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The Angular Power Spectrum of Heavy Ion Collisions

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Introduction

The angular particle distribution resultant from heavy ion collisions in the Large Hadron Collider (LHC) has a final geometry that depends on fluctuations present in the initial conditions. Hadrons scatter anisotropically with the collision axis as the preferred direction. However, around the plane perpendicular to the collision axis, particles can be considered almost isotropically distributed in pseudorapidity (η); a quantity that translates to the polar angle via $\theta = 2\arctan(\exp(-\eta))$. This study proposes to map the final particle distribution to the surface of a sphere, in a so called *Mollweide projection*, which allows for its expansion in *spherical harmonics* and the calculation of an *angular power spectrum*. The latter is sensitive to anisotropies in both polar and azimuthal (ϕ) directions, making it one more tool to probe the properties of the Quark Gluon Plasma (QGP).

Mapping Particles

Particles emitted from heavy ion collisions have their psedorapidity (η) and azimuthal angles (ϕ) measured. However, mapping particles to a sphere's surface requires the change of their coordinates: $(\eta, \phi) \to (\theta, \phi)$.

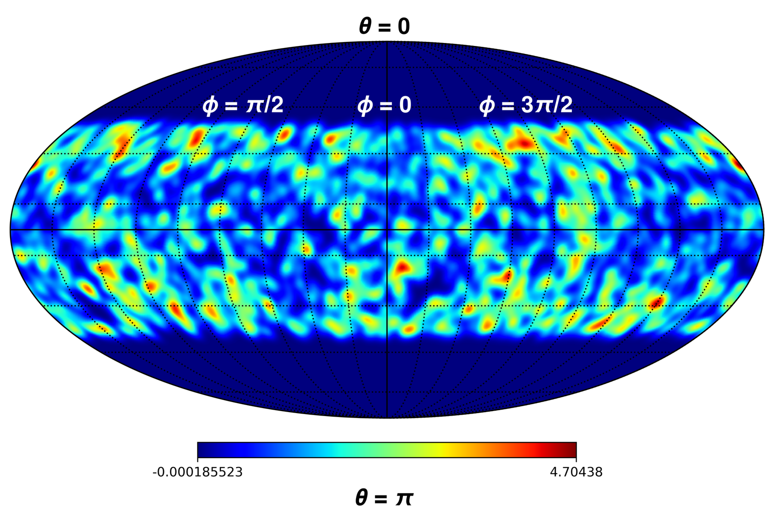

Figure 1: Resultant map of a single event in the 0-5% centrality. Numbers in the color bar result from smoothing the map: red means higher particle density, while blue stands for the opposite. Lines on the graticule are separated by 20^o.

The software HEALPix [2] (Hierarchical Equal Area isoLatitude Pixelation) partitions a spherical surface in pixels of same area. Additionally, the number of divisions depends on the chosen resolution (nside). Each particle with coordinates (θ_j, ϕ_j) maps to a pixel j on the sphere, as shown in Fig.1. The polar angle (θ) corresponds to the lines of latitude whereas ϕ matches the lines of longitude.

The map in Fig. 1 is of a single event from a central collision at $\sqrt{s_{NN}} = 2.76\text{TeV}$ within pseudorapidity range of $|\eta| < 0.9$. It can be represented by $f(\theta, \phi)$ and expanded in spherical harmonics, $Y_{lm}(\theta, \phi)$:

$$f(\theta, \phi) = \sum_{l=0}^{l_{max}} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\theta, \phi), \qquad Y_{lm} = \sqrt{\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}} P_{lm}(\cos(\theta)) e^{im\phi}, \tag{1}$$

where a_{lm} are coefficients of the decomposition, P_{lm} are the associated Legendre Polynomials and each l-mode is a *multipole moment*. Although $l_{max} \to \infty$ in the spherical harmonic expansion, it is unfeasible computationally. In this study, calculations where performed with nside = 8, which corresponds to $l_{max} = 3 \cdot \text{nside} - 1 = 23$. Such choice is bound to the event multiplicity. For each event map, one can estimate the values of a_{lm}. Through HEALPix:

$$a_{lm} = \frac{4\pi}{N_{pix}} \sum_{p=0}^{N_{pix}-1} Y^*_{lm}(\theta_p, \phi_p) f(\theta_p, \phi_p), \tag{2}$$

$$C_l = \frac{1}{2l+1} \sum_{m=-1}^{l_{max}} |a_{lm}|^2. \tag{3}$$

where N_{pix} represents the number of pixels and C_l is the angular power spectrum, given by the variance of a_{lm} at that l. The multipole moments relate to the angular scale of the distribution: lower l values correspond to larger angles and vice versa. In other words, $\alpha = \frac{180^o}{l}$, where α represents the angle in degrees.

Angular Power Spectra

The present analysis consists in mapping each event in a Mollweide projection, followed by the estimation of a_lm through Eq.2 and calculation of the angular power spectrum using Eq.3. Finally, the average over all C_l is taken. A map containing all events was created in order to possibly account for any detector failures. Each event projection is divided by such map, thus assigning weights to each pixel. The whole proceeding is repeated for each centrality from 0-5% to 35-40% in intervals of 5.

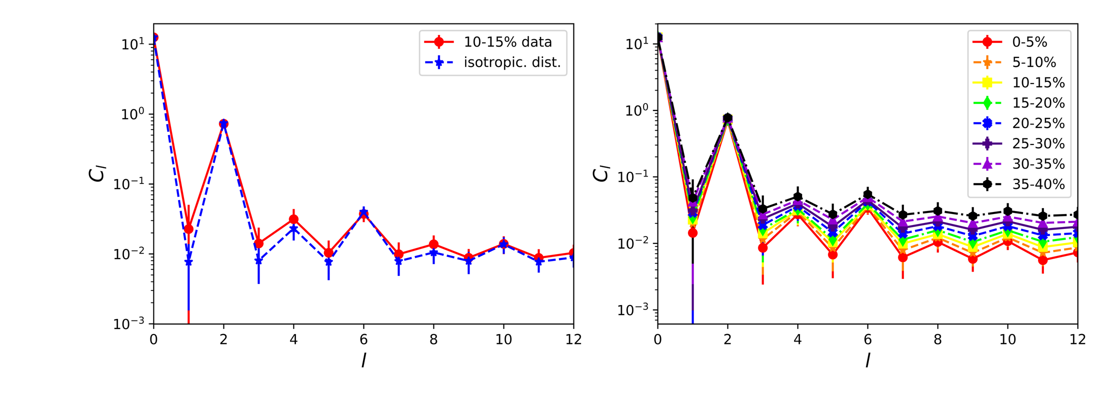

Figure 2: Left: Averaged power spectrum over 500 events for 10-15% data compared to an isotropic distribution. Right: Averaged power spectra for each centrality. Error bars correspond to 1σ deviation.

The value $C_0 = 4\pi$ in all events and centralities is a consequence of the normalization, which requires $f(\theta, \phi)$ to be divided by the event multiplicity and multiplied by N_{pix}. Distributions isotropic in θ and ϕ were created with the same multiplicities as the events for each centrality. As means of comparison, we followed the same steps in averaged C_l calculation as the data.

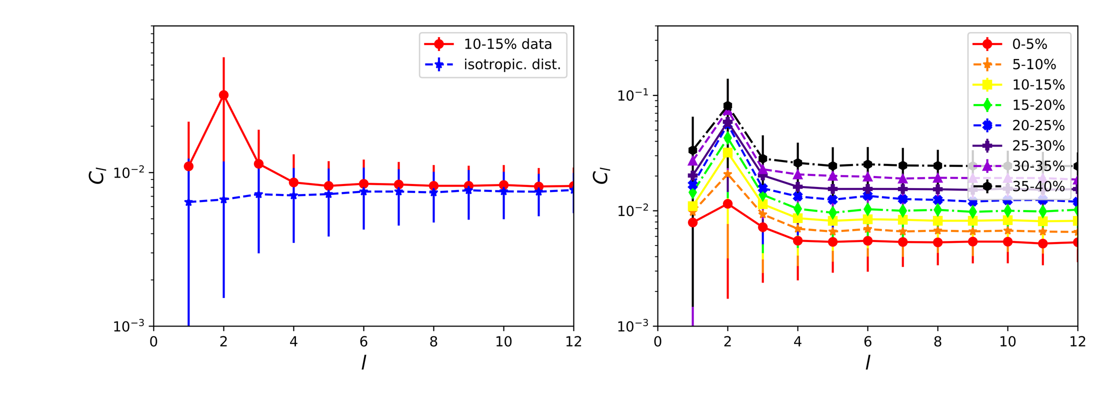

Figure 3: Left: Averaged power spectrum with $m \neq 0$ over 500 events for 10-15% data compared to an isotropic distribution. Right: Averaged power spectra with $m \neq 0$ for each centrality.

Symmetry with respect to $\theta = \frac{\pi}{2}$ in Fig. 2 causes the suppression of odd l-modes relative to the even ones. Another noteworthy trait is the average raise in power spectrum values with more peripheral centralities. Such is caused by the decrease in multiplicity as collisions approach 35-40% centrality: given that particles will be more scattered, fluctuations increase, making C_l approach higher and higher values as $l \to l_{max}$.

In both data and isotropic power spectra, there is an enhancement of even l-modes, eg. 2, 6, 10, relative to other even ones, eg. 4, 8. Because there is data within $|\eta| < 0.9$ only, these specific l-values are enhanced. An isotropic distribution taking the whole sphere should have $C_0 = 4\pi$ while all others become closer to zero as multiplicity increases. Since the same effect takes over the data power spectra, a simple technique is employed to investigate its hidden anisotropies: the exclusion of a_{l0} coefficients when calculating C_l. Given that Y_{lm} depends only on θ when $m = 0$ and the existence of a cut, a_{l0} dominate the sum in Eq. 3 for lower l-values.

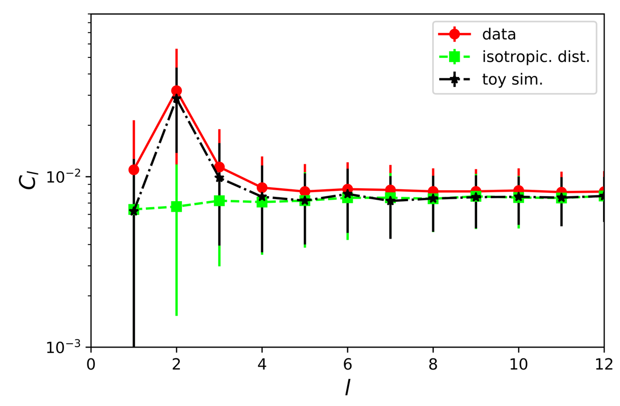

Figure 4: Comparison between averaged power spectra of MC-simluations, data and isotropic distribution at centrality 10-15%.

After estimating the a_{lm} coefficients, C_l is then calculated using Eq. 3, where the sum still goes from $m = -l$ to $m = l$, though $m = 0$ is left out. As expected, the averaged power spectrum for the isotropic distribution becomes flat: without the a_{l0} corresponding to cut effects, all the extra features were eliminated. On the other hand, the data C_l peaks in $l = 2$ and its other modes for $l < 5$ remain slightly above the isotropic C_l. The same pattern appears in all centralities, suggesting the possible existence of flow.

Flow in C_l

For simplicity, we take $f_{sim}(\theta, \phi) = 1/2\pi[1 + 2(v_2 \cos(2(\phi - \psi_2)) + v_3 \cos(3(\phi - \psi_3))]$ to be the distribution of particles in each event. We fix v_n and vary ψ_n. Using Eqs. 2 and 3 with $m \neq 0$, one can derive a formula to calculate v_2 and v_3 through the angular power spectrum:

$$|v_n|^2 = \frac{(2n+1)|a_{00}|^2 C_n^{m\neq 0}}{2|a_{nn}|^2 C_0^{full}}, \tag{4}$$

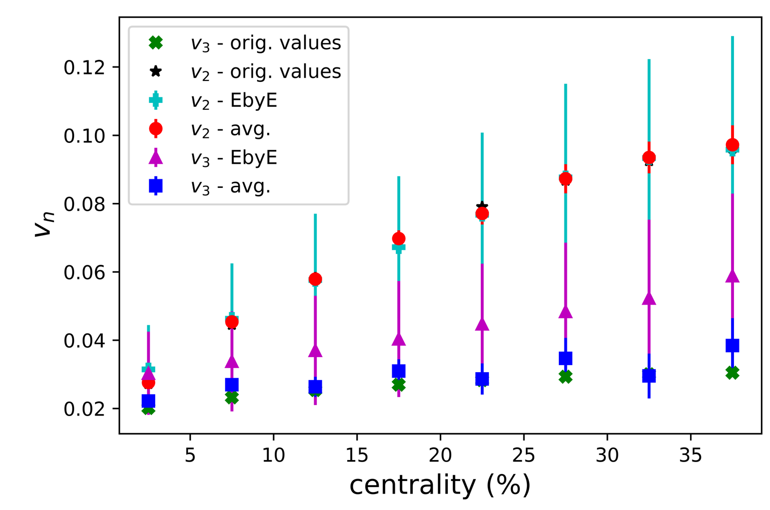

Figure 5: v_n as a function of centrality for the MC simulations. Comparison between event-by-event method, averaged method and the real values.

where $C_n^{m\neq 0}$ is the power spectrum value at $l = n$ for $m \neq 0$ (Fig. 4) and $C_0^{full} \sim 4\pi$. $C_n^{m\neq 0}$ has to be taken as the difference between the power spectrum value at that point and $C_1^{m\neq 0}$. Since there is no v_1 in $f_{sim}(\theta, \phi)$, C_1 should be zero for both cases. However, due to low multiplicities, $C_1^{m\neq 0}$ is of close order to $C_2^{m\neq 0}$ and $C_3^{m\neq 0}$. Hence, $C_n^{m\neq 0} \to C_n^{m\neq 0} - C_1^{m\neq 0}$ in Eq. 4.

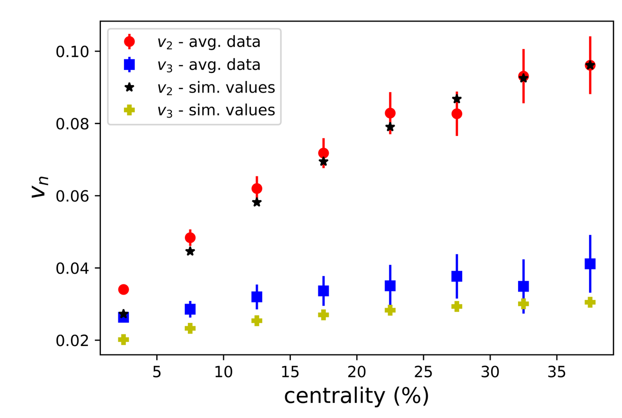

Figure 6: v_n as a function of centrality for data.

In order to compute the performance of Eq. 4, we ran simple Monte Carlo simulations, where particles were distributed according to $f_{sim}(\theta, \phi)$. Their multiplicities were the same as data for each centrality. The values of v_2 and v_3 for each centrality were inspired by [1]. Firstly, we employed an event-by-event approach to the calculation, taking the averaged v_n at the end. Secondly, we applied Eq. 4 directly to the averaged power spectra of each centrality.

One can see from Fig. 5 that while v_2 resulted in values quite close to the simulation input, v_3 had the tendency to become higher than the original. This is a result of low multiplicity fluctuations enhancing the dipole mode. An increase on the number of events taken could relieve this effect.

The same v_n calculations were performed on data, as seen in Fig. 6. It should be observed that v_n follow a similar behavior as the values inspired on measurements from [1]: the coefficients increase with centrality.

Last Remarks

- All data was taken from CERN's open data portal http://opendata.cern.ch and downloaded using the repository in https://github.com/cbourjau/alice-rs.
- **Future:**
- Analysis of apparent dipole enhancement of data in comparison with isotropic case;
- Explore further flow extraction: correct v_3, get v_4;
- Explore anisotropy in ϕ changing with θ.

References

[1] J. Adam *et al*. *Phys. Rev. Lett.*, 116:132302, Apr 2016.

[2] K. M. Górski *et al*. *The Astrophysical Journal*, 622(2):759, 2005.

Acknowledgements

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