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Cumulant ratios at nonzero temperature and density from lattice QCD

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CRC-TR 211 Strong-interaction matter under extreme conditions

(1) Motivation

Event-by-event fluctuations of conserved charges are the generic observables that could help to reveal a possible QCD critical point (Stephanov *et al.*,1999).
In its vicinity, critical phenomena predict divergent correlation lengths and susceptibilities. The strength of the divergence increases rapidly with the order of the cumulant.
The STAR Collaboration has measured cumulant ratios of net proton-number fluctuations during the Beam Energy Scan (BES) program and found an intriguing increase in fluctuations below beam energies of $\sqrt{s_{NN}} \lesssim 19.6$ GeV.

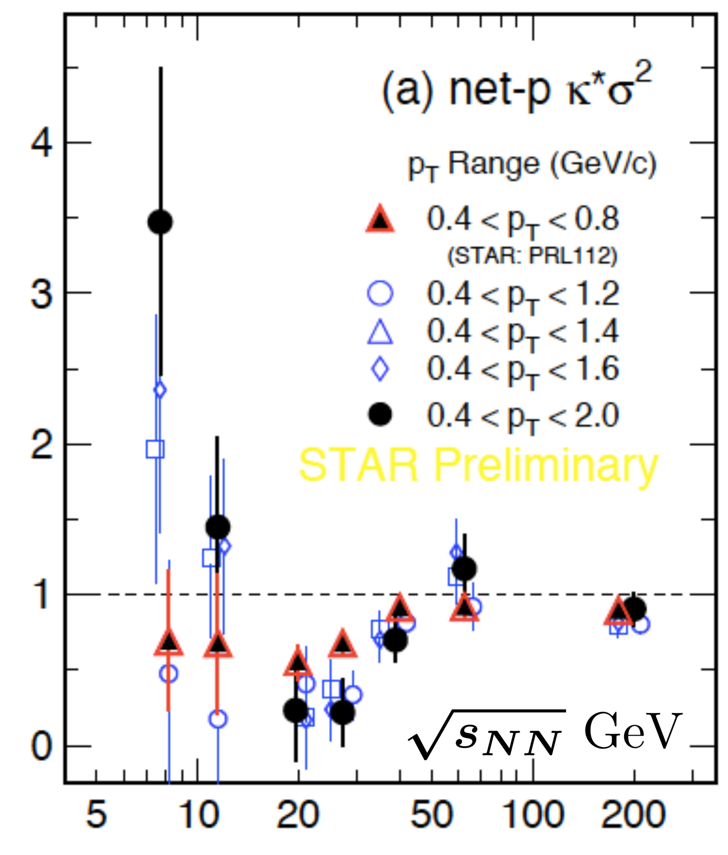

Figure 1: Preliminary STAR data for the net proton-number kurtosis, as function of the collision energy $\sqrt{s_{NN}}$ (Luo [STAR], 2014).

The question we ask here: **Can this data be understood by means of equilibrium thermodynamics of QCD?**

(2) Methodology

We perform lattice QCD calculations with (2+1)-flavor of highly improved staggered quarks (HISQ). The notorious sign problem is circumvented by performing a Taylor expansion of the pressure

$$\frac{p}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!}\chi_{ijk}^{BQS}\hat{\mu}_B^i\hat{\mu}_Q^j\hat{\mu}_S^k\,, \quad \chi_{ijk}^{BQS} \equiv \frac{\partial^i\partial^j\partial^k}{\partial\hat{\mu}_B^i\partial\hat{\mu}_Q^j\partial\hat{\mu}_S^k}\frac{p}{T^4}\,, \qquad (1)$$

in terms of the baryon (μ_B) electric charge (μ_Q) and strangeness (μ_S) chemical potentials, with $\hat{\mu}_X = \mu_X/T$. Derivatives are taken at $\mu_B = \mu_Q = \mu_S = 0$. The Taylor coefficients have been measured up to the 6^{th}-order. We demonstrated control over the resulting equation of state for $\mu_B \leq 2T$, which corresponds to $\sqrt{s_{NN}} \geq 12$ GeV (Bazavov *et al.*, 2017). Results are in good agreement with (Günther *et al.*, 2017).
Further details of the calculation:

- We either set $\mu_Q = \mu_S = 0$, or fix them such $<n_Q>/<n_B>= 0.4$ and $<n_S>= 0$ (constrained case).
- We have high statistics in the range 135 MeV $\lesssim T \lesssim$ 170 MeV.
- Quark masses are physical, *i.e.* with a ratio of $1/27$ (light / strange).
- Lattice sizes are $N_\sigma^3 \times N_\tau$, with $N_\sigma = 4N_\tau$ and $N_\tau = 6, 8, 12, 16$.

(3) Baryon number fluctuations

Definition of cumulant ratios: The coefficients in Eq. (1) (χ_{ikj}^{BQS}) are the cumulants of the conserved charges (B, Q, S), calculated at $\hat{\mu}_X = 0$. Taking appropriate derivatives of Eq. (1) with respect to $\hat{\mu}_X$, we obtain Taylor expansions for the cumulants. We consider the ratios:

- $R_{12}^B \equiv \chi_1^B/\chi_2^B = M_B/\sigma_B^2 \quad = \quad r_{12}^{B,1}(\mu_B/T) + r_{12}^{B,3}(\mu_B/T)^3 + \cdots$
- $R_{31}^B \equiv \chi_3^B/\chi_2^B = S_B\sigma_B^3/M_B \quad = \quad r_{31}^{B,0} + r_{31}^{B,2}(\mu_B/T)^2 + \cdots$
- $R_{42}^B \equiv \chi_4^B/\chi_2^B = \kappa_B\sigma_B^2 \quad = \quad r_{42}^{B,0} + r_{42}^{B,2}(\mu_B/T)^2 + \cdots$

where M_B, σ_B^2, S_B and κ_B denote the mean, variance, skewness and kurtosis of the net baryon number fluctuations. After the last equality, we have indicated the leading and next-to-leading μ_B-behavior.
Results: In the case $\mu_Q = \mu_S = 0$ we find strict relations between R_{31}^B and R_{42}^B, namely $r_{31}^{B,0} = r_{42}^{B,0}$ and $3r_{31}^{B,2} = r_{32}^{B,2}$, they are only slightly relaxed in the constrained case (see Fig. 2) (Bazaov [HotQCD] 2017).

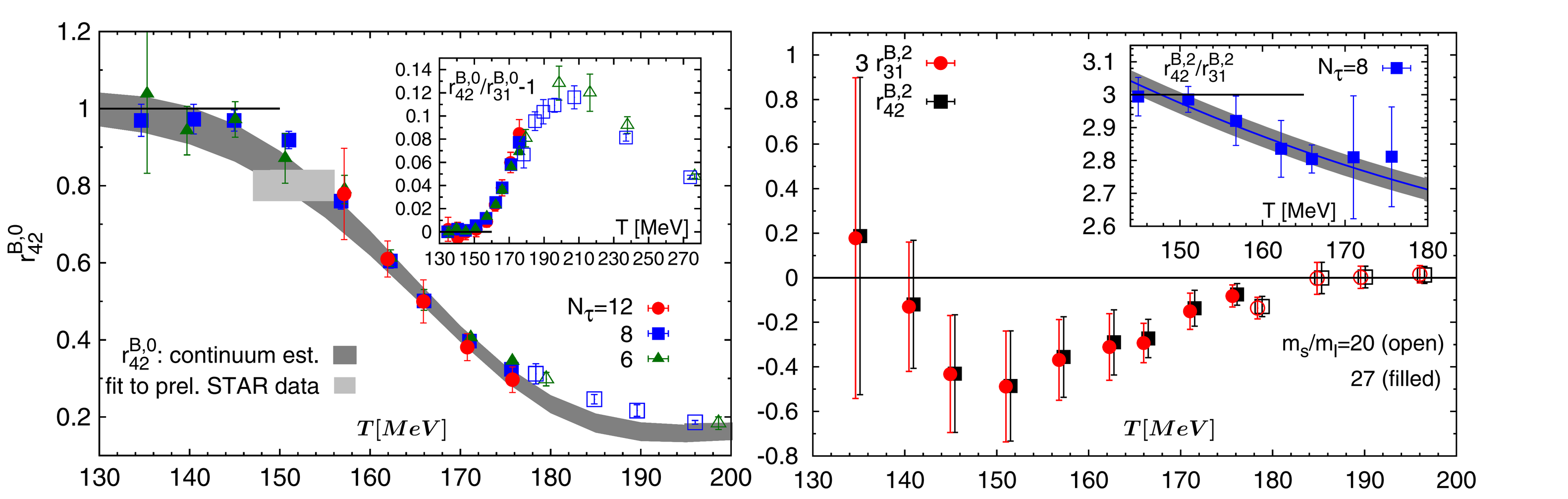

Figure 2: Leading (left) and next-to-leading (right) order of the cumulant ratios R_{31}^B and R_{42}^B as a function of the temperature.

Comparison with STAR: Although many effects influence the experimental data, which need better understanding (non-equilibrium, proton vs. baryon number, acceptance and p_t cuts, rapidity dependence), it is tempting to compare with measured proton number fluctuations. To do so, we express all ratios in term of R_{12}^B, i.e. the net baryon density. We find (see Fig. 3) that

- the STAR data respects the relations between R_{31}^B and R_{42}^B well.
- the freeze-out temperature T_f obtained from the preliminary STAR data (0.4 GeV<p_t<2 GeV) is consistent with the chiral crossover temperature.

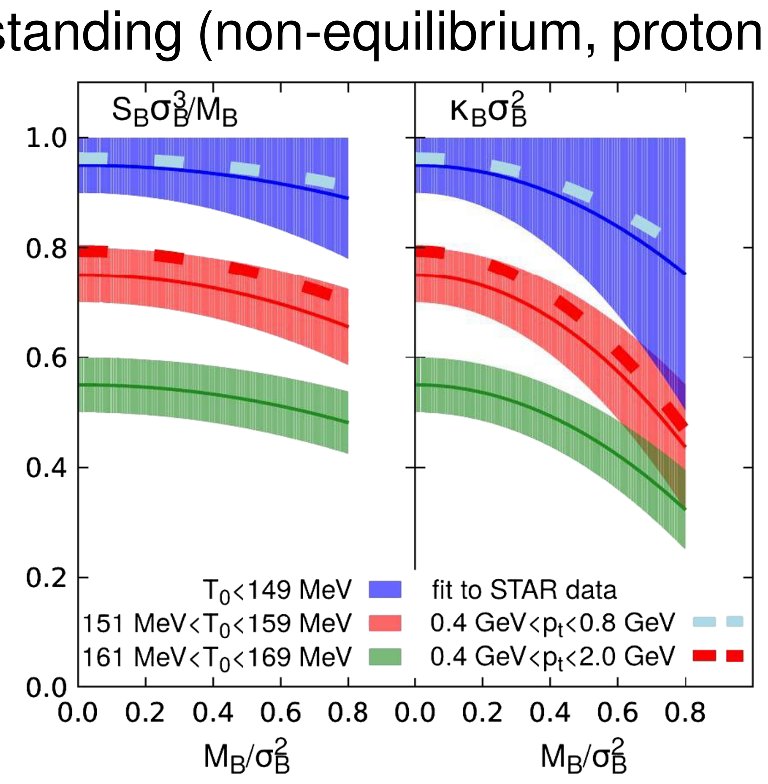

Figure 3: The ratios R_{31}^B and R_{42}^B as functions of R_{12}^B.

(4) Strangeness fluctuations

Cumulant ratios and the Hadron Resonance Gas (HRG): Since all baryons carry baryon number $B = \pm 1$. The HRG model yields constant ratios $R_{nm}^B = 1$, with $(n-m)$ even (see Fig. 2 (left)). This will not be the case for the kurtosis ratio of net strangeness:

- $R_{42}^S \equiv \chi_4^S/\chi_2^S = \kappa_S\sigma_S^2 \quad = \quad r_{42}^{S,0} + r_{42}^{S,2}(\mu_B/T)^2 + \cdots$

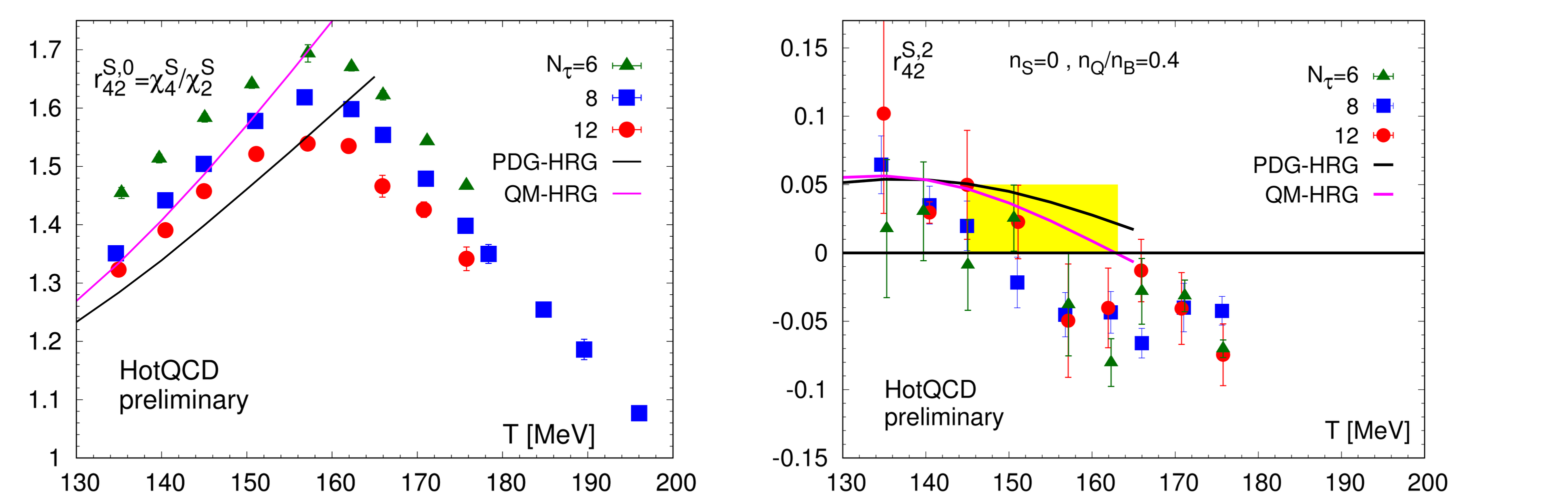

Figure 4: Leading (left) and next-to-leading order (right) of the strangeness ratio R_{42}^S as function of temperature.

- The leading order receives large contributions from $|S| = 2, 3$ baryons. At T_c fluctuations are enhanced by a factor of ~ 1.5.
- The approach to the HRG may support additional, experimentally not yet observed, strange hadrons (Bazavov, 2014; Alba 2017), predicted in Quark Model calculations (QM-HRG) and from lattice QCD (Edwards, 2013).
- The next-to-leading order corrections in μ_B remain small.

Kaon fluctuations: The experimentally measured cumulant ratios of kaon fluctuations (Adamczyk [STAR], 2017) give kurtosis ratios that are consistent with 1, due to the fact that kaons carry strangeness $|S| = 1$. Feeddown from $|S| > 1$ baryons seems to be small.

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