1) Introduction

• Trajectory of a nuclear collision depends on the time evolution of energy & net-baryon densities. At lower energies like RHIC-BES, trajectory is important for effects from the QCD critical point.
• The Bjorken formula [1] is only valid at high energies, where \( t_1 \) (the crossing time of the two nuclei) is small. This condition does not hold at lower energies [2]. For central Au+Au collisions, at \( \sqrt{s_{NN}} = 50 \) GeV \( d_1 \approx 0.5 \text{ fm/c} \) is not small but comparable to \( t_0 \), so the Bjorken formula cannot be trusted well below this energy.
• Here we extend [3] the Bjorken formula to lower energies by including the finite crossing time.

2) The Analytical Method

Fig.1a shows a collision at finite energy, where the two nuclei have finite thickness along z and cross each other during time \( [0, d_1] \), with \( d_2 = 2R_d/\gamma_f \). So the initial energy production takes place over finite widths in both t and z (the shaded area).

We study the energy density averaged over the transverse overlap area at mid space-time-rapidity (\( \eta_r = 0 \)) of central A+A collisions. Let us consider the finite width in t as shown in Fig.1b (but neglect the finite width in z). For a particle produced at time \( t \) to remain in a thin slab -d\( z \leq z \leq d \) at time \( t \), its rapidity needs to satisfy

\[
|\tan(y)| = |y| < \frac{d}{\sqrt{z^2 - d^2}}.
\]

Suppose initial particles have a finite formation time \( \tau_f \), then at any time \( t \geq \tau_f \) the average energy density within the slab is

\[
\epsilon(t) = \frac{1}{d_1} \int_0^{d_2} \int_{-d}^{d} \frac{dE}{dy} \frac{d^2 \sigma}{dy dx} \, dx \, dz.
\]

Here \( \frac{dE}{dy} \frac{d^2 \sigma}{dy dx} \) is the production rate of initial \( \frac{dE}{dy} \) (\( y = 0 \)) at time \( x \); at high energies it is much less sensitive to the uncertainty of \( \frac{d^2 \sigma}{dy dx} \), then we recover the Bjorken formula:

\[
\epsilon_B(t) = \frac{1}{\tau_f} \frac{dE}{dy} \frac{d^2 \sigma}{dy dx}
\]

Next we will take a specific form for the time (or \( x \)) profile of \( \frac{dE}{dy} \frac{d^2 \sigma}{dy dx} \). The simplest is a uniform profile (red curve in Fig.2):

\[
\text{initial energy at } y = 0 \text{ is produced uniformly over a period of time.}
\]

A more realistic profile will have \( \sim 0 \) energy produced at \( x = 0 \) & \( d_1 \) but most energy produced at \( \sim d_1/2 \) (blue curves in Fig.2).

3) Analytical Results

A uniform profile from time \( t_1 \) to \( t_2 \) (with \( t_{21} = t_2 - t_1 \)) gives

\[
\epsilon_{uni}(t) = \frac{1}{A_{t_2 - t_1}} \frac{dE}{dy} \ln \left( \frac{t_1}{\tau_f} \right), \text{ if } t \in [t_1 + \tau_f, t_2 + \tau_f];
\]

\[
\epsilon_{uni}(t) = \frac{1}{A_{t_2 - t_1}} \frac{dE}{dy} \ln \left( \frac{t_2}{\tau_f} \right), \text{ if } t \geq t_1 + \tau_f.
\]

The peak energy density is

\[
\epsilon_{uni}^{\text{max}} = \frac{1}{A_{t_2 - t_1}} \frac{dE}{dy} \ln \left( \frac{1 + t_2}{\tau_f} \right).
\]

At low energies (Fig.3a):

• peak energy density is much lower than the Bjorken formula,
• time evolution of the initial energy density is much longer,
• \( \epsilon_{uni}^{\text{max}} \sim \ln \left( \frac{\tau_0}{\tau_f} \right) \); much less sensitive to \( \tau_f \) than Bjorken’s \( \frac{1}{\tau_f} \).

At high energies (Fig.3b): solution approaches Bjorken formula.

4) Comparison with Numerical Results

We have included finite nuclear thickness to string-melting AMPT [4] to get transport results on the energy density at \( \eta_x \approx 0 \). Fig.4 confirms the key features of our analytical results:

• AMPT without finite thickness \( o = \) Bjorken formula.
• AMPT with finite thickness \( e \sim \) our extension.
• AMPT with finite thickness = AMPT with finite \( t \) but no finite \( z \).

So effect of finite width in \( z \) (neglected in our analytical method) is small, once the finite width in \( t \) is included.

5) Conclusions

• We have extended the Bjorken formula, since it neglects the finite nuclear thickness and thus breaks down at low energies.
• At \( \sqrt{s_{NN}} \ll 50 \) GeV for central Au+Au collisions, the peak energy density \( \epsilon_{uni}^{\text{max}} \) is much lower than the Bjorken formula, but the time evolution of energy density takes much longer.
• \( \epsilon_{uni}^{\text{max}} \) is much less sensitive to the uncertainty of \( \tau_f \).
• \( \epsilon_{uni}^{\text{max}} \) increases much faster with the collision energy \( \sqrt{s} \) than the Bjorken formula (see Fig.4).
• Results from the AMPT model confirm key features of our analytical solutions, now valid at both low and high energies.

References

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