Soft-gluon approximation in calculating radiative energy loss of high $p_\perp$ particles – is it well-founded?

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ABSTRACT

The soft-gluon approximation, which assumes that radiated gluon carries away a small fraction of initial parton's energy, is widely used assumption in calculating radiative energy loss of high momentum partons traversing QGP created at RHIC and LHC. Regardless of its convenience, different theoretical approaches reported significant radiative energy loss of high $p_\perp$ partons, which raised the doubts of its validity. To address this issue, we relaxed the soft-gluon approximation within DGLV formalism. Although the obtained analytical expressions are quite distinct compared to the soft-gluon case, numerically both cases lead to very similar predictions for the first order in opacity fractional energy loss. The predicted number of radiated gluons is also barely affected. Additionally, the effects on these two variables run in opposite directions, which when superposed results in nearly overlapping suppression predictions. Consequently, our results imply that, regardless of the skepticism, the soft-gluon approximation in practice works surprisingly well in DGLV formalism. We also refer to generalizing this relaxation to the dynamical QCD medium, which suggests a broader validity of the conclusions obtained here.

INTRODUCTION

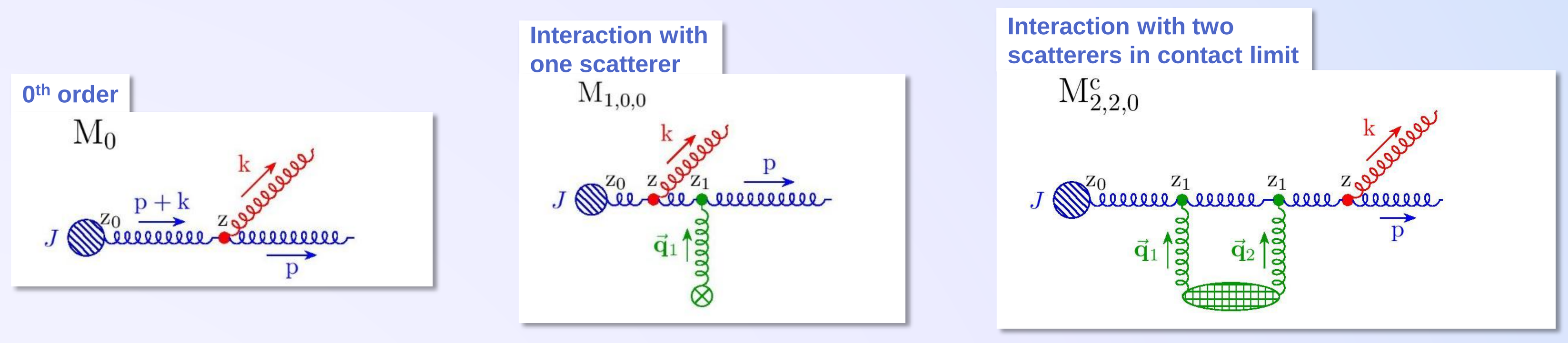
- The soft-gluon approximation (i.e. $x=\omega/E$, $E\equiv$ initial parton energy and $\omega\equiv$ radiated gluon energy) is one of the most common assumptions used in calculating radiative energy loss of high $p_\perp$ partons.
- However, different theoretical models, which also assumed this approximation, obtained significant radiative energy loss, questioning the validity of this approximation.
- Gluons are most affected by this approximation, due to color factor 9/4 compared to quarks.
- It was used in radiative part of our dynamical energy loss formalism, whose angular averaged R_{AA} predictions were successfully tested against comprehensive set of experimental data, implying formalism's reliability.
- Nevertheless, the approximation breaks down for intermediate momentum ranges (5 < $p_\perp$ <10 GeV), where experimental data are most abundant and with the smallest error-bars, and for gluons primarily.

Why is relaxing the soft-gluon approximation important?

- To establish how applicable is the approximation.
- To extend the model toward intermediate $p_\perp$ region.
- To test the reliability of our predictions in the above case.

METHODS AND DISCUSSION

- We address validity of the soft-gluon approximation within DGLV formalism, which assumes:
  - Finite size, optically thin QGP.
  - Static scattering centers, so the interactions with medium constituents are modeled by Debye colored-screened Yukawa potential.
  - Gluons, in finite temperature QGP, as massive transversely polarized plasmons with effective mass $m_g=\mu/\sqrt{2}$.
- Generalization of the results on dynamical medium is discussed.



- We relaxed the approximation for high $p_\perp$ gluon, by calculating corresponding 11 Feynman diagrams within DGLV, under the following assumptions:
  - Initial gluon propagates along the longitudinal axis
  - The soft-rescattering (eikonal) approximation
  - The first order in opacity approximation.
- The obtained analytical expression for single gluon radiation spectrum ($dN_g^{(1)}/dx$) beyond soft-gluon approximation (*bsg*):
  - Is more complicated than in soft-gluon (*sg*) case.
  - Recovers *sg* result for $x \ll 1$.
  - Is symmetric under the exchange of radiated (k) and final gluon (p).
- Finally, we compared *bsg* and *sg* numerical predictions for fractional radiative energy loss $\Delta E^{(1)}/E$, number of radiated gluons $N_g^{(1)}$, differential energy loss $dE^{(1)}/dx$, single gluon radiation spectrum and suppression R_{AA}, to assess the effect of relaxation.

THEORETICAL AND NUMERICAL RESULTS

Beyond soft-gluon approximation:

$$f_{bsg}(\mathbf{k},\mathbf{q_1},x)=\frac{(1-x+x^2)^2}{x(1-x)}\frac{(\mathbf{k}-\mathbf{q_1})^2+\chi}{\left(\frac{4x(1-x)E}{L}\right)^2+\left((\mathbf{k}-\mathbf{q_1})^2+\chi\right)^2}\left(2\frac{(\mathbf{k}-\mathbf{q_1})^2}{(\mathbf{k}-\mathbf{q_1})^2+\chi}-\frac{\mathbf{k}\cdot(\mathbf{k}-\mathbf{q_1})}{\mathbf{k}^2+\chi}-\frac{(\mathbf{k}-\mathbf{q_1})\cdot(\mathbf{k}-x\mathbf{q_1})}{(\mathbf{k}-x\mathbf{q_1})^2+\chi}\right)+$$

$$+\frac{\mathbf{k}^2+\chi}{\left(\frac{4x(1-x)E}{L}\right)^2+\left(\mathbf{k}^2+\chi\right)^2}\left(\frac{\mathbf{k}^2}{\mathbf{k}^2+\chi}-\frac{\mathbf{k}\cdot(\mathbf{k}-x\mathbf{q_1})}{(\mathbf{k}-x\mathbf{q_1})^2+\chi}\right)+\left(\frac{(\mathbf{k}-x\mathbf{q_1})^2}{((\mathbf{k}-x\mathbf{q_1})^2+\chi)^2}-\frac{\mathbf{k}^2}{(\mathbf{k}^2+\chi)^2}\right)$$

$$\chi=m_g^2(1-x+x^2)$$

Only this term remains in soft-gluon approximation and reduces to:

Soft-gluon approximation:

$$f_{sg}(\mathbf{k},\mathbf{q_1},x)=\frac{1}{x}\frac{(\mathbf{k}-\mathbf{q_1})^2+m_g^2}{\left(\frac{4xE}{L}\right)^2+((\mathbf{k}-\mathbf{q_1})^2+m_g^2)^2}2\left(\frac{(\mathbf{k}-\mathbf{q_1})^2}{(\mathbf{k}-\mathbf{q_1})^2+m_g^2}-\frac{\mathbf{k}\cdot(\mathbf{k}-\mathbf{q_1})}{\mathbf{k}^2+m_g^2}\right)$$

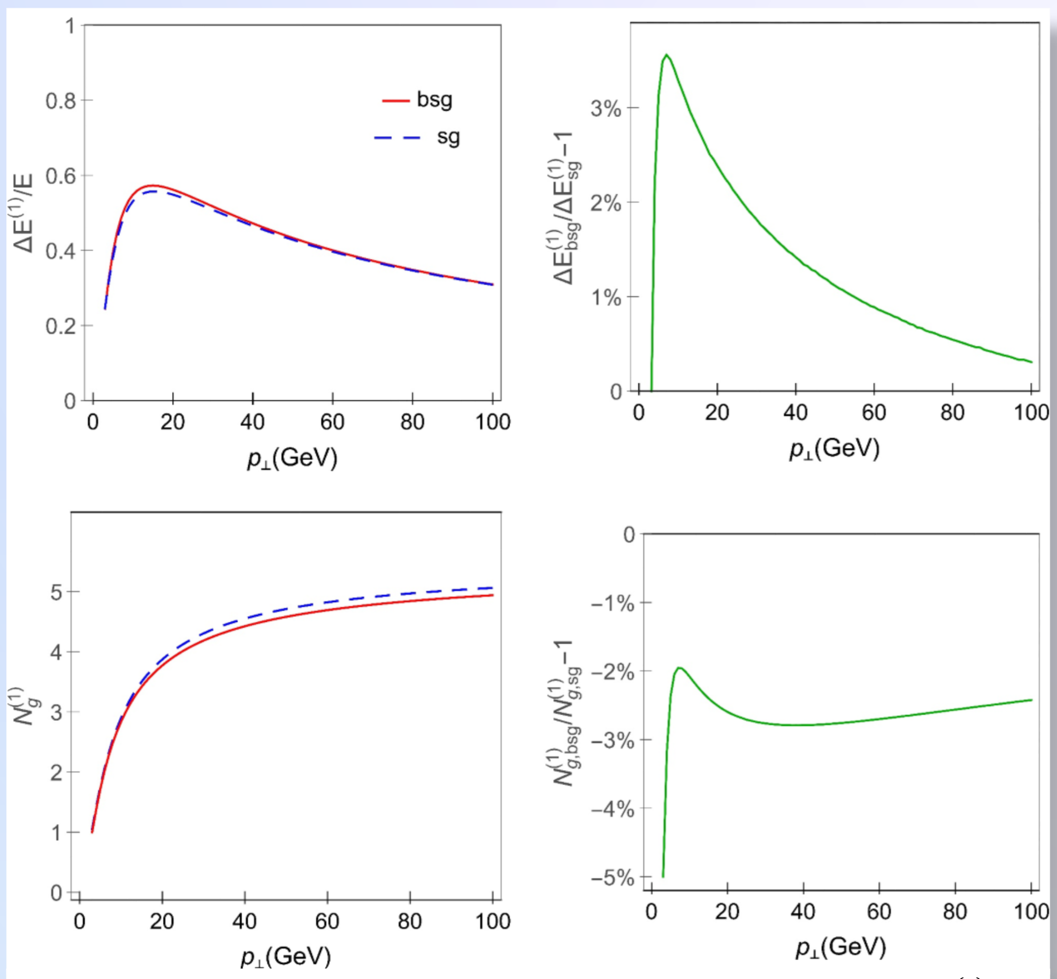


Fig.1. The effect of relaxing the soft-gluon approximation on $\Delta E^{(1)}/E$ and $N_g^{(1)}$.

Finite x has a very small effect on $\Delta E^{(1)}/E$ and $N_g^{(1)}$, and with opposite signs.

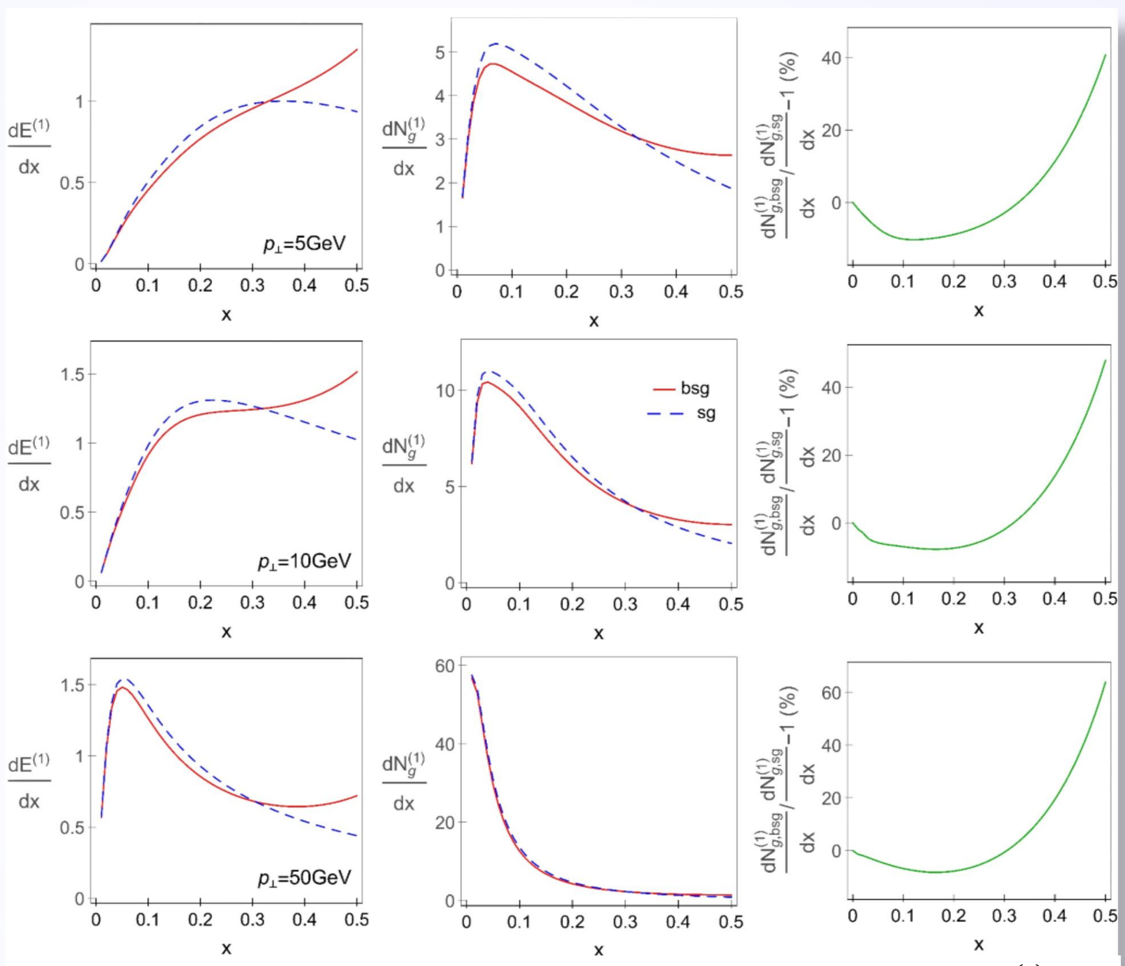


Fig.2. The effect of relaxing the soft-gluon approximation on $dE^{(1)}/dx$ and $dN_g^{(1)}/dx$.

The effect on $dE^{(1)}/dx$ and $dN_g^{(1)}/dx$ is small for $x \leq 0.4$, while enhances to notable value with increasing x above the "cross-over" point $x \approx 0.3$.

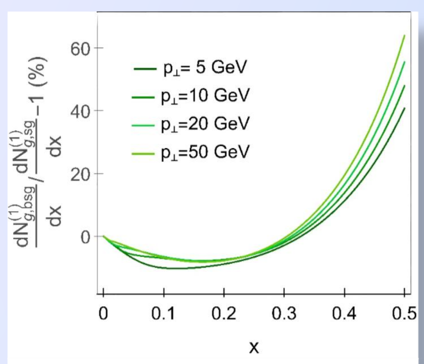


Fig.3. The effect on $dN_g^{(1)}/dx$ for different $p_\perp$.

Nearly the same effect on $dN_g^{(1)}/dx$ ($dE^{(1)}/dx$) independently of initial gluon $p_\perp$.

How does the relaxation of soft-gluon approximation affect our predictions?

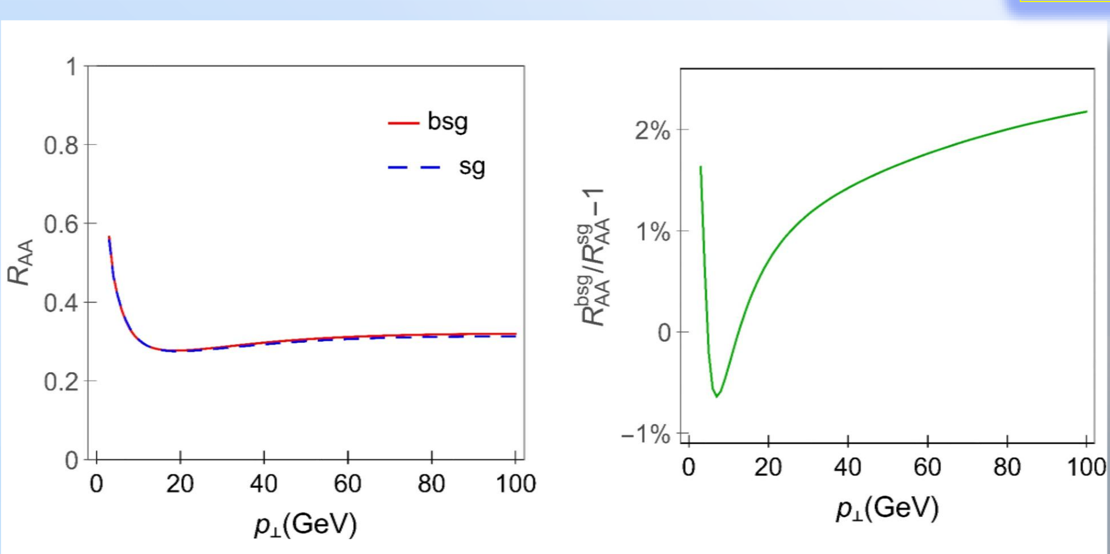


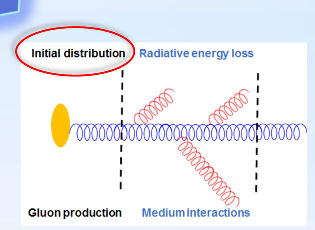
Fig.4. The effect of relaxing the soft-gluon approximation on R_{AA}.

Even smaller effect on R_{AA}!

- Why is R_{AA} barely affected by this relaxation?
- Why the differential variables discrepancies at $x \geq 0.4$ do not influence R_{AA}?

Both $\Delta E^{(1)}/E$ and $N_g^{(1)}$ non-trivially enter R_{AA}.

Interplay of the opposite effects on $\Delta E^{(1)}/E$ and $N_g^{(1)}$ is responsible for negligible effect on R_{AA}.



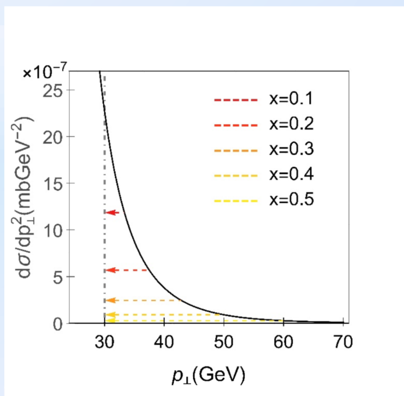


Fig.5. The initial gluon distribution constrains the relevant x region.

$x \leq 0.3$ is the relevant region for generating predictions, due to exponentially decreasing initial gluon $p_\perp$ distribution.

SUMMARY

The effect of relaxing the soft-gluon approximation is:

- Very small for:
  - Fractional radiative energy loss up to ~ 3%.
  - Number of radiated gluons up to ~ -5%, and of a opposite sign for the two variables.
- Relatively small for:
  - Differential radiative energy loss and
  - Single gluon radiation spectrum for $x<0.4$ (up to ~ 10%), whereas notable for higher x (up to ~ 60%).
- Practically the same for both $dN_g^{(1)}/dx$ and $dE^{(1)}/dx$, across the whole x region, regardless of initial gluon momentum $p_\perp$.
- Negligible for nuclear modification factor (max 2%), and practically insensitive to gluon's momentum.

The effect on R_{AA} is qualitatively the superposition of the effects on $\Delta E^{(1)}/E$ and $N_g^{(1)}$.

The relevant x region for generating the predictions, due to exponentially decreasing initial distribution, is: $x \leq 0.3$.

CONCLUSIONS AND OUTLOOK

- Few theoretical models reported considerable radiative energy loss, imposing a question: is the soft-gluon approximation well-founded?
- To that end, we relaxed the approximation for high $p_\perp$ gluons, which are most affected by it, within DGLV formalism, and although analytical results differ greatly in *bsg* and *sg* cases, numerical predictions are nearly indistinguishable.
- Consequently, high $p_\perp$ quark is even less likely to be affected by the relaxation.
- This implies that soft gluon approximation works well within DGLV formalism.
- To our knowledge, this presents the first opportunity to assess the effects of relaxing the soft-gluon approximation within DGLV formalism.
- We expect that the soft-gluon approximation remains well-founded when dynamical medium is considered as well - this still remains to be rigorously tested.

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