The sign change of the four-particle cumulant in small systems from hydrodynamics and momentum conservation

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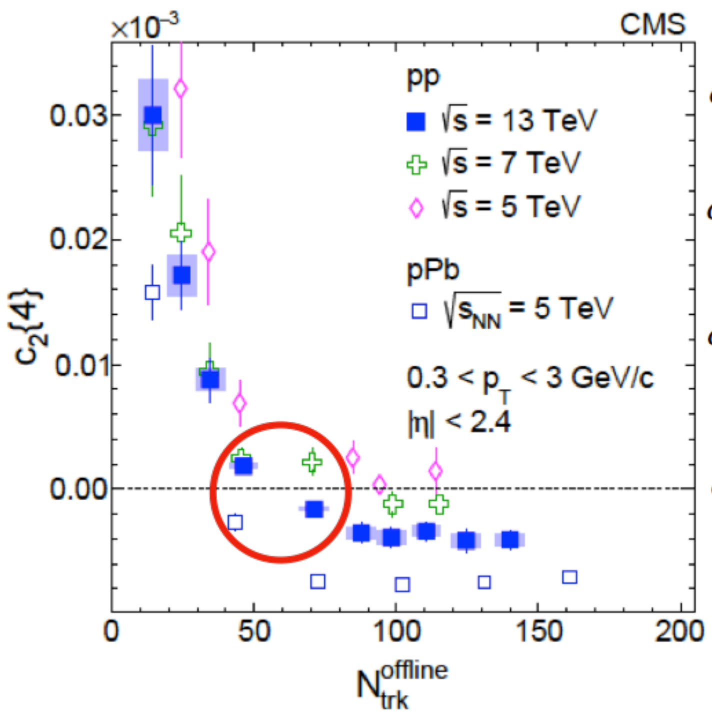
Abstract: The azimuthal cumulants, $c_2\{2\}$ and $c_2\{4\}$, originating from the global conservation of transverse momentum in the presence of hydro-like elliptic flow are calculated. We observe a sign change of $c_2\{4\}$ for small number of produced particles, which is in a qualitative agreement with the recent ATLAS measurement of multi-particle azimuthal correlations with the subevent cumulant method. Our results offer a new insight into the problem of the onset of collectivity in small systems.

(1) Introduction



$$c_2\{2\} = \langle e^{i2(\phi_1-\phi_2)}\rangle$$

$$c_2\{4\} = \langle e^{i2(\phi_1+\phi_2-\phi_3-\phi_4)}\rangle - 2\langle e^{i2(\phi_1-\phi_2)}\rangle^2$$

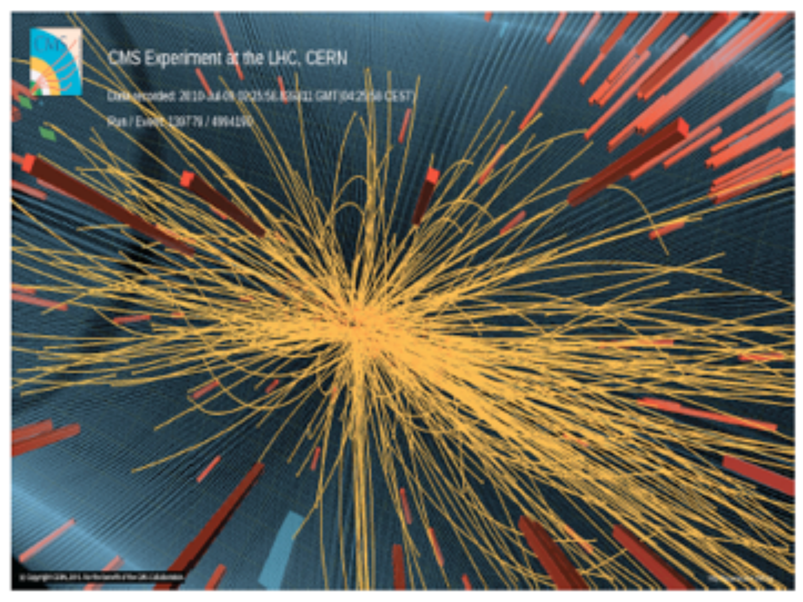
$$c_2\{6\} = \langle e^{i2(\phi_1+\phi_2+\phi_3-\phi_4-\phi_5-\phi_6)}\rangle - 9\langle e^{i2(\phi_1-\phi_2)}\rangle\langle e^{i2(\phi_1+\phi_2-\phi_3-\phi_4)}\rangle + 12\langle e^{i2(\phi_1-\phi_2)}\rangle^3$$

$$c_2\{8\} = \langle e^{i2(\phi_1+\phi_2+\phi_3+\phi_4-\phi_5-\phi_6-\phi_7-\phi_8)}\rangle - 16\langle e^{i2(\phi_1-\phi_2)}\rangle\langle e^{i2(\phi_1+\phi_2+\phi_3-\phi_4-\phi_5-\phi_6)}\rangle - 18\langle e^{i2(\phi_1+\phi_2-\phi_3-\phi_4)}\rangle^2 + 144\langle e^{i2(\phi_1-\phi_2)}\rangle^2\langle e^{i2(\phi_1+\phi_2-\phi_3-\phi_4)}\rangle - 144\langle e^{i2(\phi_1-\phi_2)}\rangle^4$$

$$(v_2\{2\})^2 = c_2\{2\},\quad (v_2\{4\})^4 = -c_2\{4\},\quad (v_2\{6\})^6 = \frac{c_2\{6\}}{4},\quad (v_2\{8\})^8 = -\frac{c_2\{8\}}{33}$$

☐Multi-particle cumulant flow $c_2\{k\}$ was designed to measure the *real* flow by reducing non-flow effects.
☐Four-particle cumulant $c_2\{4\}$ changes its sign at a N_{track}.➔the onset of collectivity in small system?

(2) Particle production under TMC



☐All produced N particles must obey the transverse momentum conservation (TMC) in a collision.
☐But ones only can experimentally measure part of them, i.e. k particles (k<N), due to the limits of acceptance and resolution.

N-particle momentum probability distribution:

$$f_N(\vec{p}_1,\ldots,\vec{p}_N) = \frac{1}{A}\delta^2(\vec{p}_1+\ldots+\vec{p}_N)f(\vec{p}_1)\cdots f(\vec{p}_N)$$

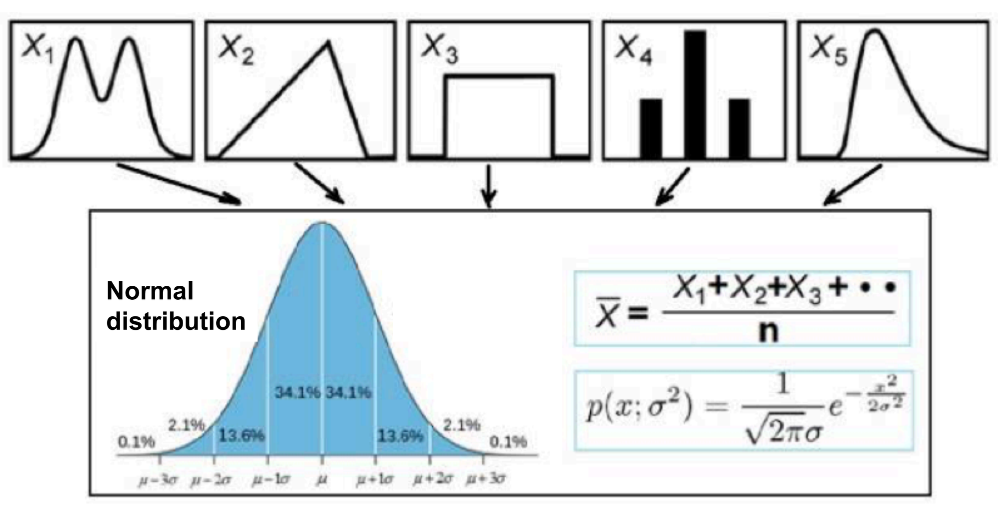
$$A = \int_F \delta^2(\vec{p}_1+\ldots+\vec{p}_N)f(\vec{p}_1)\cdots f(\vec{p}_N)d^2\vec{p}_1\cdots d^2\vec{p}_N$$

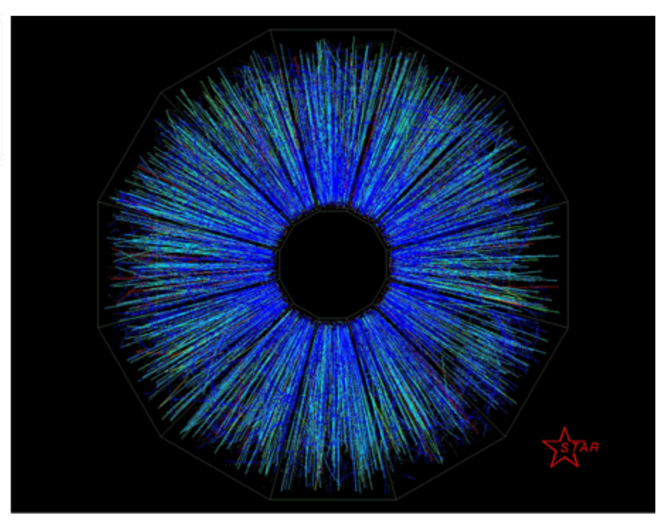
?

k-particle momentum probability distribution:

$$f_k(\vec{p}_1,\ldots,\vec{p}_k) = \frac{1}{A}f(\vec{p}_1)\cdots f(\vec{p}_k)\int_F \delta^2(\vec{p}_1+\ldots+\vec{p}_N)f(\vec{p}_{k+1})\cdots f(\vec{p}_N)d^2\vec{p}_{k+1}\cdots d^2\vec{p}_N$$

(3) Central limit theorem



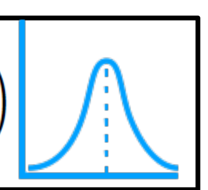


☐Central limit theorem: for large enough n, the distribution of $\bar{X}$ is close to the normal distribution with mean μ and variance σ²/n. ➔k-particle correlation function with TMC:

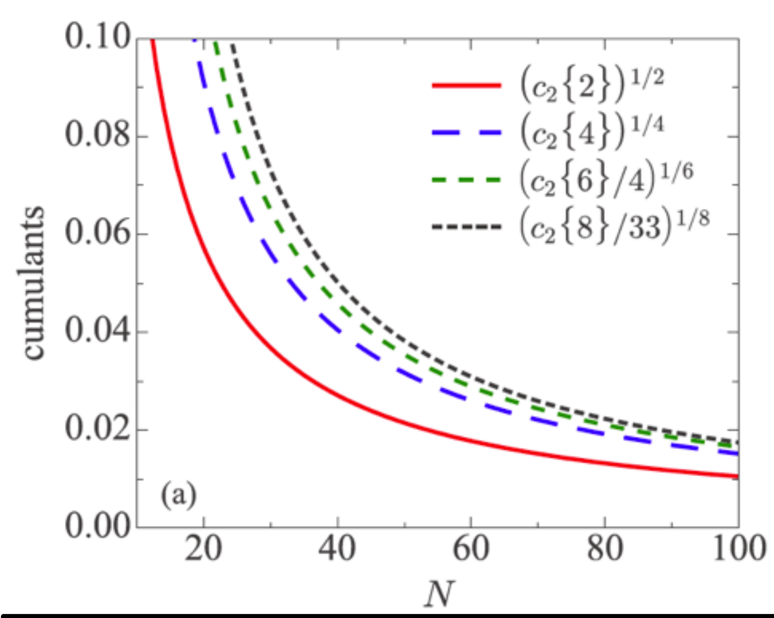
$$f_k(\vec{p}_1,\ldots,\vec{p}_k) = \frac{1}{A}f(\vec{p}_1)\cdots f(\vec{p}_k)\int_F \delta^2(\vec{p}_1+\ldots+\vec{p}_N)f(\vec{p}_{k+1})\cdots f(\vec{p}_N)d^2\vec{p}_{k+1}\cdots d^2\vec{p}_N$$

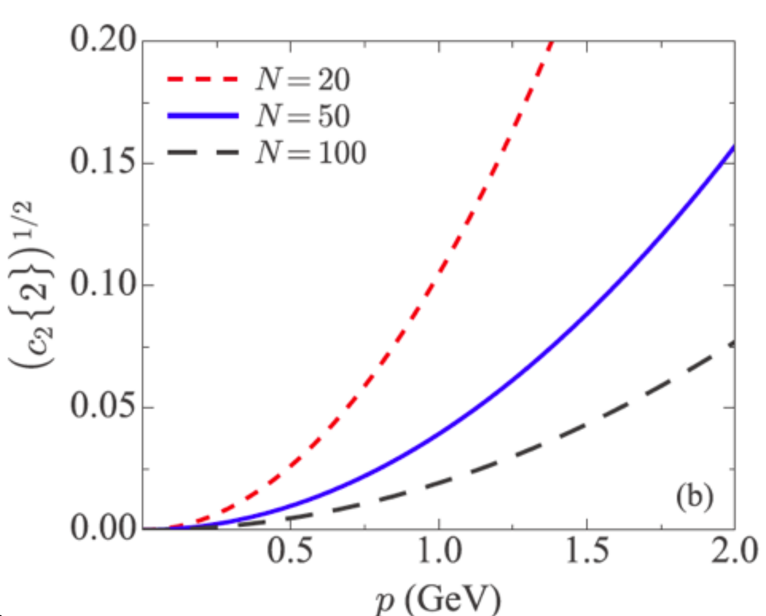
$\langle p\rangle=0$, $\sigma^2=\langle p^2\rangle$

$$f_k(\vec{p}_1,\ldots,\vec{p}_k) = f(\vec{p}_1)\cdots f(\vec{p}_k)\frac{N}{N-k}\exp\left(-\frac{(\vec{p}_1+\ldots+\vec{p}_k)^2}{(N-k)\langle p^2\rangle_F}\right)$$



(4) $c_2\{k\}$ from TMC





$$c_2\{2\}|_{p_1,p_2} \approx \frac{p_1^2p_2^2}{2(N-2)^2\langle p^2\rangle_F^2}$$

$$c_2\{4\}|_{p_1,p_2,p_3,p_4} \approx \frac{(p_1p_2p_3p_4)^2}{(N-4)^4\langle p^2\rangle_F^4}$$

$$\frac{1}{4}c_2\{6\}|_{p_1,\ldots,p_6} \approx \frac{3}{2}\frac{(p_1p_2p_3p_4p_5p_6)^2}{(N-6)^6\langle p^2\rangle_F^6}$$

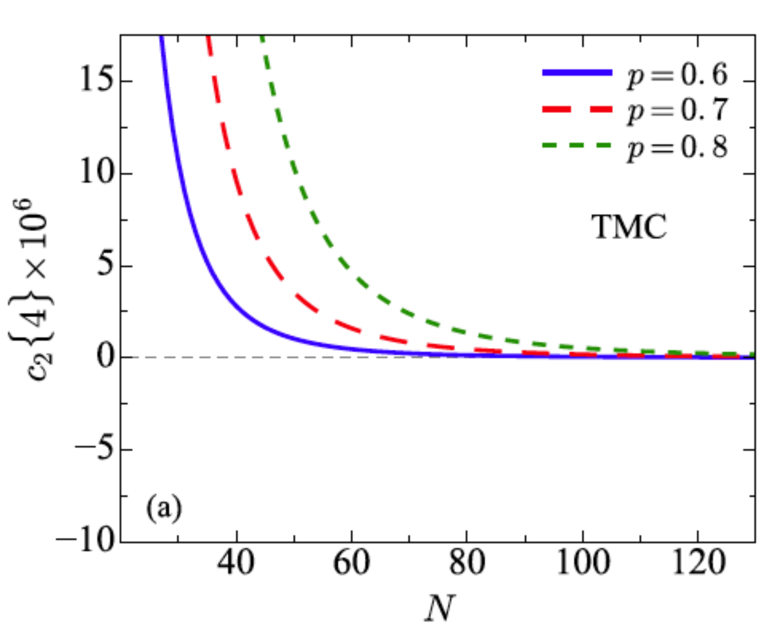
$$\frac{1}{33}c_2\{8\}|_{p_1,\ldots,p_8} \approx \frac{24}{11}\frac{(p_1p_2p_3p_4p_5p_6p_7p_8)^2}{(N-8)^8\langle p^2\rangle_F^8}$$

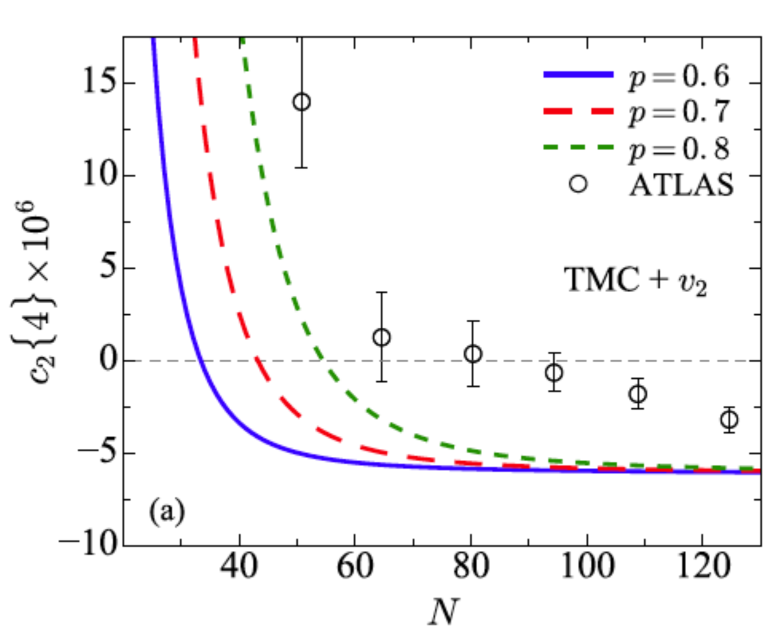
☐Cumulants from TMC are inversely proportional to N, $c_2\{k\} \propto 1/N^k > 0$.

☐The magnitudes increase with the order of cumulant due to the larger coefficients in higher order cumulants.

☐The influence of TMC on $c_2\{2\}$ is more significant for particles with higher momenta (parabolic dependence) for a smaller number of particles N [1].

(5) $c_2\{k\}$ from TMC+flow





$$c_2\{2\} \approx (v_2(p))^2 - \frac{p^2v_2(p)[2v_2(p)-\bar{v}_{2,F}]}{(N-2)\langle p^2\rangle_F} + \frac{p^4}{2(N-2)^2\langle p^2\rangle_F^2}$$

$$c_2\{4\} \approx (v_2(p))^4 - \frac{2p^2(v_2(p))^3[2v_2(p)-\bar{v}_{2,F}]}{(N-4)\langle p^2\rangle_F} + \frac{2p^4(v_2(p))^2}{(N-4)^2\langle p^2\rangle_F^2} - \frac{2p^6v_2(p)[8v_2(p)-3\bar{v}_{2,F}]}{(N-4)^3\langle p^2\rangle_F^3} + \frac{p^8[442(v_2(p))^2-360v_2(p)\bar{v}_{2,F}+27(\bar{v}_{2,F})^2]}{6(N-4)^4\langle p^2\rangle_F^4} + \frac{3p^8}{2(N-4)^4\langle p^2\rangle_F^4} - 2(c_2\{2\})^2$$

☐ The azimuthal cumulant, $c_2\{4\}$, originating from TMC + hydro-like elliptic flow v2, shows a sign change behavior[2], which qualitatively agrees with the ATLAS p+p data[3].

(6) Conclusions

☐The azimuthal cumulants, $c_2\{2\}$ and $c_2\{4\}$, originating from the global TMC in the presence of hydro-like flow are calculated.
☐TMC brings a positive azimuthal cumulant flow , $c_2\{k\} \propto 1/N^k > 0$.
☐TMC+flow can reproduce the sign change of $c_2\{4\}$, in a qualitative agreement with the recent measurement.
☐The results offers a new insight into the problem of the onset of collectivity in small systems.

References:

[1] Adam Bzdak and Guo-Liang Ma, Phys. Rev. C 97, 014903 (2018) [arXiv: 1710.00653].
[2] Adam Bzdak and Guo-Liang Ma, Phys. Lett. B 781, 117 (2018) [arXiv: 1801.01277].
[3] The ATLAS Collaboration, Phys. Rev. C 97, 024904 (2018) [arXiv:1708.03559].

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