



USP



Collective effects in nuclear collisions: theory overview

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Outline

- Hydrodynamic behavior in “simple” systems
- Relativistic hydrodynamics in heavy ion collisions
- Far-from-equilibrium hydrodynamics
- Upcoming challenges
- Conclusions

Hydrodynamic behavior in “simple” many-body systems



The ubiquitousness of fluid dynamics

Based on conservations laws + large separation of length scales

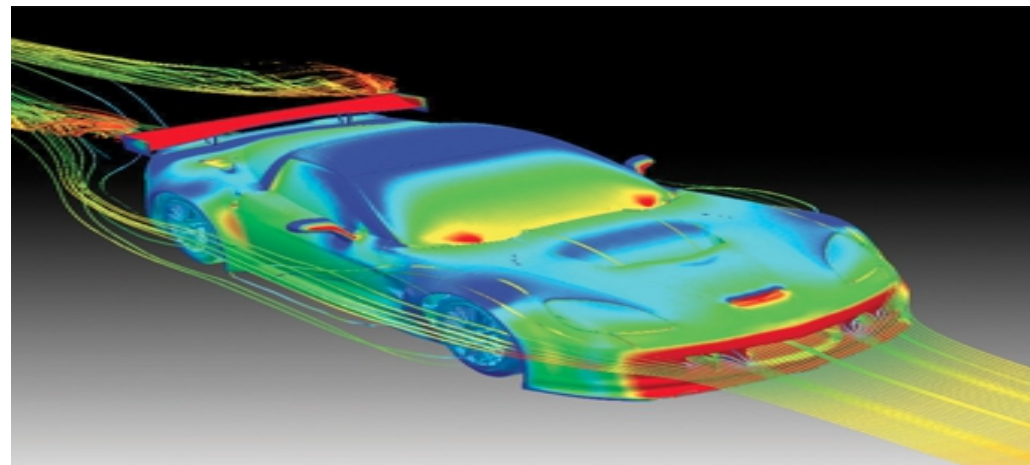
Separation of scales \rightarrow macroscopic: L microscopic: ℓ

Knudsen number:

$$K_N \sim \frac{\ell}{L} \ll 1$$



FLUID



$$L \sim 1 \text{ m}$$

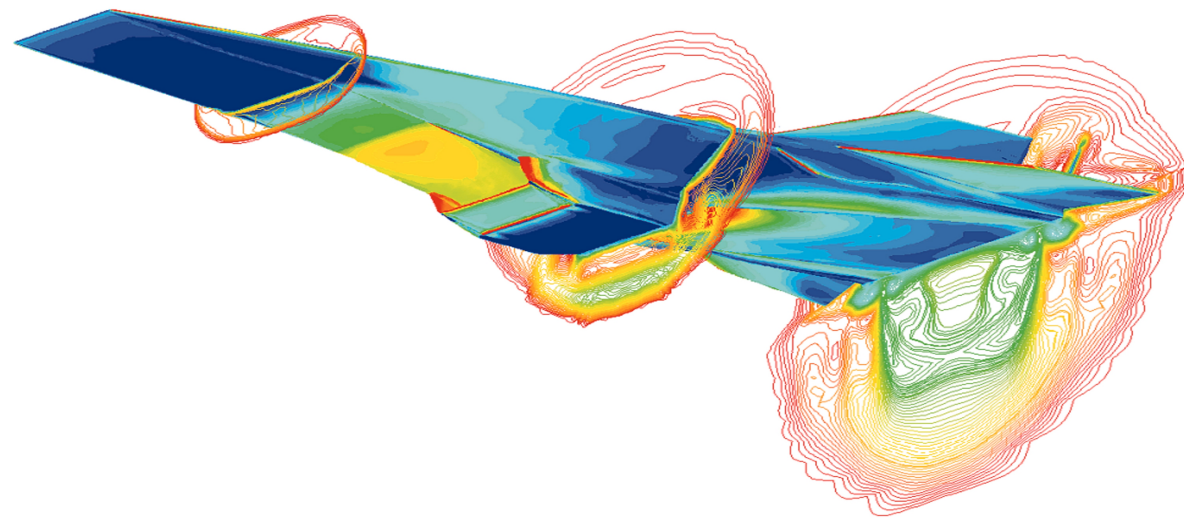
$$\ell \sim 10^{-7} \text{ m}$$

What are the limits of fluid dynamic behavior?

Macroscopic: gradient of velocity field $\rightarrow \partial V \sim 1/L$

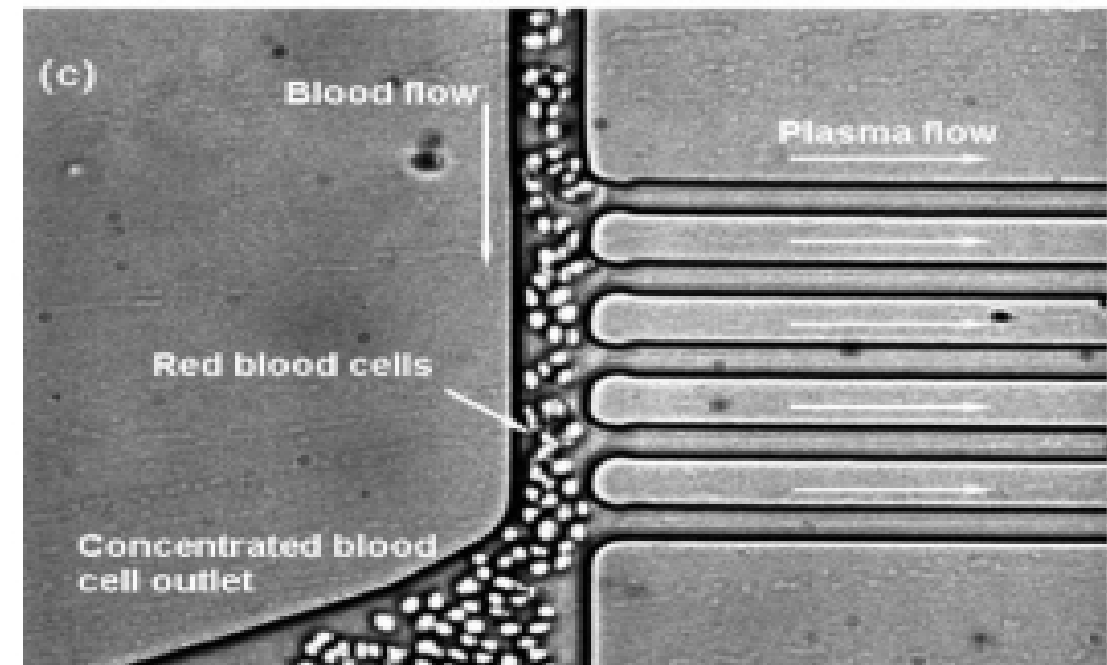
Microscopic scale (mean free path) $\rightarrow \ell_{mfp} \sim 1/(n\sigma)$

Rarefied gases $\rightarrow K_N \sim \frac{\ell}{L} \sim 1$



High-altitude flights

$L \sim 10^{-6}$ m

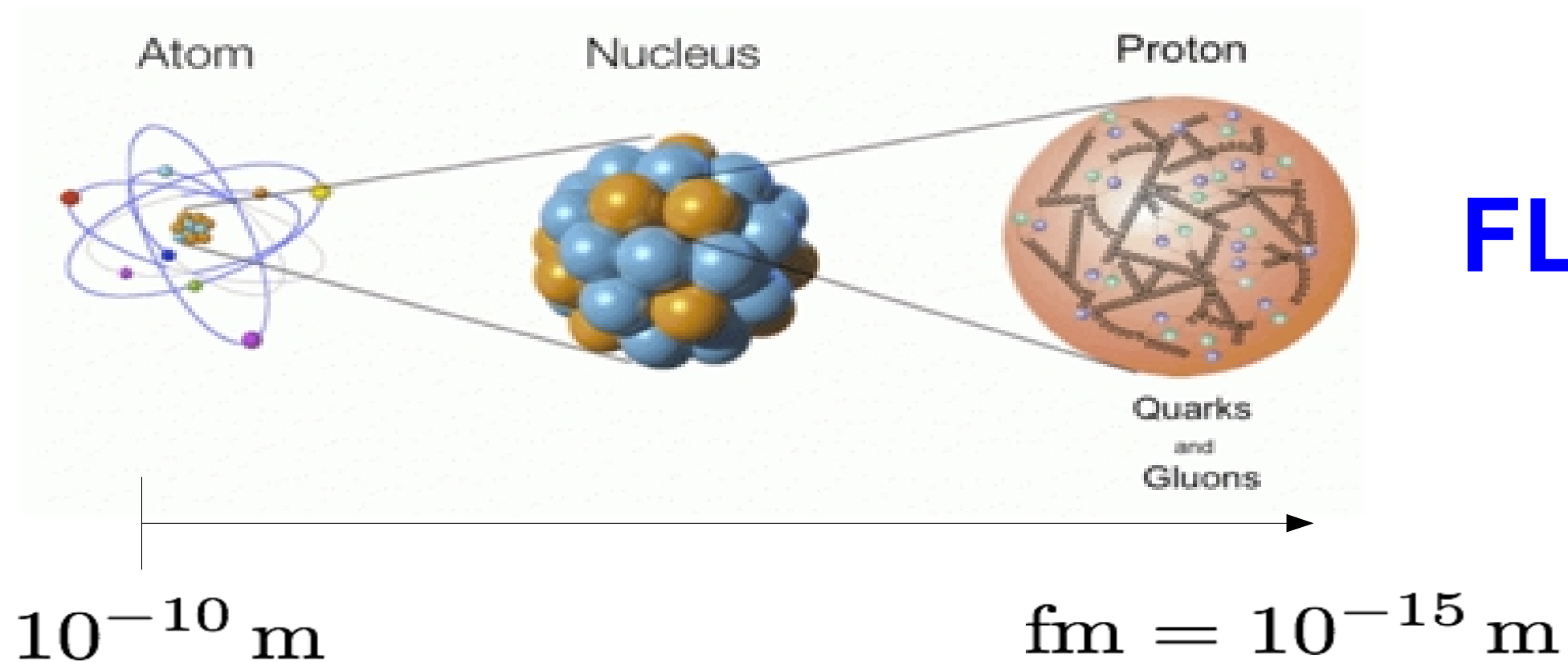


Yang et al, Lab on Chip, 2006

Microfluidic devices in medicine

The ultimate frontier in fluid dynamics

“Macro” scales → nuclear/particle physics



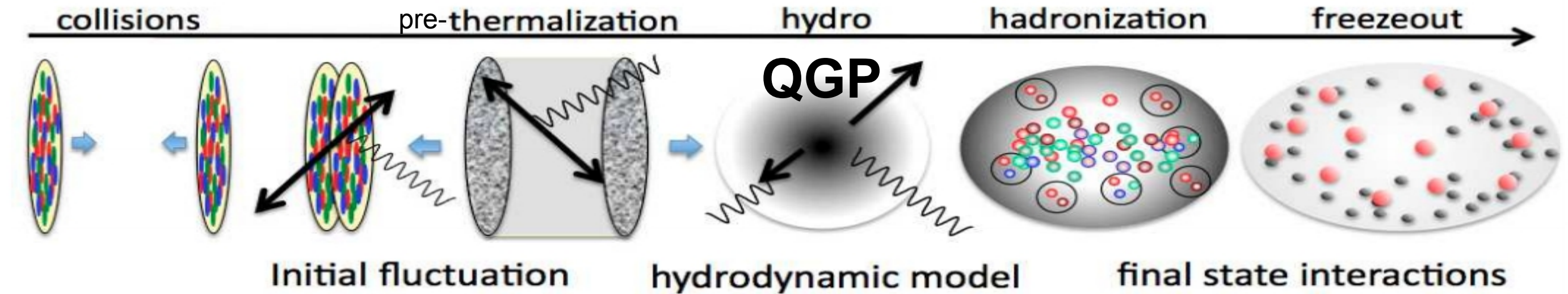
FLUID??

- At its most fundamental level: Collectivity in heavy ion collisions



Quantum mechanics + Special relativity + Many-body effects

Standard modeling of heavy ion collisions



Nonaka, Asakawa, 1204.4795

Initial state physics:

$$T^{\mu\nu} \Big|_{\tau_0} \quad J^\mu \Big|_{\tau_0}$$

- QCD classical fields/evolution
- Initial conditions

Fluid dynamics:

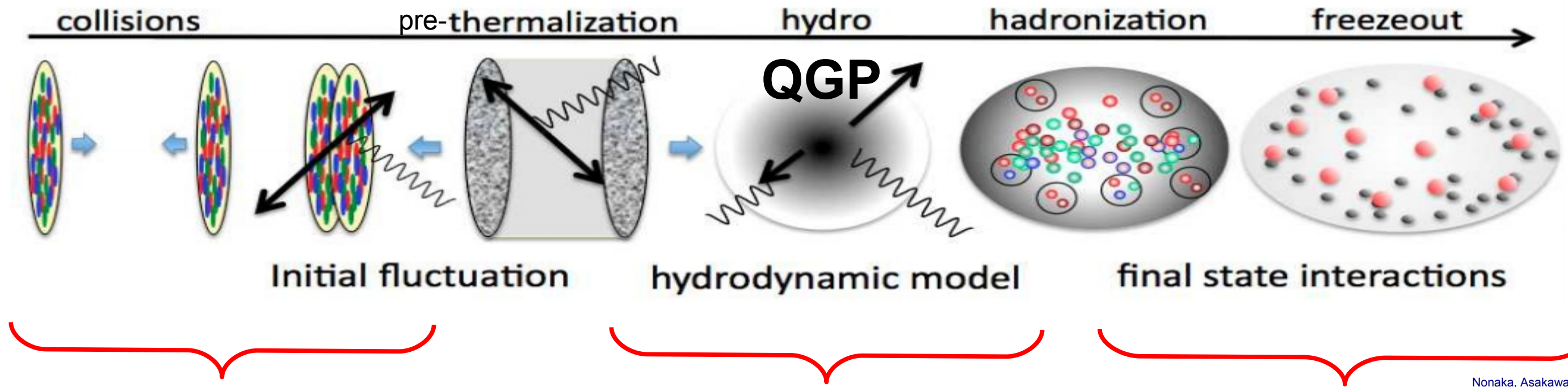
$$\nabla_\mu T^{\mu\nu} = 0 \quad \nabla_\mu J_B^\mu = 0$$

- Transport coefficients
- QCD EOS
- Conserved charges (B,S,Q)

Hadronic Interactions:

- Hadronic transport
- Hadronic resonances

Standard modeling of heavy ion collisions



Nonaka, Asakawa, 1204.4795

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Hadronic Interactions:

- Hadronic transport
- Hadronic resonances

This talk: Relativistic hydrodynamics in heavy ion collisions*

[Focus on “large” systems](#)

* For related topics, see for instance

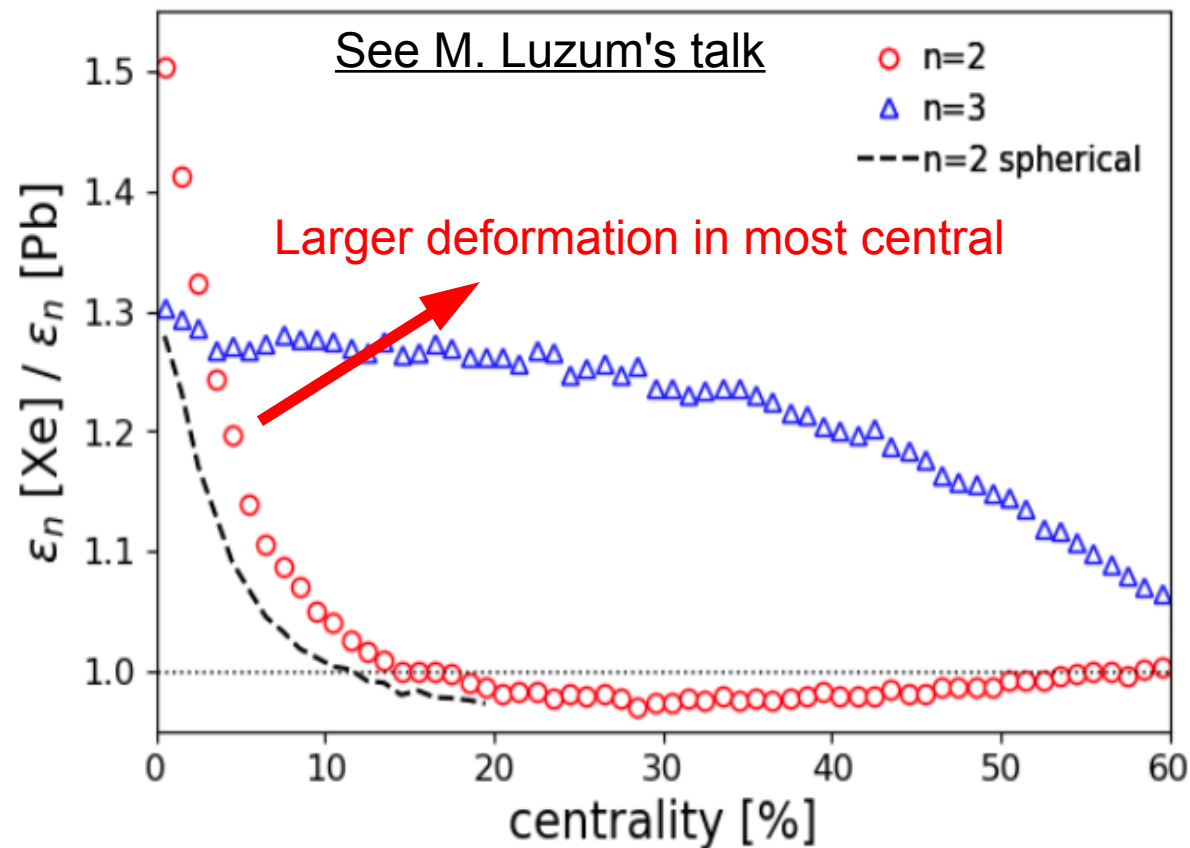
- Chiral magnetic effect + chirality → [Z. Tu's talk](#)
- Polarization + chiral → [F. Becattini's talk](#)
- Details about initial condition modeling → [A. Mazeliauskas' talk](#)
- Collectivity in small systems/different approaches → [M. Strickland's talk](#)
- Critical phenomena + hydrodynamics → [Y. Yin's talk](#)
- Medium response to jets → [Y. Tachibana's talk](#)
- EM probes at multiple energies → [J-F Paquet's talk](#)
- Kinetic theory approach to pA and AA → [A. Kurkela's talk](#)
- Anisotropic hydrodynamics → [M. Alqahtani's talk](#)

+ MANY OTHER IMPORTANT WORKS THAT COULD NOT BE REVIEWED HERE

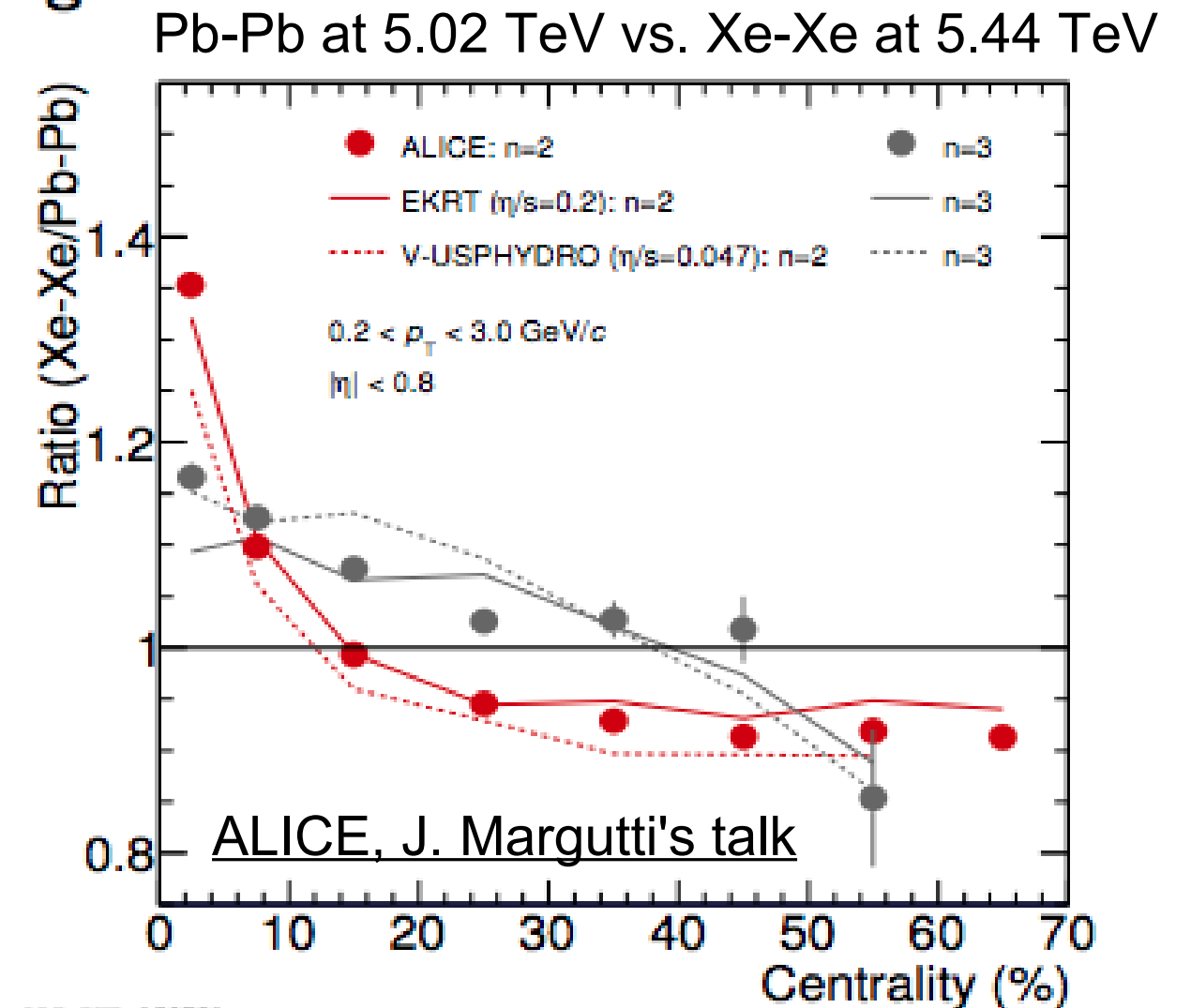
Nuclear structure effects vs. initial state in heavy ion collisions

- Xe nucleus is deformed (not exactly spherical)

Effect: $\sim 20\%$ larger $v_2\{2\}$ in central, decreasing towards peripheral

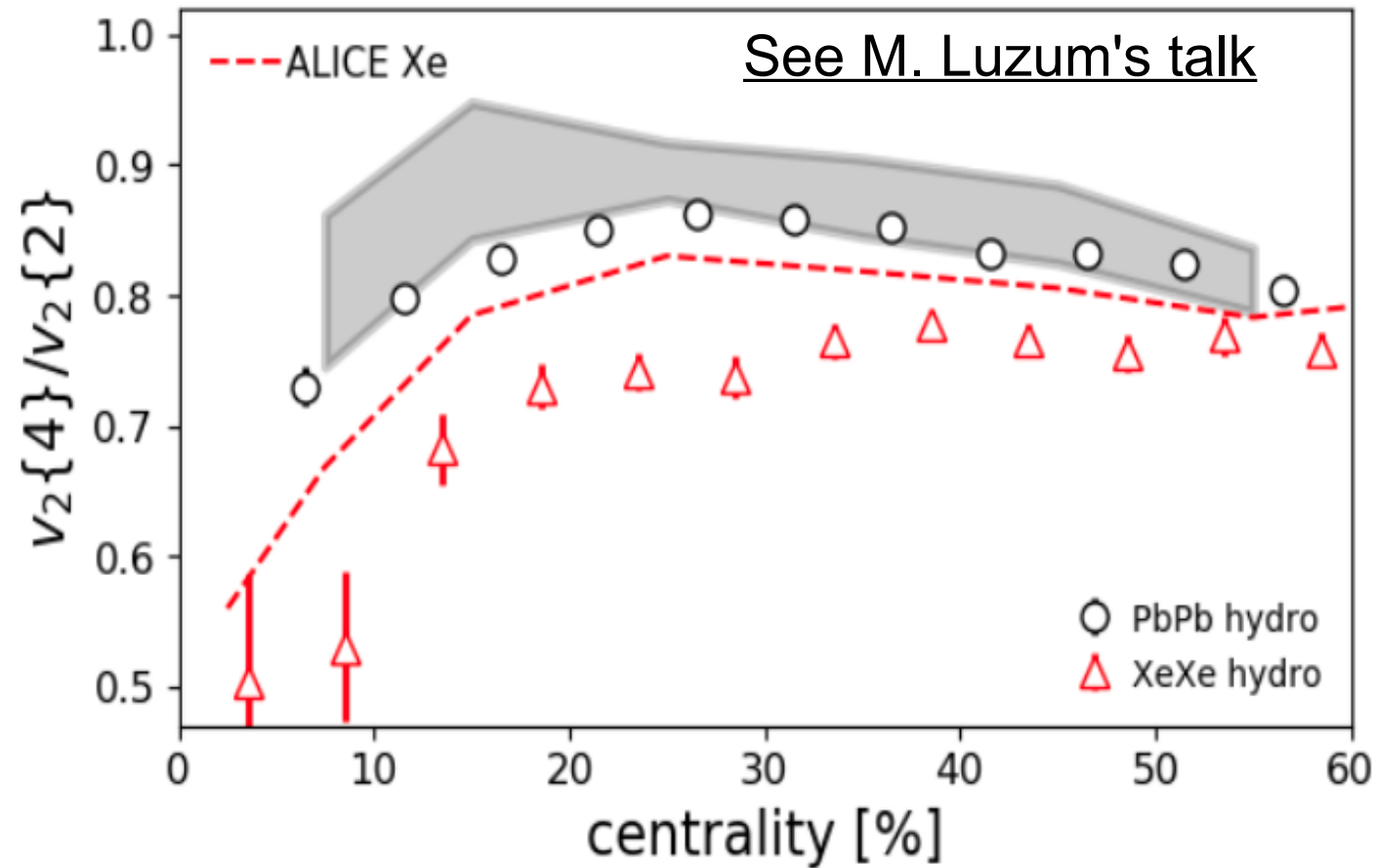


G. Giacalone et al., Phys. Rev. C 97 (2018)



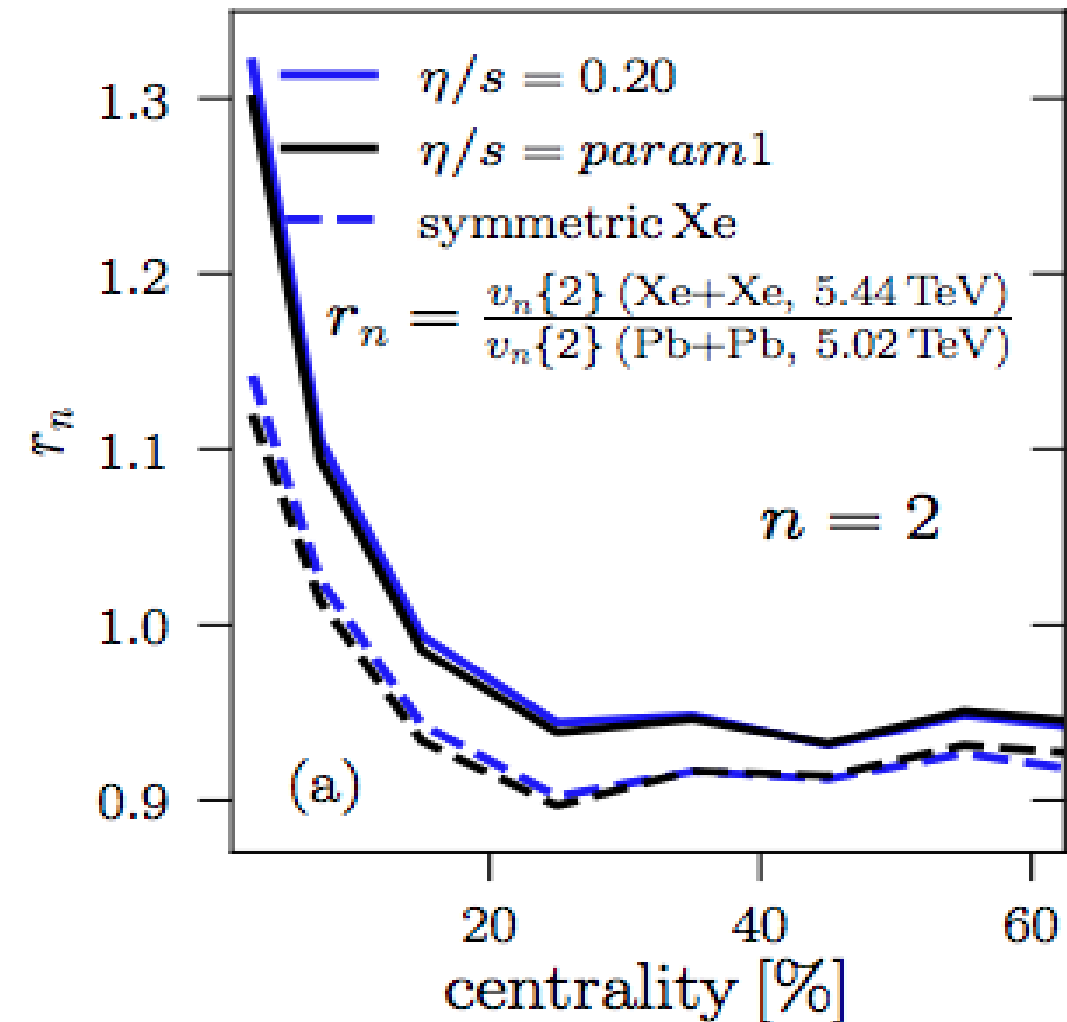
ALI-PUB-150781

Nuclear structure effects vs. initial state in heavy ion collisions



- Relative v_2 fluctuations larger in Xe-Xe
- Weak dependence on viscosity

See H. Niemi's talk



K. J. Eskola et al, PRC 97 (2018)

More comparisons with different ions at the LHC would be interesting

Dissipation in relativistic hydrodynamics

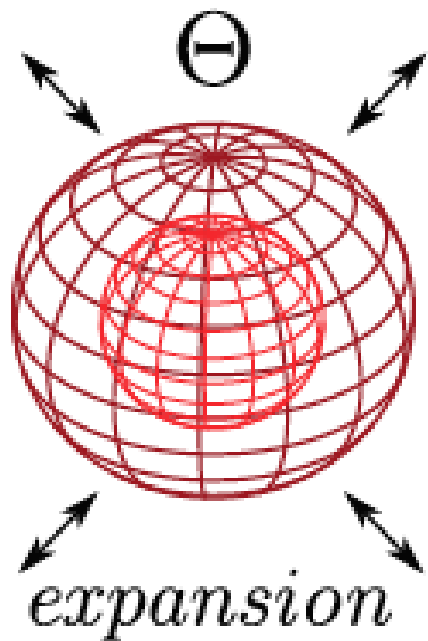
Figs. from Rezzolla and Zanotti, 2013

$$T^{\mu\nu} = T_{ideal}^{\mu\nu} + \Pi^{\mu\nu}$$

Dissipative effects

Bulk

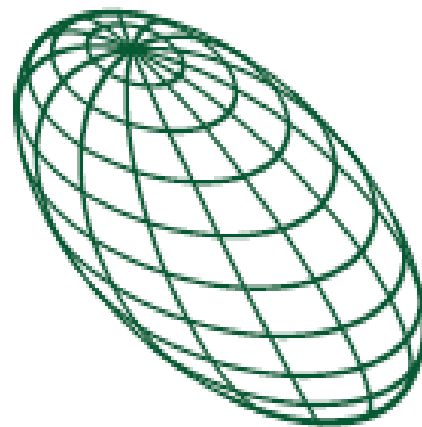
$$\Pi \sim -\zeta \Theta$$



Shear

$$\pi^{\mu\nu} \sim 2\eta\sigma^{\mu\nu}$$

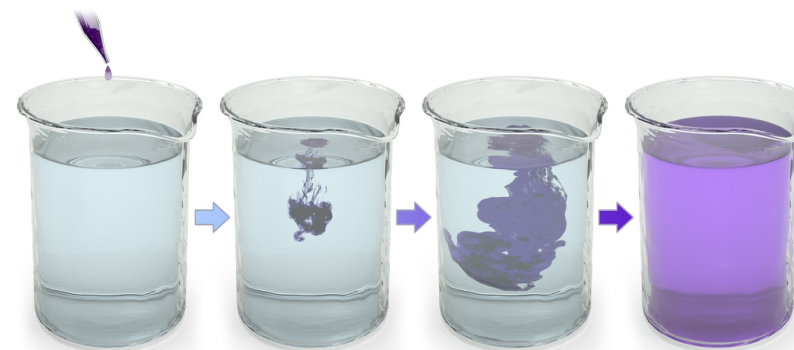
$\sigma_{\mu\nu}$



Diffusion

$$q_{\perp}^{\mu} \sim \kappa \nabla_{\perp}^{\mu} (\mu/T)$$

QCD conserved charges
(B, S, Q)



Vorticity

ω_{μ}



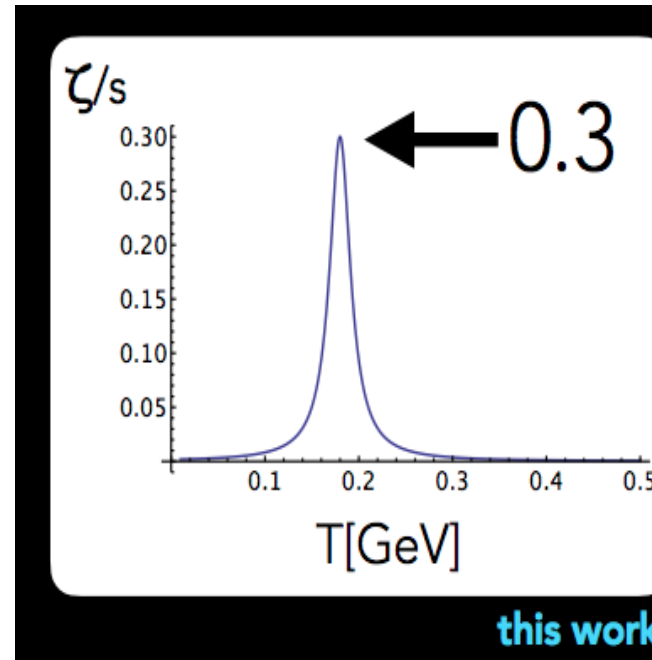
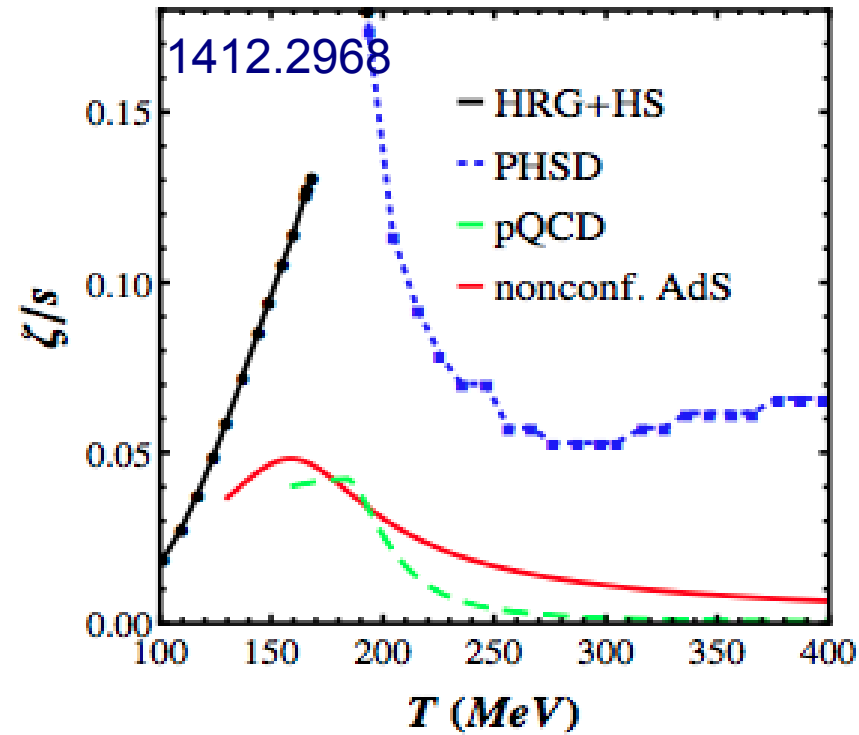
vorticity

See F. Becattini's talk

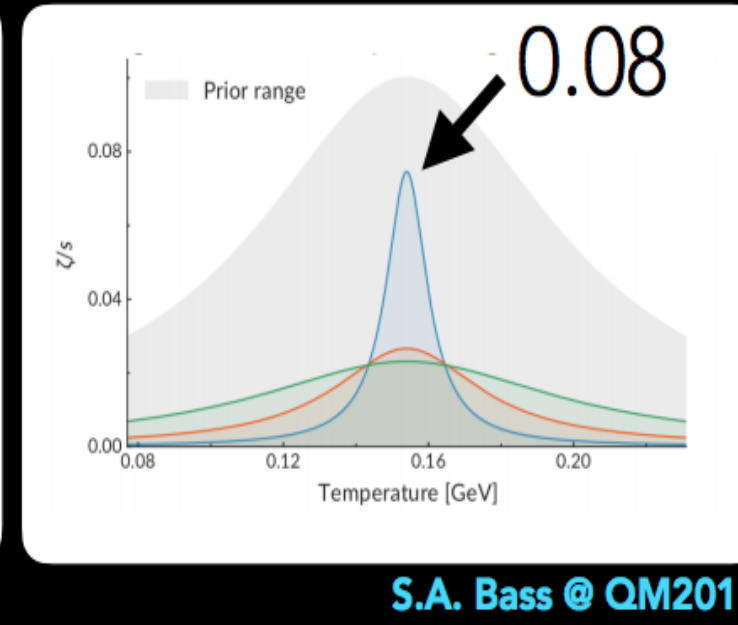
Progress on bulk viscosity in heavy ion collisions

Large theoretical uncertainty

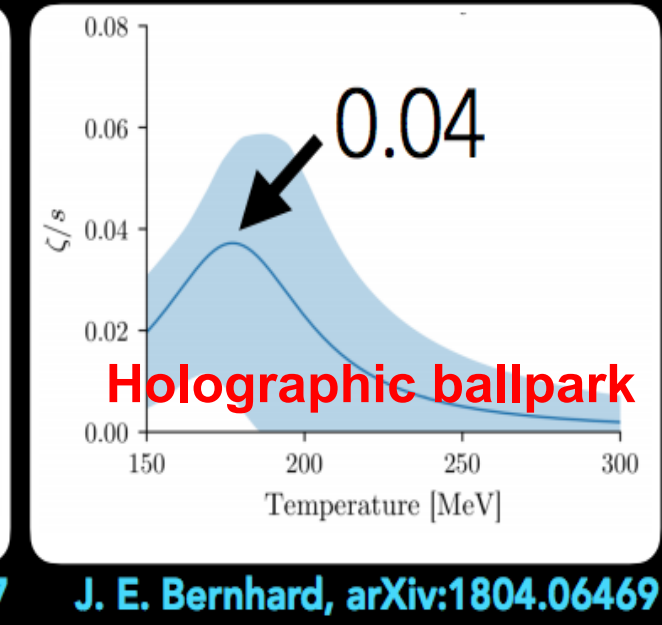
See B. Schenke's talk



this work



S.A. Bass @ QM2017



J. E. Bernhard, arXiv:1804.06469

- Peak of bulk viscosity ~ 180 MeV (feature of crossover)
- Remaining differences about the magnitude

Cavitation in pA??

1503.00531 See B. Schenke's talk

Initial conditions + δf_k^{bulk} + Details of Bayesian analysis

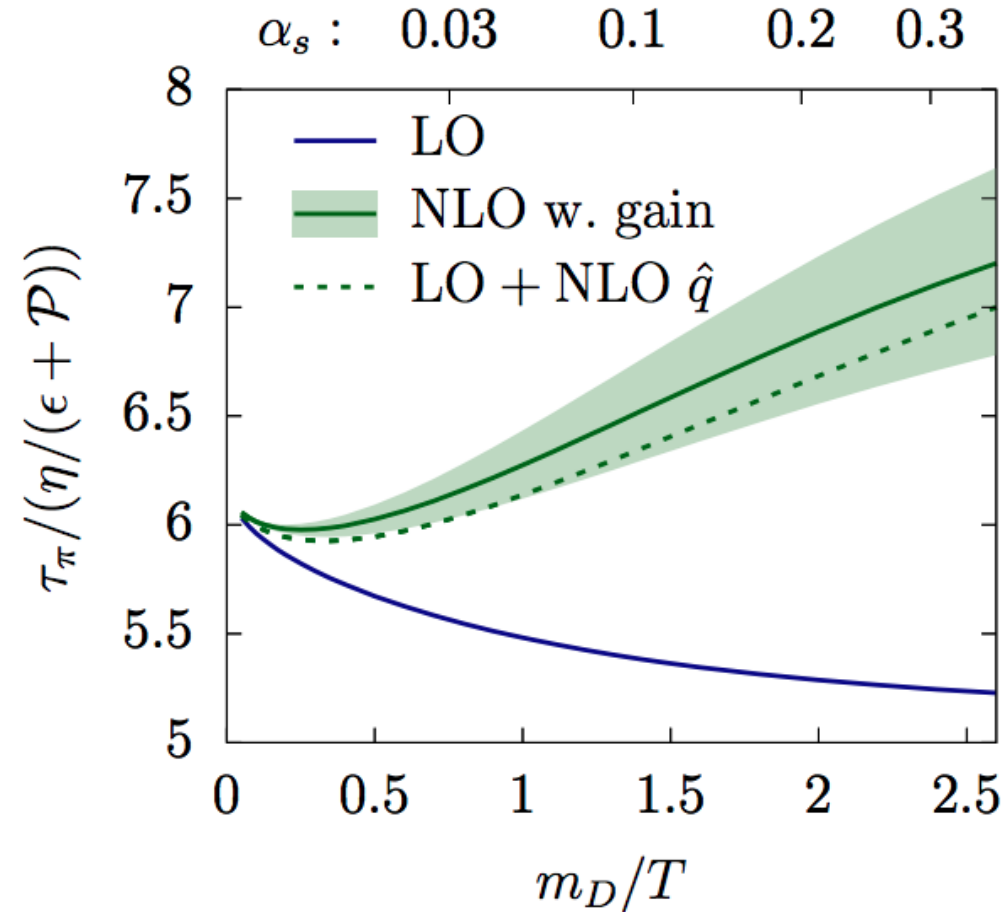
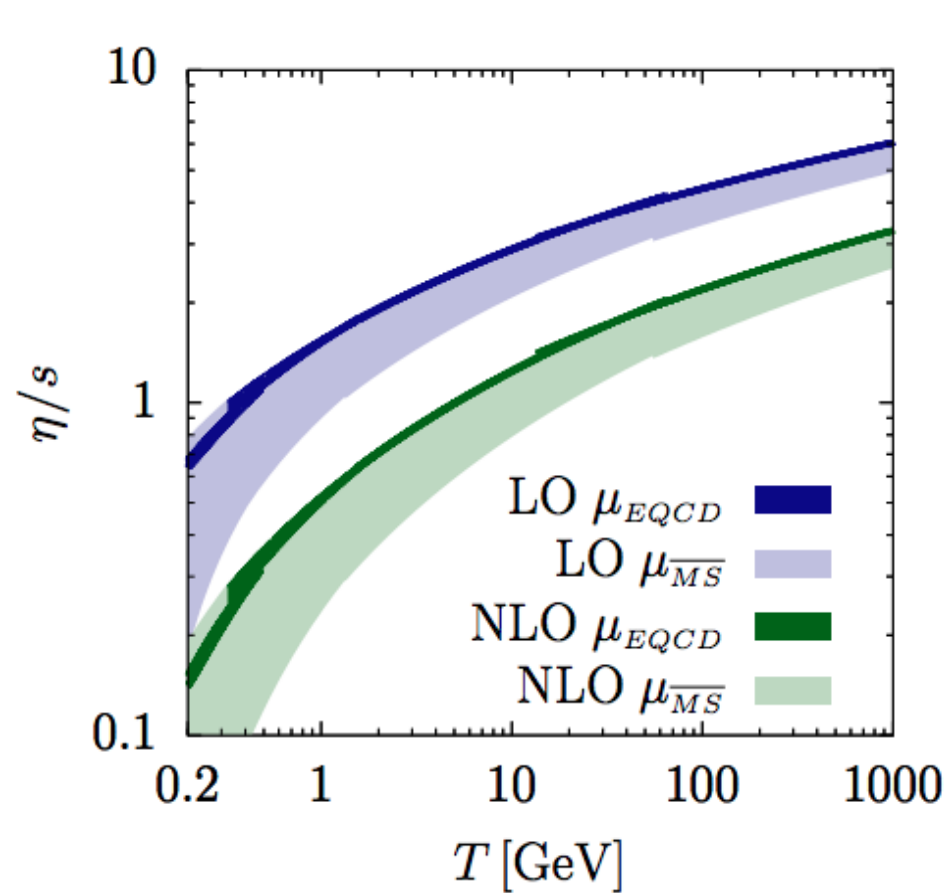
See A. Czajka's poster

See J-F Paquet @ Duke poster

Shear viscosity calculation in QCD at high T

See J. Ghiglieri's talk

Weak coupling calculation of η , τ_π in high T QCD (kinetic theory)



New bound (kinetic theory)
(ultrarelativistic particles)

$$\frac{\tau_\pi}{\eta(\epsilon + P)} \geq 5$$

Bound ~ 2 times typical
strong coupling result

(N=4 SYM and etc)

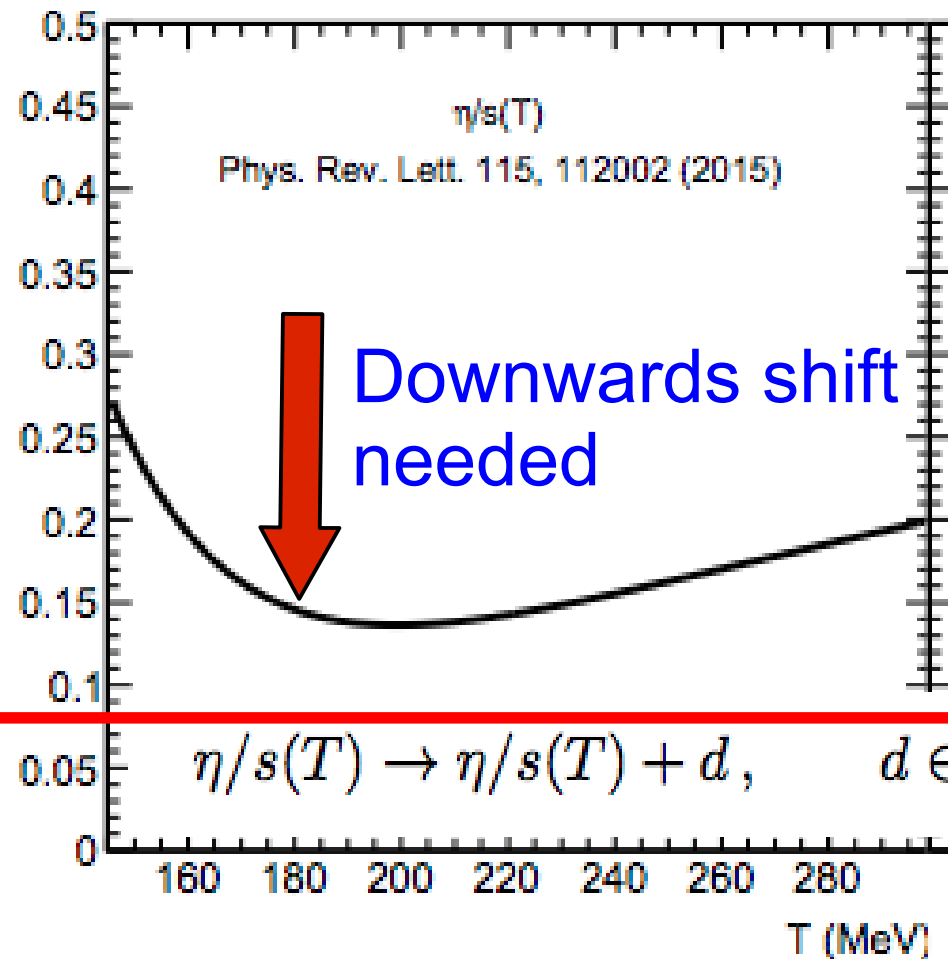
Holographic (strong coupling) transport very far from kinetic theory results

Connecting shear viscosity calculations to data

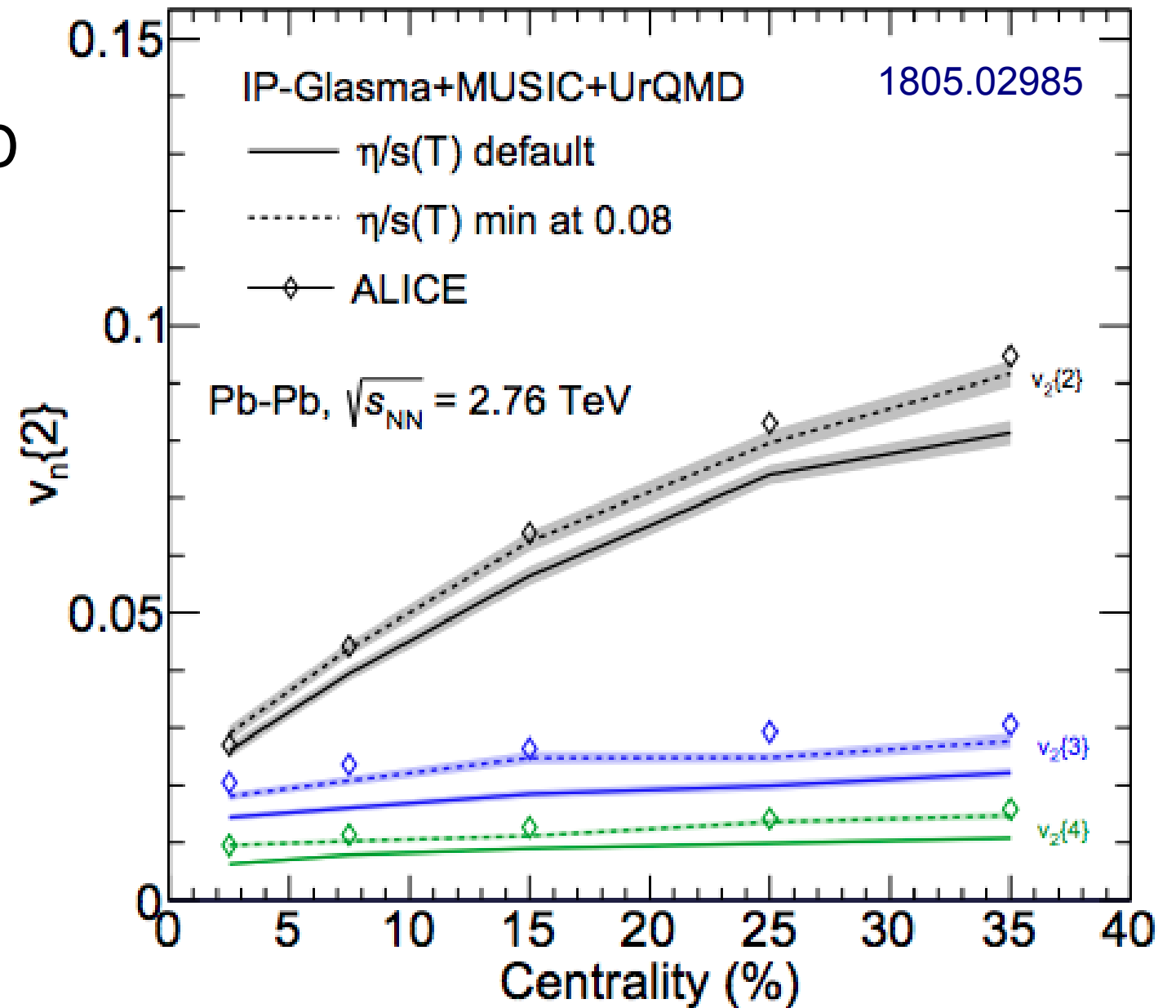
See B. Schenke's talk + A. Dubla's poster

Functional diagrammatic approach to QCD

$$\eta/s(T) = \frac{a}{\alpha_{s,HQ}^\gamma (cT/T_c)} + \frac{b}{(T/T_c)^\delta},$$



$$\eta/s(T) \rightarrow \eta/s(T) + d, \quad d \in [-0.06, 0].$$

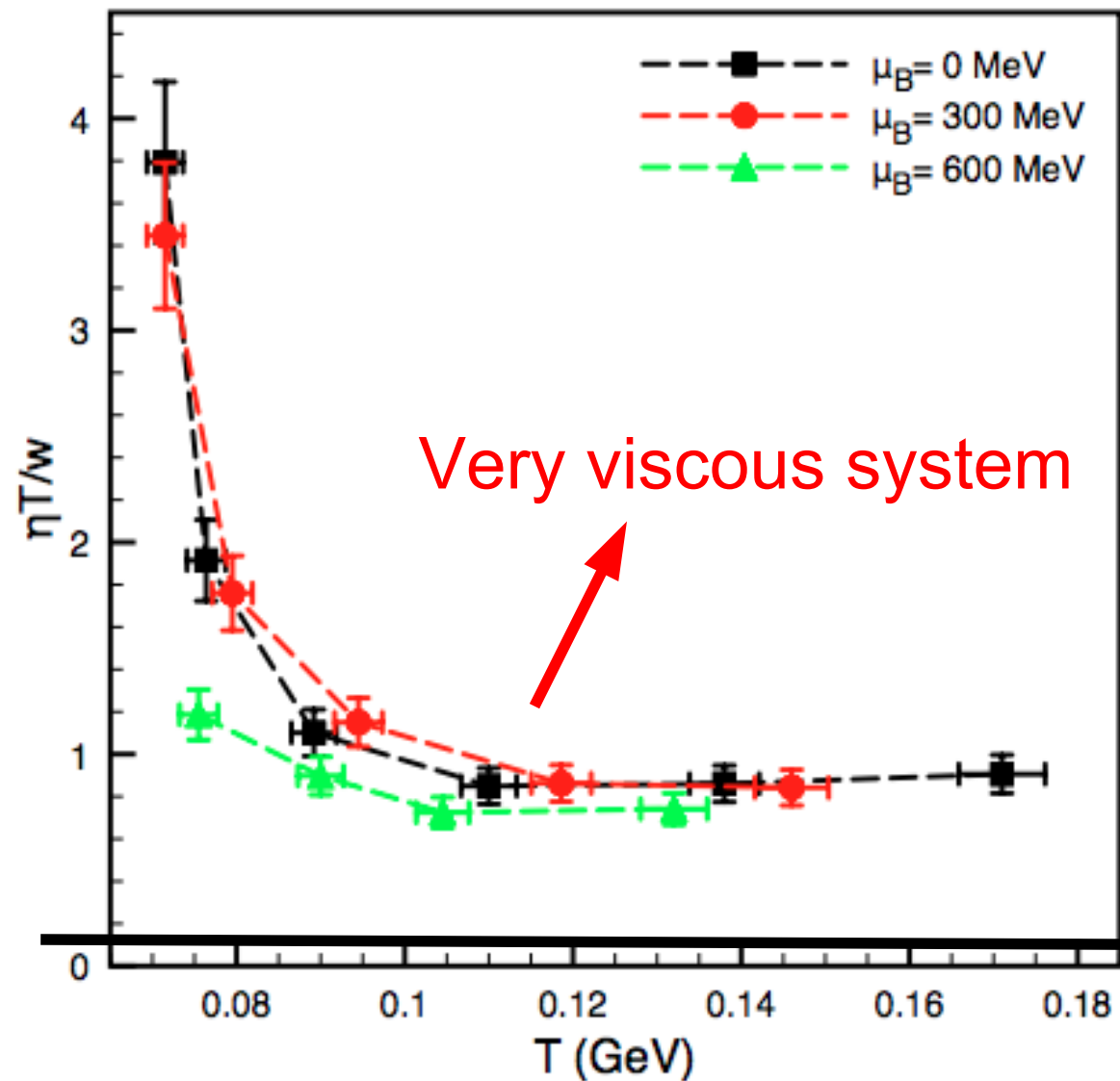


Data requires near perfect fluidity around crossover

See also C. Nonaka's poster

Shear viscosity at low temperatures (hadron gas phase)

See H. Petersen's talk, J-B Rose's talk

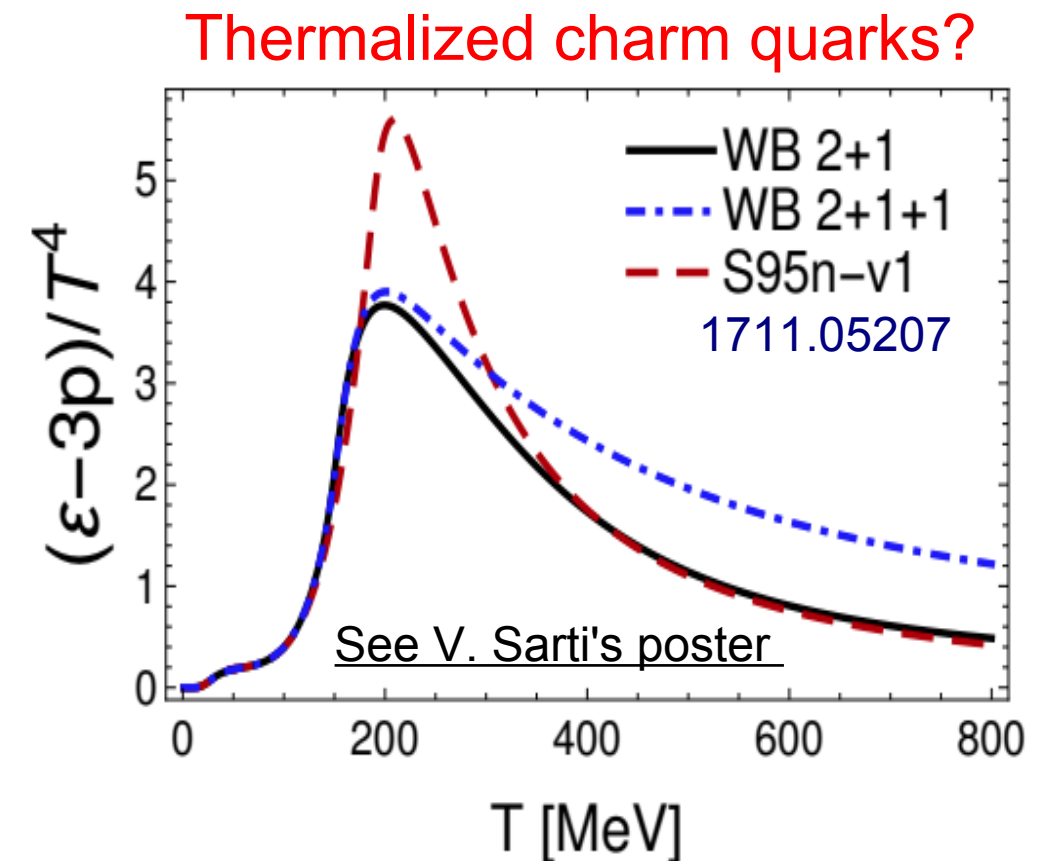
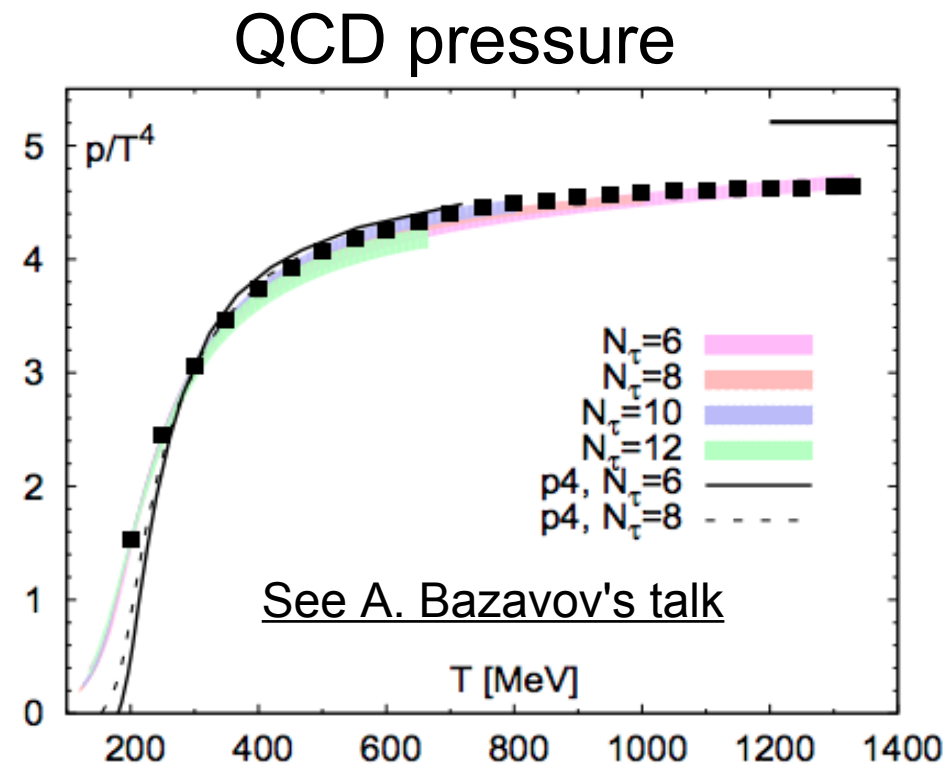


- SMASH \rightarrow agreement with Chapman-Enskog
- Details of resonance modeling matter at high T
- Very small chemical potential dependence
- This implies that viscosity is changing discontinuously from hydro models to hadronic transport

0.08

Poster by J. Hammelman
Poster by S. Ryu
Poster by R. Fries

Connection between eta/s to QCD equation of state



- Details of EOS matter for precise hydrodynamic simulations
- EOS affects the extraction of transport coefficients
- η/s can change by nearly 50%

See J. Auvinen's poster

See A. Monnai's poster

See P. Alba et al., 1711.05207

See B. Schenke's talk

Baryon diffusion

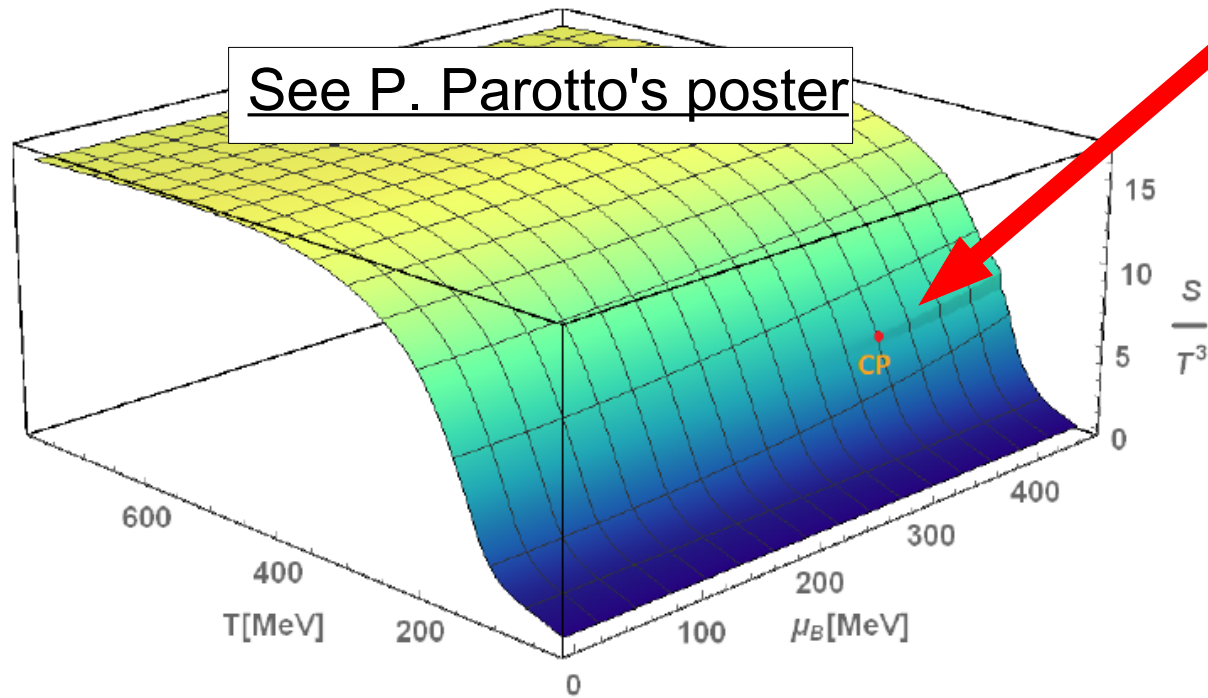


New challenges arise at large baryon density

BEST
COLLABORATION

New equation of state

See P. Parotto's poster



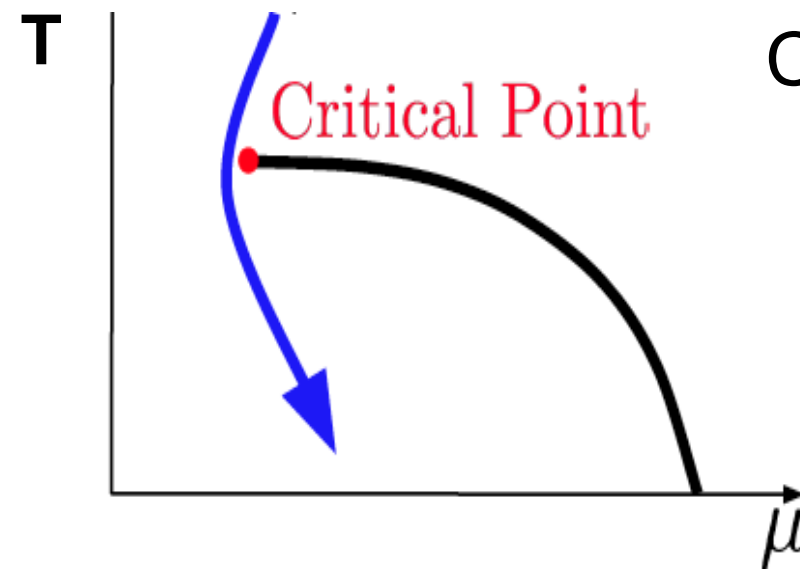
Critical point

$$\mu_{BC} = 350 \text{ MeV}$$
$$T_C \simeq 143.2 \text{ MeV}$$

- Parametrized critical point
- Compatible with lattice
- 3D Ising criticality
- Important for BES II @ RHIC

Need hydrodynamics + critical phenomena

See M. Stephanov's talk
See Y. Yin's talk



Critical fluctuations

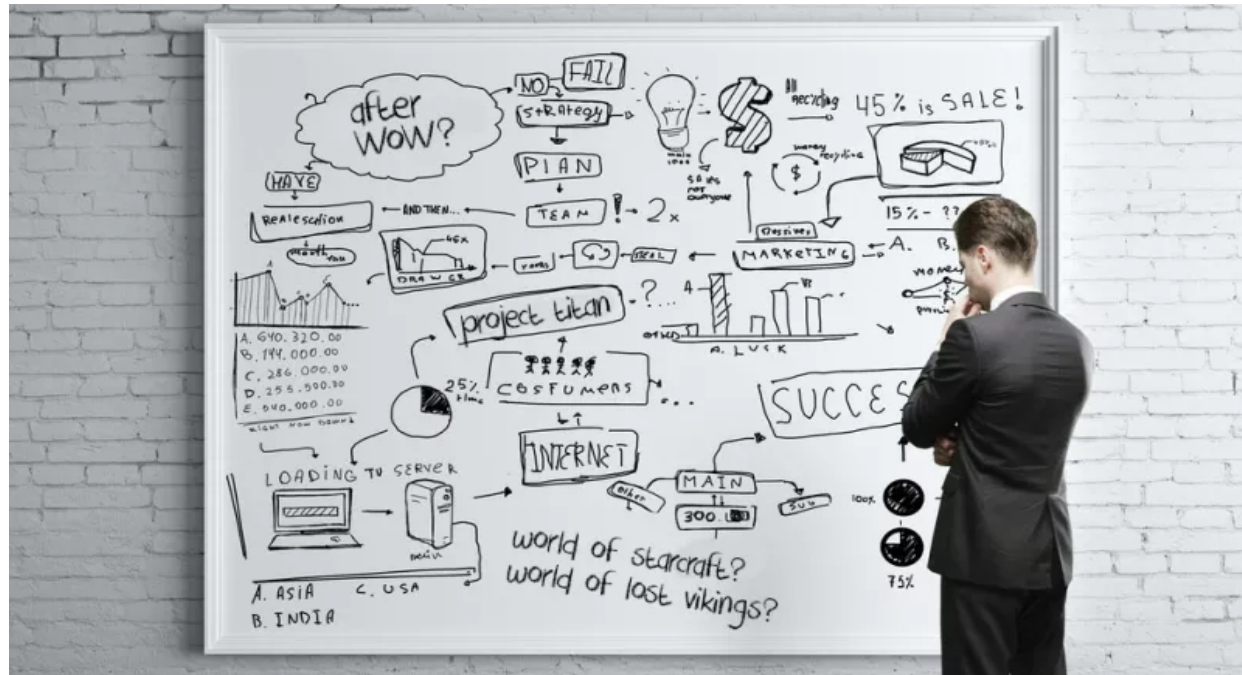
What if we miss?
Finite relaxation rate?

See F. Yan's talk

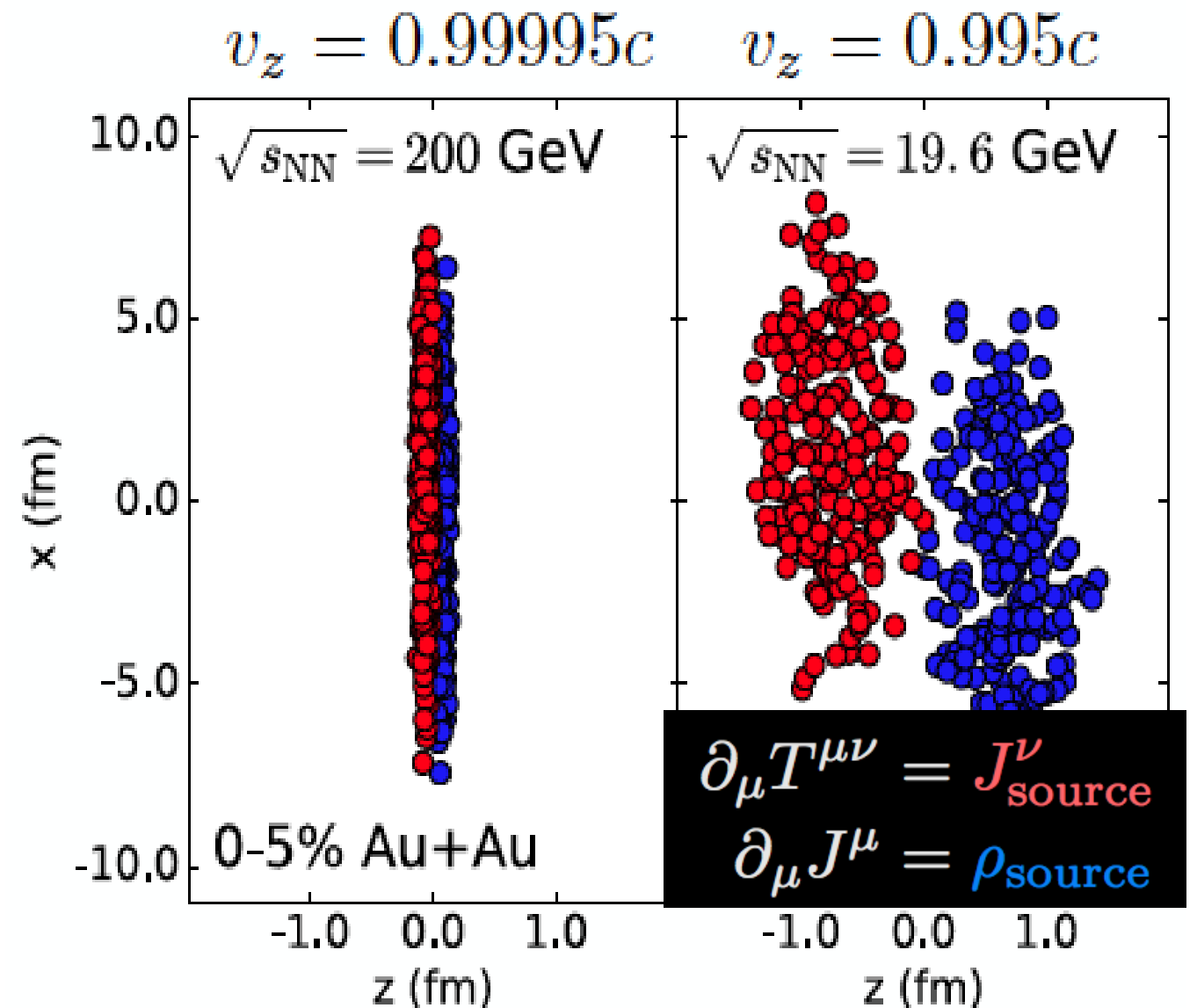
Initial conditions and hydrodynamics at low energies

- Full 3+1 event-by-event viscous hydrodynamics with baryon current effects

Back to the drawing board ...



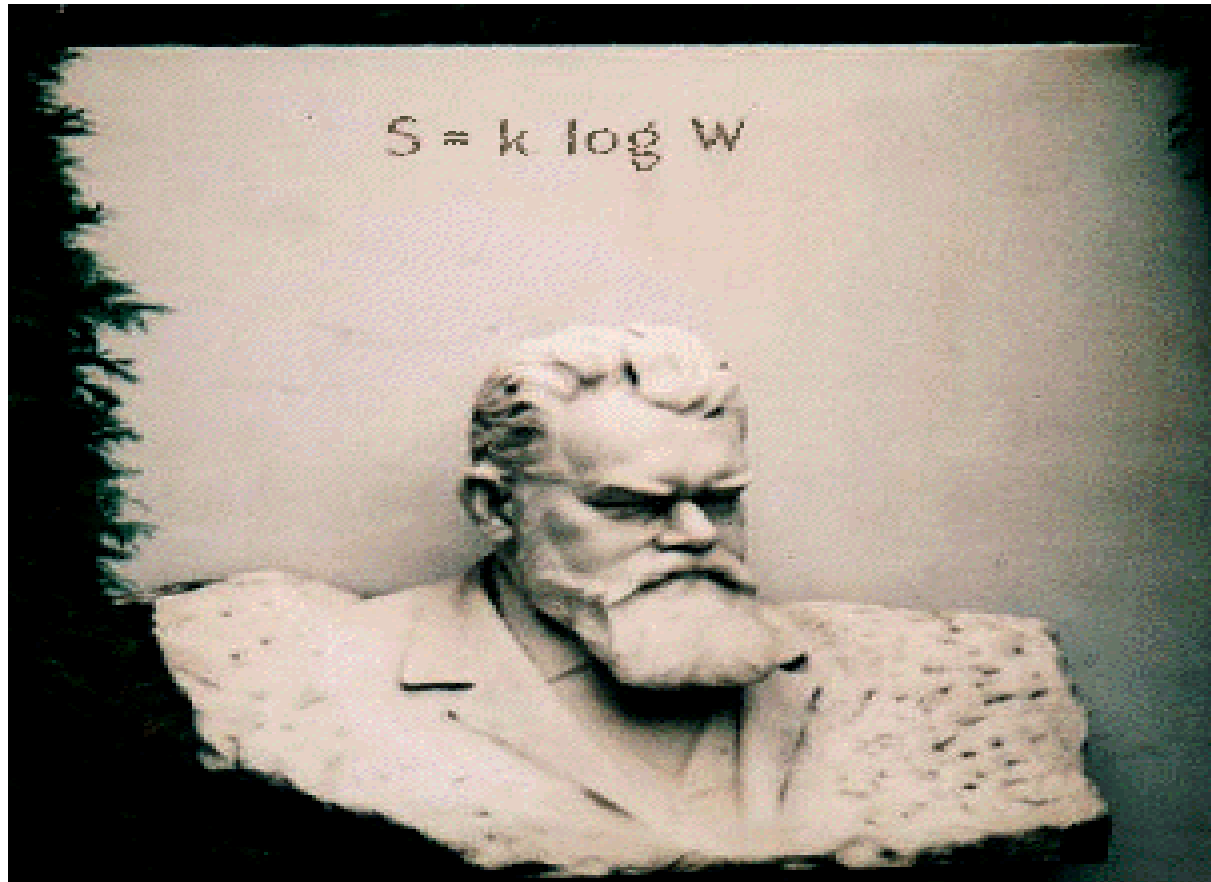
See C. Shen's talk



- Initial conditions at low \sqrt{s}
- Hydrodynamics + criticality, viscosities
- Hadronic phase at large baryon density

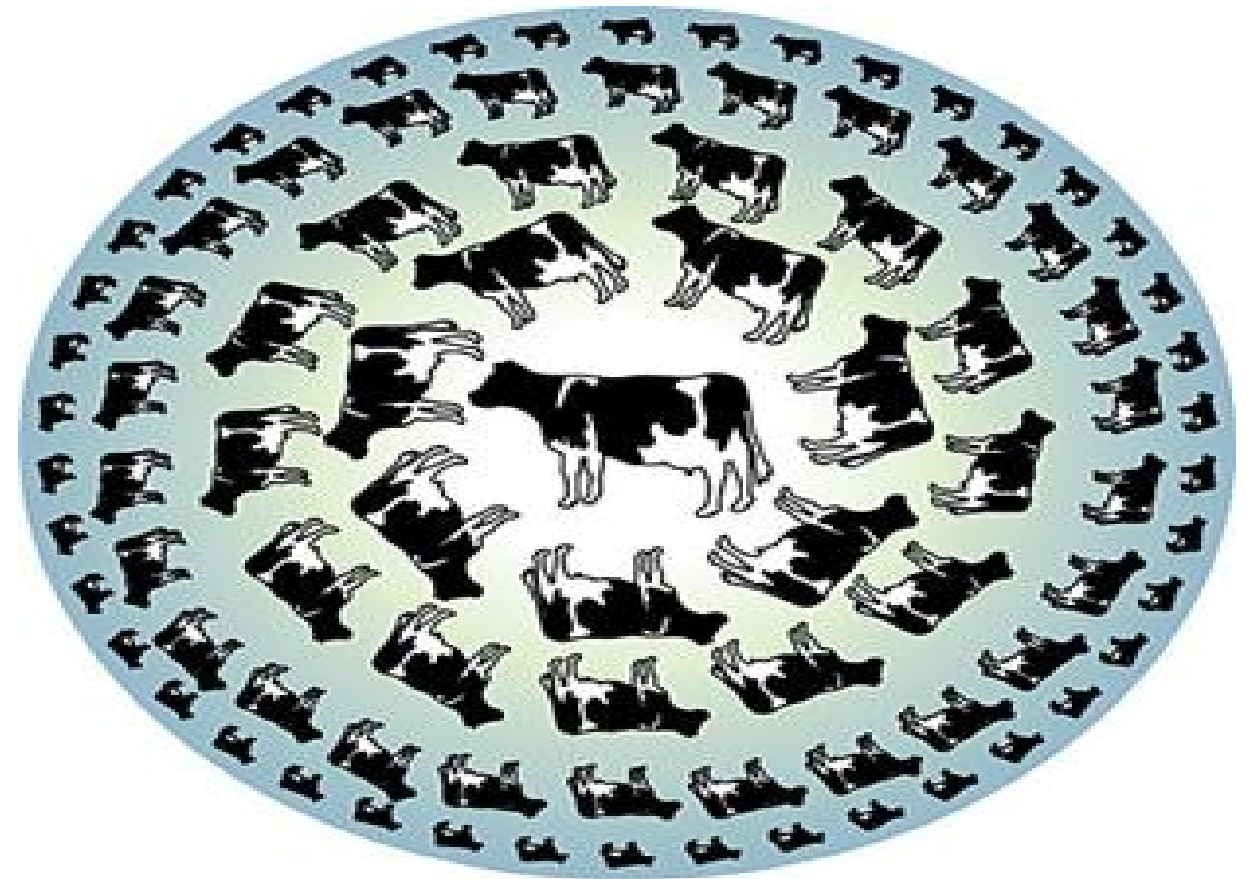
Transport coefficients: in absence of first principle calculations one resorts to

Relativistic kinetic theory



High temperature limit (weak coupling)

Holographic duality



model for strongly coupled QGP

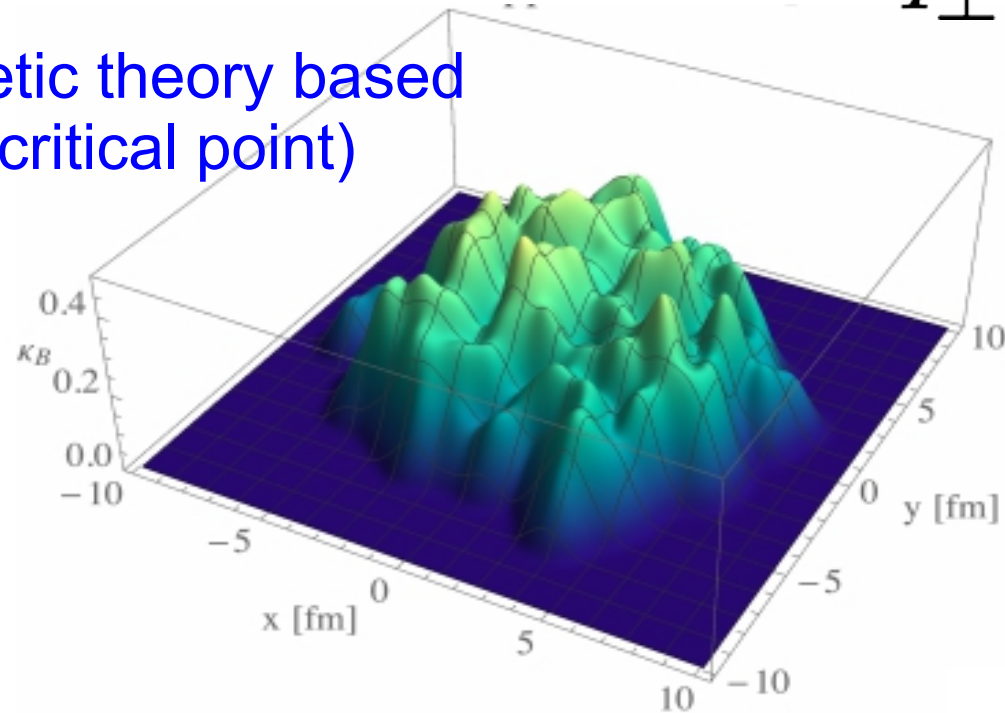
Baryon diffusion → weak vs. strong coupling

See L. Du's talk

$$q_{\perp}^{\mu} \sim \kappa \nabla_{\perp}^{\mu} (\mu/T)$$

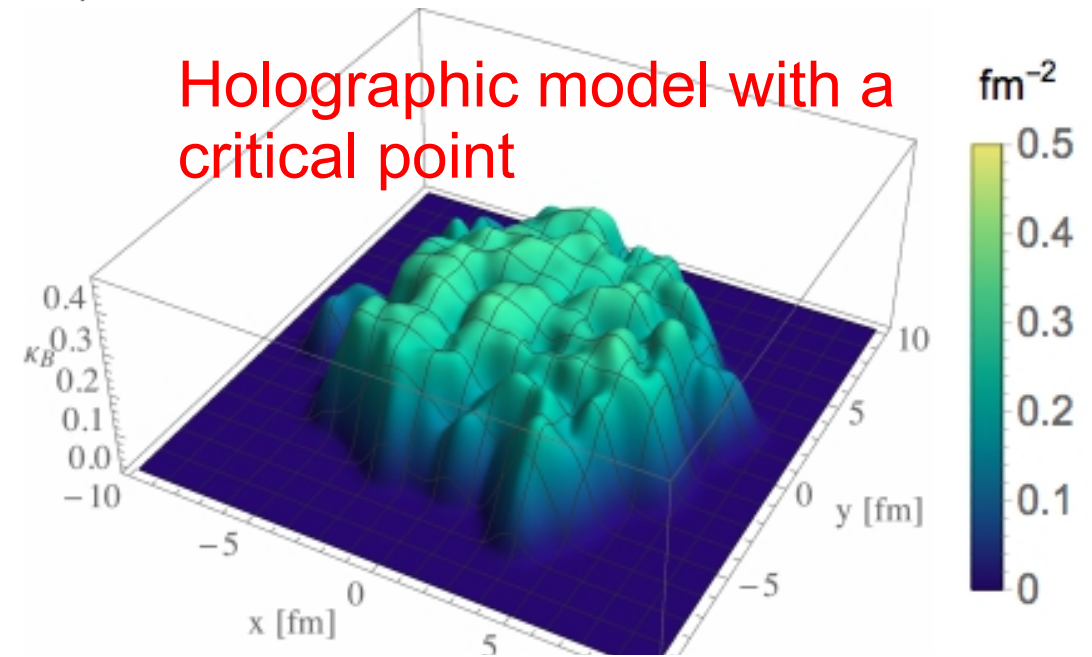
R. Rougemont et al, Phys. Rev. Lett. 115, 202301

Kinetic theory based
(no critical point)



G. S. Denicol et al, arXiv:1804.10557 [nucl-th]

Holographic model with a
critical point



Holographic model smoothes out spatial fluctuations of κ_B

Effects from other conserved
currents?

- Different conserved currents couple to each other

$$\begin{pmatrix} j_B^E \\ j_Q^E \\ j_S^E \end{pmatrix} = \begin{pmatrix} \kappa_{BB} & \kappa_{BQ} & \kappa_{BS} \\ \kappa_{QB} & \kappa_{QQ} & \kappa_{QS} \\ \kappa_{SB} & \kappa_{SQ} & \kappa_{SS} \end{pmatrix} \cdot \begin{pmatrix} \nabla^{\mu} \alpha_B \\ \nabla^{\mu} \alpha_Q \\ \nabla^{\mu} \alpha_S \end{pmatrix}$$

See C. Greiner's poster

Far-from-equilibrium hydrodynamics

From paradox to paradigm



What defines the domain of applicability of relativistic hydrodynamics?

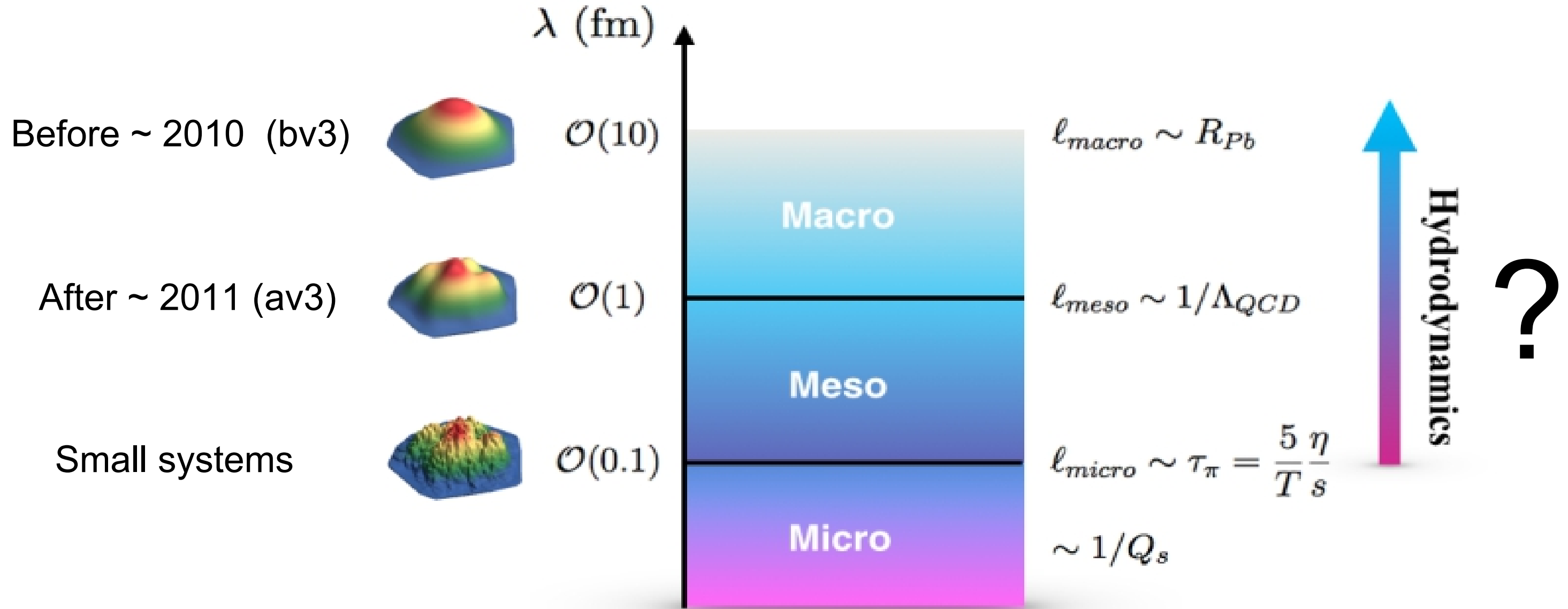


Figure from J. Noronha-Hostler, JN, M. Gyulassy, PRC 2016

What defines the domain of applicability of relativistic hydrodynamics?

For the past 100 years: Hydrodynamics has been defined as a gradient expansion

$$T^{\mu\nu} = \underbrace{T^{\mu\nu}_{ideal}}_{\text{Zero viscosity}} + \underbrace{\mathcal{O}(\partial^\mu u^\nu)}_{\text{1st order hydro Navier-Stokes}} + \underbrace{\mathcal{O}(\partial^2 u, \partial^2 T)}_{\text{2nd order hydro (BRSSS)}} + \dots$$

Zero viscosity

1st order hydro
Navier-Stokes

2nd order hydro (BRSSS)

- Well-known problems: Causality + stability

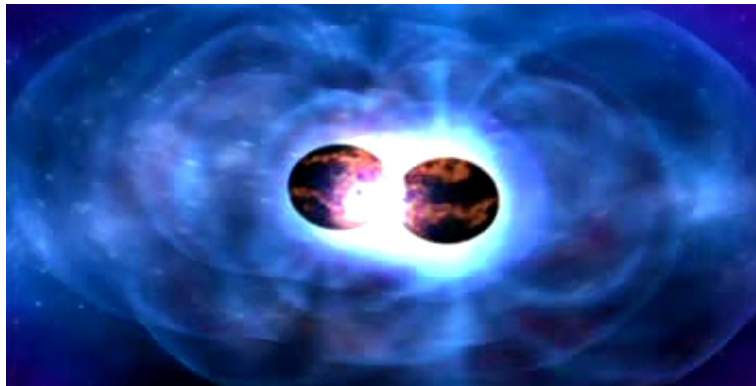
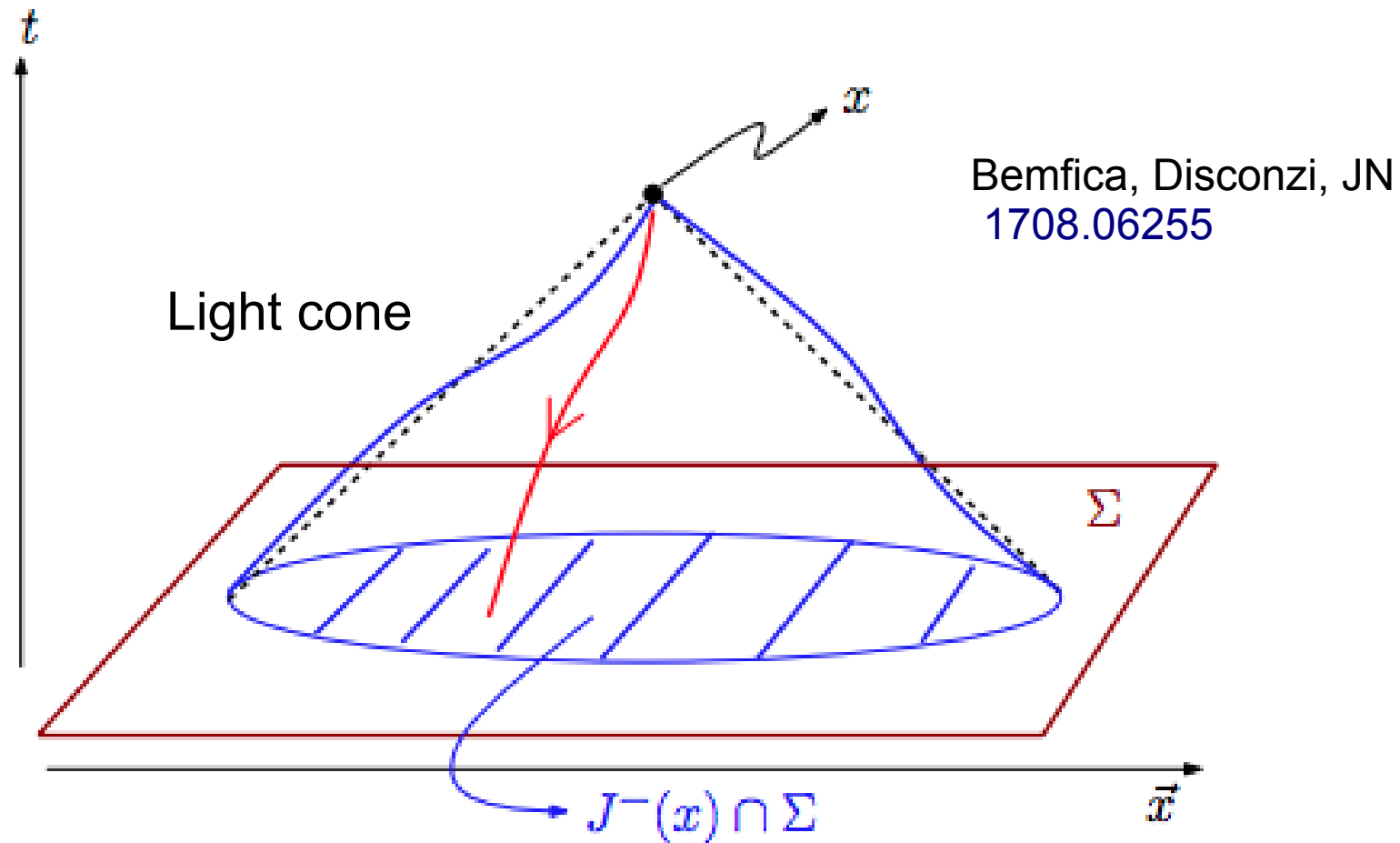
Israel, Stewart, 1970's
Hiscock, Lindblom, 1980's

See E. Grossi's poster
C. Plumberg's poster

τ_π \longrightarrow Relaxation time

Proof of causality and stability in viscous general relativistic hydrodynamics

Viscous fluid dynamics + strong gravitational fields



Viscous effects in neutron star mergers

Duez et al PRD (2004), Shibata et al. PRD (2017), Alford et al. PRL (2018)

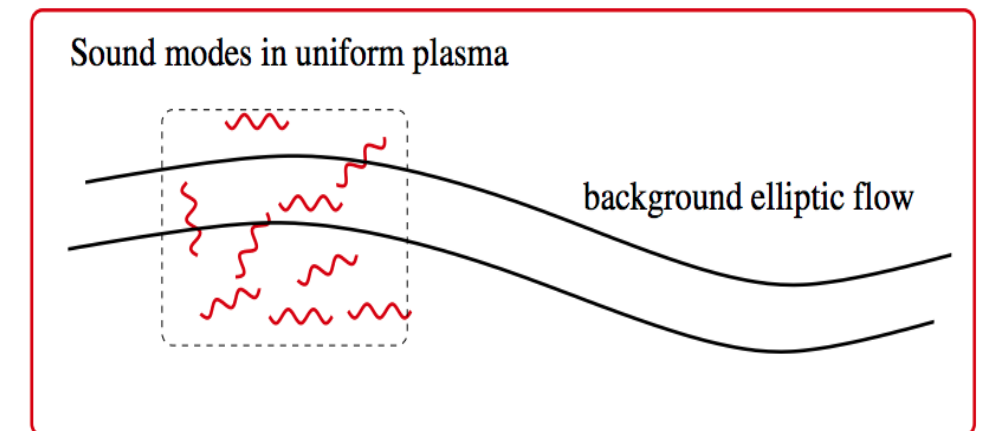
Other (less) well-known issues ...

- **Non-integer powers from thermal fluctuations**

Kovtun, Moore, Romatschke, 2011
Akamatsu, Mazeliauskas, Teaney, 2017, 2018

Thermal fluctuations in hydro: See [P. H. Lau's talk](#)
[C. Plumberg's poster](#)
[S. Pal's poster](#)

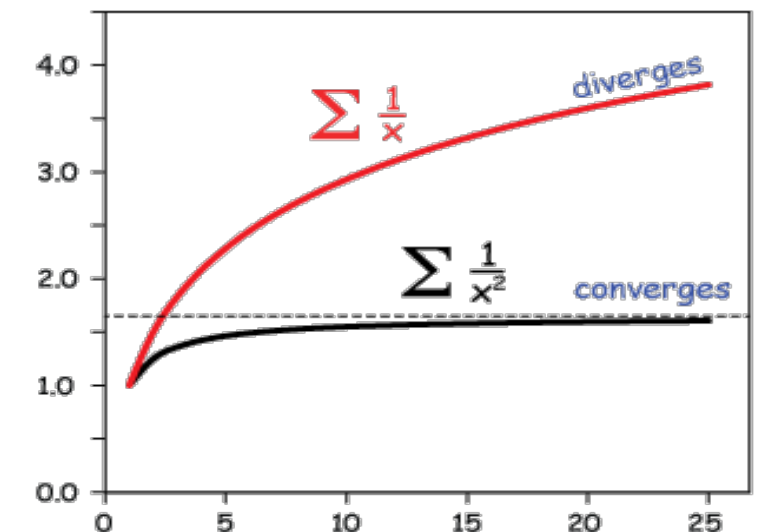
See F. Yan's talk



- **Gradient expansion diverges**

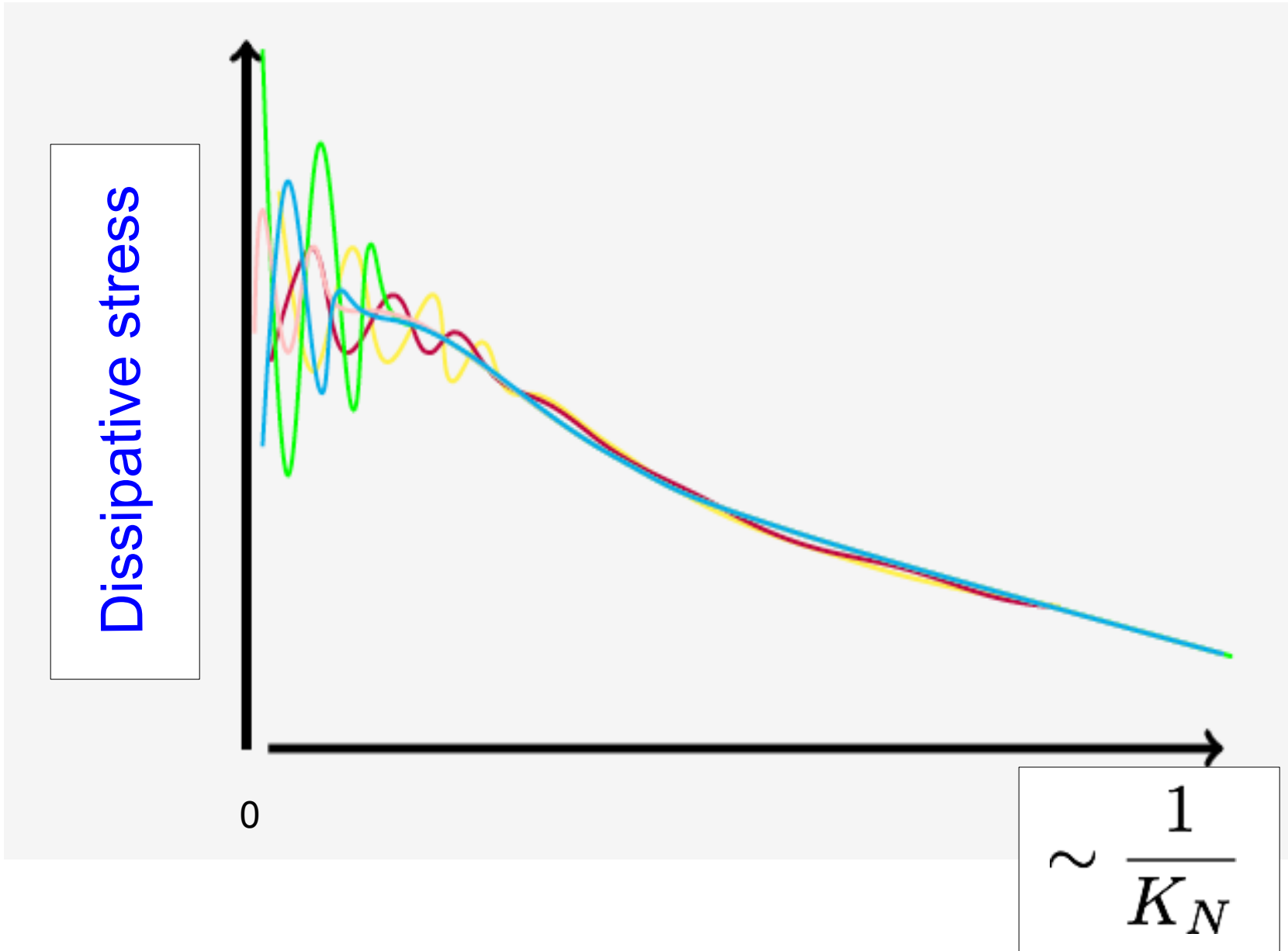
- AdS/CFT: Heller, Janik, Witaszczyk, (2013);
Heller, Buchel, JN, (2016)
- Kinetic theory: Denicol, JN, 1608.07869;
Heller, Kurkela, Spalinski, Svensson, 1609.04803

- **Truncation and systematic improvement??**



Going beyond the gradient expansion: Far-from-equilibrium hydrodynamics

Figure adapted from [B. Meiring's talk](#)

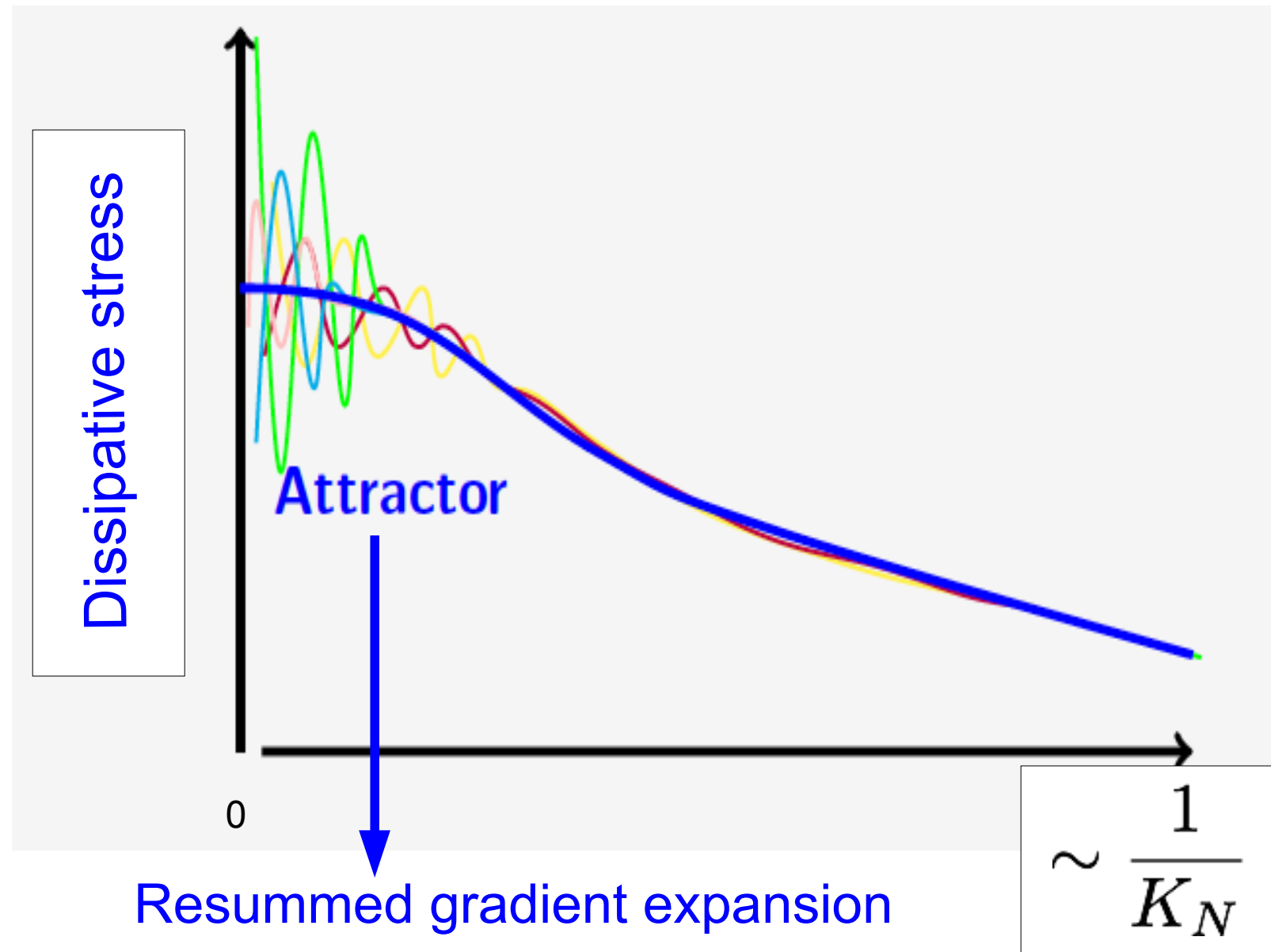


Kinetic theory
and
Holography

Going beyond the gradient expansion: Far-from-equilibrium hydrodynamics

Hydrodynamic attractor → Emergence of constitutive relations far-from-equilibrium

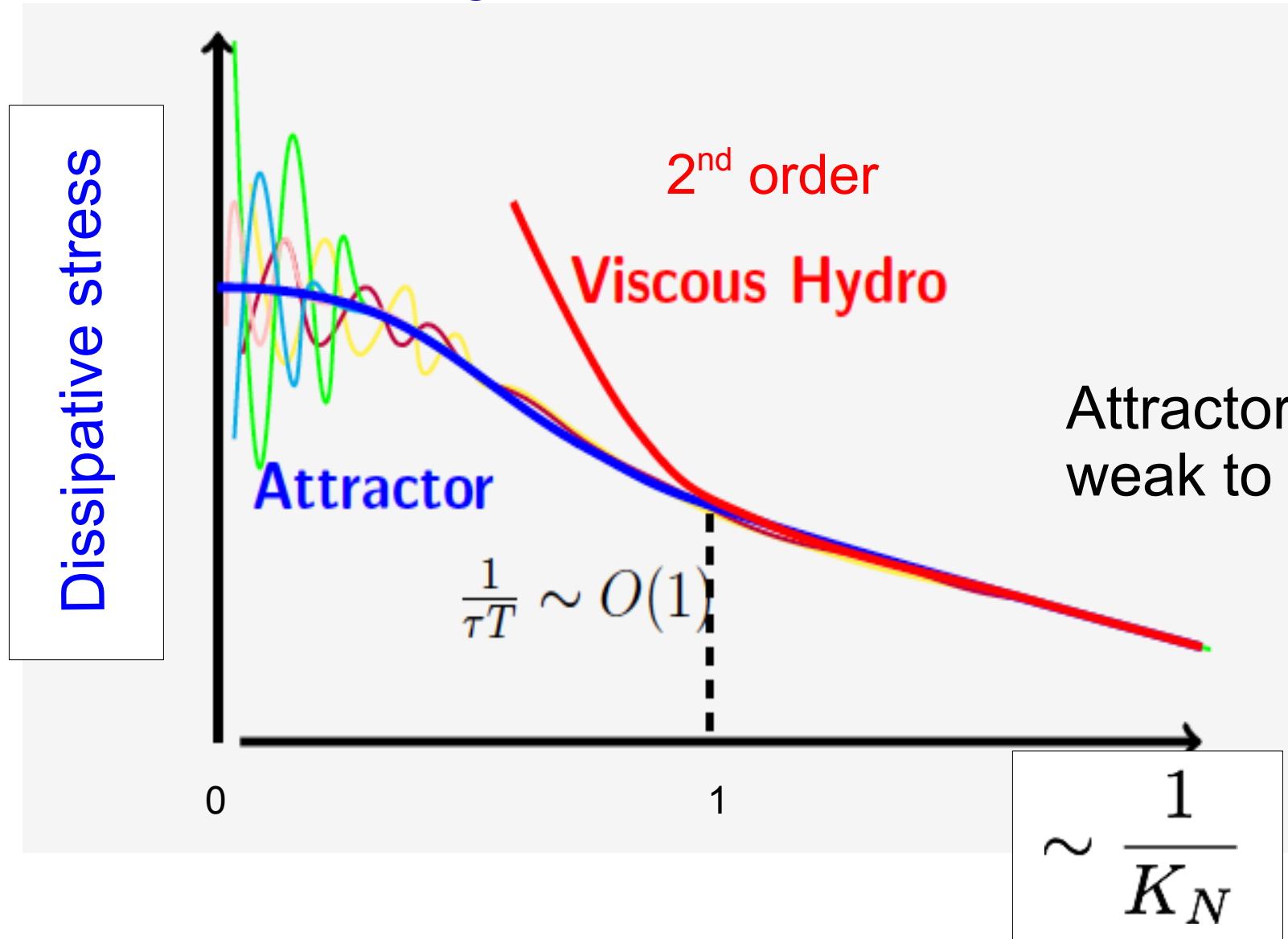
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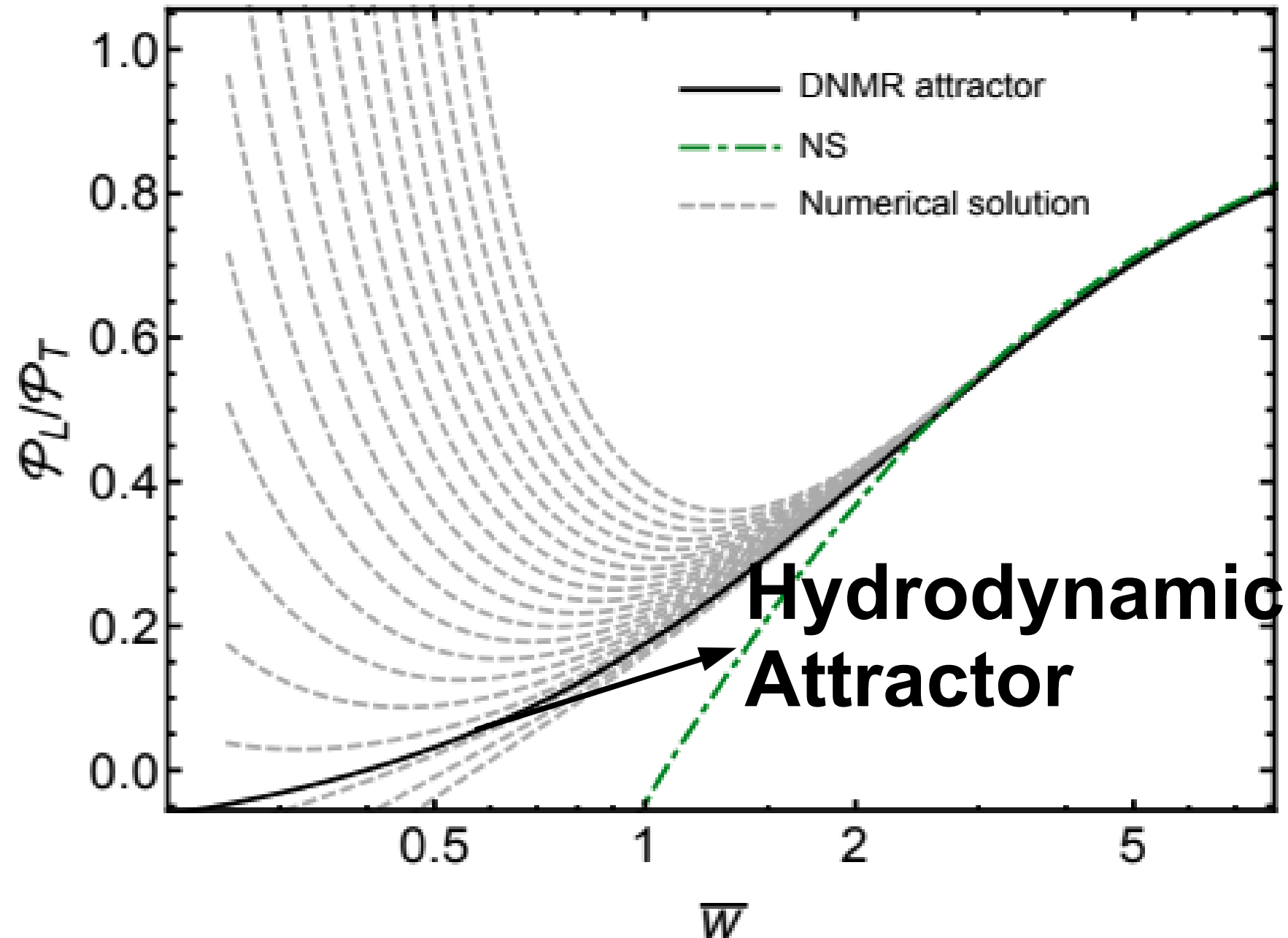


Attractor's details vary between weak to strong coupling

Originally proposed by Heller and Spalinski, PRL (2015)

Review: Florkowski, Heller, Spalinski, 2017

M. Strickland, JN, G. Denicol, PRD 2018



[See B. Meiring's talk](#)

[See L. Yan's talk](#)

[See M. Martinez' talk](#)

[See C. Chattopadhyay's talk](#)

[See U. Heinz' talk](#)

[See R. Critelli's poster](#)

[See J. C-Solana's poster](#)

[See N. C. Cruz' poster](#)

[See G. Denicol's poster](#)

[See E. Maksymiuk's poster](#)

A partial list of upcoming challenges

- Systematic mapping of full initial energy-momentum tensor to final flow harmonics

See A. Mazeliauskas' talk on initial conditions

- Comprehensive Bayesian studies across system sizes and energies

See S. Moreland's talk, J-F Paquet @ Duke's poster, J. Auvinen's poster

- Theoretical control of QCD transport coefficients

- Rigorous definition of far-from-equilibrium hydrodynamics (attractor)

→ **holography** vs. **kinetic theory**

Conclusions

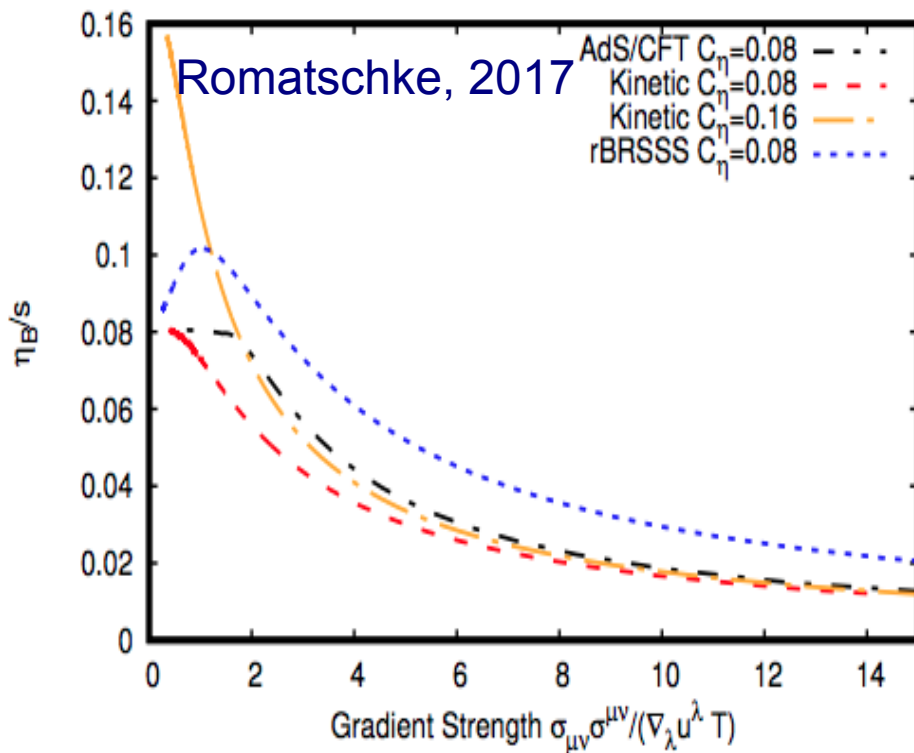
- Collectivity in heavy ion collisions define the world's state-of-the-art research in viscous relativistic hydrodynamics.
- Extensive dialog between theory and experiment.
- Theory highlights @QM2018: QCD conserved charges, transport coefficients connection to nuclear structure, applicability of hydrodynamics ...
- Far-from-equilibrium hydrodynamics: kinetic theory, string theory/holography modern mathematics, new path towards phenomenology.

EXTRA SLIDES

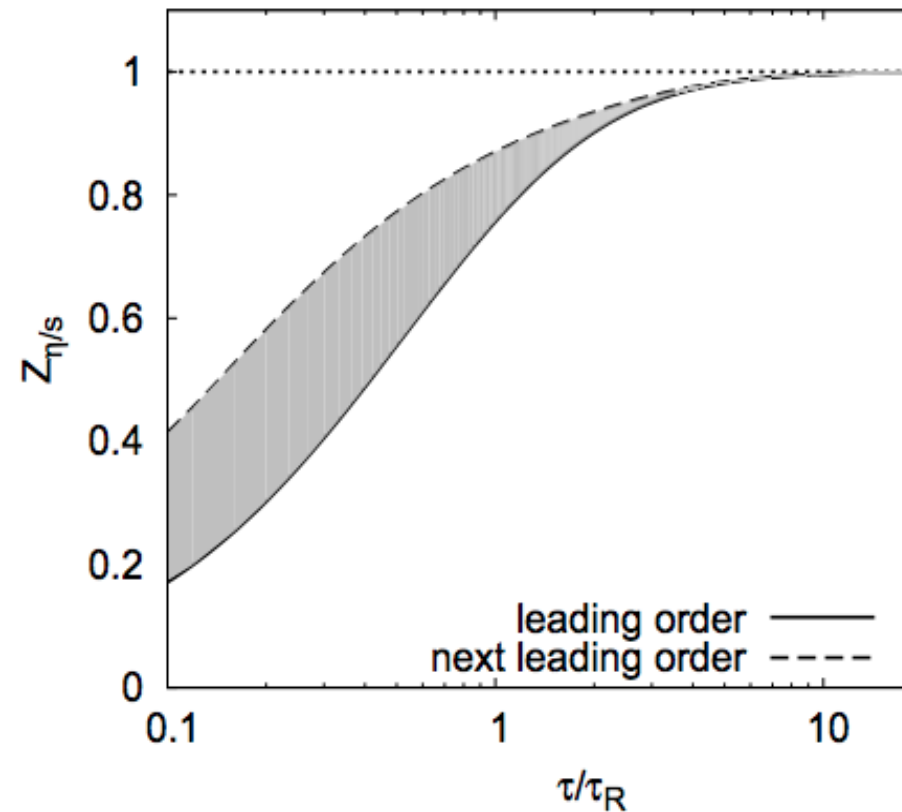
Out-of-equilibrium effects effectively absorbed into a reduced η/s

Initial idea (linear regime): Lublinski and Shuryak, 2009

Borel-Resummed Viscosity



See L. Yan's talk



Larger applicability of hydro?

$$K_N^R = \left(\frac{\eta}{s}\right)_R \frac{\sqrt{\sigma_{\mu\nu}\sigma^{\mu\nu}}}{T}$$

Fluidity in small systems??

See also M. Martinez' talk

- Renormalized (resummed) shear, bulk, and diffusion
- How does one generalize this for arbitrary flow patterns?

“Divergent series are the invention of the devil, and it is shameful to base on them any demonstration whatsoever”

N. H. Abel

Resurgence theory

Ecalte (1980), Dunne, Unsal, Schiappa, Heller, Spalinski, Basar, Aniceto, and etc

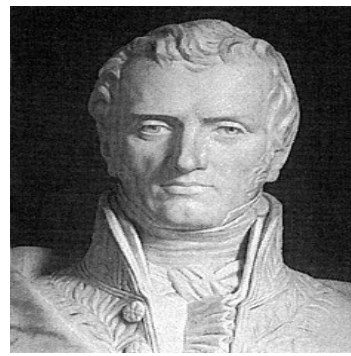
Rough idea: Think of the Knudsen number as a coupling constant in QFT

$$\pi^{\mu\nu} \sim \sum_{n=0}^{\infty} c_n^{(0),\mu\nu} (K_N)^n + c^{(1),\mu\nu} (K_N)^\beta e^{-S/K_N} + \dots$$

Resummed transport coefficients?

hydrodynamic “instanton”?

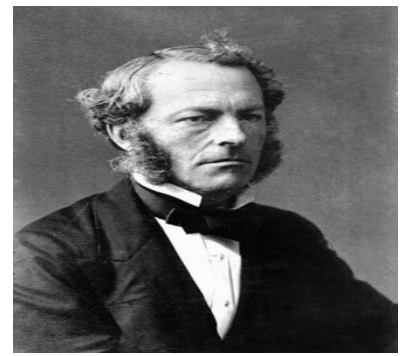
Universal behavior out-of-equilibrium?



Navier

Navier-Stokes equations

Valid when $K_N \ll 1$



Stokes
~ 1845

$$\partial_t \vec{V} + (\vec{V} \cdot \nabla) \vec{V} + \frac{\nabla P}{\rho_0} = \frac{\eta}{\rho_0} \nabla^2 \vec{V}$$

- Notoriously hard nonlinear problem to solve in three dimensions.

- Turbulence



da Vinci, 1508-1513

Millennium Prize Problem

“Existence and smoothness of the Navier-Stokes equations”



Accessing the fluctuations of the initial state

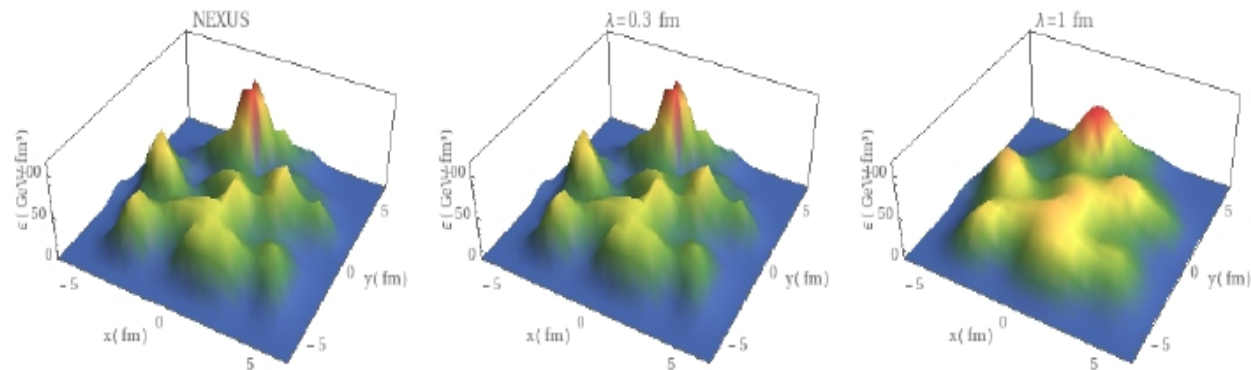
Agreement with ATLAS, JHEP 11 (2013) 183

S. McDonald et al., PRC 95, 064913 (2017)

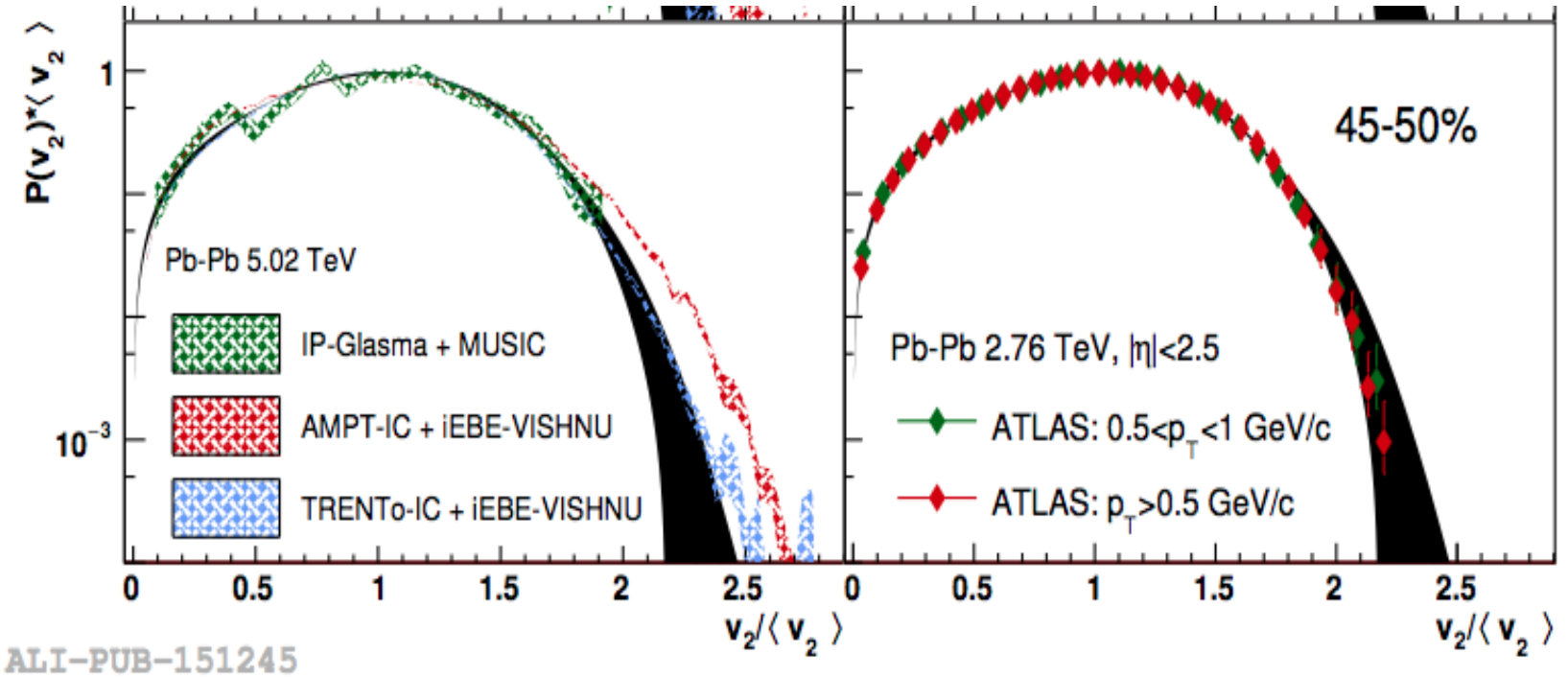
W. Zhao et al., EPJ C77 no.9, 645 (2017)

Does the v_n distribution depend on small scale structure?

Increasing smoothing scale of initial energy density



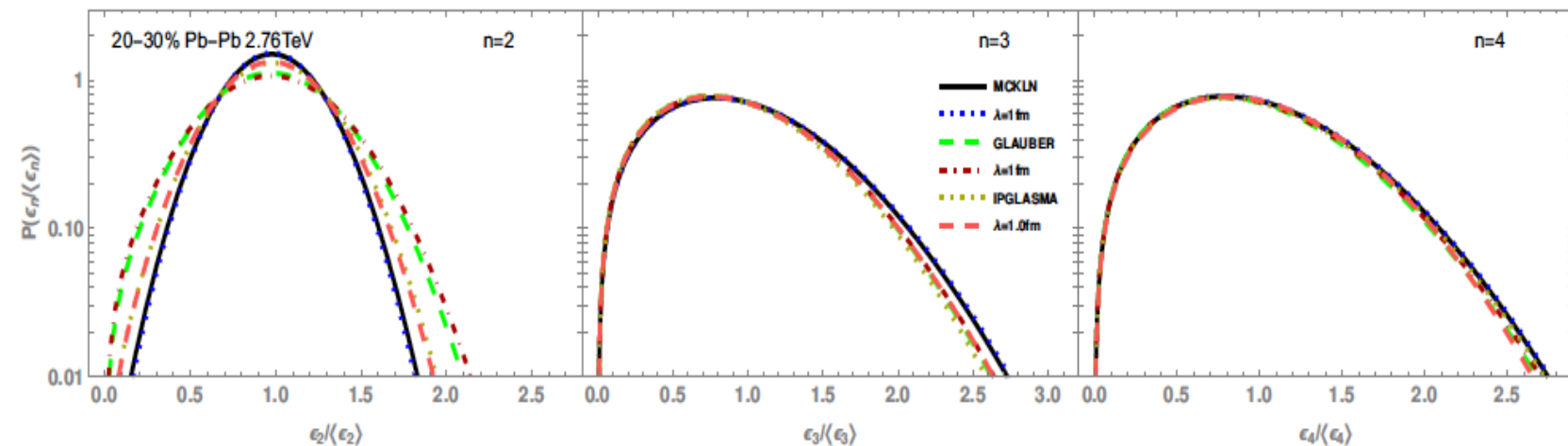
ALICE, J. Margutti's talk



ALI-PUB-151245

See F. Grassi's talk

no effect of smoothing on $P(\epsilon_n / \langle \epsilon_n \rangle)$.



- **Israel-Stewart equations**, extensively used in heavy ion collisions, already contain an infinite resummation in gradients.

DNMR (2012): Controlled behavior in Knudsen K_N , inverse Reynolds $\frac{\pi}{P}$

Anisotropic hydrodynamics: Resummation in Knudsen and 1/Reynolds

See M. Strickland's talk

See McNelis' talk See U. Heinz' talk

3+1 aHydro evolution → Phenomenological applications

See M. Alqahtani's talk

