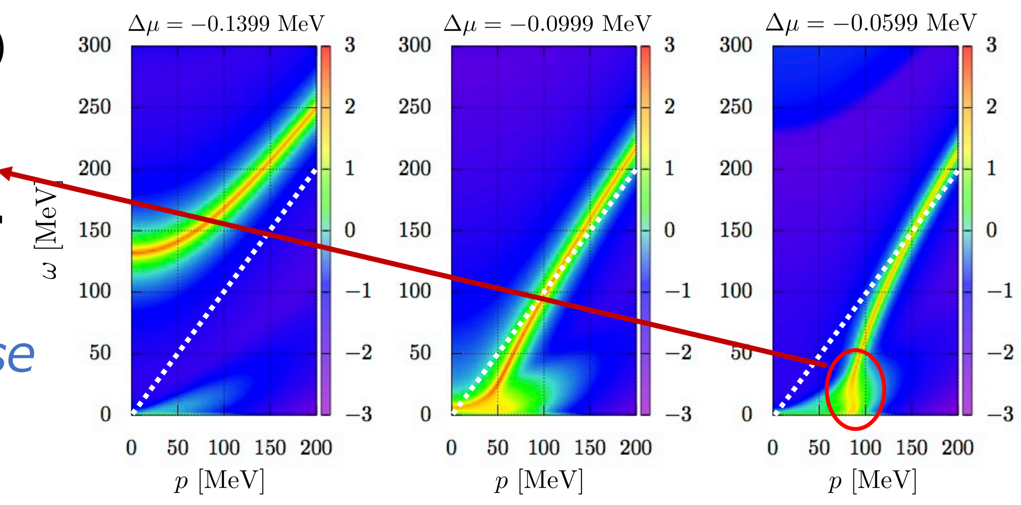
- $m_q=1.27$ MeV
  Sigma does not become soft.
  Sigma one-particle mode
  Particle-hole mode



- $m_q=5.08$ MeV
  Tachyonic mode starts to appear.
  Sigma one-particle tachyonic mode



- $m_q=8.88$ MeV (Physical point)
  Tachyonic and zero energy mode at finite p.
  → *Signal for σ inhomogeneous phase*

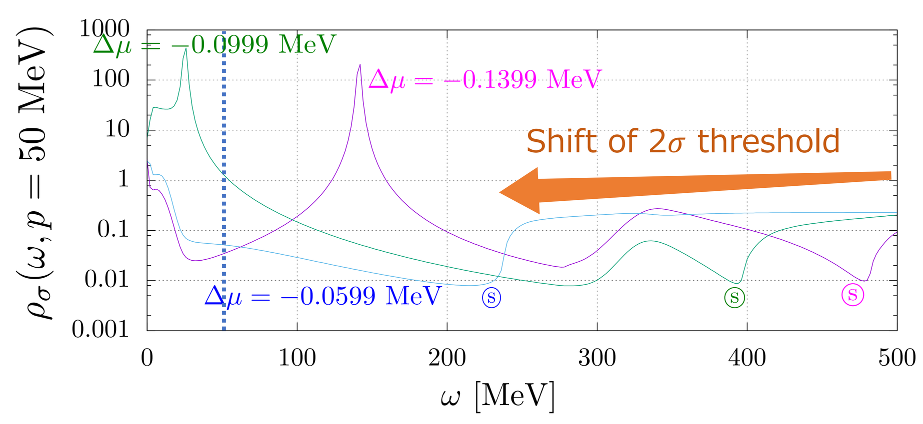


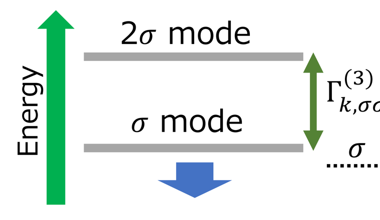
m_q

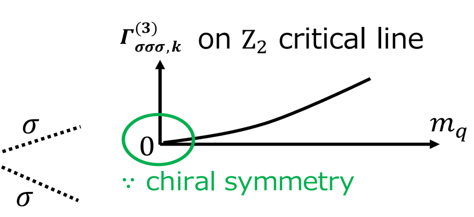
Level repulsion effect between one and two σ mode

The threshold of 2σ mode largely shifts at the same time as one sigma mode.

→ *Level repulsion between σ and 2σ mode may affect the tachyonization of σ.*







At small m_q, σ-to-2σ repulsion disappears and will not cause the tachyonization.
→ *Consistent with our results*

Summary

- We investigate the behavior of the mesonic modes around the QCD CP using FRG.
- A tachyonic mode appears in σ channel near CPs as m_q becomes large.
- A energy zero mode at p appears, which suggests the transition to a σ inhomogeneous phase.
- σ-to- 2σ level repulsion may cause the tachyonization of σ.