In-medium spectral properties of light hadrons in an arbitrary magnetic field

Snigdha Ghosh[a], Arghya Mukherjee[b], Mahatsab Mandal[b], Sourav Sarkar[a] and Pradip Roy[b]

[a]Variable Energy Cyclotron Centre, HBNI, 1/AF Bidhannagar, Kolkata 700 064, India ; [b]Saha Institute of Nuclear Physics, HBNI, 1/AF Bidhannagar, Kolkata - 700064, India

Abstract

We study the medium modification of pions and rho mesons under an arbitrary external magnetic field. The one-loop self energies of π and ρ are calculated using effective field theoretical techniques taking nucleon and pions as the loop particles respectively. The proton and charged pion propagators are modified due to the magnetic field using the full Schwinger proper time propagator. From the in-medium self energies, we obtained the effective mass and dispersion relations for π and ρ at the pole of the complete propagator calculated from the Dyson-Schwinger equation. We have also studied the spectral function as well as the detailed analytical structure of the self energy function of ρ. A non-trivial effect of magnetic field on these mesonic properties is observed.

Introduction

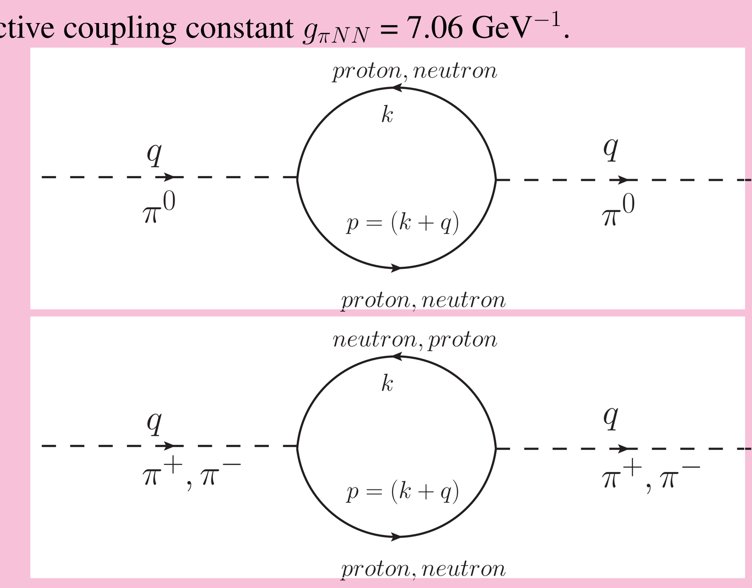
Quantum Chromodynamics in the presence of intense magnetic field reveals exotic phenomena like chiral magnetic effect, magnetic catalysis, inverse magnetic catalysis, vacuum superconductivity etc [1, 2]. Such a strong magnetic field is expected to be produced in non-central relativistic heavy ion collision(HIC) experiments at RHIC and LHC. So, the study of "strongly" interacting hot and/or dense matter under external magnetic field has become one of the most important topics of research since a decade. In particular, the study of the properties of mesons at finite temperature and/or density in an external magnetic field is important in order to extract information about chiral phase transition parameters. Moreover, the study of the in-medium spectral properties of ρ meson is essential for the HIC experiment since the dilepton production rate is directly proportional to in-medium ρ spectral function. Also, the study of the ρ meson properties in terms of its effective mass and dispersion relation is important in context of magnetic field induced vacuum superconductivity.

π and ρ self energy in the vacuum

The Lagrangian for effective πNN interaction is given by

$$\mathcal{L}_{int} = -g_{\pi NN}\bar{\Psi}\gamma^5\gamma^\mu\left(\vec{\tau}\cdot\partial_\mu\vec{\pi}\right)\Psi\ ,$$

with the effective coupling constant $g_{\pi NN}$ = 7.06 GeV^{-1}.

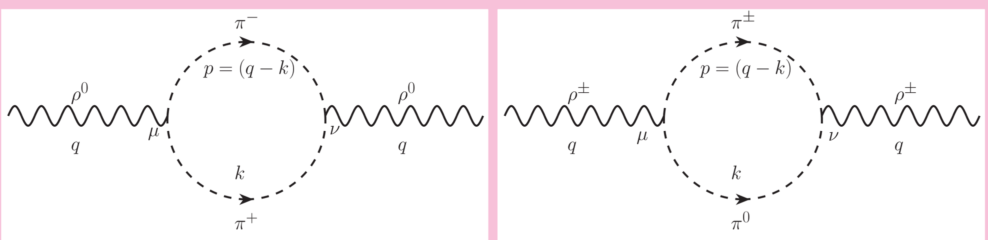


The vacuum self energy of π^0 and $\pi^\pm$ are denoted by Π_0^{vac} and $\Pi_\pm^{vac}$ and given by

$$\Pi_0^{vac}(q) = -ig_{\pi NN}^2\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\gamma^5\not{q}S_p(k)\gamma^5\not{q}S_p(k+q)\right] + p\to n$$

$$\Pi_+^{vac}(q) = \Pi_-^{vac}(-q) = -i2g_{\pi NN}^2\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\gamma^5\not{q}S_n(k)\gamma^5\not{q}S_p(k+q)\right]$$

where, $S_p(p) = -\left(\frac{\not{p}+m}{p^2-m_p^2+i\epsilon}\right)$ and $S_n(p) = -\left(\frac{\not{p}+m}{p^2-m_n^2+i\epsilon}\right)$ are the vacuum Feynman propagators for proton and neutron with masses m_p and m_n respectively.
The Lagrangian for effective $\rho\pi\pi$ interaction is given by $\mathcal{L}_{int} = -g_{\rho\pi\pi}\partial_\mu\vec{\rho}_\nu\cdot\partial^\mu\vec{\pi}\times\partial^\nu\vec{\pi}$ with the effective coupling constant $g_{\rho\pi\pi}$ = 20.72 GeV^{-2}.



The vacuum self energies of ρ^0 and $\rho^\pm$ can be written as

$$(\Pi_0^{\mu\nu}(q))_{vac} = i\int\frac{d^4k}{(2\pi)^4}\mathcal{N}^{\mu\nu}(q,k)\Delta_\pm(k)\Delta_\pm(p) \quad (1)$$

$$(\Pi_\pm^{\mu\nu}(q))_{vac} = i\int\frac{d^4k}{(2\pi)^4}\mathcal{N}^{\mu\nu}(q,k)\Delta_0(k)\Delta_\pm(p) \quad (2)$$

respectively. Here $\Delta_0(k) = \left(\frac{-1}{k^2-m_0^2+i\epsilon}\right)$ and $\Delta_\pm(k) = \left(\frac{-1}{k^2-m_\pm^2+i\epsilon}\right)$ are the vacuum Feynman propagators of π^0 and $\pi^\pm$ with masses m_0 and $m_\pm$ respectively and $\mathcal{N}^{\mu\nu}(q,k)$ is given by,

$$\mathcal{N}^{\mu\nu}(q,k) = g_{\rho\pi\pi}^2\left[q^4k^\mu k^\nu + (q.k)^2q^\mu q^\nu - q^2(q.k)(q^\mu k^\nu + k^\mu q^\nu)\right]\ ,$$

which contains the factors coming from the interaction vertices.

π and ρ self energy in the medium

In the real time formalism of thermal field theory, the thermal propagators as well as the self energies become 2×2 matrices. However, they can be diagonalized in terms of a single analytic function which is related to any one component of the corresponding 2×2 matrix, say the 11-component.
The 11-component of the proton and neutron thermal propagators are

$$S_{p,n}^{11}(p) = S_{p,n}(p) - \tilde{\eta}^p\left[S_{p,n}(p) - \gamma^0S_{p,n}^\dagger(p)\gamma^0\right]$$

whereas, the 11-component of the π^0 and $\pi^\pm$ thermal propagators are,

$$D_{0,\pm}^{11}(k) = \Delta_{0,\pm}(k) + \eta^k\left[\Delta_{0,\pm}(k) - \Delta_{0,\pm}^*(k)\right]$$

where, $\tilde{\eta}^p = \Theta(p^0)\eta_p^+ + \Theta(-p^0)\eta_p^-$ with $\eta_p^\pm = \left[e^{(p\cdot u\mp\mu)/T}+1\right]^{-1}$ and $\eta^k = \left[e^{k\cdot u/T}-1\right]^{-1}$ being the Fermi-Dirac and the Bose-Einstein distribution functions for nucleons and pions respectively. Here u is the medium four velocity which is $u^\mu \equiv (1,\vec{0})$ in the Local Rest Frame of the medium.

The complete in-medium π and ρ propagator matrices denoted respectively by, $\mathbf{D}$ and $\mathbf{D}^{\mu\nu}$ satisfy the following Dyson-Schwinger equations,

$$\mathbf{D} = \mathbf{\Delta} - \mathbf{\Delta}\mathbf{\Pi}\mathbf{D}$$

$$\mathbf{D}^{\mu\nu} = \mathbf{\Delta}^{\mu\nu} - \mathbf{\Delta}^{\mu\alpha}\mathbf{\Pi}_{\alpha\beta}\mathbf{D}^{\beta\nu}$$

where, $\mathbf{\Delta}$ and $\mathbf{\Delta}^{\mu\nu}$ are respectively the free thermal scalar and vector propagator matrices; $\mathbf{\Pi}$ and $\mathbf{\Pi}_{\alpha\beta}$ are the 1-loop thermal self energy matrices of π and ρ respectively. Each of the quantities in the above equations can be expressed in diagonal form in terms of analytic functions denoted by a bar, so that it can be written as $\bar{D} = \bar{\Delta} - \bar{\Delta}\bar{\Pi}\bar{D}$ and $\bar{D}^{\mu\nu} = \bar{\Delta}^{\mu\nu} - \bar{\Delta}^{\mu\alpha}\bar{\Pi}_{\alpha\beta}\bar{D}^{\beta\nu}$. The self energy functions $\bar{\Pi}$ and $\bar{\Pi}_{\alpha\beta}$ are related to the 11-components of $\mathbf{\Pi}$ and $\mathbf{\Pi}_{\alpha\beta}$ by the following relations,

$$\mathrm{Re}\,\bar{\Pi}(q) = \mathrm{Re}\,\Pi^{11}(q)\ \ ;\ \ \mathrm{Re}\,\bar{\Pi}_{\alpha\beta}(q) = \mathrm{Re}\,\Pi_{\alpha\beta}^{11}(q)$$

$$\mathrm{Im}\,\bar{\Pi}(q) = \epsilon(q^0)\tanh\left(q^0/2T\right)\mathrm{Im}\,\Pi^{11}(q)\ \ ;\ \ \mathrm{Im}\,\bar{\Pi}_{\alpha\beta}(q) = \epsilon(q^0)\tanh\left(q^0/2T\right)\mathrm{Im}\,\Pi_{\alpha\beta}^{11}(q)$$

where, $\epsilon(q^0)$ is the sign function.
In order to obtain the 11-component of the π and ρ self energies, one has to replace the vacuum nucleon and pion propagators by their corresponding 11-components:

$$\Pi_0^{11} = -ig_{\pi NN}^2\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\gamma^5\not{q}S_p^{11}(k)\gamma^5\not{q}S_p^{11}(k+q)\right] + p\to n$$

$$\Pi_+^{11}(q) = \Pi_-^{11}(-q) = -i2g_{\pi NN}^2\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\gamma^5\not{q}S_n^{11}(k)\gamma^5\not{q}S_p^{11}(k+q)\right]$$

$$(\Pi_0^{\mu\nu}(q))^{11} = i\int\frac{d^4k}{(2\pi)^4}\mathcal{N}^{\mu\nu}(q,k)D_\pm^{11}(k)D_\pm^{11}(p)$$

$$(\Pi_\pm^{\mu\nu}(q))^{11} = i\int\frac{d^4k}{(2\pi)^4}\mathcal{N}^{\mu\nu}(q,k)D_0^{11}(k)D_\pm^{11}(p)$$

π and ρ self energy in the medium under external magnetic field

In presence of external magnetic field (in addition to finite temperature), the neutron and π^0 propagators remain unaffected whereas the 11-component of proton and $\pi^\pm$ propagators become,

$$S_B^{11}(p) = S_B(p) - \tilde{\eta}^p\left[S_B(p) - \gamma^0S_B^\dagger(p)\gamma^0\right]$$

$$D_B^{11}(k) = \Delta_B(k) + \eta^k\left[\Delta_B(k) - \Delta_B^*(k)\right],$$

where, $S_B(p)$ and $\Delta_B(k)$ are the Schwinger proper time propagators for a charged fermion and scalar fields in momentum spece,

$$S_B(p) = i\int_0^\infty\frac{ds}{\cos(eBs)}\exp\left[is\left(p_\parallel^2 + p_\perp^2\frac{\tan(eBs)}{eBs} - m_p^2 + i\epsilon\right)\right]\times\left[\left(\cos(eBs)+\gamma^1\gamma^2\sin(eBs)\right)\left(\not{p}_\parallel + m_p\right) + \sec(eBs)\not{p}_\perp\right]$$

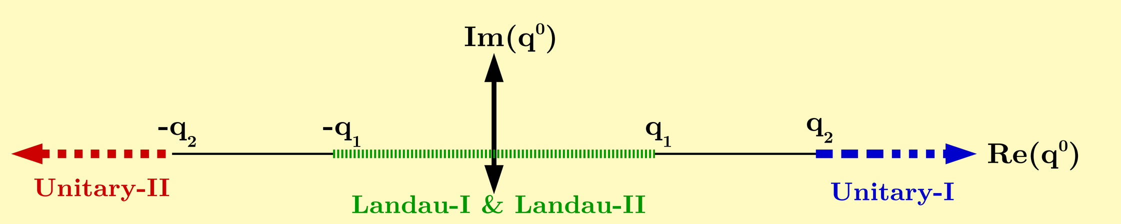
$$\Delta_B(k) = i\int_0^\infty\frac{ds}{\cos(eBs)}\exp\left[is\left(k_\parallel^2 + k_\perp^2\frac{\tan(eBs)}{eBs} - m_\pi^2 + i\epsilon\right)\right]$$

Some novelties regarding our calculation :

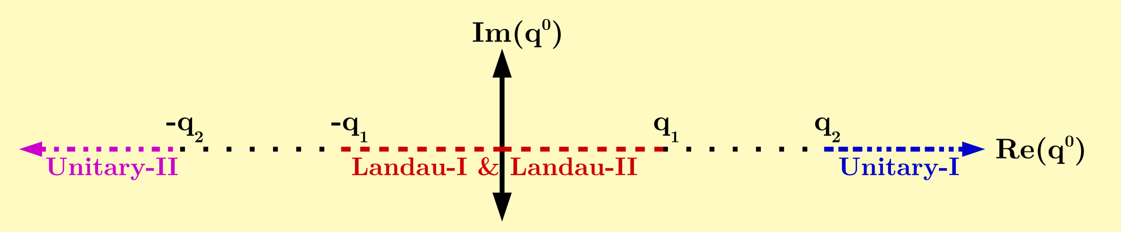
- We have not made any approximation regarding the strength of the magnetic field. Contributions from infinite number of nucleonic and pionic Landau levels are considered.
- The real part of the self energy contains the *magnetic field dependent vacuum contribution* which is usually ignored in the literature.

The detailed expression for the self energies in presence of magnetic field are lengthy and can be found in Ref. [3, 4].

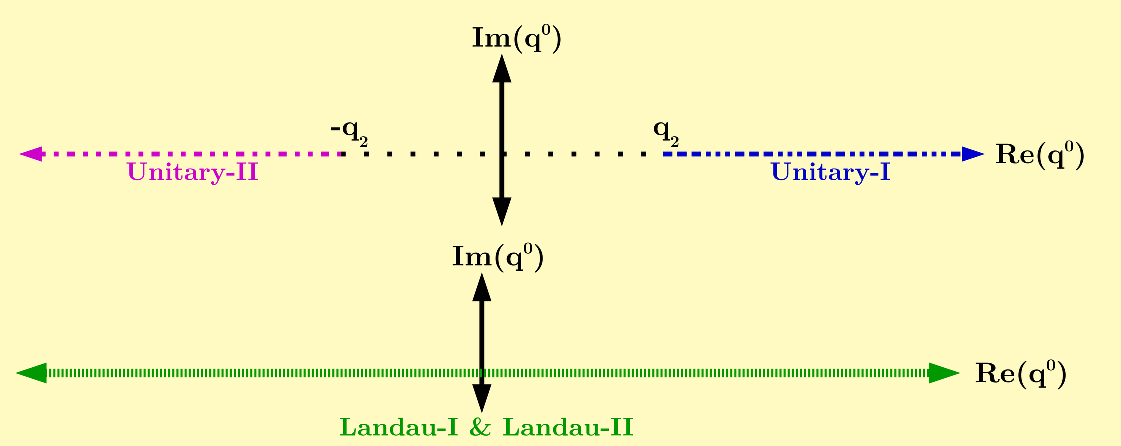
Analytic Structure of the Imaginary Parts of ρ self energy



Different cuts of the in-medium self energy function at *zero magnetic field* of the ρ in the complex q^0 plane for a given $\vec{q}$. The points correspond to $q_1 = |\vec{q}|$ and $q_2 = \sqrt{\vec{q}^2+4m^2}$.

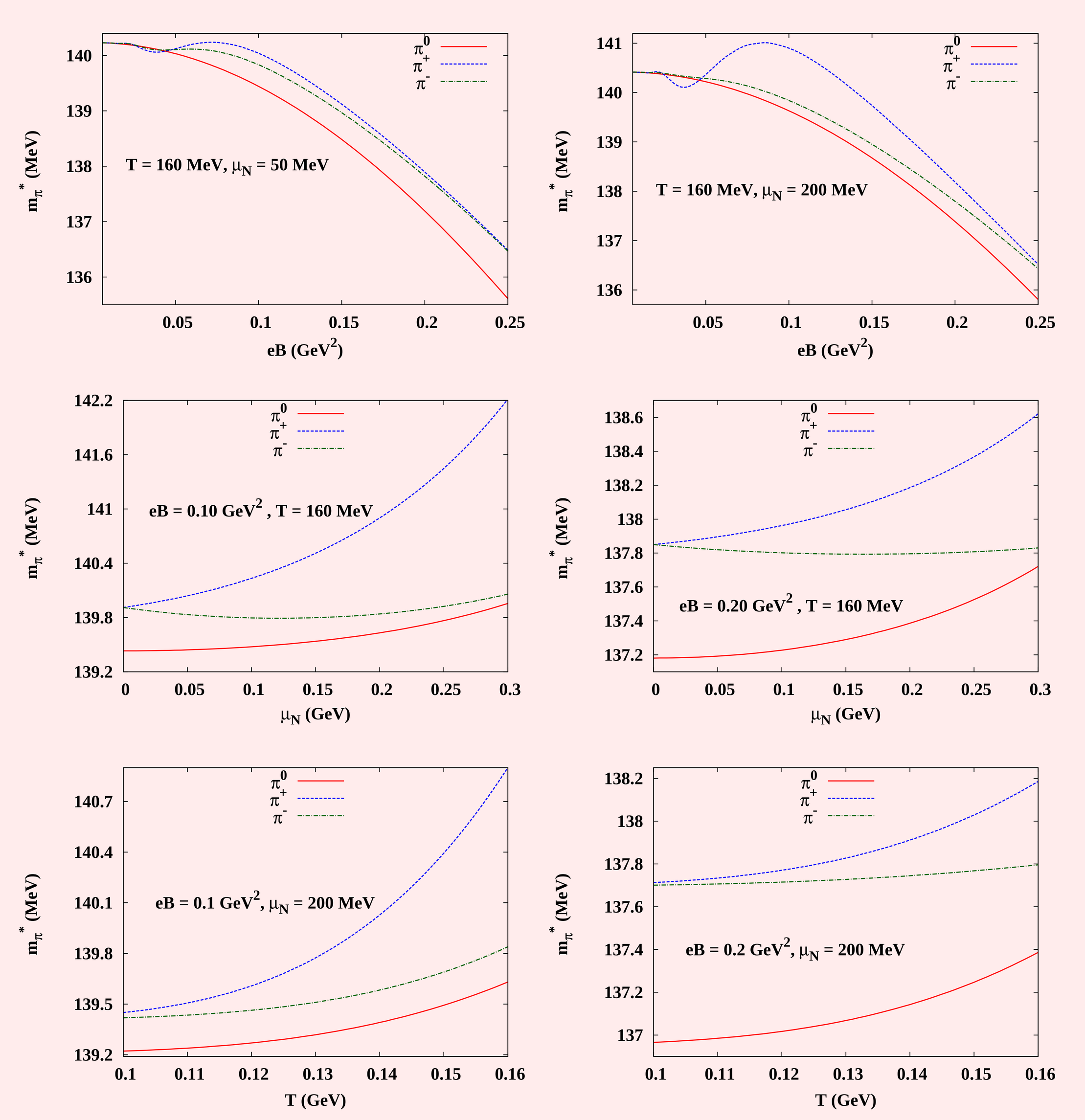


Different cuts of the in-medium self energy function under external magnetic field of the ρ^0 in the complex q^0 plane for a given $\vec{q}$. The points correspond to $q_1 = \sqrt{q_z^2 + (\sqrt{m^2+eB} - \sqrt{m^2+3eB})^2}$ and $q_2 = \sqrt{q_z^2+4(m^2+eB)}$.

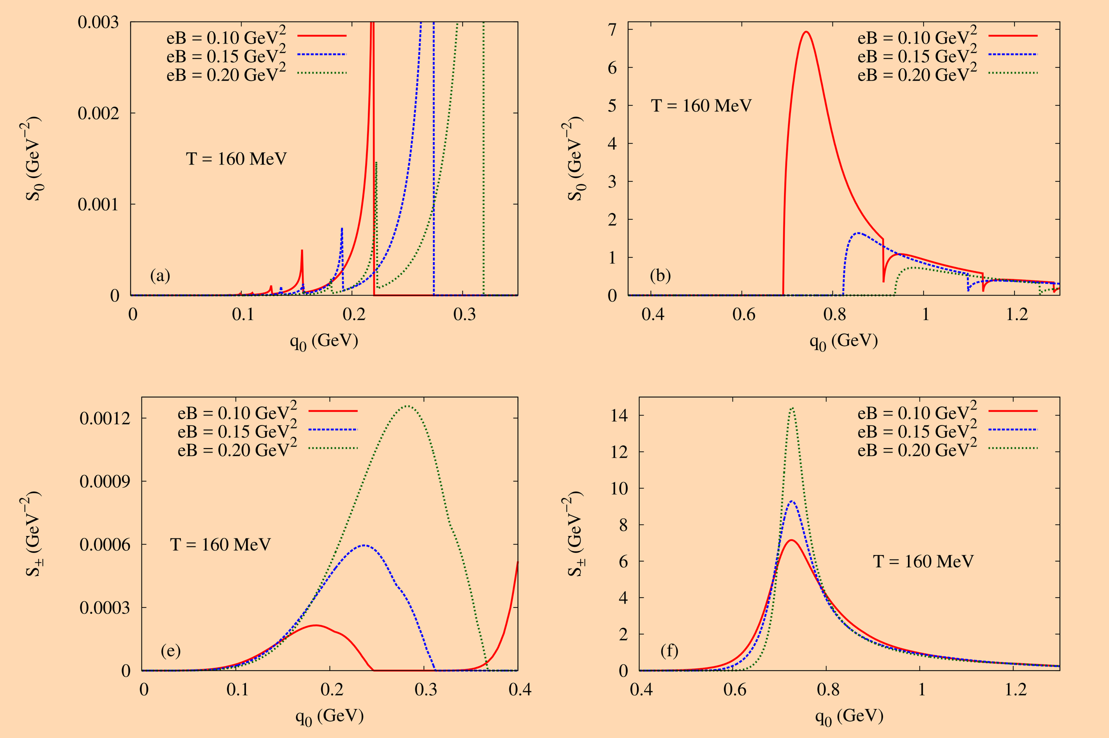


Different cuts of the in-medium self energy functions under external magnetic field of the $\rho^\pm$ in the complex q^0 plane for a given $\vec{q}$. Upper-Panel shows the Unitary cut regions with $q_2 = \sqrt{q_z^2 + (\sqrt{m^2+eB}+m)^2}$. Lower-Panel shows the Landau cut regions.

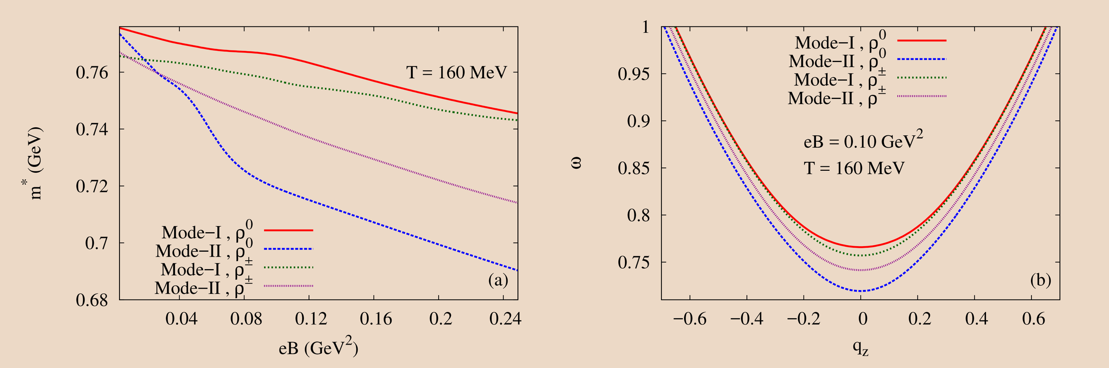
Effective mass of π



Spectral function of ρ



Effective mass and dispersion relation of ρ



Summary and Discussions

We have made comprehensive study of the self energies of π and ρ meson using effective πNN and $\rho\pi\pi$ interaction at finite temperature and arbitrary external magnetic field including all the Landau levels in the propagators of the loop particle in our calculations. We have also explicitly worked out the analytic structure of the ρ self energy at finite temperature and arbitrary non-zero external magnetic field. The kinematic domains of the imaginary part of the self energy in the complex q^0 plane are found to be different for ρ^0 and $\rho^\pm$ as well as from the $eB = 0$ case. For vanishing three momentum of the ρ, we have observed Landau cut contributions to the imaginary part of the self energy at non-zero magnetic field which is absent at zero magnetic field. While calculating the real part of the self energy, we have taken the magnetic field dependent vacuum contributions which is usually ignored in most of the works in the literature. We find that the effective masses of the charged pions possess oscillatory behaviour. However, the same oscillatory behaviour is not seen in case of the neutral pions. We have also shown that the oscillatory behaviour with finite chemical potential is not similar for π^+ and π^-. With increasing chemical potential, the oscillation in the effective mass of positive pion is found to be enhanced while that of π^- gets reduced. The in-medium spectral functions obtained from the spin-averaged self energy is found to be quite different for ρ^0 and $\rho^\pm$. The "Threshold Singularities" in the ρ^0 spectral function give rise to spike like structures which is absent in case of $\rho^\pm$. It is also shown quantitatively that, the ρ^0 meson "melts" at high magnetic field whereas $\rho^\pm$ does not.

Acknowledgement : Snigdha Ghosh acknowledges Center for Nuclear Theory, Variable Energy Cyclotron Centre and Department of Atomic Energy, Government of India for support.

References

[1] D. E. Kharzeev et al. Lect. Notes Phys. **871**, 1 (2013)
[2] D. Kharzeev and A. Zhitnitsky, Nucl. Phys. A **797**, 67 (2007)
[3] A. Mukherjee et al. Phys. Rev. D **96**, no. 1, 016024 (2017)
[4] S. Ghosh et al. Phys. Rev. D **96**, no. 11, 116020 (2017)

The 27th International Conference on Ultrarelativistic Nucleus-Nucleus Collisions, Venezia, Italy, 13-19 may 2018