Extending the Bjorken Formula to Describe Initial Energy Production at Lower Energies

Zi-Wei Lin (林子威)
Central China Normal University (China) & East Carolina University (USA)

Abstract

The Bjorken formula is very useful for estimating the initial energy density in heavy ion collisions once an initial time τ_0 is given. However, it cannot be trusted at lower energies, e.g. well below $\sqrt{s_{NN}}\sim 50$ GeV for central Au+Au collisions, when the crossing time of the two nuclei becomes comparable to τ_0. Here we extend the Bjorken formula, by including the finite time duration of the initial energy production, to obtain analytical solutions that are also valid at lower energies. Comparisons with results from a multi-phase transport (AMPT) model confirm the key features of these analytical solutions.

1) Introduction

- Trajectory of a nuclear collision depends on the time evolution of energy & net-baryon densities. At lower energies like RHIC-BES, trajectory is important for effects from the QCD critical point.
- The Bjorken formula **[1]** is only valid at high energies, where d_t (*the crossing time of the two nuclei*) is small. But this condition does not hold at lower energies **[2]**. For central Au+Au collisions, at $\sqrt{s_{NN}} = 50$ GeV d_t~0.5 fm/c is not small but comparable to τ_0, so the Bjorken formula cannot be trusted well below this energy.
- Here we extend **[3]** the Bjorken formula to lower energies by including the finite crossing time.

2) The Analytical Method

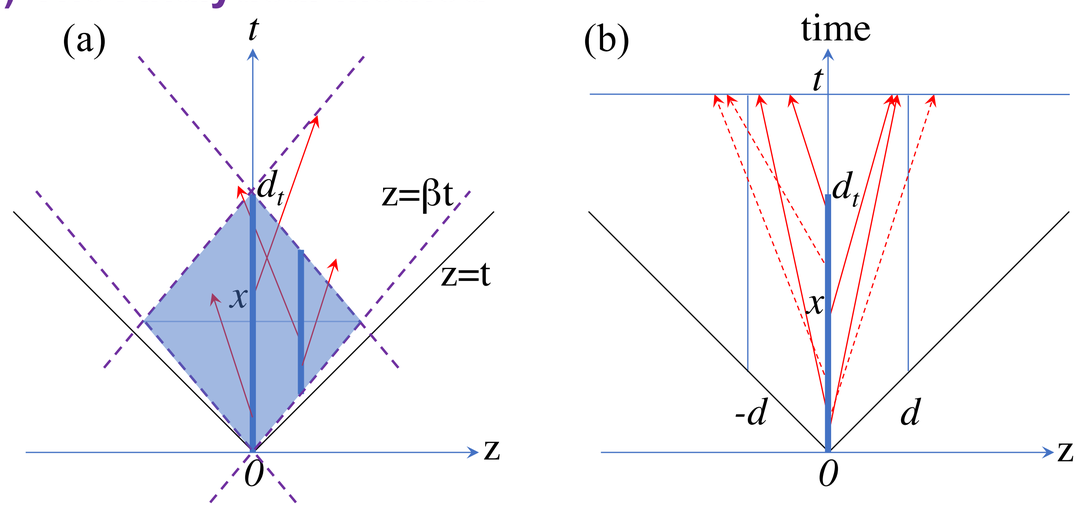

Fig.1 (a) Schematic picture of the collision of two nuclei: the initial energy production takes place over the shaded area of finite widths in *t* and *z*. **(b)** We consider this simplified picture: particles could be initially produced at any time $x \in [0, d_t]$ (*but at z=0*) & then propagate to observation time *t*.

Fig.1a shows a collision at finite energy, where the two nuclei have finite thickness along *z* and cross each other during time $[0, d_t]$, with $d_t = 2R_A/(\gamma\beta)$. So the initial energy production takes place over finite widths in both *t* and *z* (the shaded area).

We study the energy density averaged over the transverse overlap area at mid space-time-rapidity ($\eta_s = 0$) of central A+A collisions. Let us consider the finite width in *t* as shown in Fig.1b (*but neglect the finite width in z*). For a particle produced at time *x* to remain in a thin slab *-d<z<d* at time *t*, its rapidity needs to satisfy

$$|\tanh(y)| \approx |y| < \frac{d}{t-x}. \qquad (1)$$

Suppose initial particles have a finite formation time τ_F, then at any time $t \geq \tau_F$ the average energy density within the slab is

$$\epsilon(t) = \frac{1}{A_T}\int_0^{t-\tau_F} \frac{d^2E_T}{dy\,dx}\frac{dx}{(t-x)}. \qquad (2)$$

Here $\frac{d^2E_T}{dy\,dx}$ is the production rate of initial $\frac{dE_T}{dy}(y=0)$ at time x; at high energies it is $\approx \frac{dE_T}{dy}\delta(x)$, then we recover the Bjorken formula:

$$\epsilon_{Bj}(t) = \frac{1}{t}\frac{1}{A_T}\frac{dE_T}{dy} \quad \text{for } t \geq \tau_F. \qquad (3)$$

Next we will take a specific form for the time (or x) profile of $\frac{d^2E_T}{dy\,dx}$. The simplest is a uniform profile (*red curve in* Fig.2): initial energy (at $y\sim0$) is produced uniformly over a period of time. A more realistic profile will have ~0 energy produced at $x = 0$ & d_t but most energy produced at $\sim d_t/2$ (*blue curves in* Fig.2).

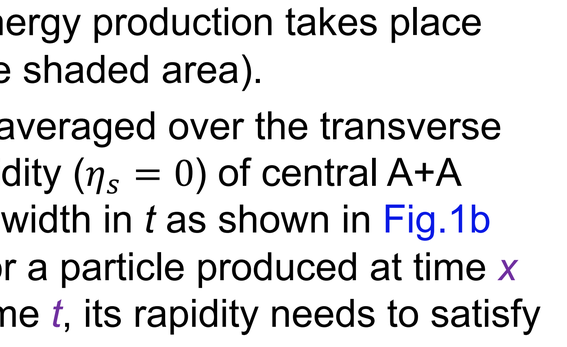

Fig.2 Time profiles for the initial energy production at $\eta_s = 0$, including the beta profiles with power n=1 to 5 (**blue**) **[3]**. Circles represent partons from AMPT for central Au+Au collisions at 11.5A GeV.

3) Analytical Results

A uniform profile from time t_1 to t_2 (with $t_{21} = t_2 - t_1$) gives

$$\epsilon_{\rm uni}(t) = \frac{1}{A_T t_{21}}\frac{dE_T}{dy}\ln\left(\frac{t-t_1}{\tau_F}\right), \text{if } t \in [t_1+\tau_F, t_2+\tau_F];$$
$$= \frac{1}{A_T t_{21}}\frac{dE_T}{dy}\ln\left(\frac{t-t_1}{t-t_2}\right), \text{if } t \geq t_2+\tau_F. \qquad (4)$$

The peak energy density is $\epsilon_{\rm uni}^{max} = \frac{1}{A_T t_{21}}\frac{dE_T}{dy}\ln\left(1+\frac{t_{21}}{\tau_F}\right)$. (5)

At low energies (Fig.3a):
- peak energy density is much lower than the Bjorken formula,
- time evolution of the initial energy density is much longer,
- $\epsilon_{uni}^{max} \sim \propto \ln\left(\frac{1}{\tau_F}\right)$: much less sensitive to τ_F than Bjorken's $\frac{1}{\tau_F}$.

At high energies (Fig.3b): solution approaches Bjorken formula.

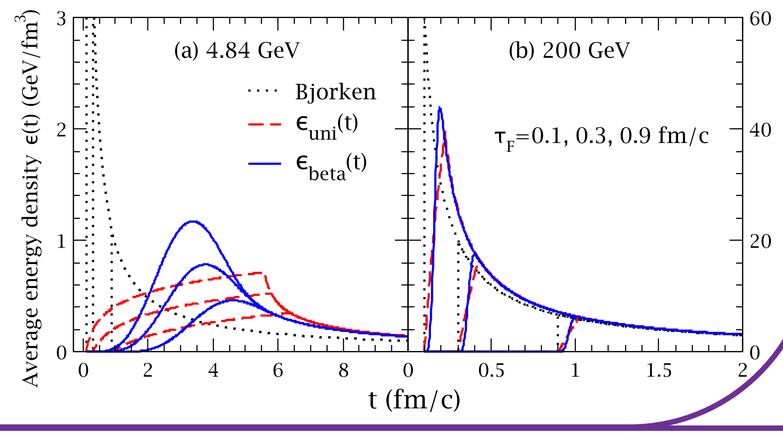

Fig.3 Energy densities as functions of time for central Au+Au collisions at **(a)** low & **(b)** high energies: Bjorken formula (**black**), uniform profile with t_1=0 and t_2=d_t (**red**), and beta profile for n=4 (**blue**).

4) Comparison with Numerical Results

We have included finite nuclear thickness to string-melting AMPT **[4]** to get transport results on the energy density at $\eta_s\sim0$. Fig.4 confirms the key features of our analytical results:

- AMPT without finite thickness ○ ~ Bjorken formula.
- **AMPT with finite thickness** ● ~ our extension.
- **AMPT with finite thickness** ~ AMPT with finite *t* but no finite *z*. So effect of finite width in *z* (*neglected in our analytical method*) is small, once the finite width in *t* is included.

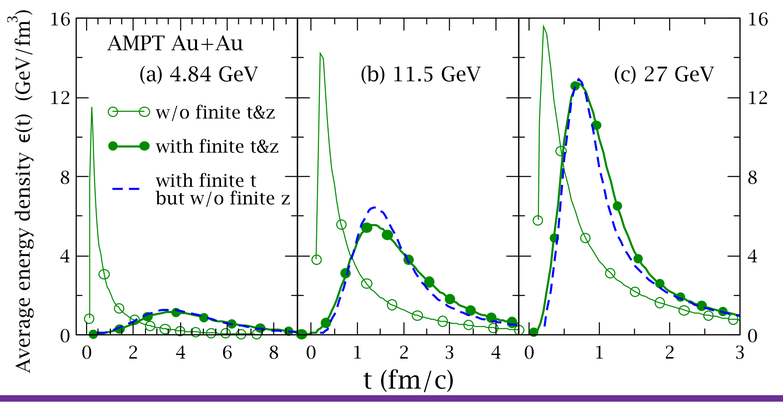

Fig.4 AMPT results of energy densities for central Au+Au collisions at **(a)** 4.84A GeV, **(b)** 11.5A GeV, **(c)** 27A GeV.

5) Conclusions

- We have extended the Bjorken formula, since it neglects the finite nuclear thickness and thus breaks down at low energies.
- At $\sqrt{s_{NN}} << 50$ GeV for central Au+Au collisions, the peak energy density ϵ^{max} is much lower than the Bjorken formula, but the time evolution of energy density takes much longer, ϵ^{max} is much less sensitive to the uncertainty of τ_F, $\epsilon^{max}(\sqrt{s})$ increases much faster with the collision energy $\sqrt{s}$ than the Bjorken formula (see Fig.4).
- Results from the AMPT model confirm key features of our analytical solutions, now valid at both low and high energies.

References

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Acknowledgements

The author acknowledges support from the National Natural Science Foundation of China Grant No. 11628508.