Thermal noise in boost-invariant matter expansion

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1 Introduction

We formulate a general theory of thermal fluctuations within causal second-order viscous hydrodynamic evolution of matter formed in relativistic heavy-ion collisions. The fluctuation is treated perturbatively on top of a boost-invariant longitudinal expansion. Phenomenological effects of thermal fluctuations on the two-particle rapidity correlations are studied numerically for a lattice QCD equation of state and for various second-order dissipative evolution equations.

2 Thermal fluctuations in boost-invariant hydro

Treating hydrodynamic fluctuation perturbatively [1, 2], so that in the linearized limit of energy density, flow velocity, and shear pressure tensor, the total energy-momentum tensor in the presence of thermal noise tensor $\Xi^{\mu\nu}$ becomes

$$\begin{aligned} T^{\mu\nu} &= \epsilon u^\mu u^\nu - p\Delta^{\mu\nu} + \pi^{\mu\nu} + \Xi^{\mu\nu}, \\ &= T_0^{\mu\nu} + \delta T_{\rm id}^{\mu\nu} + \delta\pi^{\mu\nu} + \Xi^{\mu\nu} \equiv T_0^{\mu\nu} + \delta T^{\mu\nu}. \end{aligned} \tag{1}$$

The conservation equations for the total energy-momentum tensor, $\partial_\mu T^{\mu\nu} = 0$, along with the average part, $\partial_\mu T_0^{\mu\nu} = 0$, lead to $\partial_\mu(\delta T_{\rm id}^{\mu\nu} + \delta\pi^{\mu\nu} + \Xi^{\mu\nu}) \equiv \partial_\mu(\delta T^{\mu\nu}) = 0$. The conservation equation for the average part, $\partial_\mu T_{0\,id}^{\mu\nu} = -\partial_\mu \pi_0^{\mu\nu}$, gives the evolution equation of noiseless ϵ_0 which in boost-invariant:

$$\frac{d\epsilon_0}{d\tau} = -\frac{1}{\tau}\left(\epsilon_0 + p_0 - \pi_0\right), \tag{2}$$

where $\tau^2\pi_0^{\eta\eta} \equiv -\pi_0$ is taken as the independent component of the shear pressure tensor. We have used the two commonly used second-order (dissipative) evolution equation for π_0 that are based on Müller-Israel-Stewart (MIS) formalism [3, 4, 5] and the Chapman-Enskog-like (CE) gradient expansion [6, 7]:

$$\frac{d\pi_0}{d\tau} + \frac{\pi_0}{\tau_\pi} = \frac{4\eta_v}{3\tau_\pi}\theta_0 - \lambda_\pi \theta_0 \pi_0. \tag{3}$$

where $\lambda_\pi = 4/3$ in MIS and 38/21 in CE approach; the relaxation time is set at $\tau_\pi = 2\eta_v\beta_2 = 5\eta_v/(sT_0)$.

To obtain the evolution equations for fluctuations we note that the three independent variables are $\delta\epsilon(\tau,\eta),\ \delta u^\eta(\tau,\eta),\ \delta\pi^{\eta\eta}(\tau,\eta) = -(\delta\pi^{xx} + \delta\pi^{yy})/\tau^2$. The fluctuating part of the energy-momentum conservation equation [8]:

$$\frac{\partial}{\partial\tau}(\tau\delta\epsilon) + \frac{\partial}{\partial\eta}(\tau\mathcal{U}_0\delta u^\eta) = -\delta\mathcal{V}, \tag{4}$$

$$\frac{\partial}{\partial\tau}(\tau\mathcal{U}_0\delta u^\eta) + \frac{\partial}{\partial\eta}\left(\frac{\delta\mathcal{V}}{\tau}\right) = -2\mathcal{U}_0\delta u^\eta. \tag{5}$$

Here $\mathcal{U}_0(\tau) \equiv \epsilon_0 + p_0 - \pi_0$ depends on the background variables, and $\delta\mathcal{V}(\eta,\tau) \equiv \delta p + \tau^2\delta\pi'^{\eta\eta}$ consists of the fluctuating variables.

The stochastic part of the dissipative equation for the independent component, $\delta\pi'^{\eta\eta} = \delta\pi^{\eta\eta} + \Xi^{\eta\eta} \equiv -\delta\pi'/\tau^2$, reads [8]

$$\frac{\partial\delta\pi'}{\partial\tau} + \frac{\delta\pi'}{\tau_\pi} = \frac{1}{\tau_\pi}\left[\tau^2\xi^{\eta\eta} + \frac{4\eta_v}{3s}\left(s_0\delta\theta + \delta s\theta_0\right)\right] - \lambda_\pi\left(\theta_0\delta\pi' + \delta\theta\pi_0\right). \tag{6}$$

The new noise term $\xi^{\eta\eta}$ defines the equation of motion of $\Xi^{\eta\eta}$, which for the MIS equation is

$$\dot{\Xi}^{\langle\eta\eta\rangle} = -\frac{1}{\tau_\pi}\left(\Xi^{\eta\eta} - \xi^{\eta\eta}\right) - \lambda_\pi\Xi^{\eta\eta}\theta. \tag{7}$$

The autocorrelation for $\xi^{\eta\eta}$ in the second-order dissipative hydro equation:

$$\langle\xi^{\eta\eta}(\eta,\tau)\xi^{\eta\eta}(\eta',\tau')\rangle = \frac{8\eta_v(\tau)T_0(\tau)}{3A_\perp\tau^5}\left[1 - \mathcal{A}\beta_2\pi_0\right]\delta(\tau-\tau')\delta(\eta-\eta'). \tag{8}$$

The coefficient $\mathcal{A} = 0$ in the MIS and 5/7 in CE theories, and $A_\perp = \int d\mathbf{x}_\perp$ is the usual transverse area.

3 Two-particle rapidity correlations at freezeout

We treat isothermal freeze-out at a constant background temperature T_f in the Cooper-Frye prescription. The rapidity distribution of the particle is given by

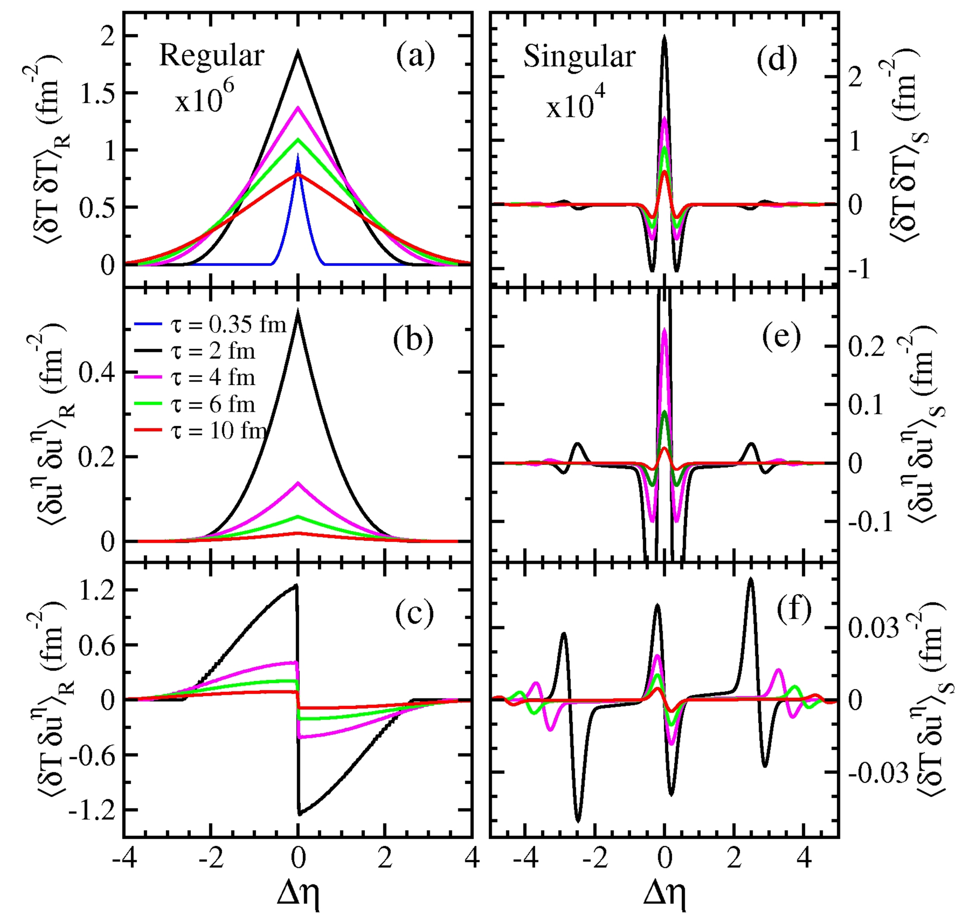
$$\begin{aligned} \frac{dN}{dy} &= \frac{g\tau_f A_\perp}{(2\pi)^3}\int d\eta\,\cosh(y-\eta)\int dp_x\,dp_y\,m_T[f_0(x,p) + \delta f(x,p)] \\ &\equiv (dN/dy)_0 + \delta(dN/dy). \end{aligned} \tag{9}$$

The two-particle rapidity correlator due to fluctuations be written as

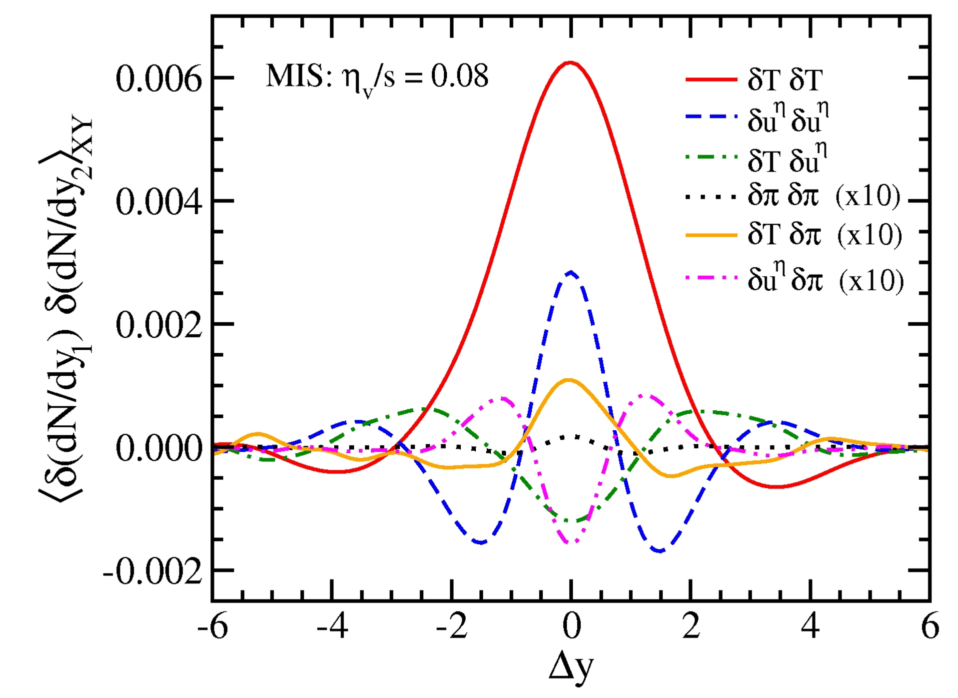
$$\begin{aligned} \left\langle \delta\frac{dN}{dy_1}\,\delta\frac{dN}{dy_2}\right\rangle &= \left[\frac{g\tau_f T_0^3 A_\perp}{(2\pi)^2}\right]^2 \int d\eta_1 \int d\eta_2 \\ &\times \sum_{X,Y}\mathcal{F}_X(y_1-\eta_1)\mathcal{F}_Y(y_2-\eta_2)\langle X(\eta_1)Y(\eta_2)\rangle. \end{aligned} \tag{10}$$

Here $(X,Y) \equiv (\delta T, \delta u^\eta, \delta\pi')$ and $\langle X(\eta_1)Y(\eta_2)\rangle$ are the two-point correlators between the fluctuating variables calculated at the freeze-out hypersurface.

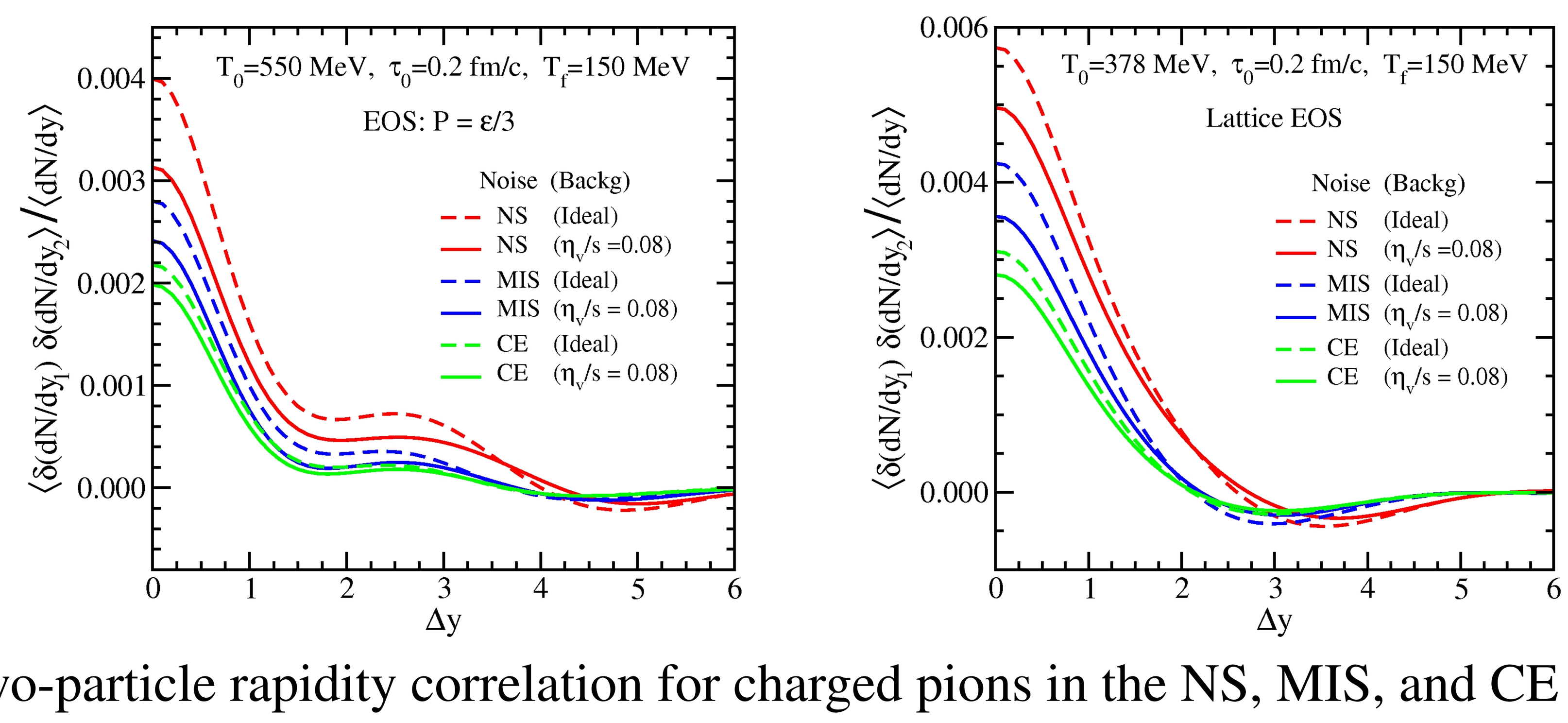
4 Results and discussions



Regular and singular parts of equal time correlators computed as a function of $\Delta\eta$ at various times in the Navier-Stokes (NS) theory. The results are for $p = \epsilon/3$ EOS with initial $T_0 = 550$ MeV and $\tau_0 = 0.2$ fm/c. The shear viscosity to entropy density ratio of $\eta_v/s = 1/4\pi$ is accounted only in the noise correlator. With increasing time, the peaks for the regular part of the correlators first increase and then decrease and spread farther in rapidity due to expansion of the fluid.



Two-particle rapidity correlations from various fluctuations for charged pions in the MIS theory computed at the freeze-out temperature $T_f = 150$ MeV. Temperature-temperature correlators dominate and correlators involving shear pressure tensor are small.



Two-particle rapidity correlation for charged pions in the NS, MIS, and CE formalisms due to thermal noise evolution with and without the inclusion of η_v/s in the background (noiseless) hydrodynamic evolution. The results are for $p = \epsilon/3$ EOS (left panel) and lattice QCD EOS (right panel).
Faster build-up of noise correlators in NS theory give largest peak at small rapidity separations. Viscosity damps the correlation peak especially in NS theory. The magnitude of the correlations are larger in lattice QCD EOS due to smaller sound velocity near T_c that slows noise propagation.

References

[1] J. I. Kapusta, B. Müller and M. Stephanov, Phys. Rev. C **85**, 054906 (2012).

[2] C. Young, *et al.*, Phys. Rev. C **91**, 044901 (2015).

[3] I. Müller, Z. Phys. **198**, 329 (1967); W. Israel and J. M. Stewart, Annals Phys. (N.Y.) **118**, 341 (1979).

[4] A. Muronga, Phys. Rev. C **69**, 034903 (2004).

[5] P. Romatschke, Int. J. Mod. Phys. E 19 (2010) 1.

[6] A. Jaiswal, Phys. Rev. C **87**, 051901 (2013).

[7] C. Chattopadhyay, A. Jaiswal, S. Pal and R. Ryblewski, Phys. Rev. C **91**, 024917 (2015).

[8] C. Chattopadhyay, R. S. Bhalerao and S. Pal, arXiv:1711.10759 [nucl-th].