Equilibration in finite gluon systems

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Introduction

The aim is to analytically solve a nonlinear kinetic equation for the time evolution of the occupation-number distribution in a finite system of **bosons** and compare it to the one for **fermions**.

The model preserves the essential features of Bose–Einstein/Fermi–Dirac statistics which are contained in the bosonic/fermionic Boltzmann equation. It is applied to the local equilibration of gluons and quarks in the early stages of relativistic heavy-ion collisions. The local equilibration time for **gluons** is calculated and compared to the one for **quarks**.

Boson equilibration model

The basic nonlinear partial differential equation for the occupation-number distribution $n \equiv n(\epsilon, t)$ is derived in [1] from the Boltzmann collision term in a finite gluon system:

$$\frac{\partial n}{\partial t} = -\frac{\partial}{\partial \epsilon}\left[v\, n\,(1+n)\right] + \frac{\partial^2}{\partial \epsilon^2}\left[D\, n\right]. \qquad (1)$$

Dissipative effects are expressed through the drift term $v(\epsilon, t)$, diffusive effects through the diffusion term $D(\epsilon, t)$. In the limit of constant transport coefficients, this kinetic equation can be solved exactly. The thermal equilibrium distribution is a stationary solution

$$n_{\mathrm{eq}}(\epsilon) = \frac{1}{e^{(\epsilon-\mu)/T} - 1}$$

with the chemical potential $\mu < 0$ in a finite bosonic system. In the model with constant transport coefficients, the equilibrium temperature is given as $T = -D/v$, with $v < 0$ since the drift is towards smaller energies.

Exact solution

The analytical solution of Eq. (1) is derived through the nonlinear transformation [1]

$$n(\epsilon, t) = -\frac{D}{vP(\epsilon, t)}\,\frac{\partial P(\epsilon, t)}{\partial \epsilon}.$$

It reduces the boson Eq. (1) to a linear diffusion equation for $P(\epsilon, t)$ that is solvable and can be retransformed to $n(\epsilon, t)$ as [1]

$$n(\epsilon, t) = \frac{1}{2v}\,\frac{\int_{-\infty}^{+\infty} \frac{\epsilon - x}{t} F(\epsilon - x, t) G(x)\, dx}{\int_{-\infty}^{+\infty} F(\epsilon - x, t) G(x)\, dx} - \frac{1}{2}$$

with a gaussian function $F(\epsilon - x, t)$ and a function $G(x)$ that contains the initial conditions. For schematic initial conditions ($Q_s \approx 1$ GeV)

$$n_{\mathrm{i}}(\epsilon) = N_{\mathrm{i}}\, \theta(1 - \epsilon/Q_s)\, \theta(\epsilon) \qquad (2)$$

that are appropriate [2, 3] for the gluon system in a relativistic heavy-ion collision such as Au-Au or Pb-Pb at RHIC or LHC energies, exact solutions can be obtained.

Gluon equilibration

The solutions of the bosonic local equilibration problem can be evaluated [1] in closed form for the simplified initial condition (2). Results and $n_{\mathrm{eq}}(\epsilon)$ (dotted) are shown at four timesteps until the thermal tail is reached:

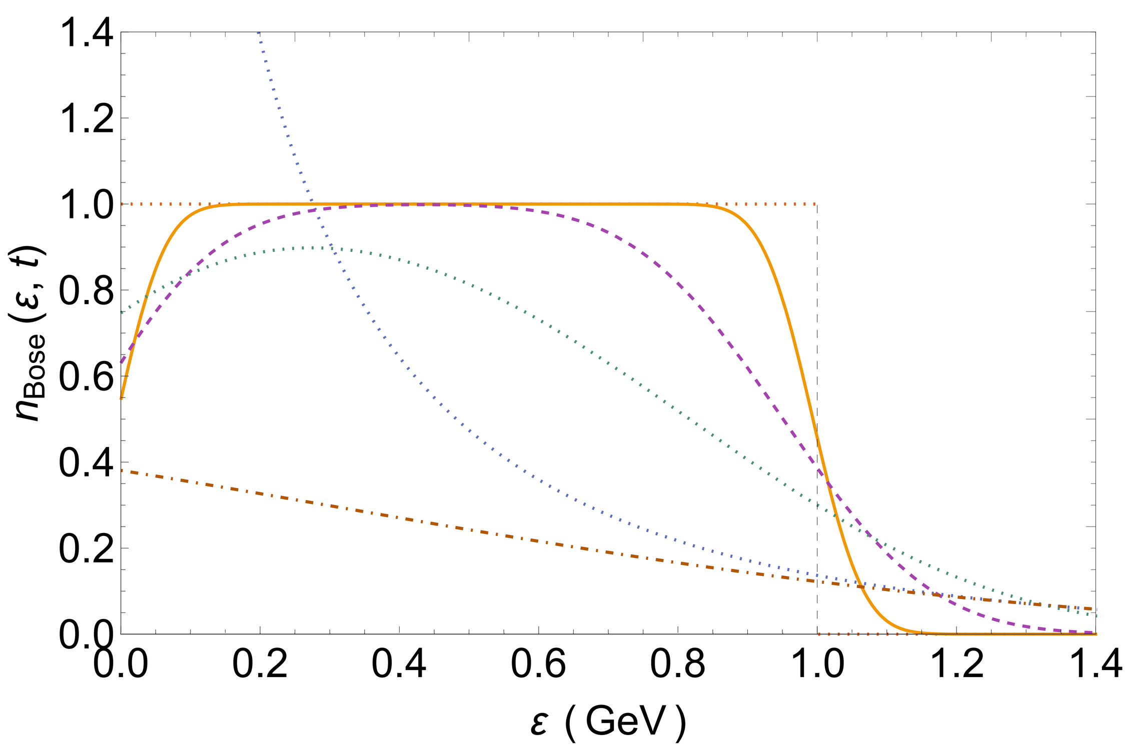

Figure 1: Local equilibration of massless gluons.

Local equilibration time

An explicit expression for the equilibration time follows from an asymptotic expansion of the error functions occurring in the solution as

$$\tau_{\mathrm{eq}}^{\mathrm{Bose}} = 4D/[(1 + 2N_{\mathrm{i}})^2\, v^2].$$

The Bose equilibration time for $N_{\mathrm{i}} = 1$ and transport coefficients D, v is almost an order of magnitude shorter than the corresponding equilibration time in a fermion system, which was found to be $\tau_{\mathrm{eq}}^{\mathrm{Fermi}} = 4D/v^2$ in [4],

$$\tau_{\mathrm{eq}}^{\mathrm{Fermi}} / \tau_{\mathrm{eq}}^{\mathrm{Bose}} = 9.$$

This is due to the different statistical properties of bosons and fermions, with the latter obeying Pauli's principle that slows down the thermalization.

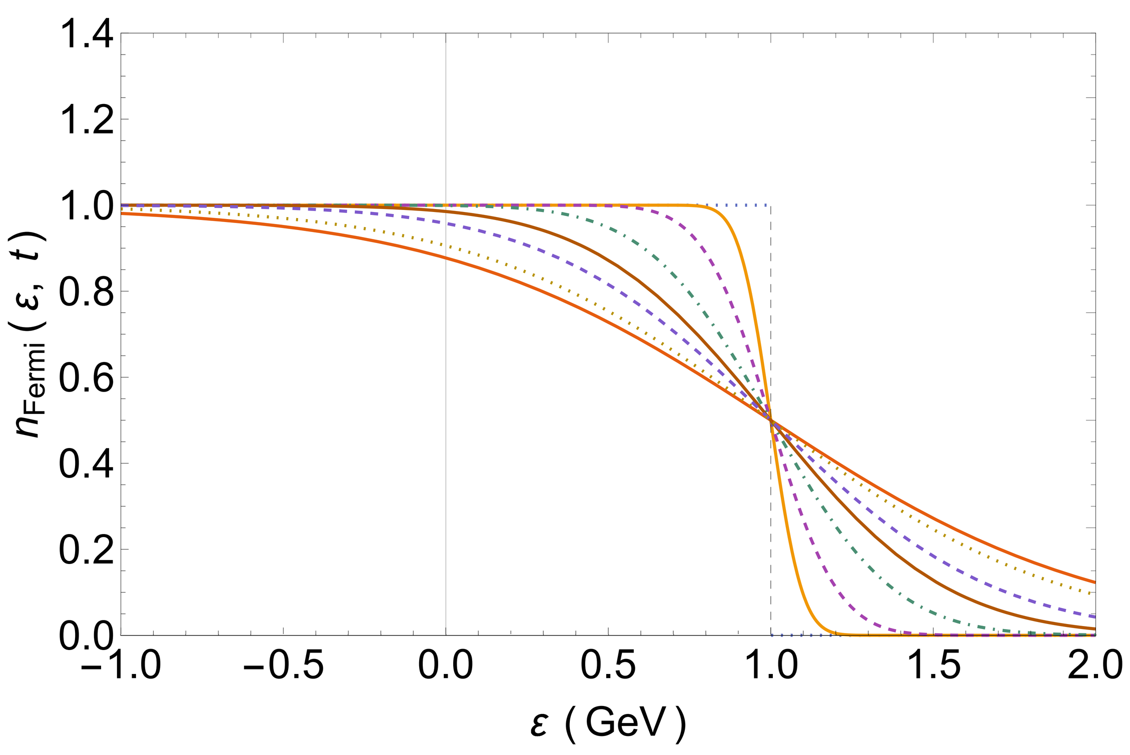

Figure 2: Local equilibration of valence quarks.

Parameters for 5.02 TeV Pb-Pb:

$T = 500$ MeV, local temperature at $t = \tau_{\mathrm{eq}}$

$\tau_{\mathrm{eq}}^{\mathrm{Fermi}} = 1$ fm/c, local equilibration time

$\tau_{\mathrm{eq}}^{\mathrm{Bose}} = 0.11$ fm/c, τ_{eq} for gluons

$D = 3 \times 10^{23}\ \mathrm{GeV}^2/\mathrm{s}$, diffusion coefficient

$v = -6 \times 10^{23}$ GeV/s, drift coefficient

The short local equilibration time for gluons is a decisive prerequisite for the application of hydrodynamics.

Antiparticle creation

Different from the nonlinear diffusion equation for gluons, Eq. (1), the corresponding equation [4] for the occupation of fermionic states $n_{\mathrm{F}}(\epsilon, t)$ conserves the total particle number $N_{\mathrm{th}}^{\mathrm{F}}(t)$ (here for constant density of states $g(\epsilon)$) when antiparticle production from the filled Dirac sea is taken into account,

$$N_{\mathrm{th}}^{\mathrm{F}}(t) = \int_0^{\infty} n_{\mathrm{F}}(\epsilon, t)\, g(\epsilon)\, d\epsilon - \int_{-\infty}^{0} [1 - n_{\mathrm{F}}(\epsilon, t)]\, g(\epsilon)\, d\epsilon = \mathrm{const}.$$

Particle number conservation for fermions (quarks) including antiquark creation differs fundamentally from the bosonic case, where gluons can in principle disappear from the (nonequilibrium or equilibrium) statistical part of the distribution and move into the condensed state (BEC).

Conclusion

Due to the nonlinearity of the basic bosonic diffusion equation (1), the sharp edges of the initial gluon distribution at $Q_s = 1$ GeV are continously smeared out and local equilibrium of the gluon distribution with a thermal tail in the ultraviolett region is rapidly attained [1] within $\tau_{\mathrm{eq}}^{\mathrm{Bose}} \approx 0.1$ fm/c, dot-dashed curve in Fig. 1.

In the infrared, gluons can move into the condensed BEC state at $p = 0$ and hence, the integral of the distribution function for conserved total gluon content (with the appropriate density of states) is smaller than that of the initial distribution. Although this is accounted for implicitly by the solution of the nonlinear diffusion equation (1), due to inelastic processes and the nonconservation of particle number in relativistic collisions BEC formation appears unlikely to actually occur in heavy-ion collisions.

The thermal equilibration time for gluons turns out to be nine times shorter than the equilibration time for fermions (quarks) as a consequence of the statistical properties of bosons versus fermions. Because of the larger color factor, gluons will equilibrate locally even faster. This result can be viewed as one of the main reasons for the very short local equilibration time in relativistic heavy-ion collisions that are governed by gluons in their initial stage.

References

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