Abstract

The expression for the statistical average value of the axial current is derived on the basis of the covariant Wigner function. In the resulting formula, the rotation speed and acceleration come in combination with the chemical potential. The limit of zero mass and zero temperature is investigated. For zero mass limiting case, the axial current is described by a smooth function at a temperature higher than the Unruh temperature. It is shown that at zero temperature, the axial current, as a function of the rotation speed and chemical potential, turns out to be zero in a two-dimensional plane region.

Background and methods

- According to Chiral Vortical Effect (CVE) axial current flows along vorticity  \( j^a_\mu = \left( \frac{T^2}{6} + \frac{\mu^2}{2m^2} \right) \omega^a_\mu \)
- CVE can be obtained by many different ways [1-5].
- Thermal vorticity tensor [4,6] contains information about local vorticity and temperature gradients in the media with local thermodynamic equilibrium.

\( \omega^a_\mu = -\frac{1}{2} (\partial_\mu \beta^a - \partial_\nu \beta^a_\nu) \)

The method to describe kinetic properties of the media, which allows to take into account quantum effects, is based on the covariant Wigner function. For the spin 1/2 particles this is a spinorial matrix, expressed by mean value of the operators of Dirac fields [6]. In [6] an ansatz of local equilibrium distribution functions for massive particles with spin in the media in a state of nonuniform motion was introduced

\[ X(x, p) = \left( \exp[\beta_\mu p^\mu - \zeta] \exp[-\frac{1}{2} \omega_{\mu\nu} \Sigma^\mu \nu + iT] \right)^{-1} \]

Axial Current, massive fermions

After summation of the entire series in thermal vorticity from Wigner function the next general formula in the limit \( \frac{\mu}{m} \gg 1 \) and \( \frac{\mu}{T} \gg 1 \) follows

\[ \langle j^a \rangle = \frac{1}{2 \pi^3} \int d^3 p \left\{ n_F(E_p - \mu - \frac{\omega}{2} + \frac{a}{2}) - n_F(E_p - \mu + \frac{\omega}{2} + \frac{a}{2}) + n_F(E_p + \mu - \frac{\omega}{2} + \frac{a}{2}) - n_F(E_p + \mu + \frac{\omega}{2} + \frac{a}{2}) \right\} \delta \left( \frac{\omega}{2} - \frac{a}{2} \right) \] for \( \omega \parallel a \) that is, the acceleration occurs along the rotation axis \( \Omega \) is a modulus of rotational speed and \( \eta = \frac{\omega}{2} \) is the unit vector in the direction of the rotation speed and \( \Omega \) is the Fermi-Dirac distribution.

- The axial current is expressed in terms of the integral of the Fermi-Dirac distribution.
- It is interesting to note that the speed of rotation and acceleration are manifested in combination with the chemical potential, as additional real and imaginary chemical potentials of special kind.

Zero temperature limit

The axial current and CVE are significant in calculating the polarization of baryons [9]. However, in different approaches carriers of axial charge differ, so these are quarks in [9] and baryons in [8], that is, particles with different masses. In this connection, it is useful to consider the effects associated with a finite mass in the axial current. From the general formula in the limit \( T = 0 \) and \( a_\mu = 0 \) follows

\[ \langle j^a \rangle = \frac{1}{2 \pi^3} \int d^3 p \left\{ n_F(p - \mu - m) - n_F(p - \mu + m) + n_F(p + \mu - m) - n_F(p + \mu + m) \right\} \delta \left( \frac{\omega}{2} - \frac{a}{2} \right) \]

- The formula for the axial current is greatly simplified in the limit of zero temperature and zero acceleration. As can be seen, it is expressed in terms of the sum of Heaviside functions.
- It can be shown that in region \( \Omega < 2(m - |\mu|) \) the axial current is zero.
- Under condition \( \Omega \gg m \) or \( \mu \gg m \), it asymptotically tends to its limit at zero mass, just as it should be.

\[ j^a = \begin{cases} j^a_0 = 0 & \text{if } \Omega \ll 2(m - |\mu|) \\ j^a_0 = \frac{\mu}{m} \left( 1 - \frac{\Omega}{2m} \right) & \text{if } \Omega \gg m \end{cases} \]

Massless limit

In the limiting case of massless fermions \( m = 0 \), momentum integrals in general formula can be defined analytically

\[ \langle j^a \rangle = \frac{1}{2 \pi^3} \int d^3 p \left\{ n_F(p - \mu) - n_F(p + \mu) \right\} e^{ia \cdot \omega} \]

\[ \omega^a \]

- Leads to CVE in the linear approximation (as was shown previously in [4]).
- Existence of border temperature \( T_U = \frac{\hbar}{2 \pi} \alpha \) for \( \Omega \parallel \alpha \) or \( \omega^a \parallel \alpha \) it equals Unruh temperature, \( T_U = \frac{\hbar}{2 \pi} \alpha \) while in general case \( T_U \rightarrow T_U(\Omega, \alpha, \theta) \).
- Discontinuities below Unruh temperature \( T < T_U \), due to the contributions with integer part \( \frac{\hbar}{2 \pi} \left[ \frac{\alpha}{\Omega} + \frac{1}{2} \right] \). Corresponds to the statement in [7], that Unruh temperature is an absolute lower bound temperature for accelerated medium.
- The term \( \left( -\frac{\omega^2}{4 \pi^2} \right) \omega^a \) coincides with [1].

References