



Measurements of Fluctuations in Nuclear Collisions

Rosi Reed
Lehigh University



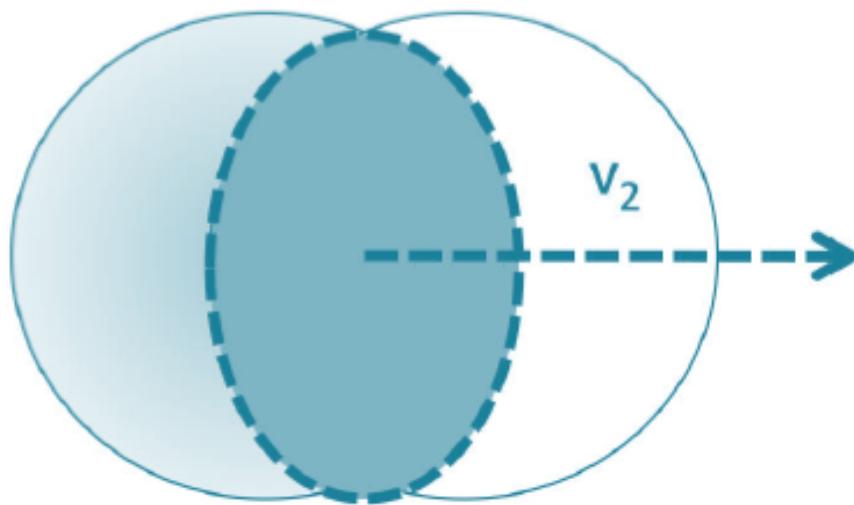
Introduction

Fluctuations in high-energy particle collisions have played an important role in understanding QCD over the last decade

The system fluctuates in many aspects, especially

- Initial State
- Hydrodynamics
- Hadronization
- Near a Critical Point

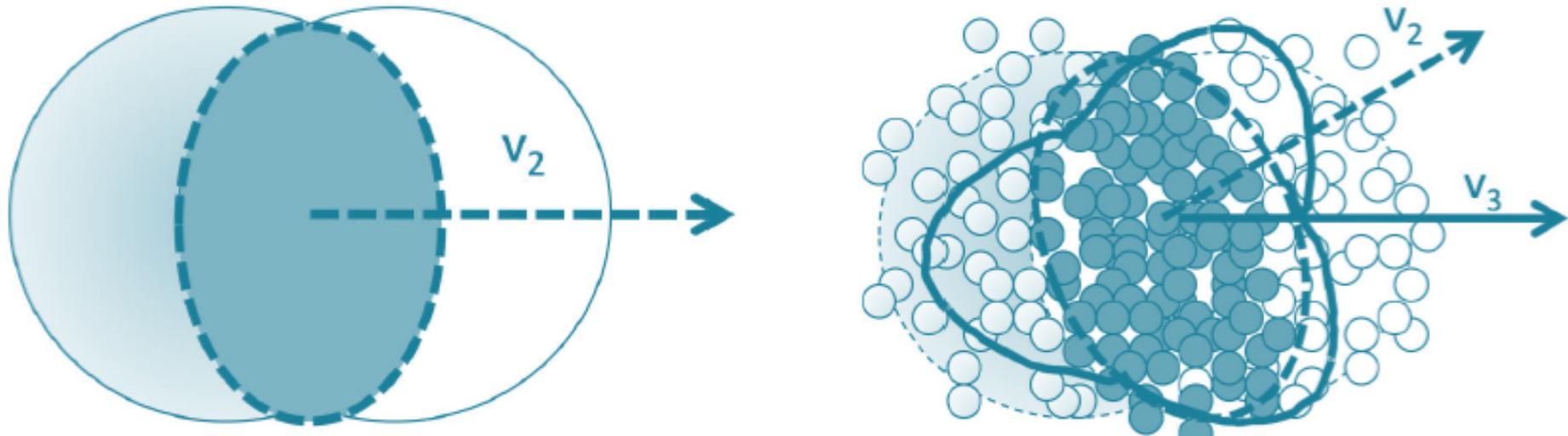
Heavy-Ion Collision before 2010



2-particle azimuthal correlation structures arise from two contributions:

- Elliptic flow → anisotropic hydrodynamic expansion of the medium from an anisotropic initial state
- Non-flow → resonances and jets and ...

Heavy-Ion Collision 2010+



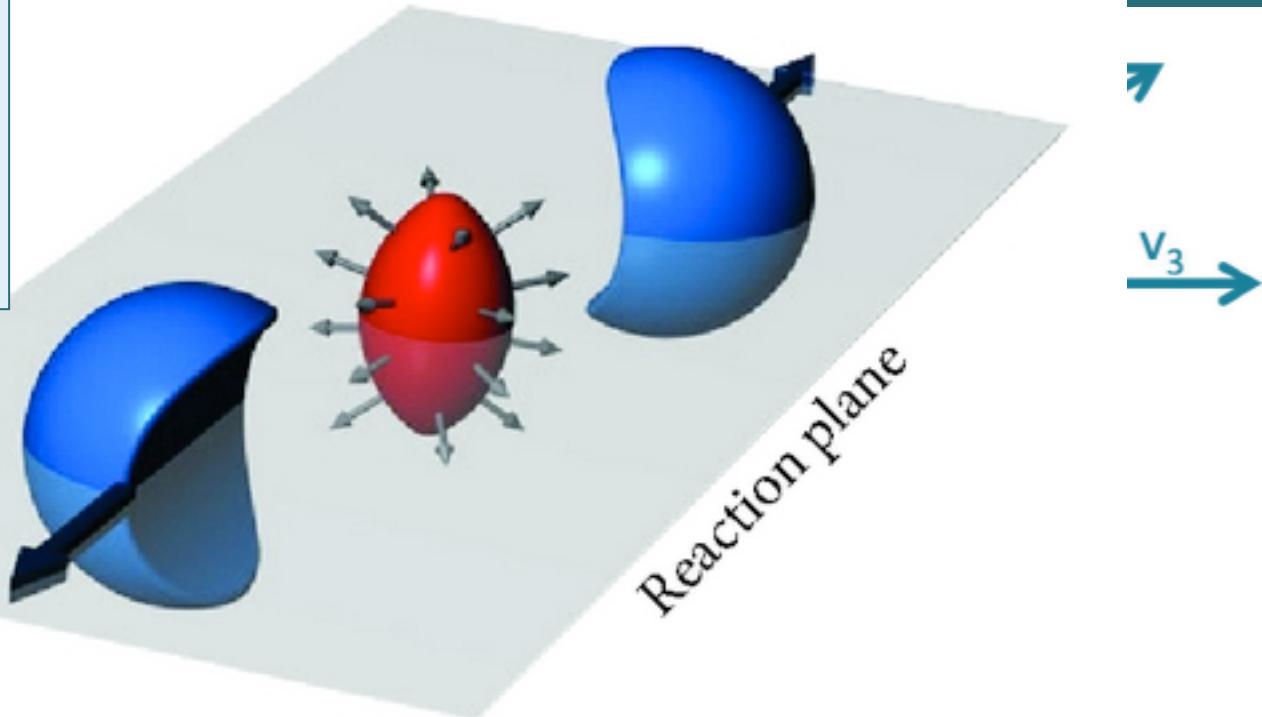
2-particle azimuthal correlation structures arise

- Elliptic flow
- Non-flow
- **Event-by-Event Fluctuations**

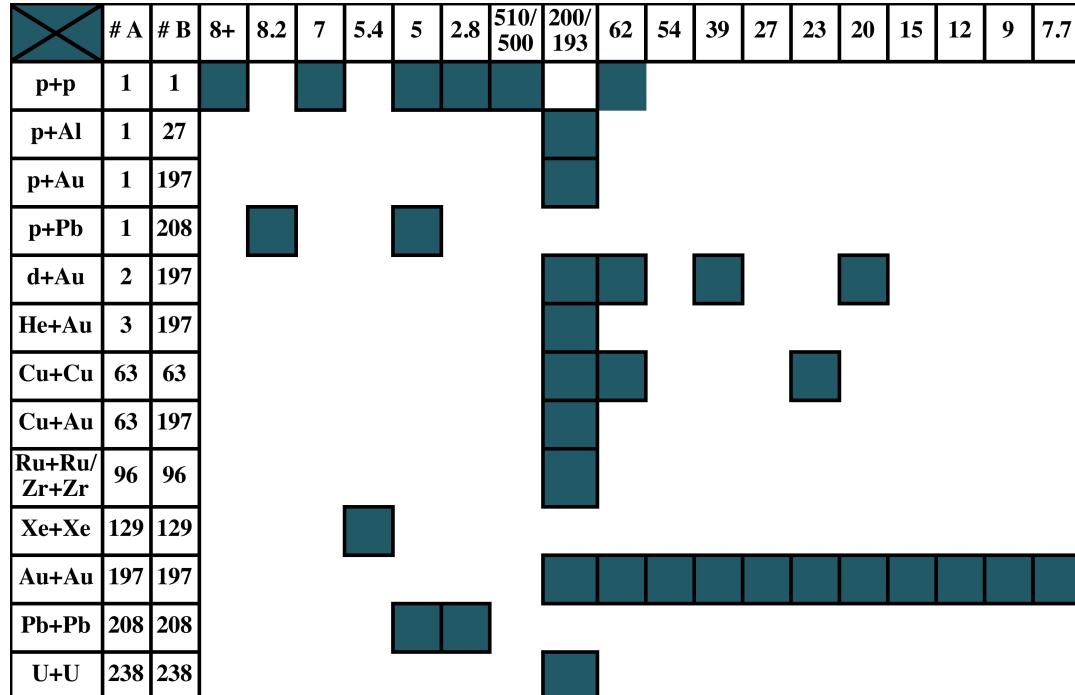
Heavy-Ion Collision 2010+

Lesson: We should be careful not to be fooled by a compelling picture!

2-par
• Elli
• No
• **Event-by-Event Fluctuations**



Wealth of Species Data



Size

Including 1-2 GeV per nucleon -C+C, Ar+KCl, pp, dp and pNb from Hades, Ar+Sc from NA61

Nearly two decades of data have allowed us to scan allowed space in many directions

Energy

Heavy Ion Experimental Controls

Vary the geometry
(different collision species)



Same size and life time
(same collision energy)

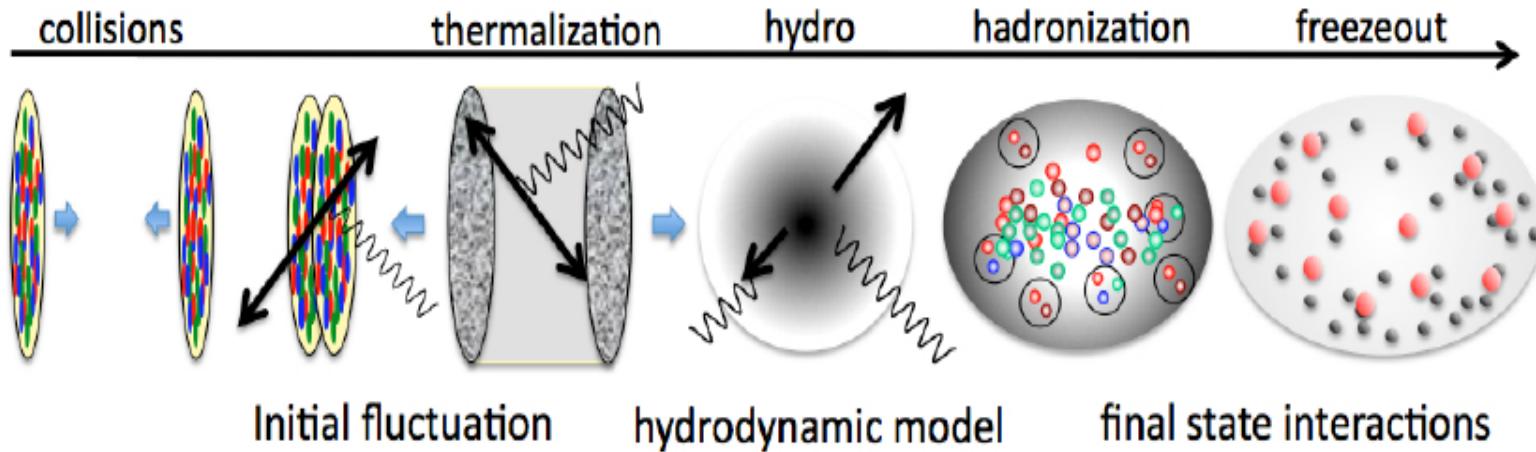
Fix the geometry
(same collision species)



Vary size and life time
(different collision energy)

Construct observables sensitive to one aspect of the system and take ratios to isolate physics mechanism in question

Evolution of a Heavy Ion Collision



Fluctuations develop at every stage of the collision

- How do we differentiate them?
- What do we learn about QCD?

Nonaka, Chiho et al. PTEP 2012 (2012) 01A208

Initial State Fluctuations

The initial conditions are the **major unknown** for the hydrodynamic evolution

- No 1st principle treatment of non-equilibrium QCD to calculate the initial stage
 - Efforts with CGC, etc are underway
- Initial state profile is constrained from experimental data
- Required to extract QGP transport properties

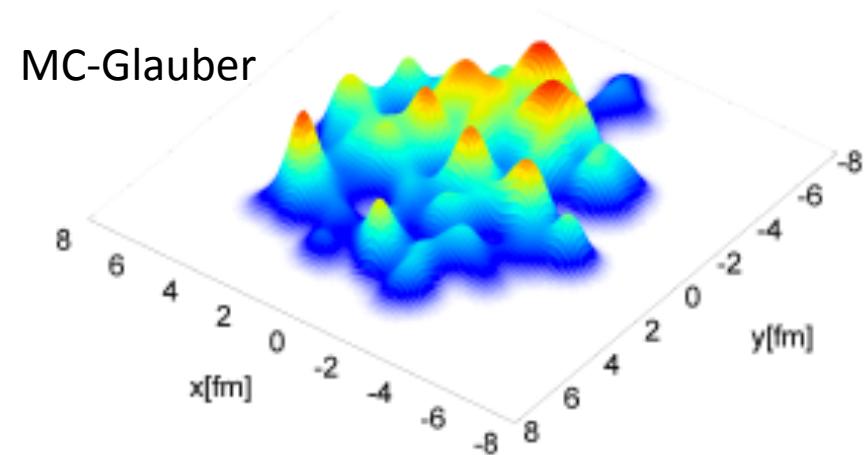
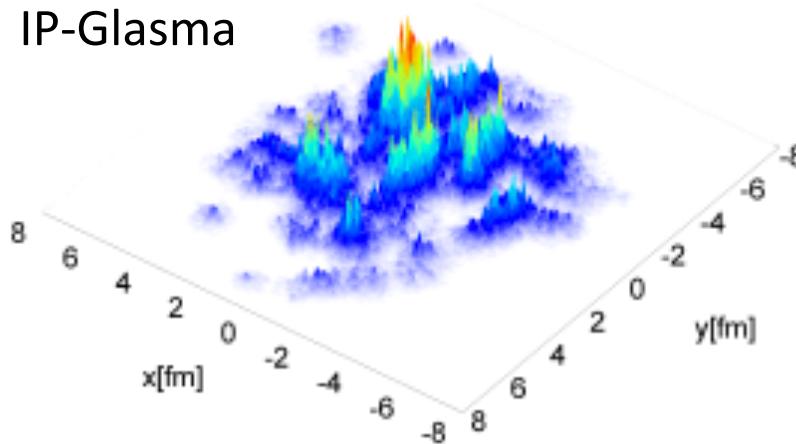
Initial state spatial anisotropy is approximately beam energy independent

Viscous attenuation is beam energy dependent

Initial State Fluctuations

Hydrodynamics translates initial geometry → final state

- Test hydro hypothesis by varying initial state geometry via different species



Phys.Rev.Lett. 108 (2012) 252301

Reminder - Measuring Flow

Measure symmetry plane Ψ_{RP} and correlate other measured particles

$$v_n = \langle \cos n(\phi - \Psi_{RP}) \rangle$$

Reaction plane is not precisely the event plane

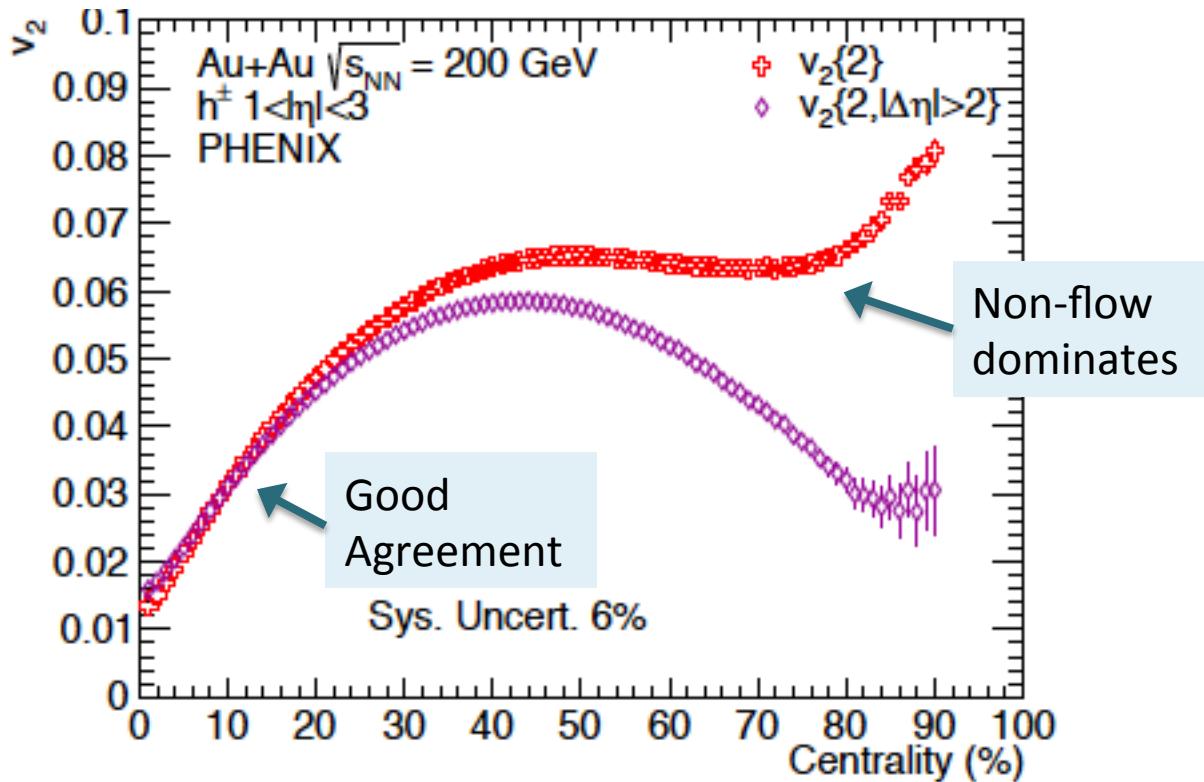
Fluctuations are ill-defined for the event plane → Use multi-particle correlations

$$c_2\{2\} = \langle e^{i2(\phi_1-\phi_2)} \rangle = v_2^2 + \delta_2$$

$\delta_2 = \text{Non-flow}$

Many techniques for removal
(η -gap, higher order terms,)

v_2 via Multiparticle Cumulants



We see here
that the η gap
method allows
removal of
non-flow

Reminder - Measuring Flow

The cumulant method can be extended to any number of particles

- Influence of non-flow is small for ≥ 4 particles
- Influence of fluctuations is small

Assume that the non-flow δ is negligible due to selection

$$c_2\{2\} = \langle e^{i2(\phi_1-\phi_2)} \rangle = v_2^2 + \delta_2 \quad \longrightarrow \quad c_2\{2\} = \langle e^{i2(\phi_1-\phi_2)} \rangle = v_2^2$$

Look at 4 (or more) particles

$$c_2\{4\} = \langle e^{i2(\phi_1+\phi_2-\phi_3-\phi_4)} \rangle - 2\langle e^{i2(\phi_1-\phi_2)} \rangle^2 = -v_2^4\{4\}$$

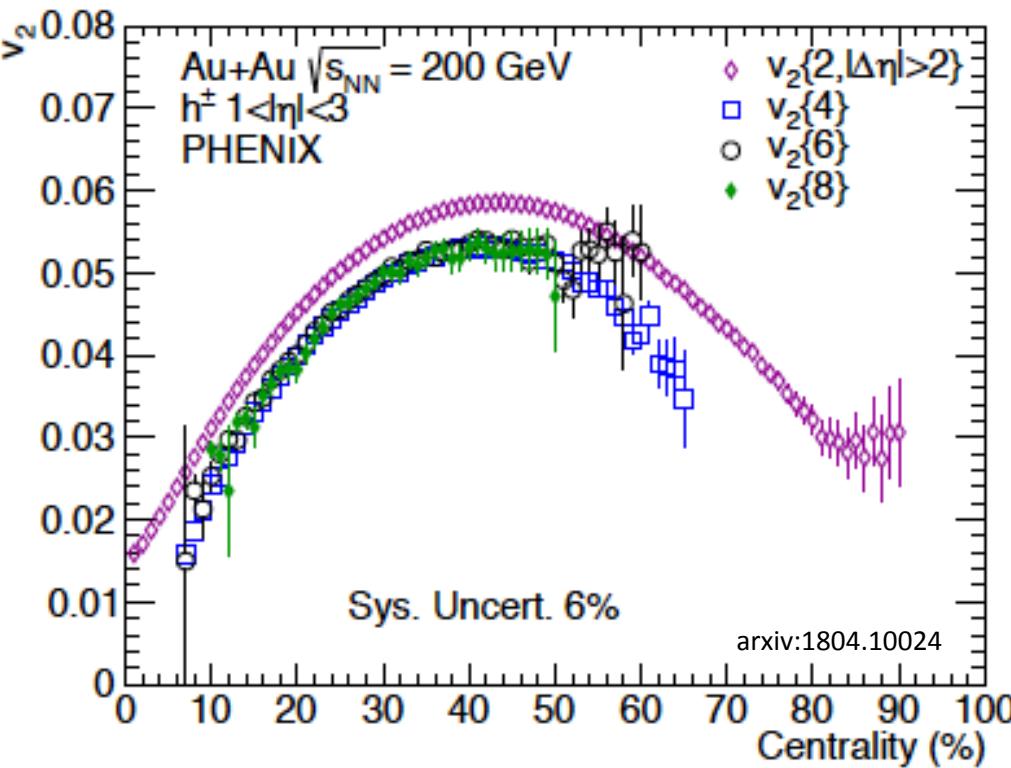
If no 4-particle correlations exist, $c_2\{4\} = 0$

- As cumulant order increases \rightarrow non-flow influence decreases

$$v_2^2\{2\} = \langle v_2 \rangle^2 + \sigma_v^2 \quad v_2^2\{4\} \approx \langle v_2 \rangle^2 - \sigma_v^2$$

Higher order cumulants allow us to access the fluctuations

v_2 via Multiparticle Cumulants

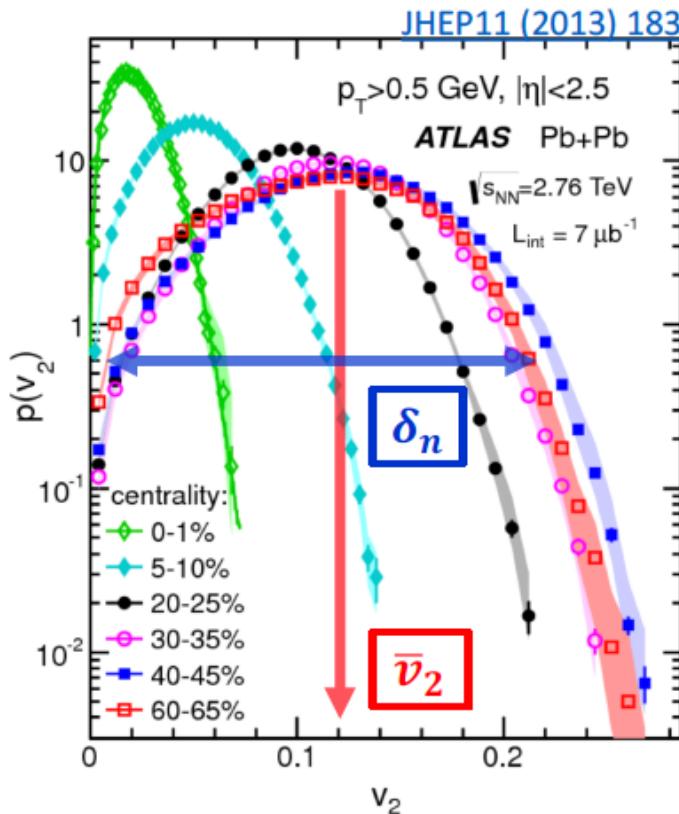


$$v_2\{2\} > v_2\{4\} \approx v_2\{6\} \approx v_2\{8\}$$

Consistent with expectations given Gaussian fluctuations and small variance

What is the width of each distribution for a given centrality class?

Event-by-Event Flow Fluctuations

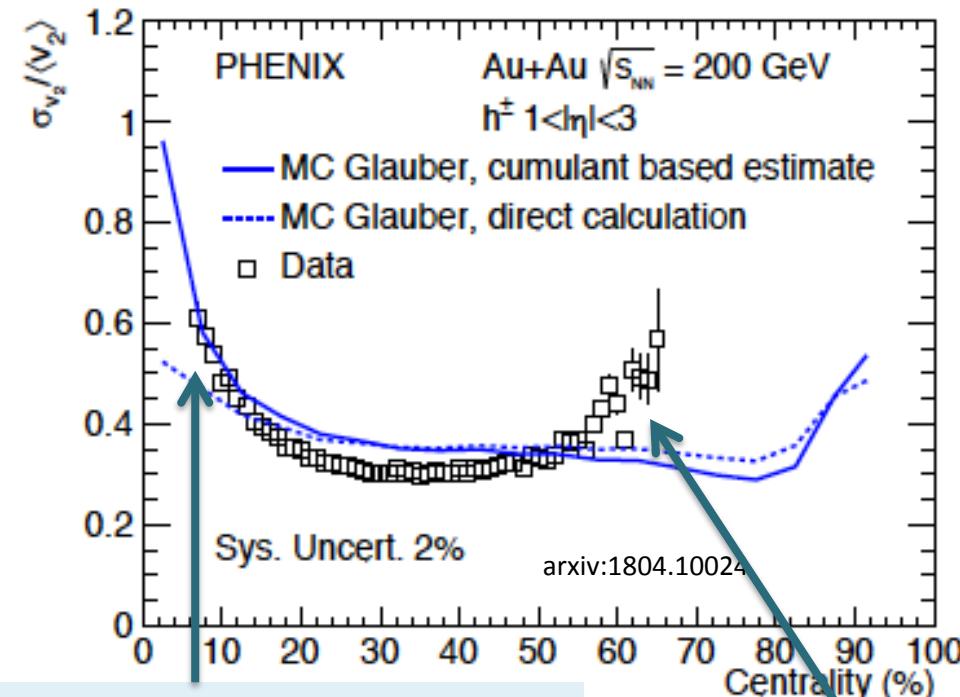


- Flow fluctuates event-by-event
 - Initial geometry + hydro evolution
- Recall, higher order cumulants suppress non-flow
- Many sources → Gaussian fluctuation
This is why event shape engineering works!



Mingliang Zhou
T 15:00

v_2 Relative Fluctuations



Small variance assumption
not valid for central events

Non-linearity in hydro

$$v_2^2\{2\} = \langle v_2 \rangle^2 + \sigma_v^2$$

For small variance:

$$v_2^2\{4\} \approx \langle v_2 \rangle^2 - \sigma_v^2$$

Compare with $\sigma_E / \langle \epsilon_2 \rangle$ from Glauber

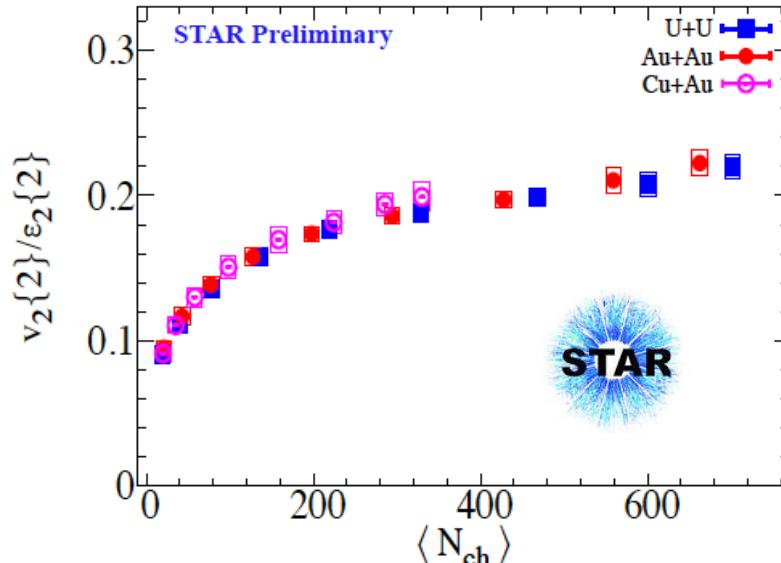
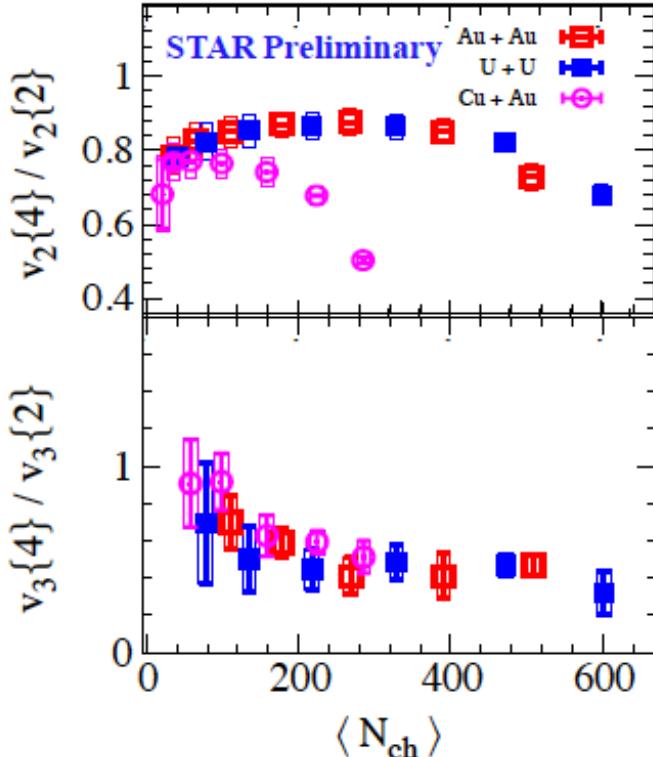
For central events, the higher terms matter and should be included

ϵ_2 fluctuations do not match v_2 fluctuations in peripheral events due to the nonlinearity of the hydro response

Kurt Hill
T 15:40

J. Noronha-Hostler et al Phys.
Rev. C 93, 014909 (2016)

Change Species – Keep vs_{NN}



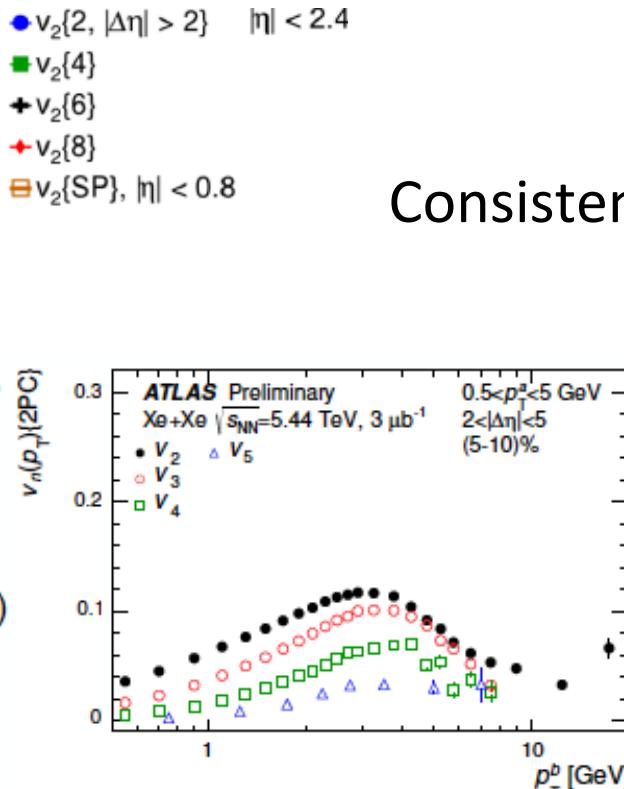
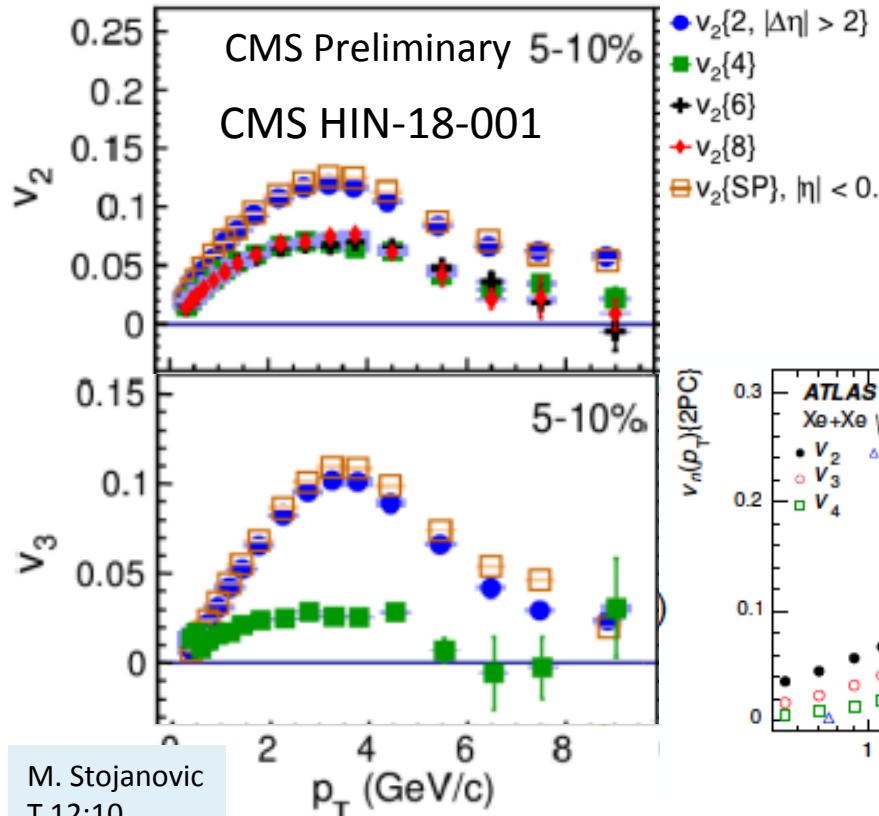
Au+Au
U+U
Cu+Au

Linear hydro response in central events
 $\epsilon_2 \leftrightarrow v_2$

- $v_n\{4\}/v_n\{2\} = 1 \rightarrow$ minimal fluctuations
- $v_n\{4\}/v_n\{2\} = 0 \rightarrow$ increased fluctuations

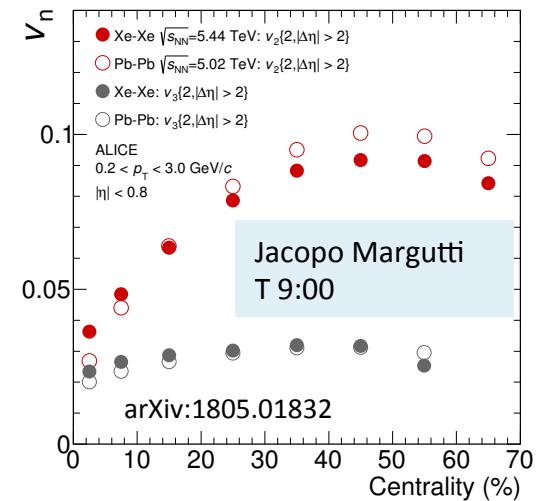
Niseem Magdy
T 11:30

Xe-Xe collisions

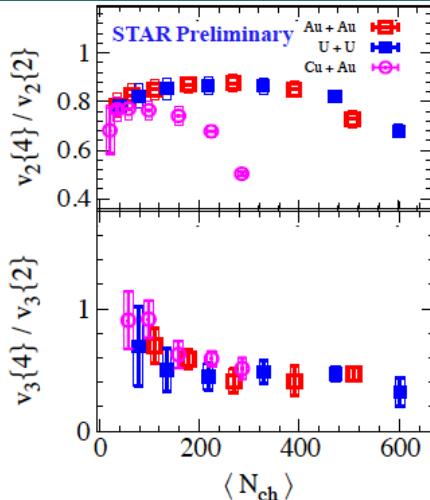
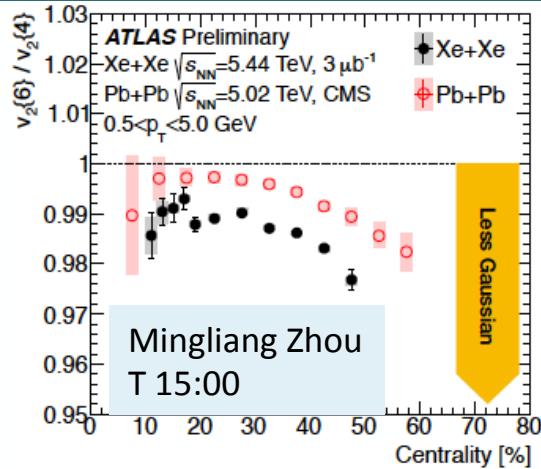


$\sqrt{s_{NN}} = 5.44$ TeV

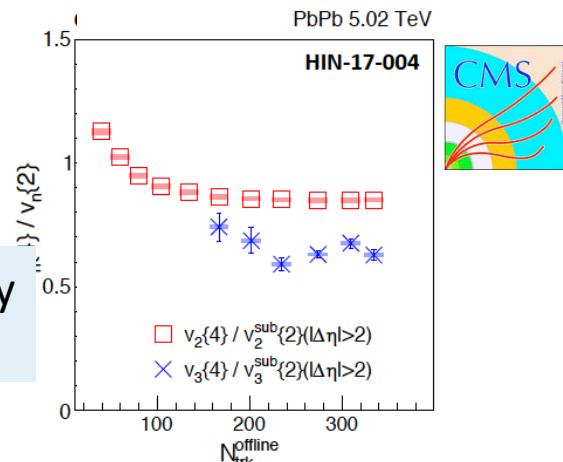
Consistent with Hydro Picture!



Flow Fluctuations vs Species



Niseem Magdy
T 11:30



Ratio of $v_n\{x\}/v_n\{x-2\}$ is sensitive to flow fluctuations

- v_2 is dominated by collision geometry, $v_{n>2}$ dominated by fluctuations
- $v_2\{6\}/v_2\{4\} \leq 1$ in Xe+Xe < Pb+Pb → Less Gaussian

Interesting comparison of U+U (highly deformed!) to Xe+Xe

Further constraints on initial state (and hydrodynamics)

$$\frac{v_n\{4\}}{v_n\{2\}} = \sqrt{\frac{\langle v_n \rangle^2 - \sigma_n^2}{\langle v_n \rangle^2 + \sigma_n^2}}$$

Flow Fluctuations Summary

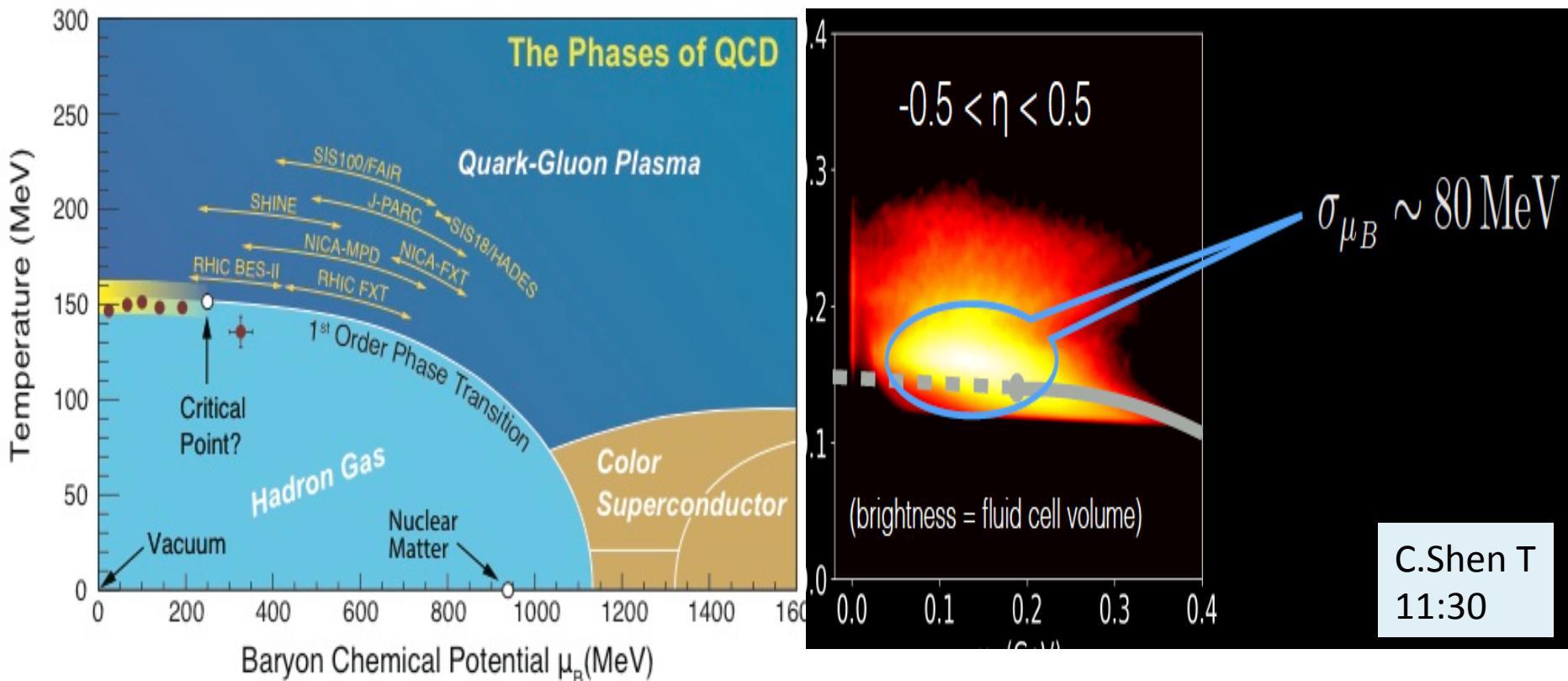
- For all energies, $v_n\{x\} \approx v_n\{x+2\}$
 - LHC energies have a hint of difference
- Precision flow fluctuation measurements allow us to constrain the initial state
 - Varying both energy and species
- Non-linear hydrodynamics show that in more peripheral events the relationship between ε and v is not one-to-one
- Higher order moments are more sensitive to fluctuations and thus more sensitive to the differences in initial state

$T > T_c$ $T \gtrsim T_c$ $T \lesssim T_c$ $T < T_c$

Flow Fluctuations → Fluctuations in Conserved Quantities



QCD Phase Diagram



Net Baryon Fluctuations

- Thermodynamic susceptibilities χ
 - Describe the response of a thermalized system to changes in external conditions

$$\chi_n^B = \frac{\partial^n (P / T^4)}{\partial (\mu_B / T)^n}$$

Calculated via Lattice QCD

- Related to event-by-event fluctuations of the number of conserved charges
- Baryon multiplicity distributions

$$\Delta N_B = N_B - N_{\bar{B}}$$

Measured

$$\langle \Delta N_B \rangle = VT^3 \chi_1^B$$

$$\langle (\Delta N_B - \langle \Delta N_B \rangle)^2 \rangle = VT^3 \chi_2^B = \sigma^2$$

Relation between experiment and theory
(There are higher order terms)

Search for the QCD critical point

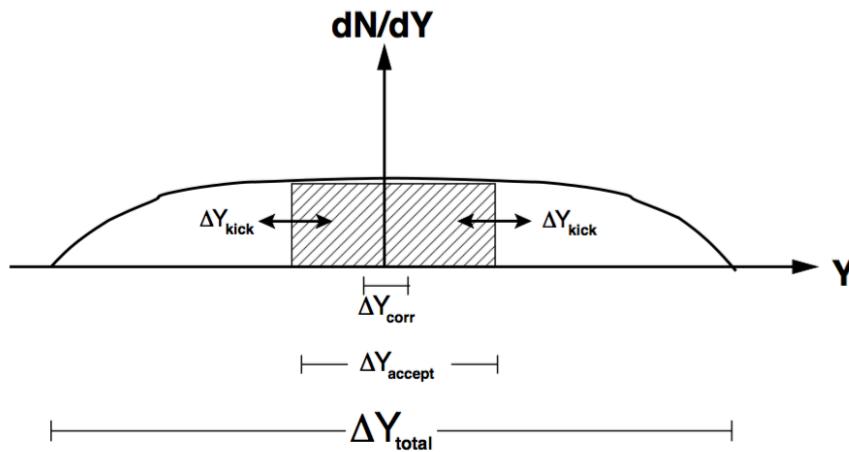
- **Local fluctuations** of conserved numbers increase near a critical point
 - Among the most promising signatures
 - Characteristic feature of a CP is the divergence of the correlation length (ξ) & magnitude of fluctuations
 - Fluctuating system with finite size and lifetime complicate our ability to measure ξ
- Non-Gaussian distributions, higher moments → increased sensitivity to ξ
 - Kurtosis is proportional to the 7th power of ξ
 - $K \propto \langle (N - \langle N_B \rangle)^4 \rangle$
 - Higher order moments are also more prone to experimental measurement issues such as efficiency or resolution

Net Baryon Fluctuations

Skewness
and Kurtosis

$$S\sigma = \chi_3^B / \chi_2^B$$

$$\kappa\sigma^2 = \chi_4^B / \chi_2^B$$



What
acceptance
window is
just right?

No Signal

0%



Poisson
fluctuations

Net-baryon Acceptance:

Maximum Signal

No Signal

100%



Zero fluctuations
(baryon # conservation)

Skewness and Kurtos

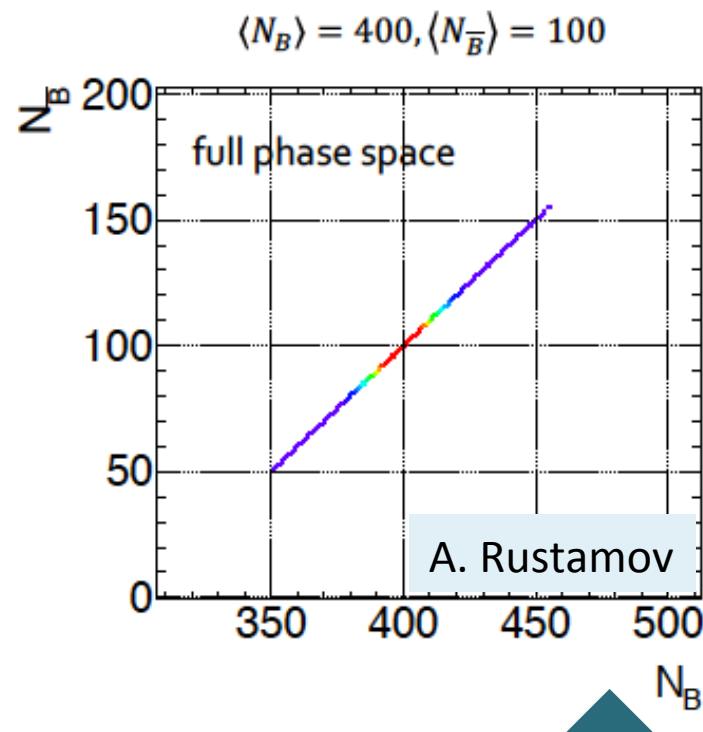
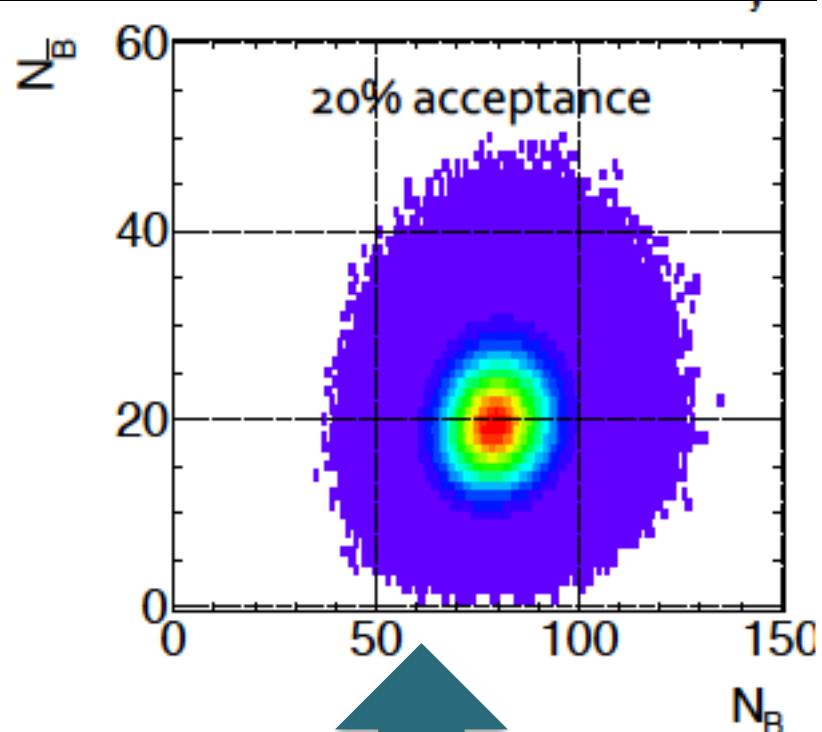
$$S\sigma = \chi_3^B / \chi_2^B$$

$$\kappa\sigma^2 = \chi_4^B / \chi$$

No Signal

0%

Poisson
fluctuations



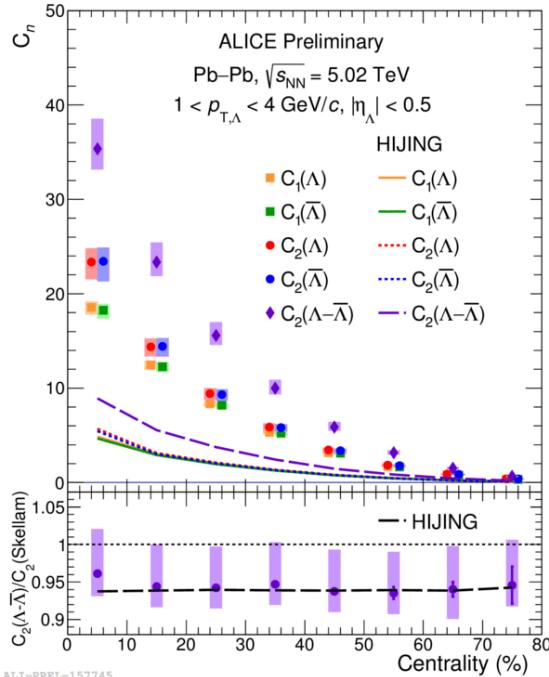
No Signal
100%

Zero fluctuations
(baryon # conservation)

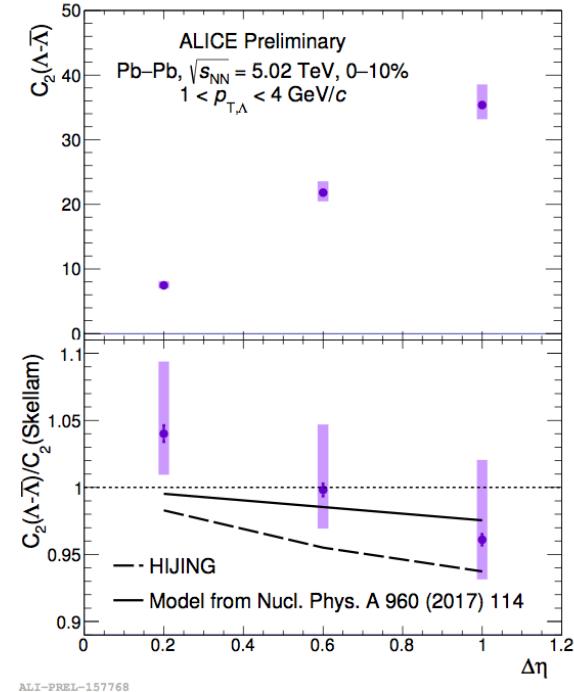
Λ fluctuations

- Λ allows us to explore correlated fluctuations of baryon number and strangeness
 - Improve understanding of net-baryon fluctuations
 - Different contributions from resonances, etc, than in net-proton measurement
- Results from Λ s can be combined with net-proton or net-kaon results
 - This brings us closer to net-baryon or net-strangeness fluctuations

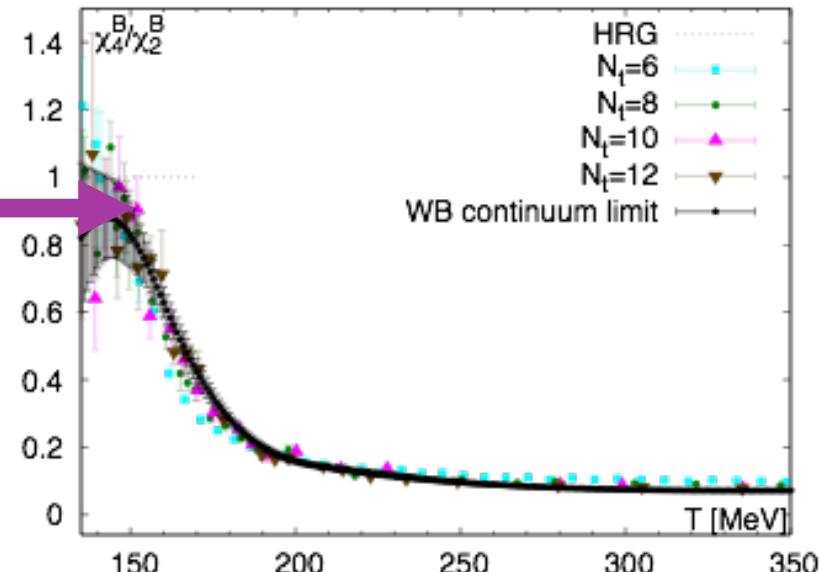
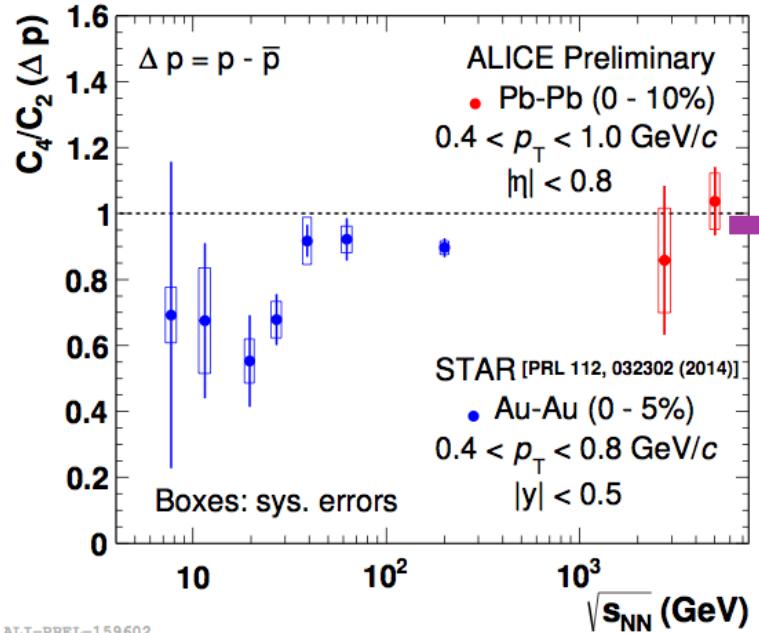
Λ Fluctuations



- $C_1(\Lambda) = \langle N_\Lambda \rangle$
- $C_2(\Lambda) = \langle (N_\Lambda - \langle N_\Lambda \rangle)^2 \rangle$
- First + second order net- λ moments extracted at the LHC
- Deviation from Skellam for $\Delta\eta$ dependence consistent with baryon number conservation expectations
- HIJING does not explain strangeness well



Higher order net-proton fluctuations at LHC



ALI-PREL-159602

- Temperature consistent with thermal fits to particle yields $T_{fo} \sim 153 \text{ MeV}$
- No observation of non-thermal fluctuations at high energy in lower orders as expected → driven by conservation laws → Consistent with Lattice QCD expectation ($\mu_B \approx 0$)
- Solid baseline for search for critical fluctuations at higher orders

F. Bellini N. Behera
W 12.50 W 11:50

Higher order net-proton fluctuations at RHIC

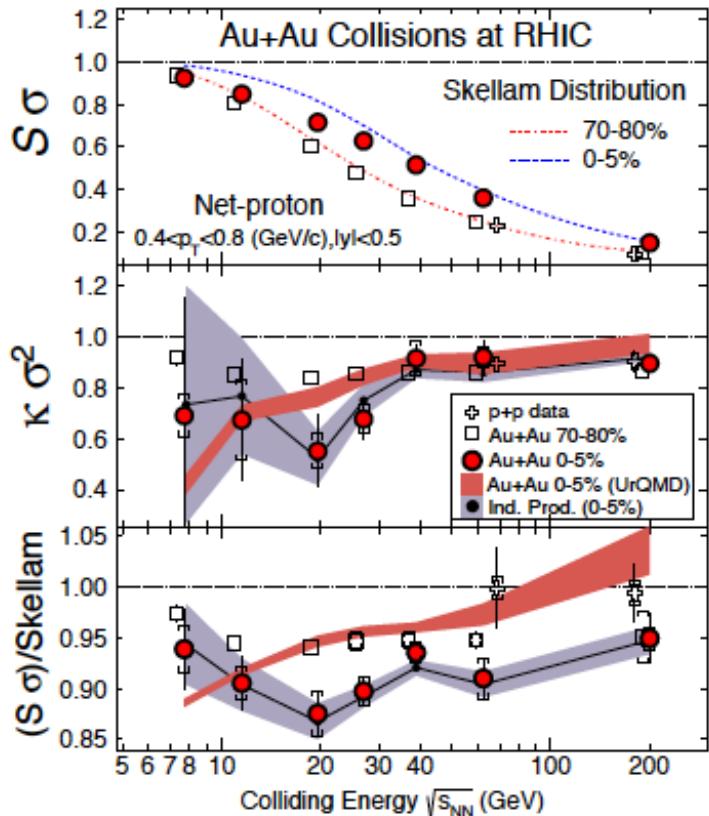
- The Skellam expectations → system of totally uncorrelated, statistically random particle production
- p+p collisions is similar to peripheral Au+Au collisions for $\sqrt{s_{NN}} = 62.4$ and 200 GeV
- For $\sqrt{s_{NN}}$ below 39 GeV, differences are observed between the 0-5% and 70-80% central Au+Au collisions
 - Deviations from Skellam expectations most significant for 19.6 and 27 GeV
- The results are closer to unity for $\sqrt{s_{NN}} = 7.7$ GeV
- To understand NB number conservation and acceptance UrQMD without a CP for 0-5% is plotted

Skewness



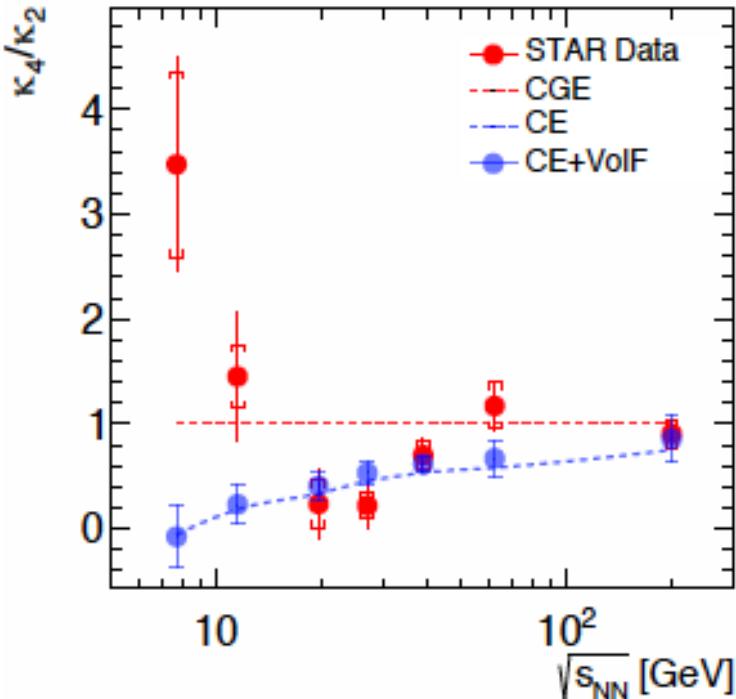
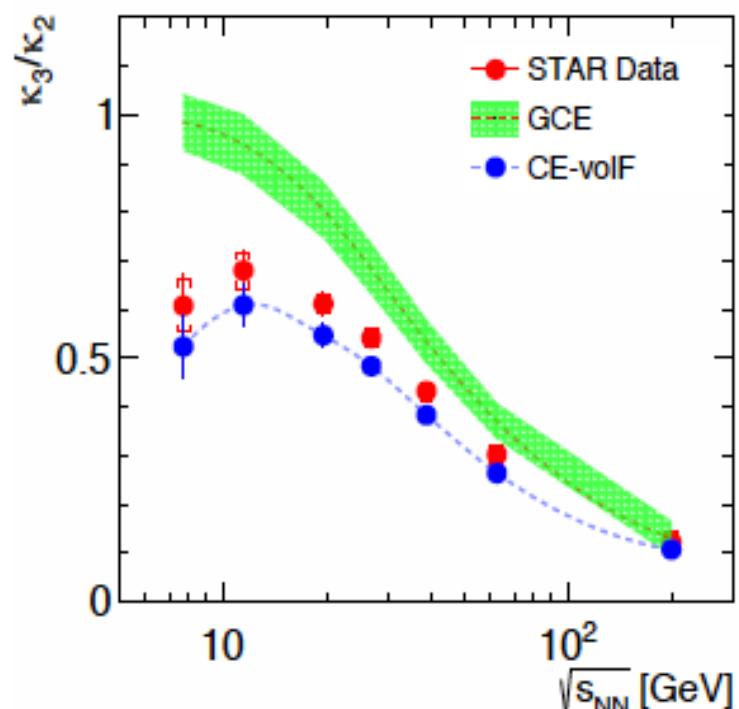
Kurtosis

Compared
to Skellam
expectation



Phys.Rev.Lett. 112 (2014) 032302

Model Comparisons RHIC Net Proton Data

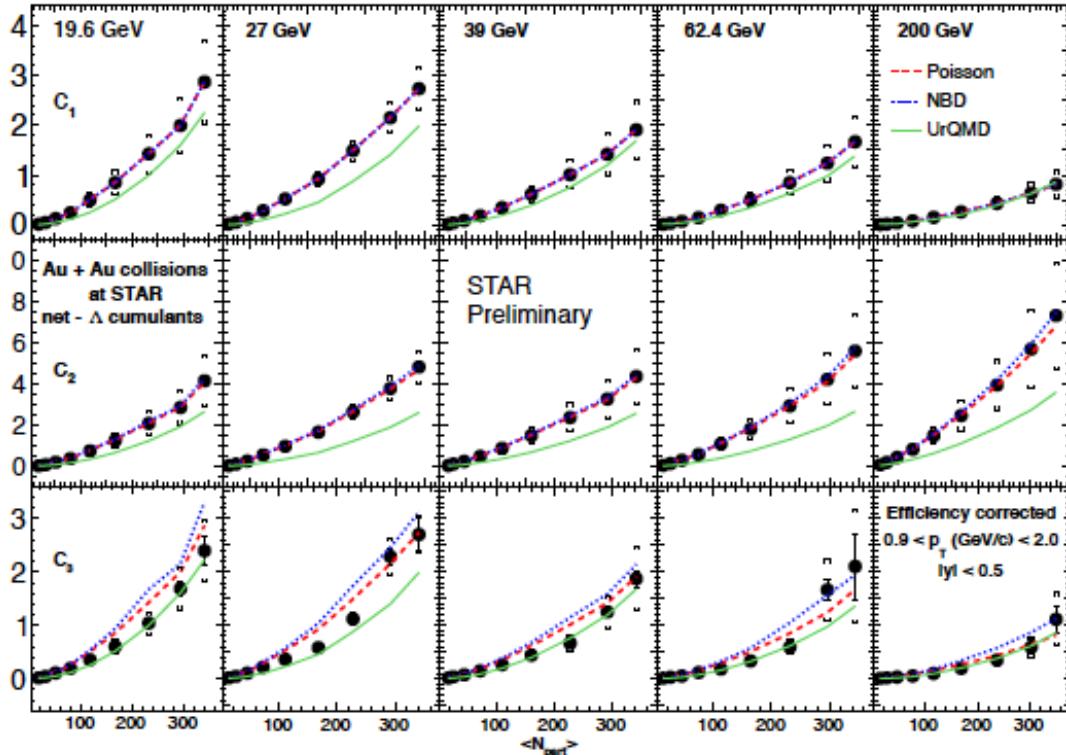


Above 11.5 GeV Canonical Ensemble (CE) suppression for cumulants accounts for measured deviations from GCE

- Impact of conservation laws and volume fluctuations on data have to be considered
- Qualitative differences emerge above 4th order cumulants!

A. Rustamov

Net- Λ cumulants at RHIC



Consistent with
Poisson/NBD
baselines.



- C_1 and C_2 are above UrQMD results
- C_3 has closer agreement with UrQMD

2nd Order Off-Diagonal Cumulants

Off-Diagonal cumulants → additional constraints on freeze out

The correlation between net-protons and net-kaons changes sign with beam E

T. Nonaka
T 12:50



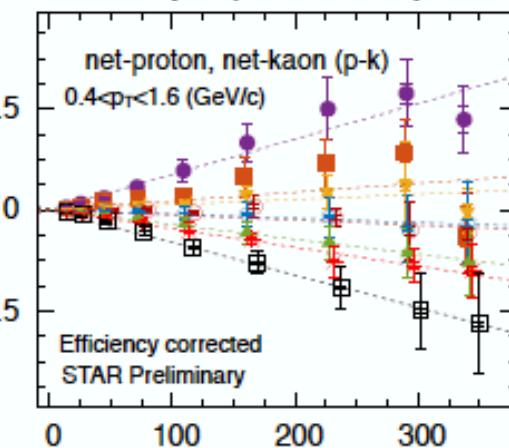
$$\begin{pmatrix} \sigma_Q^2 & \sigma_{Q,p}^{1,1} & \sigma_{Q,k}^{1,1} \\ \sigma_{p,Q}^{1,1} & \sigma_p^2 & \sigma_{p,k}^{1,1} \\ \sigma_{k,Q}^{1,1} & \sigma_{k,p}^{1,1} & \sigma_k^2 \end{pmatrix}$$

$$\sigma_{x,y}^2 = \langle xy \rangle - \langle x \rangle \langle y \rangle$$

$$C_{x,y} = \frac{\sigma_{x,y}^{1,1}}{\sigma_y^2}$$

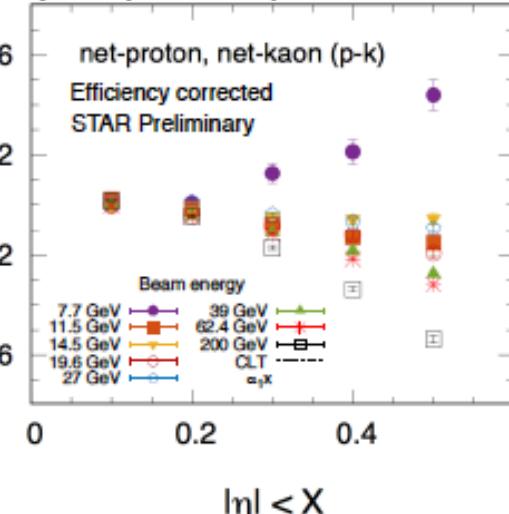
$$\text{Co-variance } (\sigma^{1,1})$$

Centrality dependence, $|\eta| < 0.5$



$$\text{Co-variance } (\sigma^{1,1})$$

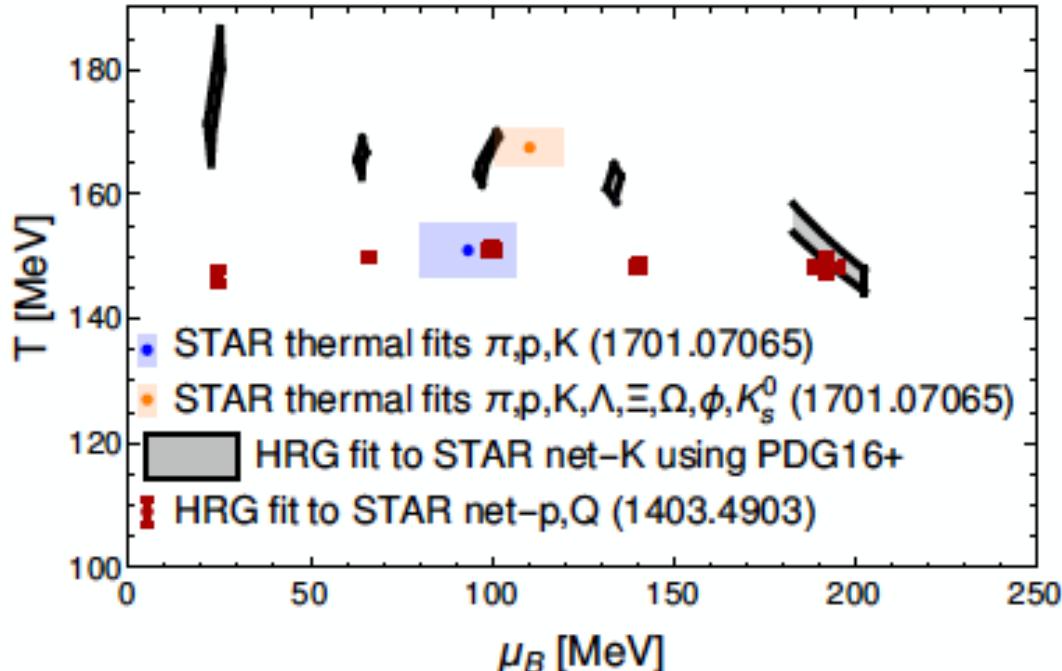
η acceptance dependence, 0-5%



- A. Majumder and B. Muller, *Phys. Rev. C* 74 (2006)
- A. Bazavov et al. *Phys. Rev. D86* (2012) 034509
- A. Chatterjee et al. *J. Phys. G: Nucl. Part. Phys.* 43 (2016) 125103
- Z Yang et al. *Phys. Rev. C* 95 014914 (2017)

Net- Λ cumulants

J. Noronha-Hostler, C. Ratti, P. Parotto,
R. Bellwied, arXiv:1805.00088

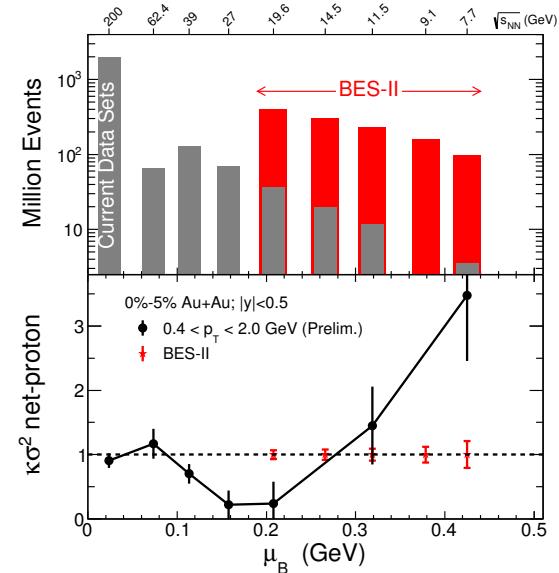
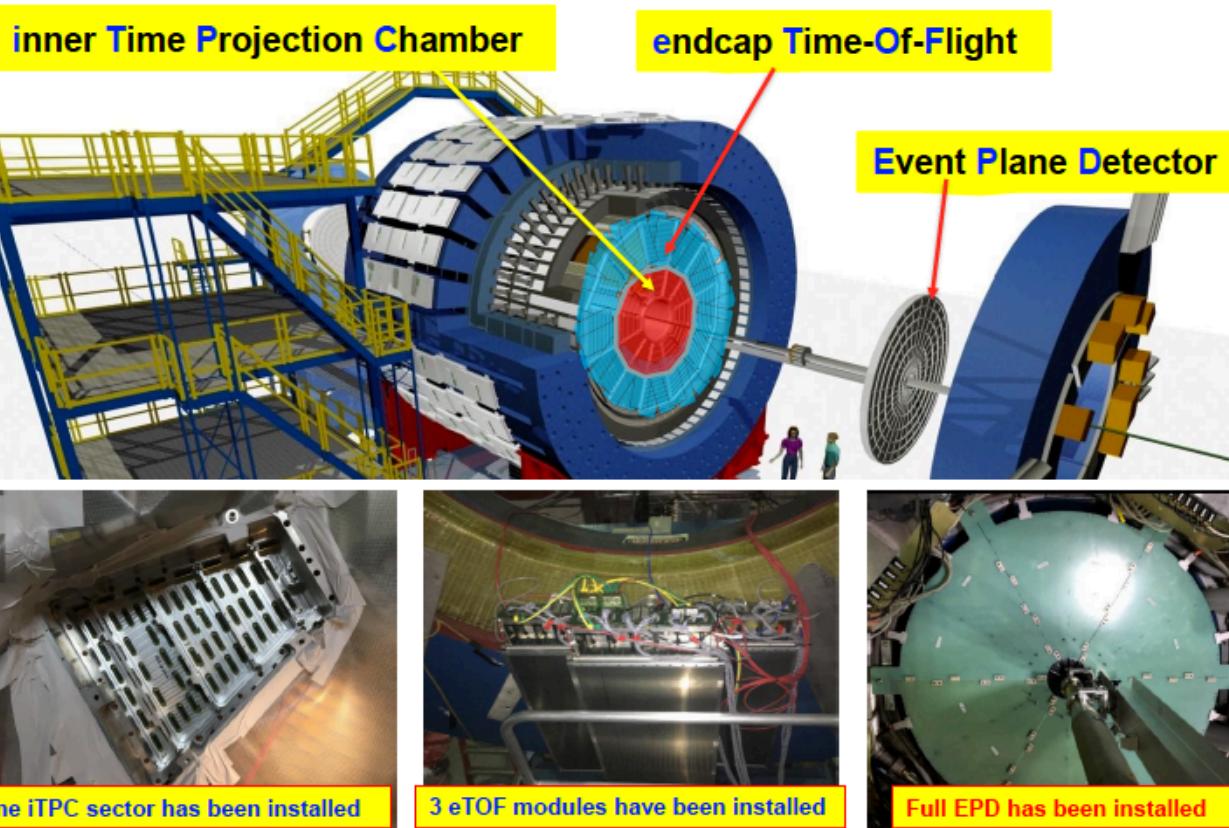


- Strange hadrons freeze-out earlier than light flavor hadrons?
- Net- Λ cumulants might provide additional constraints on freeze-out conditions

Λ BESI Results → BESII

- Net- Λ cumulants up to 3rd-order
 - Consistent with Poisson/NBD baselines
 - The result of C_2/C_1 is closer to those of HRG with kaon freeze-out condition rather than light flavor hadrons.
- Second-order off-diagonal cumulants
 - Q-k and Q-p correlations are in excess of the UrQMD results
- BESII will extend these measurements
 - Au+Au Collision Energies
 - $\sqrt{s_{NN}} = 7.7\text{-}200 \text{ GeV}$ Collider mode
 - $\sqrt{s_{NN}} = 3.0\text{-}7.7 \text{ GeV}$ Fixed Target mode
 - Search for Critical Point → STAR+RHIC Upgrades

BESII STAR Upgrades



Upgrades increase
statistics and
acceptance!

Qian Yang
T 10:00

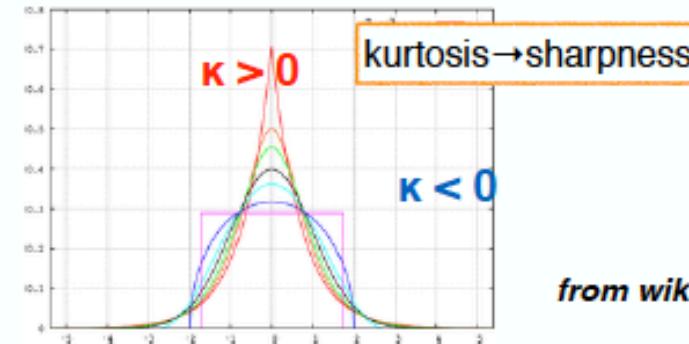
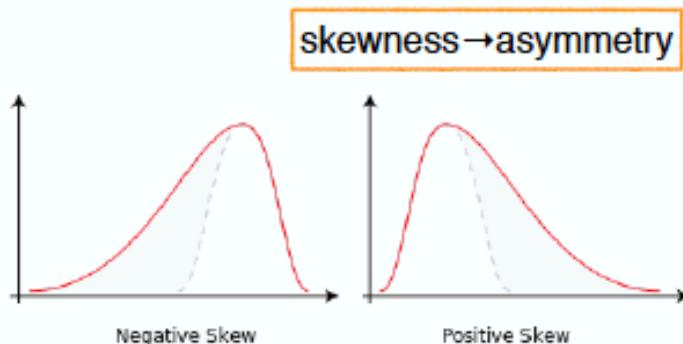
Conclusions

- Flow Fluctuation analyses put additional constraints on initial state and hydrodynamic models
 - Dial geometry in via species choice
 - Change viscosity via collision energy
 - We should try to measure the same quantities!!
- Fluctuations in conserved quantities
 - Critical Point determination
 - Connection with statistical models
 - Lattice QCD \longleftrightarrow Data connection

Back-Up

Higher-order fluctuation

- ◆ Moments and cumulants are mathematical measures of “shape” of a distribution which probe the fluctuation of observables.
 - ✓ Moments: mean (M), standard deviation (σ), skewness (S) and kurtosis (κ).
 - ✓ S and κ are *non-gaussian* fluctuations.



- ✓ Cumulant \Leftrightarrow Moment

$$\langle \delta N \rangle = N - \langle N \rangle$$

$$C_1 = M = \langle N \rangle$$

$$C_2 = \sigma^2 = \langle (\delta N)^2 \rangle$$

$$C_3 = S\sigma^3 = \langle (\delta N)^3 \rangle$$

$$C_4 = \kappa\sigma^4 = \langle (\delta N)^4 \rangle - 3 \langle (\delta N)^2 \rangle^2$$

- ✓ Cumulant : additivity

$$C_n(X + Y) = C_n(X) + C_n(Y)$$

→ proportional to volume

Symmetric and Asymmetric Cumulants

Cumulants can be used to construct higher order observables with different sensitivities to different processes

- Symmetric Cumulants prove correlations between magnitudes of harmonics of different order

(Phys.Rev.C.89, 064904)

$$sc_{n,m}\{4\} = \langle v_n^2 v_m^2 \rangle - \langle v_n^2 \rangle \langle v_m^2 \rangle$$

- Asymmetric Cumulant is also sensitive to the correlation between harmonics

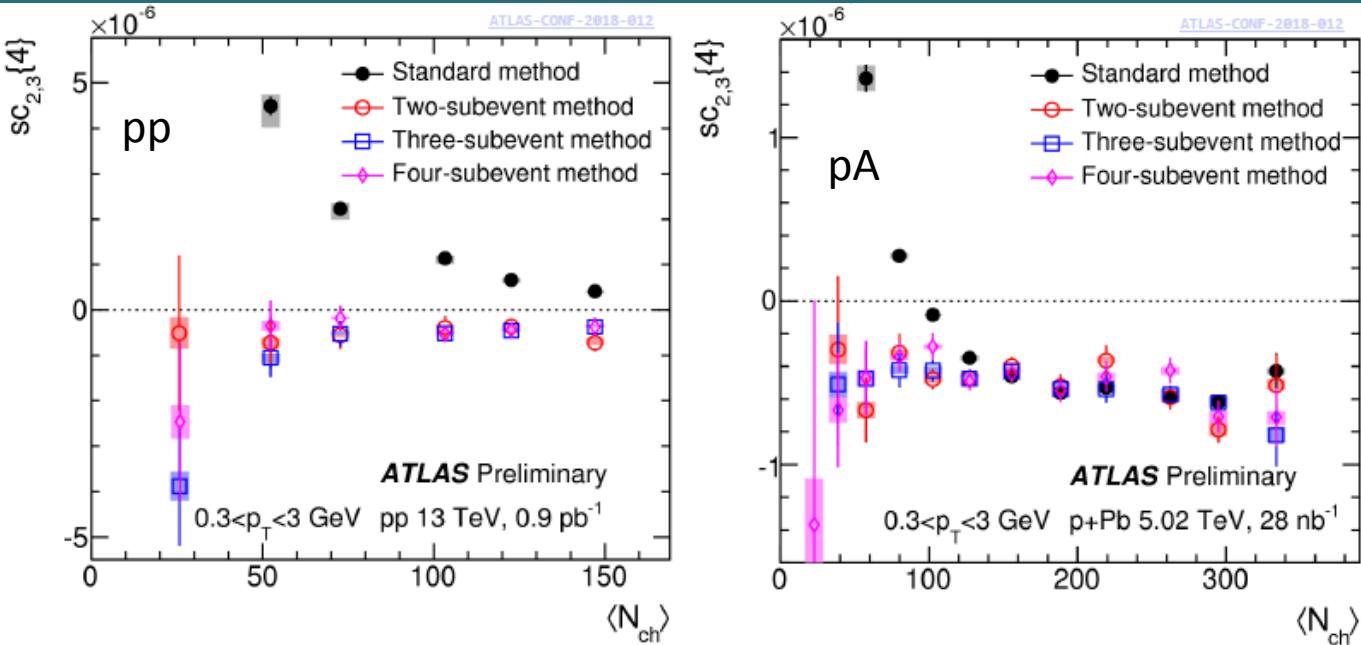
(Phys.Rev.C.90, 024905)

$$ac_2\{3\} = \langle v_2^2 \cos 4(\phi_2 - \phi_4) \rangle$$

Symmetric Cumulants

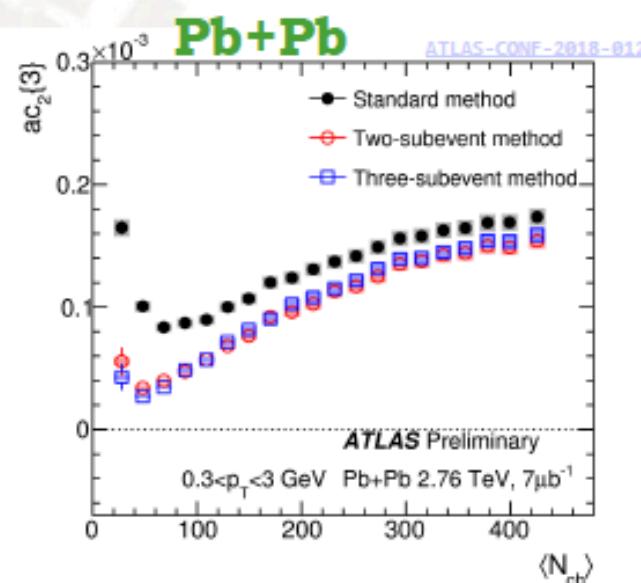
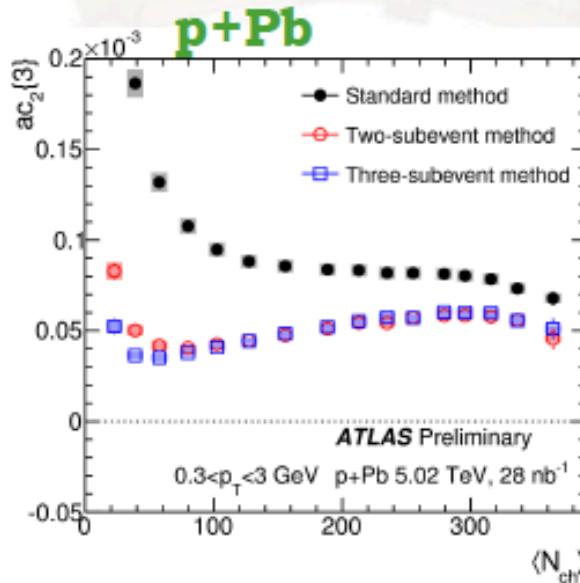
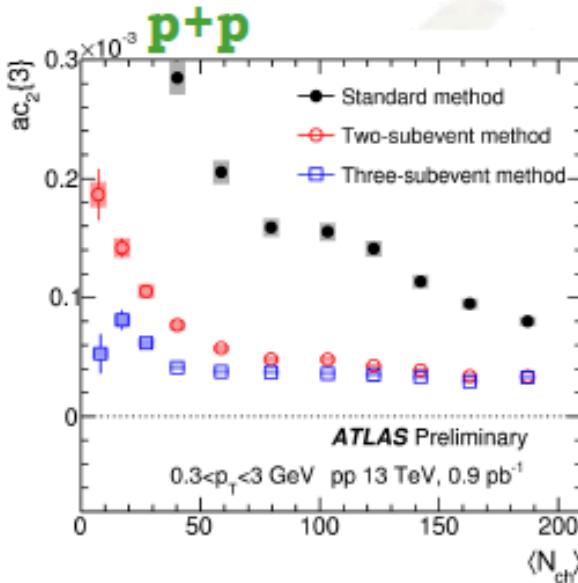
Standard method is dominated by non-flow in pp → subevent method

Anti-correlation between v_2 and v_3
Correlation pattern is the same for
 $0.5 < p_T < 5 \text{ GeV}$



Above $\langle N_{ch} \rangle = 140$ all methods give consistent results → Long Range Correlations

Asymmetric Cumulants



Asymmetric cumulants measure the positive correlation between v_2 and v_4
Do all methods converge at high $\langle N_{ch} \rangle$?

Testing Hydrodynamics via System Geometry

Sylvia Morrow, Tuesday 15/05/2018, 11:10

arXiv:1805.02973, submitted to Nature Physics

Hydrodynamics translates initial geometry into final state

Test hydro hypothesis by varying initial state

	ε_2	ε_3
p+Au	0.24	0.16
d+Au	0.57	0.17
$^3\text{He}+\text{Au}$	0.48	0.23

$$\varepsilon_2^{\text{p+Au}} < \varepsilon_2^{\text{d+Au}} \approx \varepsilon_2^{\text{He+Au}}$$

$$\varepsilon_3^{\text{p+Au}} \approx \varepsilon_3^{\text{d+Au}} < \varepsilon_3^{\text{He+Au}}$$

