

# Contributions of multi-parton distributions to multi-particle azimuthal correlations

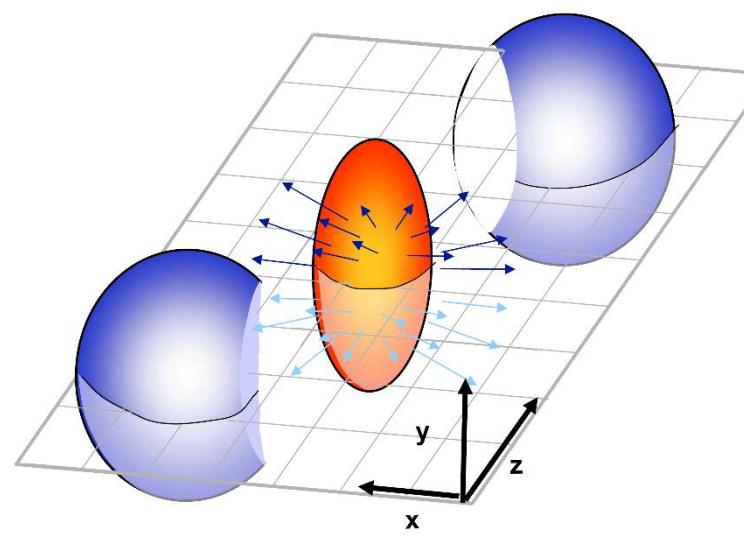
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## Introduction

What is the Elliptic flow?

- anisotropic flow at heavy ion collisions  
anisotropy of particle production
- Measuring elliptic flow  $v_2 \rightarrow$  two particle azimuthal correlations

CMS Collaboration, Phys. Lett. B 724 (2013) 213



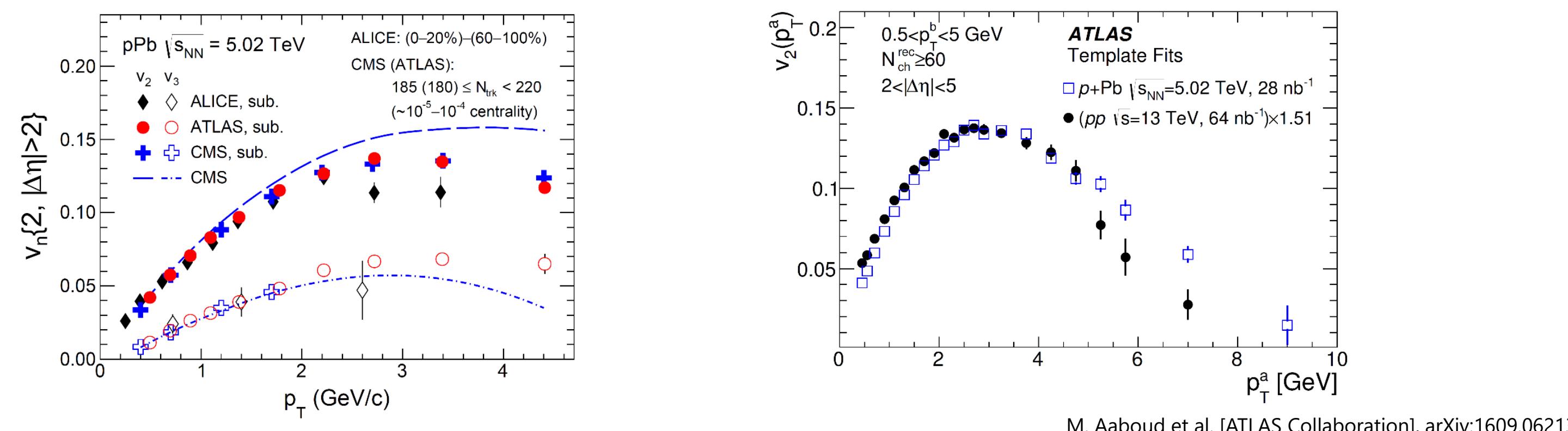
$$\frac{1}{N_{\text{trig}}} \frac{dN_{\text{pair}}}{d\Delta\phi} = \frac{N_{\text{assoc}}}{2\pi} \left[ 1 + \sum_n 2V_{n\Delta} \cos(n\Delta\phi) \right]$$

$$v_2(p_t) = \frac{V_{2\Delta}(p_t, p_t^{\text{ref}})}{\sqrt{V_{2\Delta}(p_t^{\text{ref}}, p_t^{\text{ref}})}}$$

- Theoretical explanations → Relativistic fluid dynamics

### Elliptic flow in small systems

- In  $pA$  and  $pp$  collision, un-negligible  $v_2$  is measured. ← almost central collisions



- Small system: nucleon-nucleus and nucleon-nucleon scatterings  
→ validation of the fluid dynamics is opaque  
some explanations from fluid dynamics  
and pQCD based theories (like Color Glass Condensate frameworks)

E. Avsar, C. Flensburg, Y. Hatta, J.-Y. Ollitrault, and T. Ueda, Phys. Lett. B702, 394 (2011),  
T. Lappi, B. Schenke, S. Schlichting and R. Venugopalan, JHEP 01 (2016) 061,  
A. Kovner and M. Lublinsky, Phys. Rev. D 83, 034017 (2011), E. Levin and A. H. Rezaeian, Phys. Rev. D 84, 034031 (2011)

## Multi-parton distributions

### Multi-parton scattering (MPS)

- The differential cross section of multi-particle production

$$\frac{d^m N}{d^2 \mathbf{p}_{\perp 1} \cdots d^2 \mathbf{p}_{\perp m}} \Big|_{\text{MPS}} \propto \int \prod_{i=1}^m d^2 \mathbf{b}_{\perp i} d^2 \mathbf{x}_{\perp i} e^{i \mathbf{p}_{\perp i} \cdot \mathbf{x}_{\perp i}} F_p(\mathbf{b}_{\perp 1}, \dots, \mathbf{b}_{\perp m}) F_A(\mathbf{b}_{\perp 1}, \dots, \mathbf{b}_{\perp m}, \mathbf{x}_{\perp 1}, \dots, \mathbf{x}_{\perp m})$$

M. Diehl, D. Ostermeier and A. Schafer, JHEP 1203, 089 (2012)

- Large nucleus approximation

$$F_A(\mathbf{b}_{\perp 1}, \dots, \mathbf{b}_{\perp m}, \mathbf{x}_{\perp 1}, \dots, \mathbf{x}_{\perp m}) \approx \prod_{i=1}^m \exp(-\Lambda_{\text{nuc}}^2 \mathbf{b}_{\perp i}^2) S_Y(\mathbf{x}_{\perp i}, \mathbf{b}_{\perp i})$$

Correlations of the two particles?

→ **multi-parton distribution  $F_p$**  generate correlations!!

- Multi-parton distribution

→ Approximate by a convolution of Single parton distribution  $f_p$

$$F_p(\mathbf{b}_{\perp 1}, \dots, \mathbf{b}_{\perp m}) \approx \int d\mathbf{b}_{\perp} f_p(\mathbf{b}_{\perp}) \prod_{i=2}^m f_p(\mathbf{b}_{\perp} - \mathbf{b}_{\perp 1} + \mathbf{b}_{\perp i})$$

M. Diehl, D. Ostermeier and A. Schafer, JHEP 1203, 089 (2012)

- Examples ( $m = 2, 4$ )

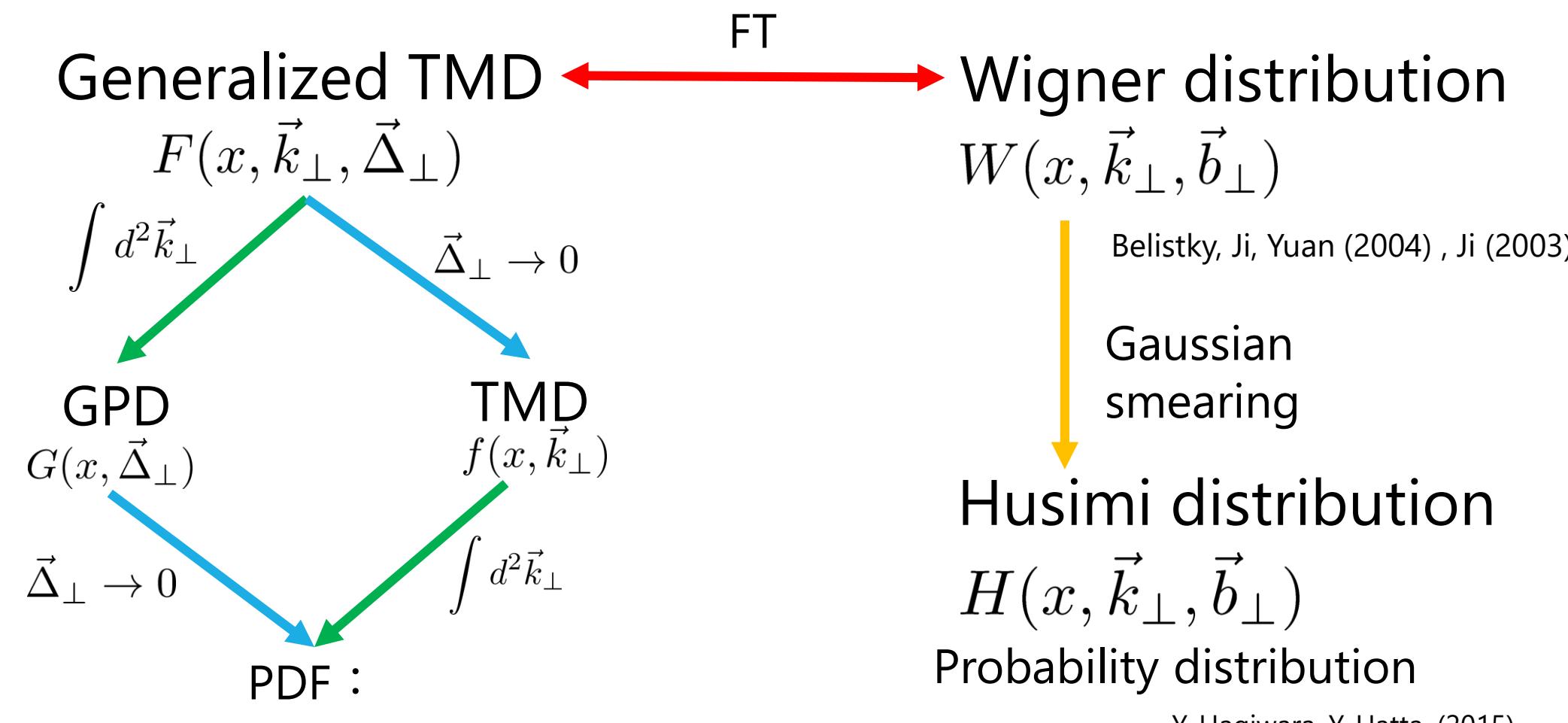
$$f_p(\mathbf{b}_{\perp}) = \exp\left(-\frac{\mathbf{b}_{\perp}^2}{B}\right)$$

$$F_p(\mathbf{b}_{\perp 1}, \mathbf{b}_{\perp 2}) \propto \exp\left(-\frac{(\mathbf{b}_{\perp 1} - \mathbf{b}_{\perp 2})^2}{2B}\right)$$

$$F_p(\mathbf{b}_{\perp 1}, \mathbf{b}_{\perp 2}, \mathbf{b}_{\perp 3}, \mathbf{b}_{\perp 4}) \propto \exp\left(-\frac{1}{4B} \sum_{i < j} (\mathbf{b}_{\perp i} - \mathbf{b}_{\perp j})^2\right)$$

## Wigner distribution

### Relations of Parton distributions



### The gluon Wigner distribution

- Wigner distribution: phase space distribution of partons

The gluon Wigner distribution at small  $x$

$$xW_g(x, \vec{k}_{\perp}, \vec{b}_{\perp}) = \frac{2N_c}{\alpha_S} \int \frac{d^2 \vec{r}_{\perp}}{(2\pi)^2} e^{i \vec{k}_{\perp} \cdot \vec{r}_{\perp}} \left( \frac{1}{4} \nabla_{\vec{b}_{\perp}}^2 - \nabla_{\vec{r}_{\perp}}^2 \right) S_Y(\vec{r}_{\perp}, \vec{b}_{\perp})$$

Y. Hatta, B. W. Xiao, F. Yuan Phys. Rev. Lett. 116, 202301 (2016)

$S_Y(\vec{r}_{\perp}, \vec{b}_{\perp})$ : S-matrix of the dipole-nucleon scattering  
 $Y = \ln(1/x)$ : rapidity

$$S_Y(\mathbf{x}_{\perp i}, \mathbf{b}_{\perp i}) = S_0(x_{\perp i}, b_{\perp i}) + 2 \cos 2(\phi_x - \phi_b) \mathbf{S}_2(\mathbf{x}, \mathbf{b}) + \dots$$

Azimuthal angle correlation between  $\vec{b}_{\perp}$  and  $\vec{k}_{\perp}$

### Pictures of the gluon Wigner distribution

Y. Hagiwara, Y. Hatta and T. Ueda, Phys. Rev. D 94, no. 9, 094036 (2016)

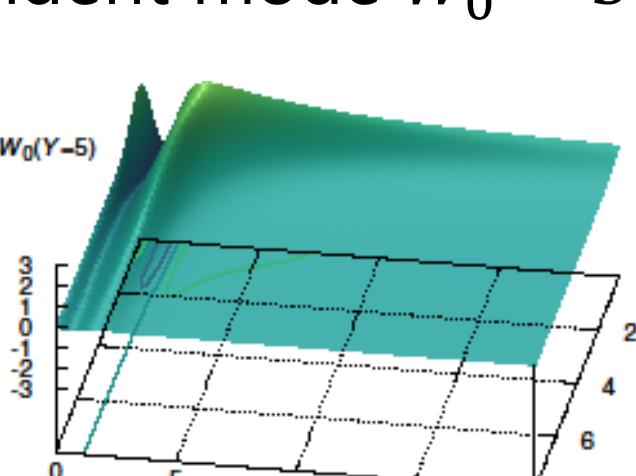
- Balitsky-Kovchegov equation : rapidity evolution of the S-matrix

$$\partial_Y S_Y(\vec{x}_{\perp}, \vec{y}_{\perp}) = \frac{\alpha_S}{2\pi} N_c \int d^2 \vec{z}_{\perp} \frac{(\vec{x}_{\perp} - \vec{y}_{\perp})^2}{(\vec{x}_{\perp} - \vec{z}_{\perp})^2 (\vec{z}_{\perp} - \vec{y}_{\perp})^2} \{ S_Y(\vec{x}_{\perp}, \vec{z}_{\perp}) S_Y(\vec{z}_{\perp}, \vec{y}_{\perp}) - S_Y(\vec{x}_{\perp}, \vec{y}_{\perp}) \}$$

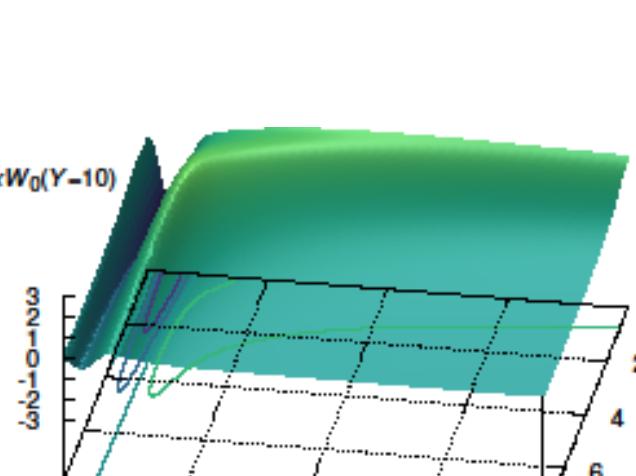
Angular independent mode  $W_0 \sim S^0$

$Y = \ln(1/x)$ : rapidity

$Y=5$

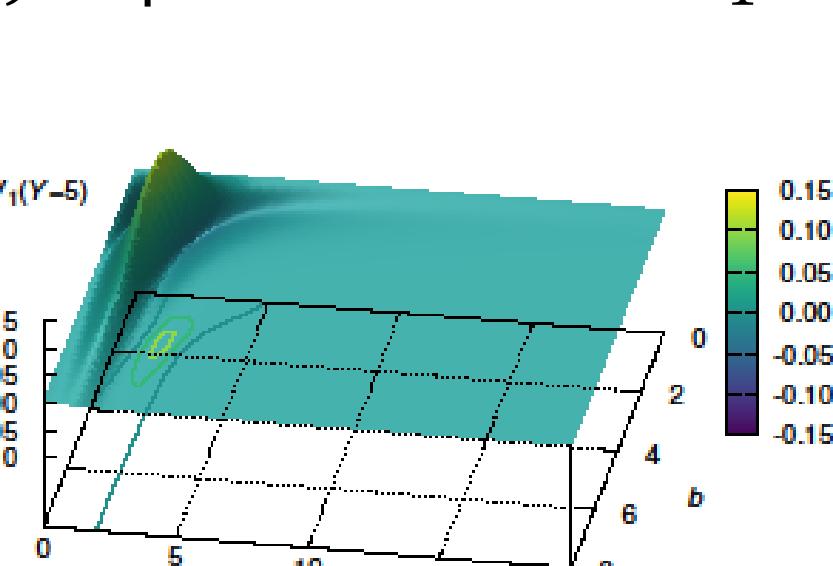


$Y=10$

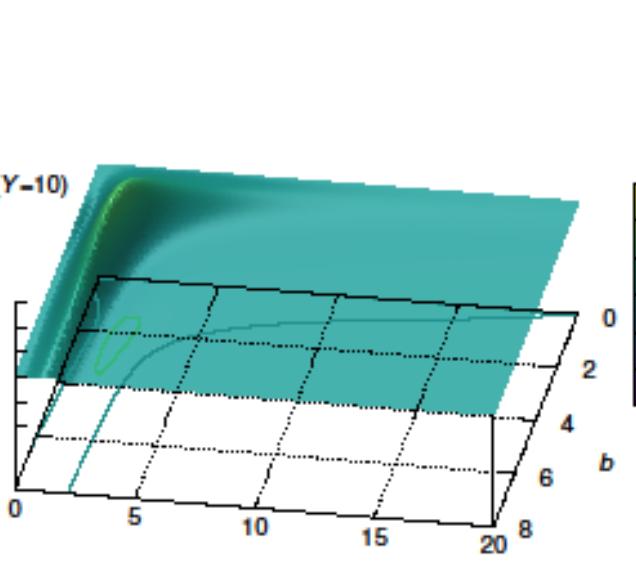


$\cos 2(\phi_k - \phi_b)$  dependent mode  $W_1 \sim \tilde{S}$

$Y=5$



$Y=10$



## Elliptic flow

### MPS and the elliptic flow

- $n$ th-moment of the  $m$ -particle correlation

K. Dusling, M. Mace and R. Venugopalan, Phys. Rev. Lett. 120, no. 4, 042002 (2018)  
K. Fukushima and Y. Hidaka, JHEP 1711, 114 (2017)

$$\kappa_n\{m\} := \prod_{i=1}^m \int \frac{d^2 \mathbf{p}_{\perp i}}{(2\pi)^2} e^{in(-1)^{i+1} \phi_i} \frac{d^m N}{d^2 \mathbf{p}_{\perp 1} \cdots d^2 \mathbf{p}_{\perp m}}$$

$$\kappa_n\{2m\} = D_n(2\pi)^{2m} \int \prod_{i=1}^{2m} d^2 \mathbf{b}_{\perp i} F_p(\mathbf{b}_{\perp 1}, \dots, \mathbf{b}_{\perp 2m})$$

$$\times \cos(n \sum_{i=0}^{2m} (-1)^i \phi_i) \prod_{i=1}^{2m} \exp(-\Lambda_{\text{nuc}}^2 \mathbf{b}_{\perp i}^2) \int_0^\infty dr_i e^{-r_i^2 p_{\text{up}}} A_n(p_{\text{up}} r_i) S_n(r_i, b_i)$$

$$A_0(x) = J_1(x) \quad , \quad S_0(r, b) = \int_0^{2\pi} d\phi S_Y(r, b, \cos(2\phi)) \quad ,$$

$$A_2(x) = \frac{2 - 2J_0(x) - xJ_1(x)}{x} \quad , \quad \mathbf{S}_2(\mathbf{r}, \mathbf{b}) = \int_0^{2\pi} d\phi S_Y(r, b, \cos(2\phi)) \cos(2\phi) \quad J_n : \text{Bessel function}$$

with  $S_Y(\mathbf{x}_{\perp i}, \mathbf{b}_{\perp i}) = S_0(x_{\perp i}, b_{\perp i}) + 2 \cos 2(\phi_x - \phi_b) \mathbf{S}_2(\mathbf{x}, \mathbf{b}) + \dots$

← From Wigner distribution

### MPS and the elliptic flow

- Relation to  $v_2$

$$c_n\{2\} = \langle e^{in(\phi_1 - \phi_2)} \rangle = \frac{\kappa_2\{2\}}{\kappa_0\{2\}}$$

$$c_n\{4\} = \langle e^{in(\phi_1 + \phi_2 - \phi_3 - \phi_4)} \rangle - 2c_n\{2\}^2 = \frac{\kappa_2\{4\}}{\kappa_0\{4\}} - 2 \frac{\kappa_2\{2\}^2}{\kappa_0\{2\}^2}$$

$$v_n\{2\} = \sqrt{c_n\{2\}} \quad v_n\{4\} = (-c_n\{4\})^{\frac{1}{4}}$$

### Numerical calculation of $v_2$

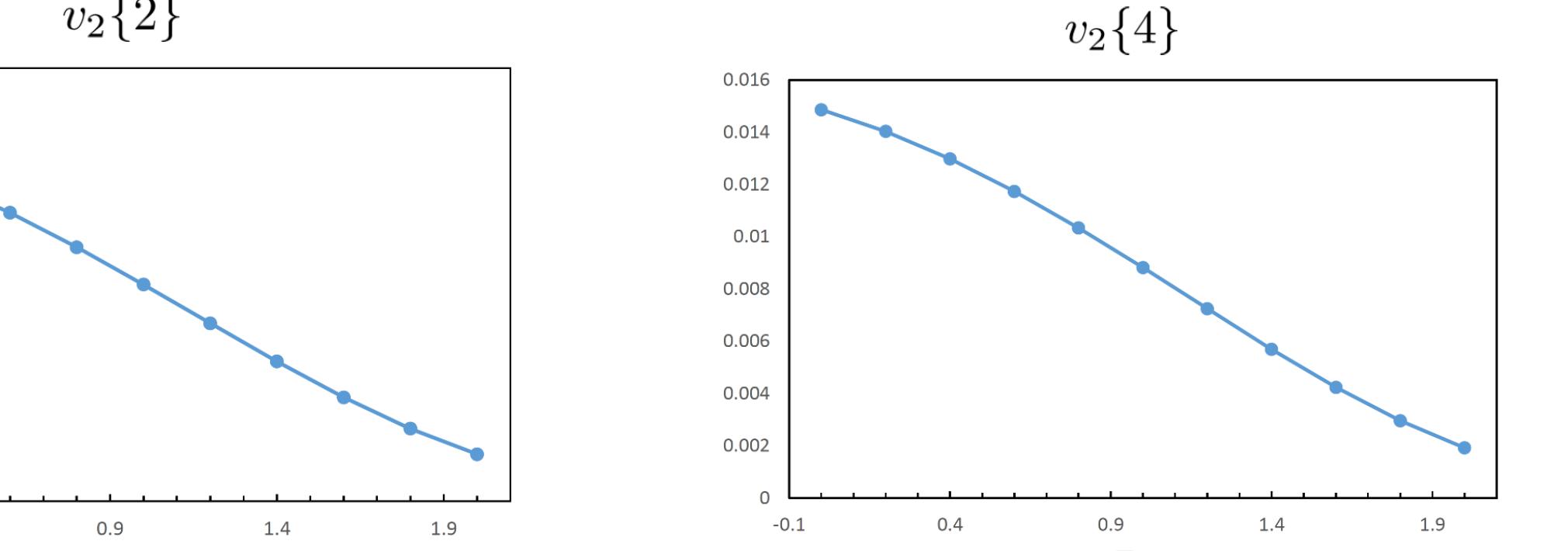
- Results

Monte-Carlo integration method

$Y = \ln(1/x)$ : rapidity

$\tau := \alpha_s Y$

$v_2\{4\}$



Magnitudes of the  $v_2$  are 1/10 from the measured ones

## Conclusion

- Investigating the effect of multi-parton distributions and the Wigner distributions to the elliptic flow
- Qualitatively the  $v_2\{m\}$  are generated and their magnitudes are ten times smaller than the measured ones
- Finite color effects?