The anisotropic non-equilibrium hydrodynamic attractor

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Abstract

In Ref. [1] we determined the dynamical attractors associated with anisotropic hydrodynamics (aHydro) and the DNMR equations for a 0+1d conformal system using kinetic theory in the relaxation time approximation. We compared our results to the non-equilibrium attractor obtained from exact solution of the 0+1d conformal Boltzmann equation, Navier-Stokes theory, and second-order Mueller-Israel-Stewart theory. We demonstrated that the aHydro attractor equation resums an infinite number of terms in the inverse Reynolds number. The resulting resummed aHydro attractor possesses a positive longitudinal to transverse pressure ratio and is virtually indistinguishable from the exact attractor. This suggests that an optimized hydrodynamic treatment of kinetic theory involves a resummation not only in gradients (Knudsen number) but also in the inverse Reynolds number. We also demonstrated that the DNMR result provides a better approximation to the exact kinetic theory attractor than Mueller-Israel-Stewart theory.

Viscous Hydrodynamics (vHydro)

Bjorken symmetry and conformal invariance may be used to show that the energy-momentum conservation laws, obtained from the first moment of the Boltzmann equation, can be reduced to a single equation

$$\tau \frac{d \log \epsilon}{d\tau} = -\frac{4}{3} + \frac{\Pi}{\epsilon}\,, \tag{1}$$

involving the energy density ϵ and $\Pi = \Pi^{\varsigma}_{\varsigma}$. In second-order hydrodynamic theories, such as Mueller-Israel-Stewart (MIS) [2] and Denicol-Niemi-Molnar-Rischke (DNMR) [3], one uses the 14-moment approximation for the single particle distribution function to obtain the most simple form of a differential equation for Π, which can be written in the following form

$$\dot{\Pi} = \frac{4\eta}{3\tau\tau_\pi} - \beta_{\pi\pi}\frac{\Pi}{\tau} - \frac{\Pi}{\tau_\pi}\,, \tag{2}$$

where $\dot{} = d/d\tau$ and for RTA $\beta_{\pi\pi} = 38/21$ and $\tau_\pi = \tau_{\rm eq}$ in the complete second order DNMR calculation while in MIS $\beta_{\pi\pi} = 4/3$ and $\tau_\pi = 6\tau_{\rm eq}/5$. By solving Eqs. (1) and (2) one can determine the dynamical evolution of a viscous fluid described by second order hydrodynamics and investigate the emergence of hydrodynamic attractor behavior, as done in Ref. [4].

Anisotropic Hydrodynamics (aHydro)

In the 0+1d case, anisotropic hydrodynamics (aHydro) requires only one anisotropy direction and parameter, $\hat{\mathbf{n}}$ and ξ [5]. This leads to an Ansatz of the form [6]

$$f(\tau,\mathbf{x},\mathbf{p}) = f_{\rm eq}\!\left(\frac{1}{\Lambda(\tau,\mathbf{x})}\sqrt{p_T^2 + [1+\xi(\tau,\mathbf{x})]p_L^2}\right), \tag{3}$$

where Λ can be interpreted as the local "transverse temperature". For a conformal system, one finds that the energy density, transverse pressure, and longitudinal pressure factorize, resulting in $\epsilon = \mathcal{R}(\xi)\epsilon_0(\Lambda)$, $\mathcal{P}_T = \mathcal{R}_T(\xi)P_0(\Lambda)$, and $\mathcal{P}_L = \mathcal{R}_L(\xi)P_0(\Lambda)$ with the $\mathcal{R}$ functions being particular special functions. Using Landau matching, one has $\epsilon = \mathcal{R}(\xi)\epsilon_0(\Lambda) = \epsilon_0(T)$, which results in $T = \mathcal{R}^{1/4}(\xi)\Lambda$. With this, Eq. (1) is the same as in viscous hydrodynamics. To obtain the additional equation of motion for required, we use the second moment of the Boltzmann distribution $I^{\mu\nu\lambda} = N_{\rm dof}\int dP\, p^\mu p^\nu p^\lambda f$,. From the Boltzmann equation in the relaxation-time approximation (RTA), the equation of motion for this moment is $\partial_\alpha I^{\alpha\mu\nu} = \frac{1}{\tau_{\rm eq}}(u_\alpha I^{\alpha\mu\nu}_{\rm eq} - u_\alpha I^{\alpha\mu\nu})$. Taking the zz projection of this equation minus one-third of the sum of its xx, yy, and zz projections gives our second equation of motion

$$\frac{1}{1+\xi}\dot{\xi} - \frac{2}{\tau} + \frac{\mathcal{R}^{5/4}(\xi)}{\tau_{\rm eq}}\xi\sqrt{1+\xi} = 0\,, \tag{4}$$

which can be used to define the evolution of the anisotropy parameter. One can rewrite Eq. (4) in terms of the shear stress tensor component Π. Using $\overline{\Pi}(\xi) \equiv \frac{\Pi}{\epsilon} = \frac{1}{3}\left[1 - \frac{\mathcal{R}_L(\xi)}{\mathcal{R}(\xi)}\right]$ one obtains

$$\frac{\dot{\Pi}}{\epsilon} + \frac{\Pi}{\epsilon\tau}\left(\frac{4}{3} - \frac{\Pi}{\epsilon}\right) - \left[\frac{2(1+\xi)}{\tau} - \frac{\mathcal{H}(\xi)}{\tau_{\rm eq}}\right]\overline{\Pi}'(\xi) = 0\,, \tag{5}$$

with $\mathcal{H}(\xi) \equiv \xi(1+\xi)^{3/2}\mathcal{R}^{5/4}(\xi)$. Written in this form, we can see that the aHydro second-moment equation sums an infinite number of terms in the expansion in the inverse Reynolds number. Note, importantly, that Eq. (5) reduces identically to Eq. (2) in the small-ξ limit.

Finding the non-equilibrium attractor

We follow [4] and introduce the dimensionless "time" variable

$$w \equiv \tau T(\tau)\,, \tag{6}$$

with which one may define the *amplitude*

$$\varphi(w) \equiv \tau\frac{\dot{w}}{w} = 1 + \frac{\tau}{4}\partial_\tau \log\epsilon\,, \tag{7}$$

which is related to Π as follows

$$\frac{\Pi}{\epsilon} = 4\left(\varphi - \frac{2}{3}\right). \tag{8}$$

The change of variables from $\{\epsilon,\Pi\} \to \{w,\varphi\}$ is convenient because it allows one to express the coupled set of first-order ODEs for $\{\epsilon,\Pi\}$ in terms of a single first-order ODE for $\varphi(w)$ [4].

vHydro

For standard viscous hydrodynamics one obtains,

$$\overline{w}\varphi\varphi' + 4\varphi^2 + \left[\overline{w} + \left(\beta_{\pi\pi} - \frac{20}{3}\right)\right]\varphi - \frac{4c_{\eta/\pi}}{9} - \frac{2}{3}(\beta_{\pi\pi} - 4) - \frac{2\overline{w}}{3} = 0\,, \tag{9}$$

where $\overline{w} = w/c_\pi$. To connect these equations with the RTA Boltzmann one must set $c_{\eta/\pi} = 1/5$.

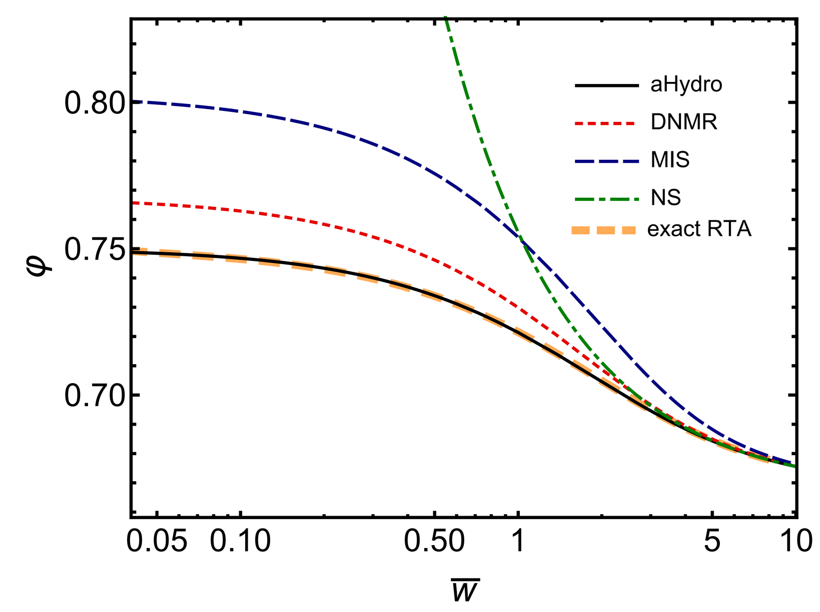
aHydro

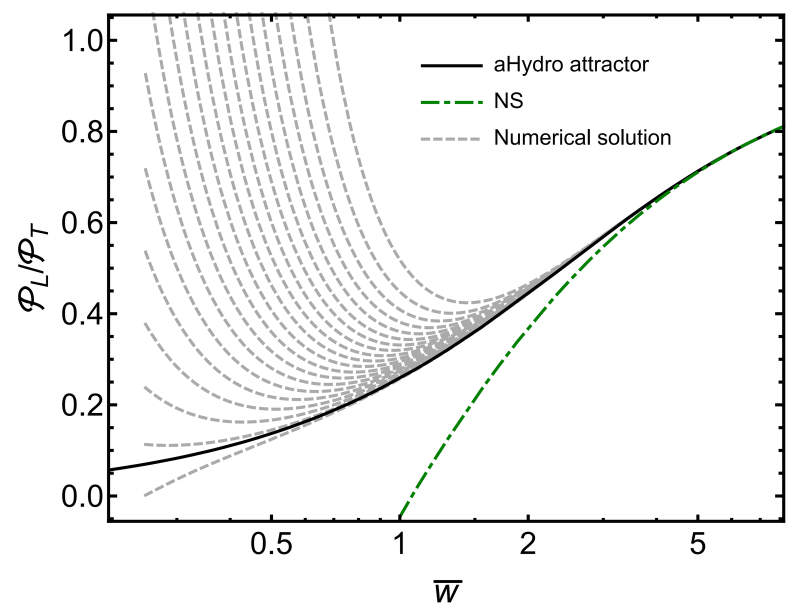
In the case of aHydro, this procedure gives

$$\overline{w}\varphi\frac{\partial\varphi}{\partial\overline{w}} = \left[\frac{1}{2}(1+\xi) - \frac{\overline{w}}{4}\mathcal{H}(\xi)\right]\overline{\Pi}'\,. \tag{10}$$

Numerical solution

To obtain the attractor, one solves either Eq. (9) or (10) subject to a boundary condition at $\overline{w} = 0$ which can be determined using the "slow-roll approximation" [4]. We then compare the result to the attractor obtained from the exact solution of the 0+1d Boltzmann equation in RTA [7].





Conclusions

- We obtained the dynamical attractors associated with the aHydro, MIS, and DNMR versions of viscous hydrodynamics.
- We demonstrated that the aHydro dynamical equations resum an infinite number of terms in the inverse Reynolds number. As a direct consequence of this all-order resummation, we found that
  - The resulting aHydro attractor was naturally restricted to $1/2 < \varphi < 3/4$ which guarantees the positivity of both the longitudinal and transverse pressures.
  - The resulting aHydro attractor was virtually indistinguishable from the attractor emerging from exact solution of the RTA Boltzmann equation.
- Numerical solutions for a variety of different initial conditions approach the attractor within a time $\tau_{\rm attractor}$. In LHC heavy-ion collisions, one expects initial temperatures $T_0 \lesssim 500$ MeV at $\tau_0 = 0.25$ fm/c and $\eta/s \sim 0.2$, which translates into $\tau_{\rm attractor} \gtrsim 1.3$ fm/c with the lower bound holding in the hot center of the fireball on average.
- The dynamics of the system prior to $\tau_{\rm attractor}$ is non-universal. In fact, at very early times, the whole set of non-hydrodynamic modes should contribute to the evolution of the system.

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