Relaxation Times for Chiral Transport Phenomena and Spin Polarization in Strongly Coupled Plasma

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Abstract

We compute the dynamical relaxation times for chiral transport phenomena in strongly coupled regime using the AdS/CFT correspondence. These relaxation times should be a useful proxy for the dynamical time scale for achieving equilibrium spin-polarization of quasi-particles in the presence of magnetic field and fluid vorticity. We identify the Kubo relations for these relaxation times and clarify some previous issues regarding time-dependence of the Chiral Vortical Effect. We also study the consequences of imposing time-reversal invariance on parity-odd random noise fluctuations that are related to chiral transport coefficients by the fluctuation-dissipation relation. We find that time-reversal invariance dictates the equality between some of the chiral transport coefficients as well as their relaxation times.

Introduction and Summary

The full anomalous transport phenomena with one right-handed chiral fermion species in the anomaly frame or no-drag frame consist of four transport coefficients in leading order of derivative[1]

$$T^{\mu\nu}_{chiral} = \sigma^B_\epsilon (B^\mu u^\nu + B^\nu u^\mu) + \sigma^V_\epsilon(\omega^\mu u^\nu + \omega^\nu u^\mu),$$

$$J^\mu_{chiral} = \sigma_B B^\mu + \sigma_V \omega^\mu,$$

$$\sigma_B = \frac{\mu}{4\pi^2}, \quad \sigma_V = \frac{\mu^2}{8\pi^2}, \quad \sigma^B_\epsilon = \frac{\mu^2}{8\pi^2}, \quad \sigma^V_\epsilon = \frac{\mu^3}{6\pi^2}.$$

We correspondingly introduce the four new relaxation times in the next order in time derivative

$$T^{\mu\nu}_{chiral} = \sigma^B_\epsilon (B^\mu - \tau^B_\epsilon (u\cdot\nabla)B^\mu)u^\nu + \sigma^V_\epsilon(\omega^\mu - \tau^V_\epsilon(u\cdot\nabla)\omega^\mu)u^\nu + (\mu \leftrightarrow \nu),$$

$$J^\mu_{chiral} = \sigma_B(B^\mu - \tau_B(u\cdot\nabla)B^\mu) + \sigma_V(\omega^\mu - \tau_V(u\cdot\nabla)\omega^\mu).$$

In this work, we show that $\sigma^B_\epsilon = \sigma_V$, $\tau^B_\epsilon = \tau_V$. by using the time-reversal (T) invariance and we confirm this in our numerical computation in the AdS/CFT correspondence.

Kubo relations for the relaxation times

Writing $\langle \boldsymbol{J}^i(k)\boldsymbol{J}^j(-k)\rangle_R = i\tilde\sigma_B(k)\epsilon^{ijl}\boldsymbol{k}^l \quad \langle \boldsymbol{J}^i(k)T^{0j}(-k)\rangle_R = i\tilde\sigma_V(k)\epsilon^{ijl}\boldsymbol{k}^l$
$\langle T^{0i}(k)\boldsymbol{J}^j(-k)\rangle_R = i\tilde\sigma^B_\epsilon(k)\epsilon^{ijl}\boldsymbol{k}^l, \quad \langle T^{0i}(k)T^{0j}(-k)\rangle_R = i\tilde\sigma^V_\epsilon(k)\epsilon^{ijl}\boldsymbol{k}^l$

The retarded correlation functions including the relaxation times

$$\tilde\sigma_B(k) = \sigma_B(1+i\omega\tau_B) - \frac{n}{\epsilon+p}\Big(\sigma^B_\epsilon(1+i\omega\tau^B_\epsilon) + \sigma_V(1+i\omega\tau_V)$$
$$-\sigma^V_\epsilon(1+i\omega\tau^V_\epsilon)\frac{n}{\epsilon+p}\frac{\omega}{\omega+i\gamma_\eta\boldsymbol{k}^2}\Big)\frac{\omega}{\omega+i\gamma_\eta\boldsymbol{k}^2},$$

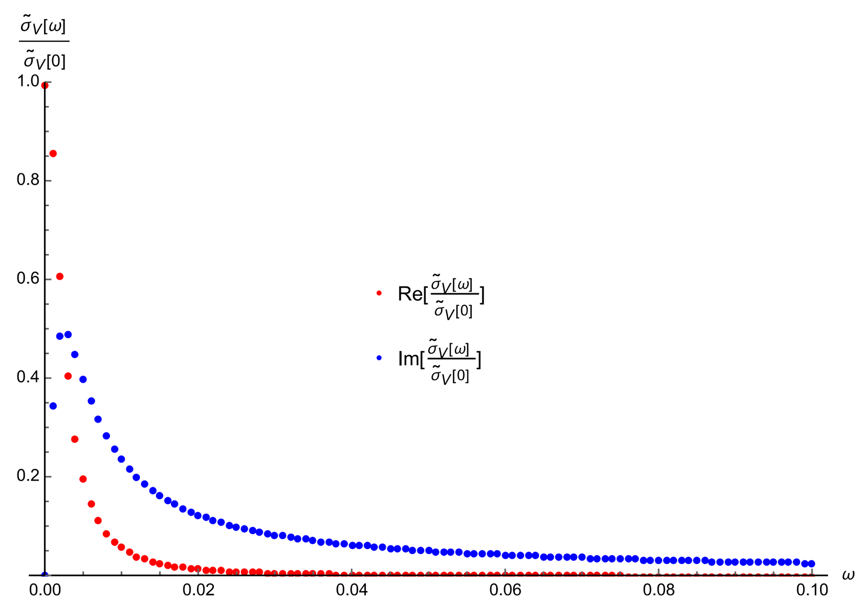
$$\tilde\sigma_V(k) = \left(\sigma_V(1+i\omega\tau_V) - \frac{n}{\epsilon+p}\sigma^V_\epsilon(1+i\omega\tau^V_\epsilon)\frac{\omega}{\omega+i\gamma_\eta\boldsymbol{k}^2}\right)\frac{i\gamma_\eta\boldsymbol{k}^2}{\omega+i\gamma_\eta\boldsymbol{k}^2},$$

$$\tilde\sigma^B_\epsilon(k) = \left(\sigma^B_\epsilon(1+i\omega\tau^B_\epsilon) - \frac{n}{\epsilon+p}\sigma^V_\epsilon(1+i\omega\tau^V_\epsilon)\frac{\omega}{\omega+i\gamma_\eta\boldsymbol{k}^2}\right)\frac{i\gamma_\eta\boldsymbol{k}^2}{\omega+i\gamma_\eta\boldsymbol{k}^2}$$

$$\tilde\sigma^V_\epsilon(k) = \sigma^V_\epsilon(1+i\omega\tau^V_\epsilon)\frac{-\gamma_\eta^2\boldsymbol{k}^4}{(\omega+i\gamma_\eta\boldsymbol{k}^2)^2},$$

Holographic Numerical Study

In the zero frequency limit, all four chiral transport coefficients decouple from each other, and these give rising to the correct Kubo relations for the leading order chiral transport coefficients. On the other hand, in the zero momentum limit with non-zero frequency, $\langle JT\rangle_R$, $\langle TJ\rangle_R$ and $\langle TT\rangle_R$ correlators all vanish due to the diffusion pole structure. Our holographic numerical study indeed shows that $\langle JT\rangle_R$ goes to zero in the zero momentum limit, as found in previous literature[2] and it is required by the conservation of energy-momentum Ward identity.



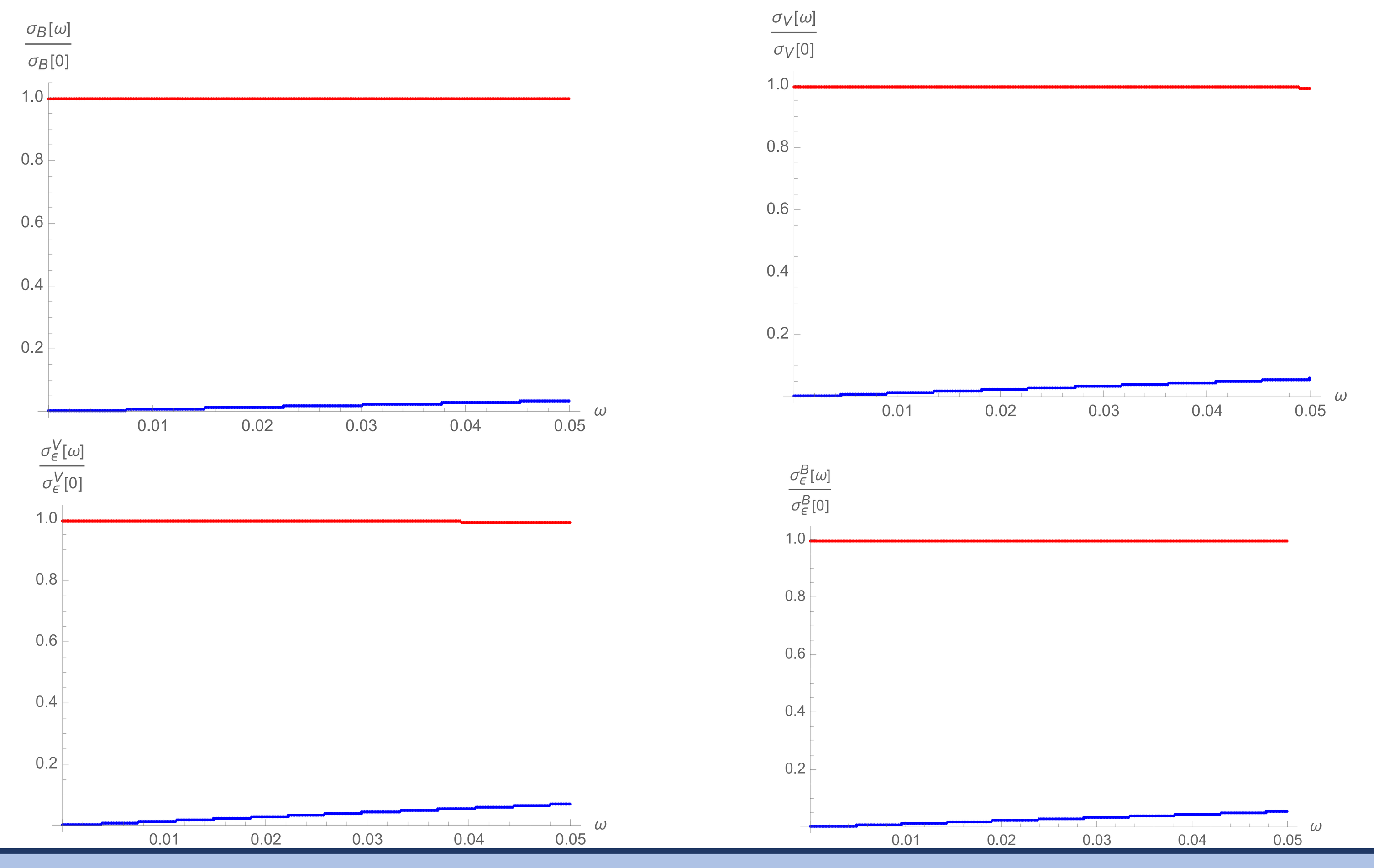
But this diffusion pole structure must be taken care of first before taking appropriate to zero momentum limit and the remaining imaginary linear term gives rising to the relaxation time

$$\sigma_V(\omega) \equiv \lim_{\boldsymbol{k}\to 0}\left(\frac{\omega+i\gamma_\eta\boldsymbol{k}^2}{i\gamma_\eta\boldsymbol{k}^2}\tilde\sigma_V(k) + \frac{n}{\epsilon+p}\sigma^V_\epsilon(1+i\omega\tau^V_\epsilon)\frac{\omega}{\omega+i\gamma_\eta\boldsymbol{k}^2}\right)$$

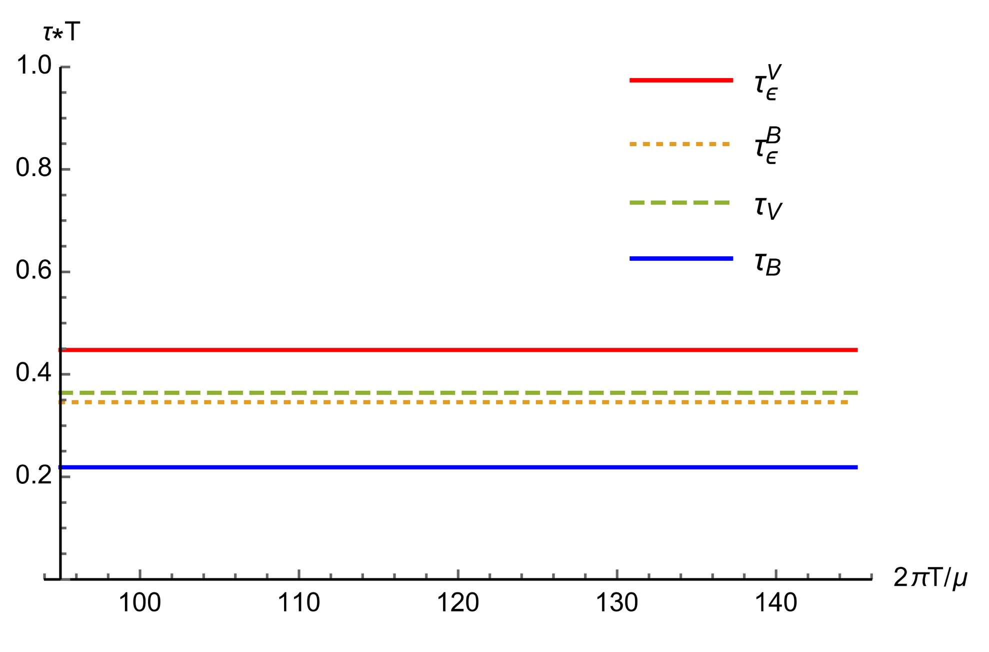
$$\tau^V_\epsilon = -\frac{1}{\sigma^V_\epsilon}\lim_{\omega\to 0}\frac{1}{\omega}\lim_{\boldsymbol{k}\to 0}\mathrm{Im}\left[\frac{(\omega+i\gamma_\eta\boldsymbol{k}^2)^2}{-\gamma_\eta^2\boldsymbol{k}^4}\tilde\sigma^V_\epsilon(k)\right]$$

$$\tau_V = \frac{1}{\sigma_V}\lim_{\omega\to 0}\frac{1}{\omega}\lim_{\boldsymbol{k}\to 0}\mathrm{Im}\left[\frac{\omega+i\gamma_\eta\boldsymbol{k}^2}{i\gamma_\eta\boldsymbol{k}^2}\tilde\sigma_V(k) + \frac{n}{\epsilon+p}\sigma^V_\epsilon(1+i\omega\tau^V_\epsilon)\frac{\omega}{\omega+i\gamma_\eta\boldsymbol{k}^2}\right]$$

Results



Results



This figure shows the extracted relaxation times of the chiral transport phenomena as a function of dimensionless parameter. Up to numerical uncertainty, the result confirms our expectation of $\tau^B_\epsilon = \tau_V$ dictated by the time-reversal invariance.

Conclusion and Outlook

1. In this work, we compute the dynamical time scale of chiral transport phenomena, that characterizes how fast an off-equilibrium condition relaxes to the equilibrium configuration that is dictated by chiral anomaly, in the strongly coupled regime using the AdS/CFT correspondence.
2. We also find interesting consequences of imposing time-reversal invariance on the P-odd random noise fluctuation correlation functions, that are related to chiral transport coefficients via fluctuation-dissipation relation.
3. It would be interesting to compute these and the other second order chiral transport coefficients in weakly coupled regime in perturbative QCD.

References

[1] M. A. Stephanov and H. U. Yee, "No-Drag Frame for Anomalous Chiral Fluid," Phys. Rev. Lett. 116, no. 12, 122302(2016).
[2] K. Landsteiner, E. Megias and F. Pena-Benitez, "Frequency dependence of the Chiral Vortical Effect," Phys. Rev. D 90, no. 6, 065026(2014).
[3] S. Li and H. U. Yee, " Relaxation Times for Chiral Transport Phenomena and Spin Polarization in Strongly Coupled Plasma," arXiv:1805.04057 [hep-th].

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