High-Temperature QCD: theory overview

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Quark Matter 2018, Venice, Italy, 14-19 May 2018

OUTLINE

- **The QCD transition and equation of state at zero baryon density**

- **Results at non-zero baryon density**

- **Properties of the thermal medium**

I apologize for many important aspects of QCD at high-T which will not be discussed in my talk, like for instance the predictions for axion cosmology coming from the study of topological properties in the deconfined phase

Lattice QCD

Best present source of information for equilibrium properties of QCD at high T

IDEA (K.G.Wilson, 1974):

discretize the QCD lagrangian on a space-time lattice and compute numerically its partition function by sampling its Euclidean path-integral measure (if positive)

Theoretical, algorithmic and machine developments have permitted:

- **simulations of $N_f = 2+1$ and $N_f = 2+1+1$ QCD with physical quark masses**
- **lattice spacings fine enough for reliable continuum extrapolations**
- **cross-checks among different collaborations/discretizations/methods**

Some relevant exceptions:

- **non-equilibrium properties**
- **QCD at finite baryon density (sign problem)**

Results on the finite T crossover

Crossover nature of the "transition" known since a few years

Y. Aoki *et al.* Nature 443, 675 (2006) [hep-lat/0611014].

Temperature of the transition (from the chiral condensate):

S. Borsanyi *et al.* JHEP 1009, 073 (2010) $T_c = 155(6)$ **MeV (stout link stag. discretization, $a_{min} \simeq 0.08$ fm)**

A. Bazavov *et al.*, PRD 85, 054503 (2012) $T_c = 154(9)$ **MeV (HISQ/tree stag. discretization, $a_{min} \simeq 0.1$ fm)**

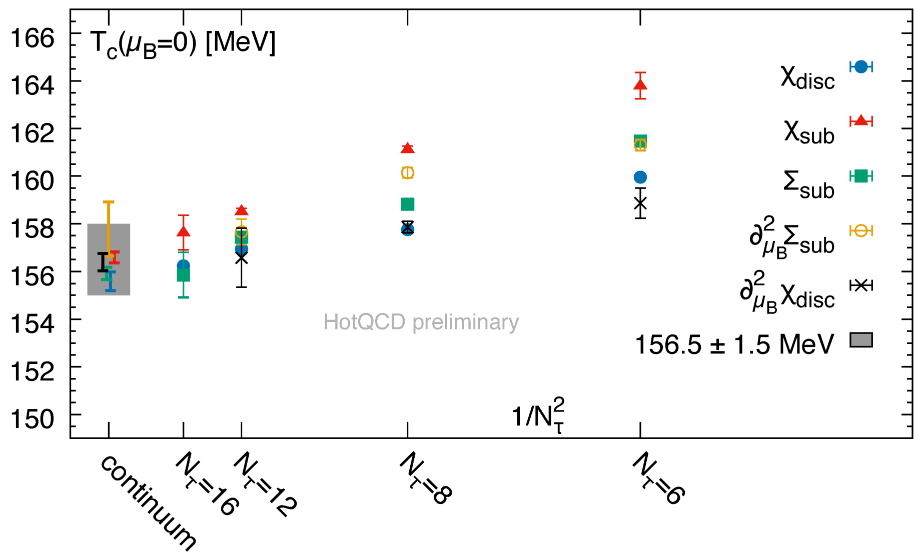
Various refinements in recent years

Update at QM2018

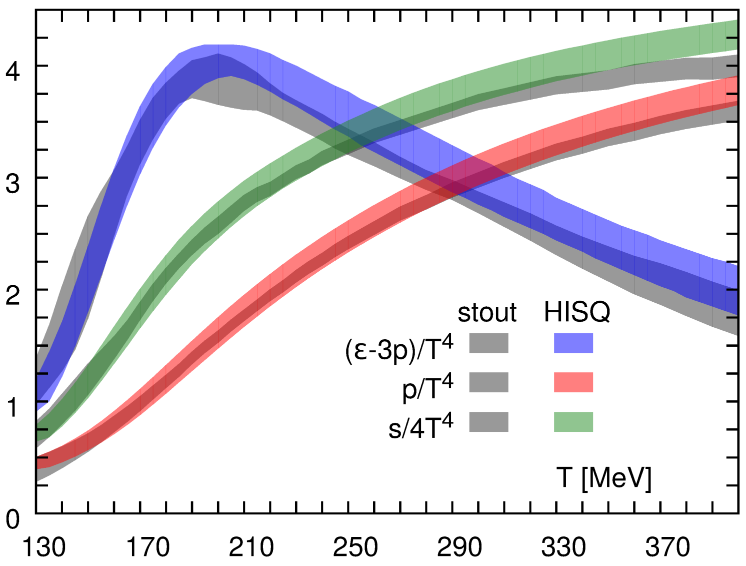
$\longrightarrow$ **talk by P. Steinbrecher**

$$T_c \simeq 156.5 \pm 1.5$$

$$T = 1/(N_t a) \quad \rightarrow \quad a \propto 1/N_t$$

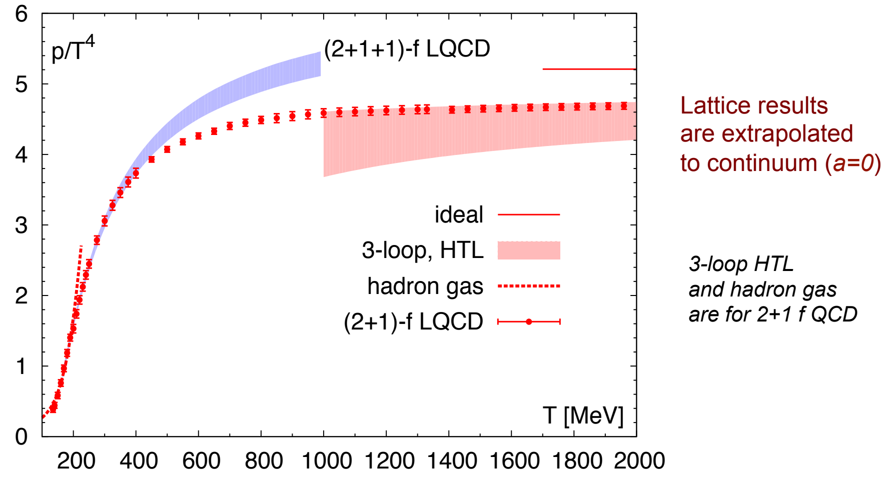


Finite T Equation of State: convergence towards continuum achieved since a few years



EoS at high temperatures

2+1 flavor: Bazavov, PP, Weber, PRD97 (2018) 014510
2+1+1 flavor (with charm quark): Borsányi et al (BW Coll.), Nature 539 (2016) 69



Charm quark contribution to QCD pressure is significant for *T>400* MeV
The pressure is well describe by weak coupling calculations for *T>1000* MeV

LEFT: comparison of the EoS for $N_f = 2+1$ QCD

S. Borsanyi *et al* **arXiv:1309.5258**

A. Bazavov *et al.* **arXiv:1407.6387**

RIGHT: $N_f = 2+1$ vs $N_f = 2+1+1$: high T and effects of charm quark inclusion

S. Borsanyi *et al.*, **arXiv:1606.07494**

A. Bazavov, P. Petreczky and J. H. Weber, arXiv:1710.05024

$\rightarrow$ talk by A. Bazavov at QM2018   comparison with perturbation theory OK

Challenge for the future:

Bring simulations with Wilson fermions at the same level of accuracy as staggered fermions: continuum limit and physical quark masses

Main difficulty: because of the explicit breaking of chiral symmetry, one needs to go to much smaller lattice spacings

Promising ongoing approaches:

- Gradient flow thermodynamics

H. Suzuki, arXiv:1304.0533; M. Asakawa *et al* arXiv:1312.7492; arXiv:1610.07810

- Thermodynamics with twisted Wilson fermions

F. Burger *et al* arXiv:1510.02262

- QCD with twisted temporal boundary conditions (moving frame)

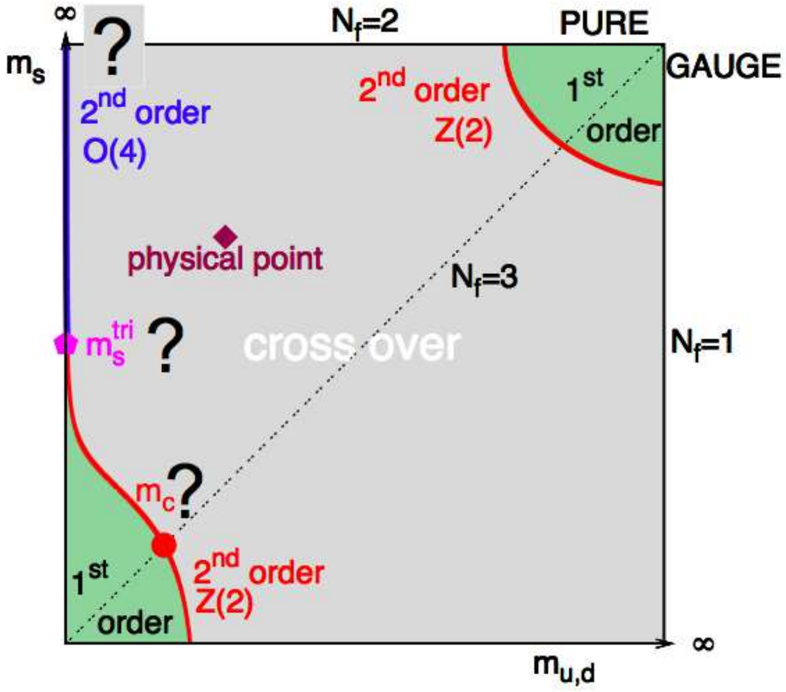
L. Giusti, H. B. Meyer, 2013; L. Giusti, M. Pepe, 2014; M. Dalla Brida, L. Giusti, M. Pepe, arXiv:1710.09219

- Thermodynamics from non equilibrium methods

M. Caselle, A. Nada and M. Panero, arXiv:1801.03110

QCD transition as a function of the quark masses

Locating critical points can be difficult even without a sign problem



Columbia plot sketched mostly on the basis of universality arguments

(Pisarski, Wilczek 1984)

1. **first order for sure around the quenched limit**
2. **for $N_f \geq 3$ massless flavors the effective chiral model does not have a fixed point $\rightarrow$ first order**
3. **for $N_f = 2$ massless flavors, if $U_A(1)$ not effectively restored, could be in the $O(4)$ universality class or first order**

Determination of critical masses delimiting first regions reveals a hard task as one tries to approach the continuum limit

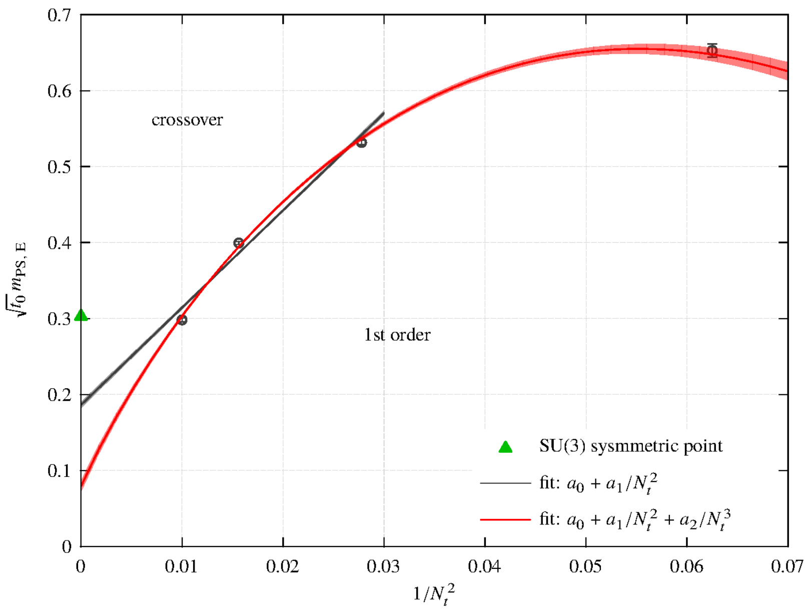
The continuum limit of the critical masses for $N_f = 3$ and $N_f = 4$ bends towards zero. Second order chiral point, contrary to universality??

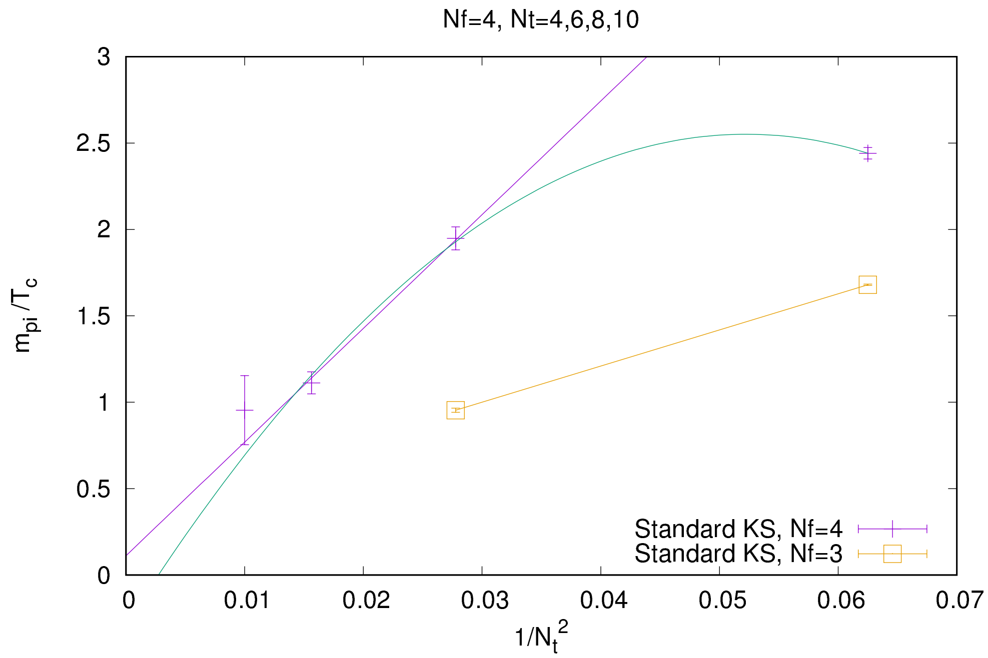
G. Endrodi, Z. Fodor, S. D. Katz and K. K. Szabo, arXiv:0710.0998

P. de Forcrand, S. Kim and O. Philipsen, arXiv:0711.0262

X. Y. Jin, Y. Kuramashi, Y. Nakamura, S. Takeda and A. Ukawa, arXiv:1411.7461, arXiv:1612.05371

P. de Forcrand, MD,, arXiv:1702.00330





arXiv:1612.05371, $N_f = 3$, Wilson fermions

arXiv:1702.00330, $N_f = 4$, staggered fermions

Universality is known to fail in models containing gauge degrees of freedom!

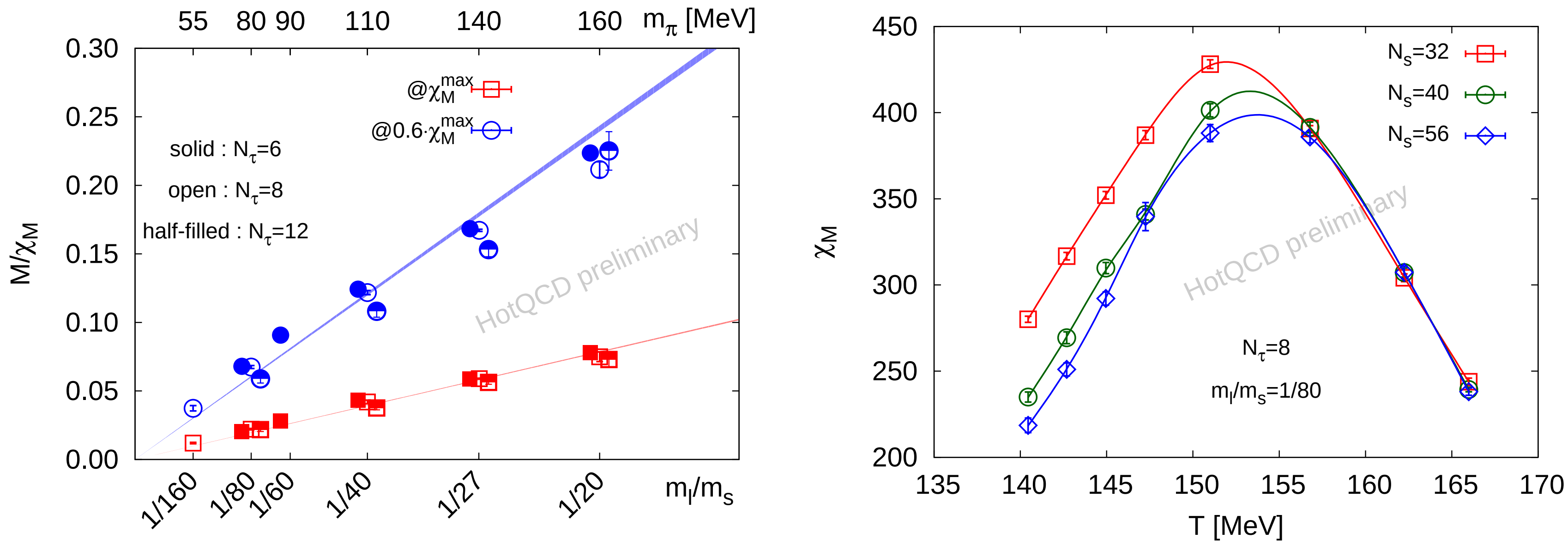
(A. Pelissetto, A. Tripodo and E. Vicari, arXiv:1706.04365, arXiv:1711.04567)

On the other hand, the expected second order in the chiral limit for $N_f = 2$ has been debated: some works have claimed first order (but on coarse lattices)

MD, A. Di Giacomo, C. Pica, hep-lat/0503030; C. Bonati *et al*, arXiv:1408.5086; O. Philipsen and C. Pinke, arXiv:1602.06129

studies with $N_f = 2\mathrm{massless} + 1$ have claimed $O(4)$-$O(2)$ (S. Ejiri *et al.*, arXiv:0909.5122)

New results at QM2018 ($\rightarrow$ talk by A. Lahiri) m_s fixed, $m_l \rightarrow 0$



- Critical temperature in the chiral limit $T_c = 138(5)$ MeV
- LEFT: chiral condensate/susceptibility $\sim (m_l/m_s)/\delta$, $\delta \simeq 0.21$, compatible with $O(4)$ - $O(2)$ (but also with many other relevant universality classes: Z_2, tricritical ...)
- RIGHT: 1st order excluded down to $m_\pi \simeq 80$ MeV

Problems in lattice QCD at $\mu_B \neq 0$

$$Z(\mu_B, T) = \mathrm{Tr}\left(e^{-\frac{H_{\mathrm{QCD}} - \mu_B N_B}{T}}\right) = \int \mathcal{D}U e^{-S_G[U]} \det M[U, \mu_B]$$

$\det M[\mu_B]$ **complex** $\Longrightarrow$ **Monte Carlo simulations are not feasibile.**

This is usually known as the sign problem

The two approximate solutions for small μ_B/T which are presently mostly used are:

- **Taylor expansion of physical quantities around $\mu = 0$**

  Bielefeld-Swansea collaboration 2002; R. Gavai, S. Gupta 2003

- **Simulations at imaginary chemical potentials (plus analytic continuation)**

  Alford, Kapustin, Wilczek, '99; Lombardo '00; de Forcrand, Philipsen, '02; MD, Lombardo '03.

Other: Reweighting Barbour et al. 1998; Z. Fodor and S. Katz, 2002, **Canonical approach** see poster by M.P. Lombardo

Better solutions are being developed but still not fully operative

(Langevin simulations, Lefschetz thimble simulations, density of states method, ...)

Phase diagram OK for sign-free theories, like QCD at finite isospin density or $SU(2)$

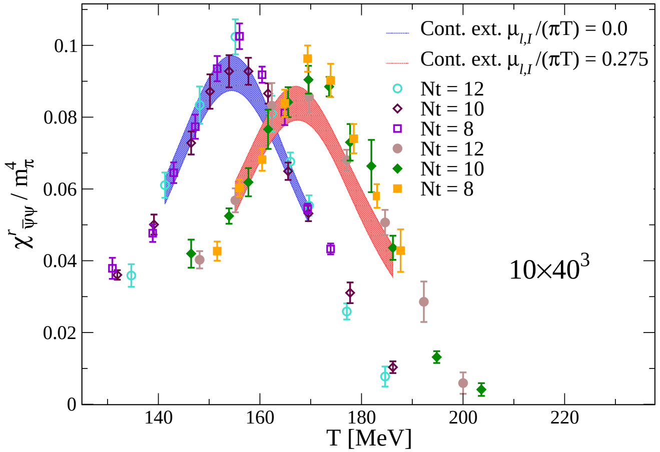
B. B. Brandt, G. Endrodi and S. Schmalzbauer, arXiv:1712.08190; $\rightarrow$ poster by A. Kotov.

Reliable results achievable for small μ_B/T. Example: Dependence of T_c on μ_B

$$\frac{T_c(\mu_B)}{T_c} = 1 - \kappa \left(\frac{\mu_B}{T_c}\right)^2 + O(\mu_B^4)$$

Curvature κ can be determined by following explicitly how T_c moves at imaginary μ_B and then continuing to real μ_B (right)

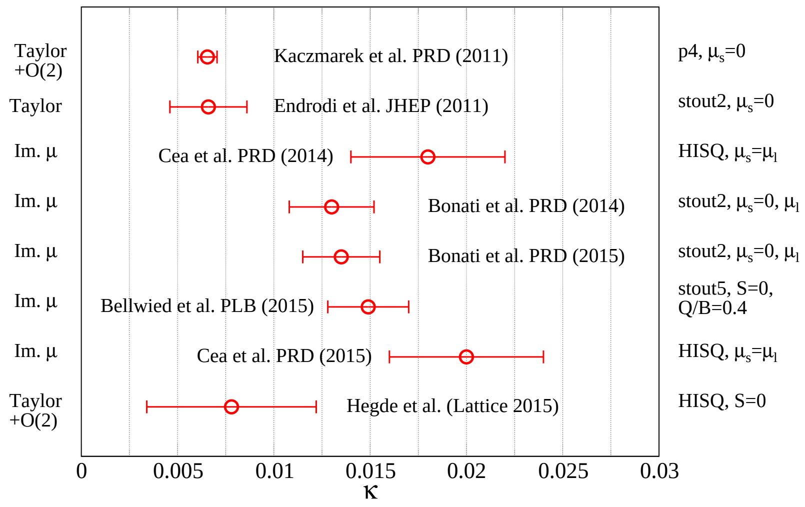
or by finding $dT_c/d\mu_B^2$ implicitly in terms of derivatives at $\mu_B = 0$ (Taylor expansion)



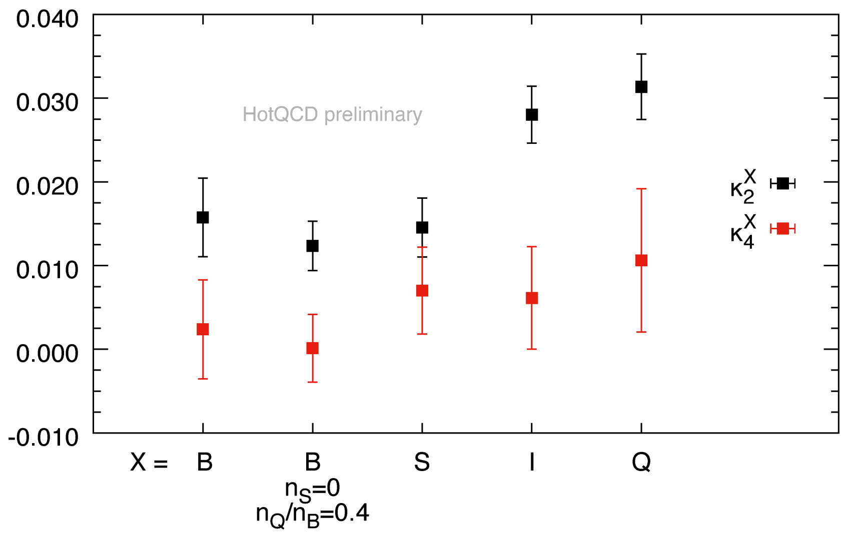
$\chi_{\bar\psi\psi}$, from Bonati et al. arXiv:1507.03571

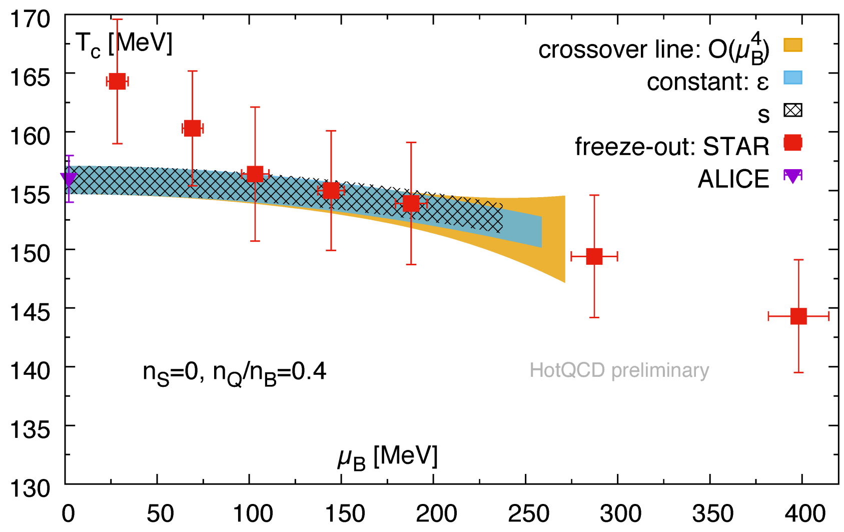
This is the situation 3 years ago from lattice simulations at or close to the physical point.

Slight tendency of Taylor expansion results to stay lower.



New results on κ from Taylor expansion at QM2018

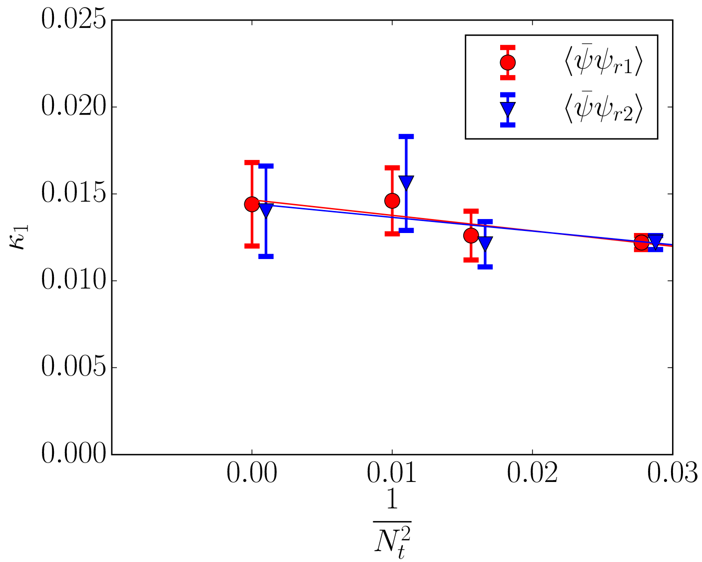




→ talk by P. Steinbrecher

$$\frac{T_c(\mu_X)}{T_c} = 1 - \kappa_2^X \left(\frac{\mu_X}{T_c}\right)^2 - \kappa_4^X \left(\frac{\mu_X}{T_c}\right)^2 + O(\mu_X^6)$$

- $N_f = 2+1$ **QCD by HISQ staggered quarks, various setup of chemical potentials**
- **curvature determined from the maximum of the disconnected chiral susceptibility**
- **imposing strangeness neutrality** $\kappa = 0.0123(30)$



$\rightarrow$ **poster by F. Negro** **C. Bonati *et al* arXiv:1805.02960**

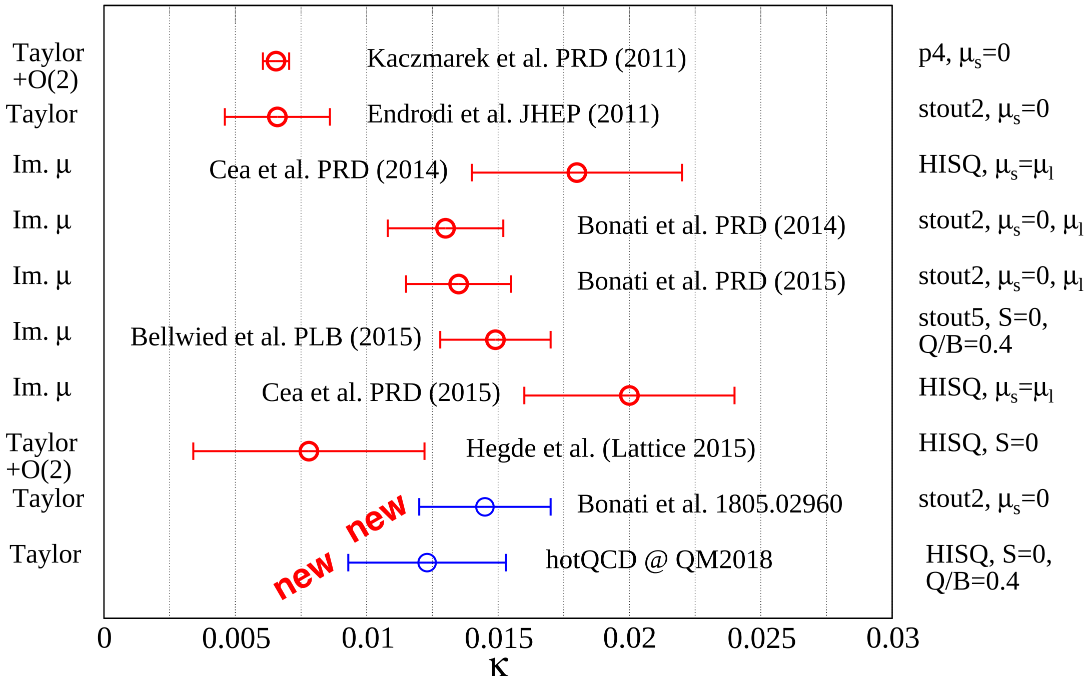
- $N_f = 2+1$ QCD by stout staggered quarks, $\mu_u = \mu_d = \mu_B/3$, $\mu_s = 0$.

- $\kappa = 0.0142(25)$ determined from the renormalized chiral condensate

Quantitative agreement of the most recent determinations confirms the reliability of analytic continuation and Taylor expansion

Functional methods yield similar results

B. J. Schaefer, J. Wambach, nucl-th/0403039; J. Braun, B. Klein, B. J. Schaefer, arXiv:1110.0849; J. M. Pawlowski, F. Rennecke, arXiv:1403.1179; C. S. Fischer, J. Luecker and C. A. Welzbacher, arXiv:1405.4762



Generalized Quark Number Susceptibilities

The dependence on the chemical potentials is encoded in the generalized susceptibilities χ_{ijk}:

$$\frac{P}{T^4}(\mu_u,\mu_d,\mu_s) = \frac{P}{T^4}(0,0,0) + \sum_{i+j+k=even} \frac{\chi_{ijk}(T)}{i!j!k!}\hat{\mu}_u^{(i)}\hat{\mu}_d^{(j)}\hat{\mu}_s^{(k)}$$

$$\chi_{ijk}(T) = \frac{1}{VT^4}\frac{\partial^{(i+j+k)}F(T,\mu)}{\partial\hat{\mu}_u^{(i)}\partial\hat{\mu}_d^{(j)}\partial\hat{\mu}_s^{(k)}}\Bigg|_{\mu_u=\mu_d=\mu_s=0}$$

where μ_i are quark number chemical potentials

$$\begin{aligned}\mu_u &= \mu_B/3 + 2\mu_Q/3\\ \mu_d &= \mu_B/3 - \mu_Q/3\\ \mu_s &= \mu_B/3 - \mu_Q/3 - \mu_S\,.\end{aligned}$$

They can be systematically determined by lattice simulations

- **They encode information on fluctuations of conserved charges, direct comparison with experiment, e.g. to determine freeze-out conditions in alternative to HRG**

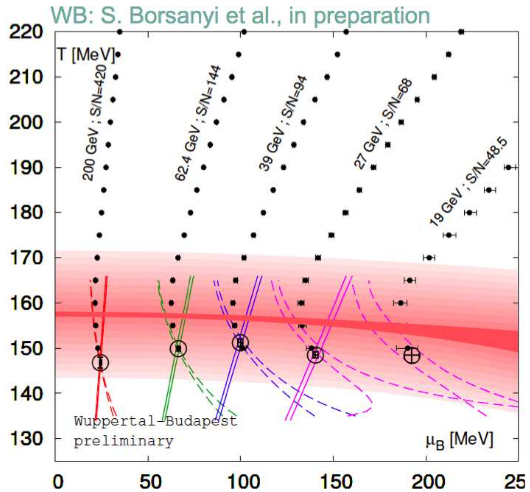
  A. Bazavov *et al.*, arXiv:1208.1220; arXiv:1509.05786 S. Borsanyi *et al.*, arXiv:1305.5161

  New results at QM2018: → talk by C. Ratti (figure on the left)

  Freeze-out points stay on the lower bound of the freeze-out region

  Freeze-out for strange hadrons 15-20 MeV higher

  → poster by C. Schmidt



- **They are an input in an increasing number of Lattice-based effective models**

  G. A. Almasi *et al* arXiv:1805.04441 and talk at QM2018 by K. Redlich

  S. Liu, R. Rapp, arXiv:1612.09138, arXiv:1711.03282 and talk at QM2018 by S. Liu

  V. Vovchenko *et al* arXiv:1711.01261 (CEM model) and talk at QM2018 by V. Vovchenko

  talk by J. Steinheimer, poster by A. Motornenko

  P. Huovinen and P. Petreczky, arXiv:1708.00879 and poster at QM2018 by P. Huovinen

  poster by P. Moreau

  P. Parotto *et al*, arXiv:1805.05249 and poster by P. Parotto

Strategies to determine χ_{ijk} on the lattice:

- **Taylor expansion approach: computed as expectation values at $\mu = 0$**

  C. R. Allton *et al* hep-lat/0204010, hep-lat/0501030;

  R. V. Gavai and S. Gupta, hep-lat/0303013, hep-lat/0412035;

  A. Bazavov *et al.*, arXiv:1701.04325 going up to order μ_B^6

  Numerical difficulties in approaching higher orders

- **Analytic continuation approach Determine χ_{ijk} as a function of imaginary μ's and then obtain higher order χ's by fitting lower order ones**

  MD and M. P. Lombardo, hep-lat/0406012 first computation of susceptibilities from quark numbers

  MD and F. Sanfilippo arXiv:0904.1400 extension to multiple chemical potentials

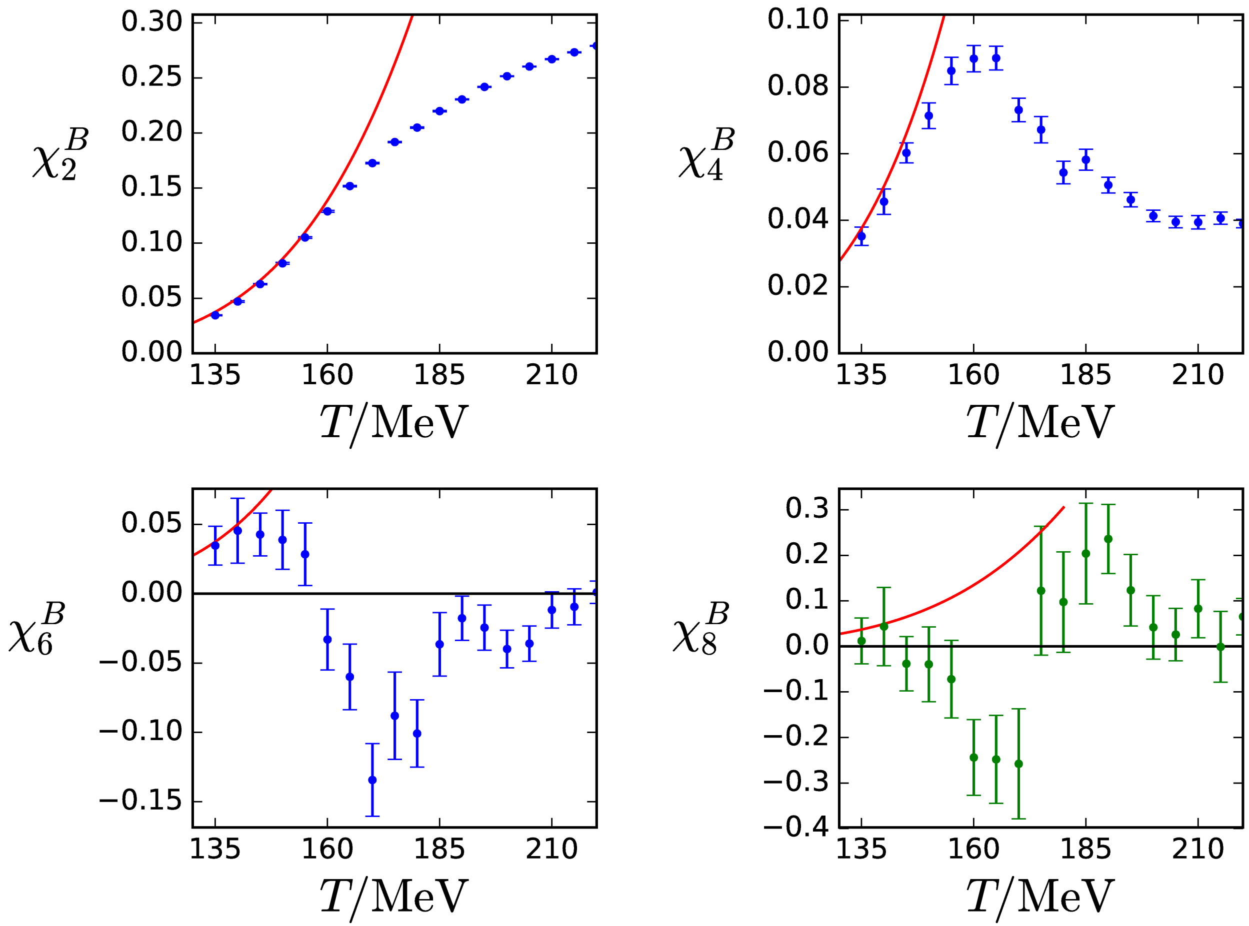
  P. de Forcrand and T. Takaishi, arXiv:1002.0890 global fit to different order susceptibilities

  J. Guenther *et al* arXiv:1607.02493 $O(\mu_B^6)$ achieved

  MD, G. Gagliardi and F. Sanfilippo arXiv:1611.08285 global fit to different order and multiple chemical potentials, $O(\mu_B^8)$ achieved

New results at QM2018

arXiv:1805.04445 $\to$ talks by J. Guenther, A. Pasztor and S. Borsanyi



global fit to different orders and one imaginary chemical potential μ_B

$O(\mu_B^8)$ **achieved using a prior**

One possible use of the generalized susceptibilities is to estimate the radius of convergence of the series to bound the location of the critical endpoint

C. Allton *et al* **hep-lat/0305007**

R. Gavai, S. Gupta, hep-lat/0412035, arXiv:0806.2233

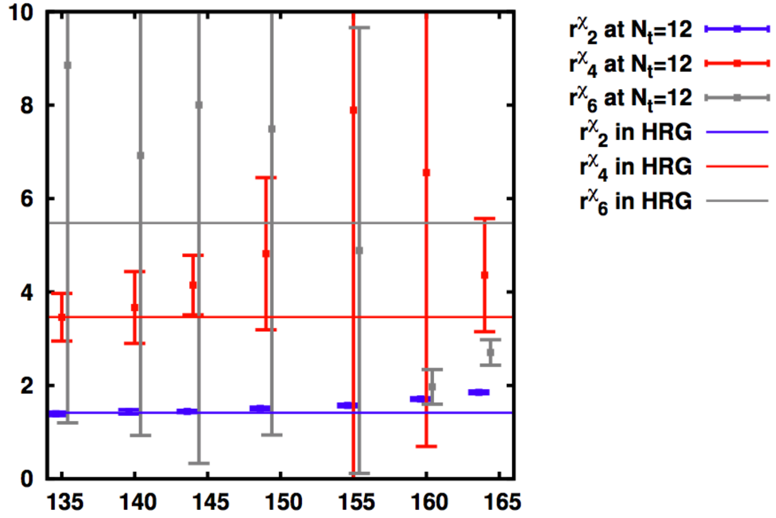
$$\rho^{f}_{n,m} = \left(\frac{\chi^B_n/n!}{\chi^B_m/m!} \right)^{\frac{1}{(m-n)}} \quad \rho^{\chi}_{n,m} = \left(\frac{\chi^B_n/(n-2)!}{\chi^B_m/(m-2)!} \right)^{\frac{1}{(m-n)}}$$

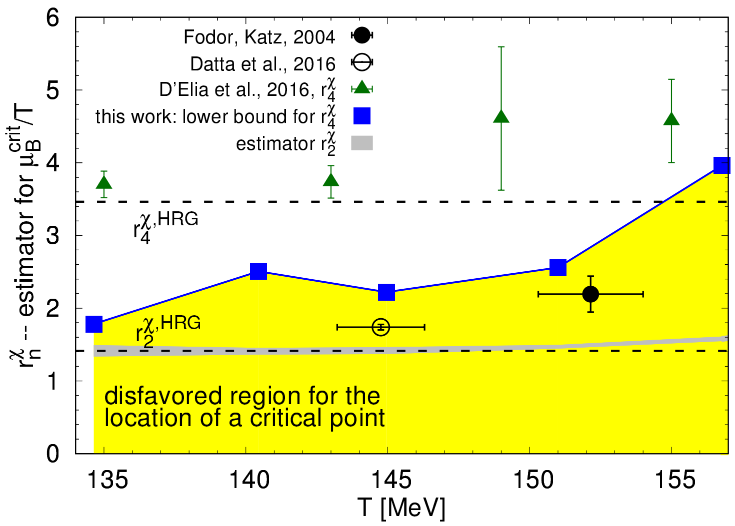
Recent estimates/lower bounds for the critical endpoint obtained in this way

S. Datta, R. V. Gavai and S. Gupta, arXiv:1612.06673

MD, G. Gagliardi and F. Sanfilippo, arXiv:1611.08285

A. Bazavov *et al*, **arXiv:1701.04325**





from the talk by Attila Pazstor at QM2018 **from A. Bazavov *et al*, arXiv:1701.04325**

- **Recent estimates of the radius at the physical point: no evidence of convergence**

- **Are we able to estimate it at all?**

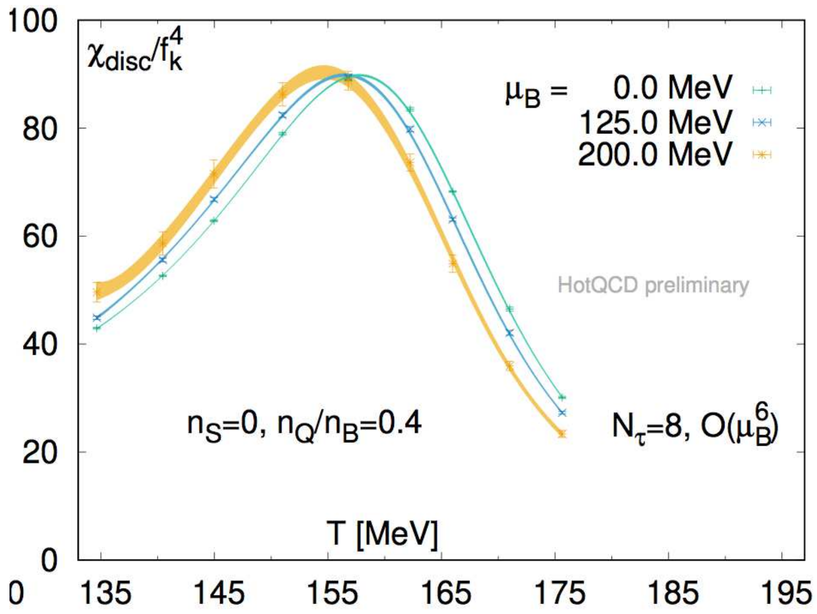
Are we just seeing the regular part? How far does an hypothetical critical scaling region extends and influences physics at $\mu_B = 0$? Is the critical behavior hidden at much higher order of the series?

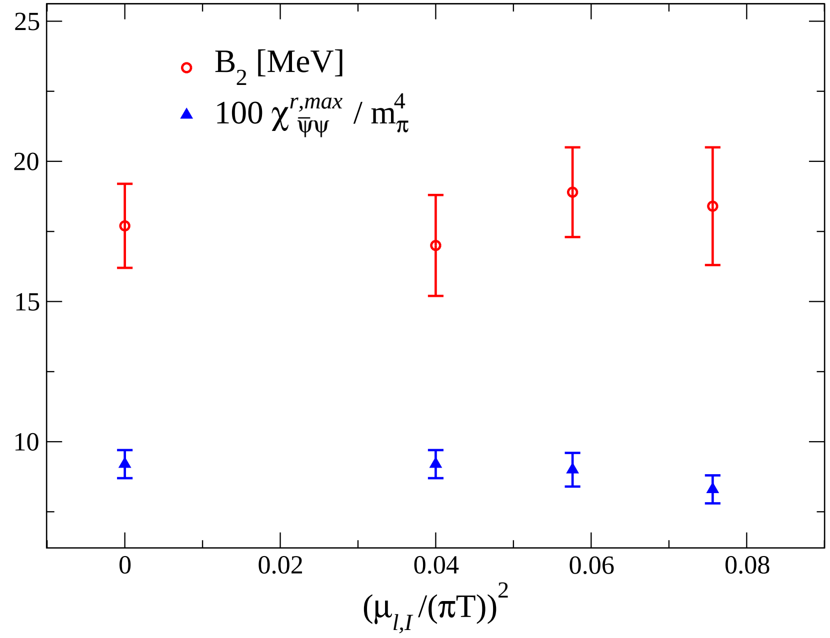
- **Interesting guide may come from LQCD-based models**

V. Vovchenko, J. Steinheimer, O. Philipsen and H. Stoecker, arXiv:1711.01261 [hep-ph] (CEM model)

P. Parotto *et al*, arXiv:1805.05249 and poster by P. Parotto

The fact that we might be completely insensitive to a possible critical endpoint is supported by the fact that the strength of the transition seems independent of μ_B around $\mu_B = 0$.



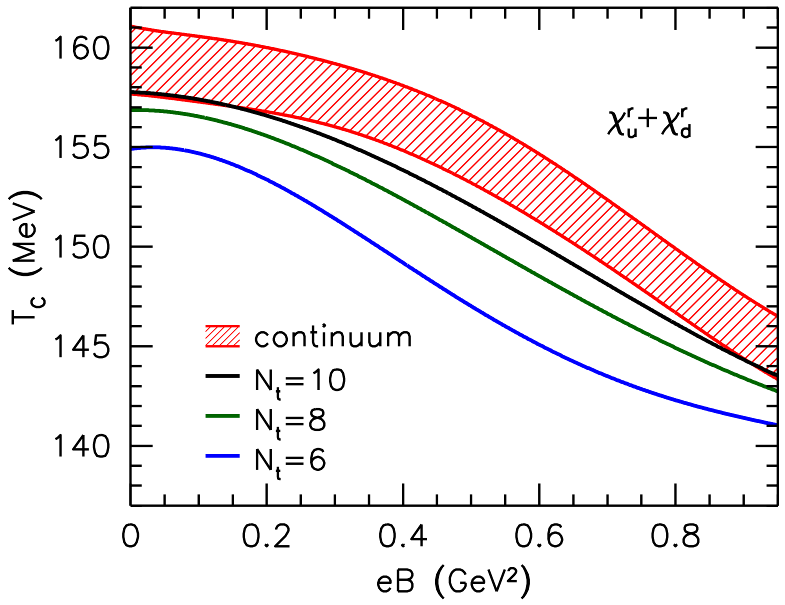


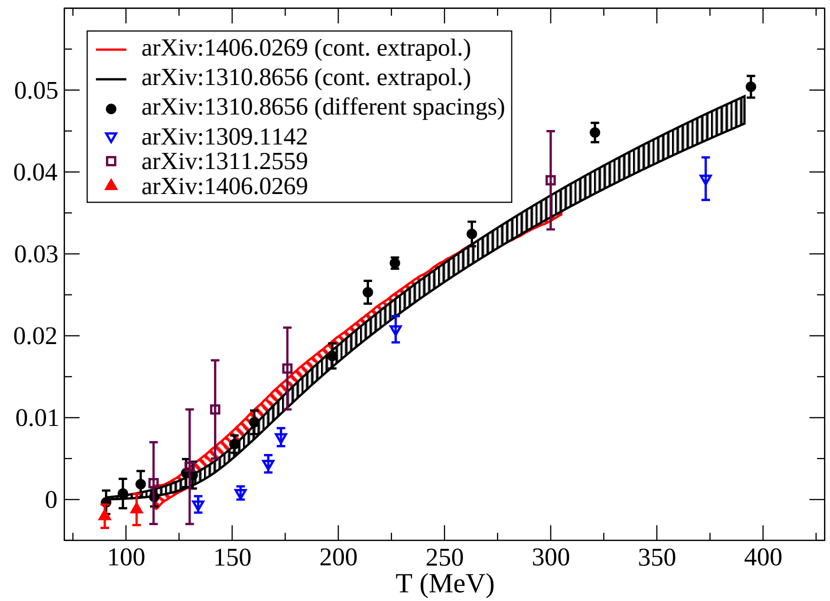
talk by P. Steinbrecher

height and width of χ_q, Bonati *et al*, arXiv:1507.03571

Similar results shown in the talk by S. Borsanyi

Other extensions of the QCD phase diagram





No sign problem with a magnetic background. T_c **decreases** (figure from G. S. Bali et al., arXiv:1111.4956), **magnetic susceptibility strongly increases above** T_c (from arXiv:1502.06047); **static quark potential anisotropic** (C. Bonati et al., arXiv:1403.6094, arXiv:1607.08160)

- **NEWS: the decrease of** T_c **takes place even at large quark mass and in absence of inverse magnetic catalysis** poster by F. Manigrasso at XQCD2018 next week

- **It would be phenomenologically interesting to extend to finite angular velocity** (preliminary study by A. Yamamoto, arXiv:1303.6292)

Properties of the thermal medium

- **What are the trasport properties of the medium?**

- **Chiral symmetry is restored above T_c**

  $U_A(1)$ **axial symmetry is effectively restored for $T \gtrsim 200$ MeV**

  A. Tomiya *et al*, **arXiv:1612.01908; H. Fukaya arXiv:1712.05536, T. Bhattacharya** *et al.*, **arXiv:1402.5175;**

  T. W. Chiu *et al.* **arXiv:1311.6220**

  Is the restoration visible in the spectrum?

- **What are the screening properties of the medium and the fate of heavy-quark bound states?**

Transport coefficients: difficulties and prospects

Many model and pQCD computations at QM2018

$\rightarrow$ talk by J. Ghiglieri, Perturbation Theory

$\rightarrow$ talk by S. Liu, model computation based on LQCD-EoS

In principle, Euclidean correlators give direct access to relevant spectral functions

$$G_E(\tau) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi}\, \rho_E(\omega)\, \frac{\cosh[\omega(\frac{\beta}{2} - \tau)]}{\sinh[\frac{\omega\beta}{2}]}$$

Difficulties: solve the integral equation with a finite number of determinations of G_E.

Increasing high precision and number of points for G_E finally will constrain the systematic uncertainties: this program is presently approachable in quenched QCD

H. B. Meyer, arXiv:1104.3708; N. Astrakhantsev, V. Braguta and A. Kotov, arXiv:1701.02266, arXiv:1804.02382: latest quenched studies of bulk and shear viscosity $\rightarrow$ **poster by A. Kotov**

A. Francis et al., 1508.04543 (heavy quark diffusion coefficient) ($192^3 \times 48$ lattice with $a \simeq 0.01$ fm)

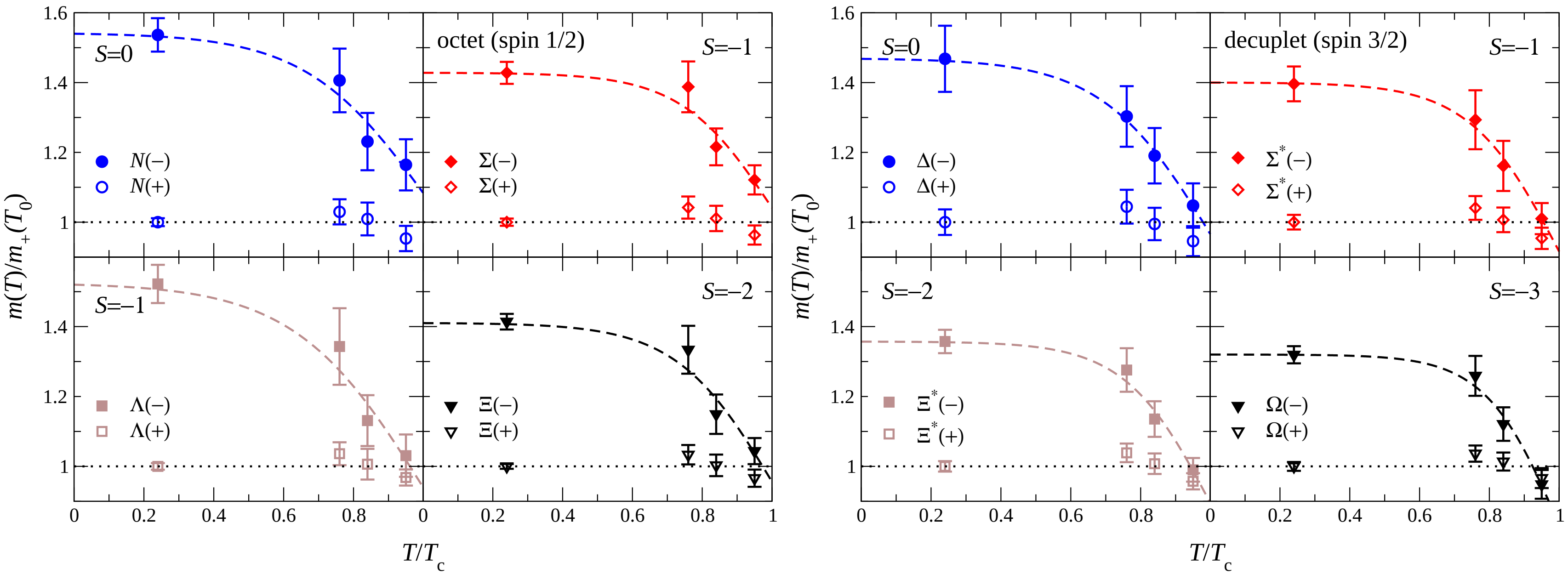
Some limited success in QCD with dynamical quarks

$\rightarrow$ **electric conductivity** G. Aarts *et al*, arXiv:1307.6763, arXiv:1412.6411

- multilevel updating schemes not available yet in full QCD

domain decomposition algorithms under way: M. Cè, L. Giusti and S. Schaefer, arXiv:1601.04587

Hadron spectrum and chiral symmetry across T_c



Chiral symmetry restoration in the baryon sector: parity doublets become degenerate at $T \sim T_c$ G. Aarts *et al*, arXiv:1710.08294 and talk by C. Allton. See also talk by C. Sasaki

Should one take the effect into account in HRG models?

Talk by C. Rohrhofer: Restoration of chiral symmetry observed in spatial correlators in the vector channel. Additional emergent $SU(2N_f)$ observed!

Heavy quark bound states are more sensitive to the fate of the confining properties of the medium. These are being studied with increasing precision and refinement:

- **Fate of confinement across T_c studied at the level of flux tube disappearance**

  P. Cea, L. Cosmai, F. Cuteri and A. Papa, arXiv:1710.01963

  P. Bicudo, N. Cardoso and M. Cardoso, arXiv:1702.03454

- **Heavy $Q\bar{Q}$ free energies and screening properties explored up to high T**

  A. Bazavov *et al* arXiv:1804.10600

- **Investigation of screening properties and heavy quark free energies at finite μ_B**

  M. Andreoli *et al* arXiv:1712.09996 and poster by A. Rucci

- **First approach to a determination of the in-medium potential in full QCD**

  $\rightarrow$ talk by A. Rothkopf

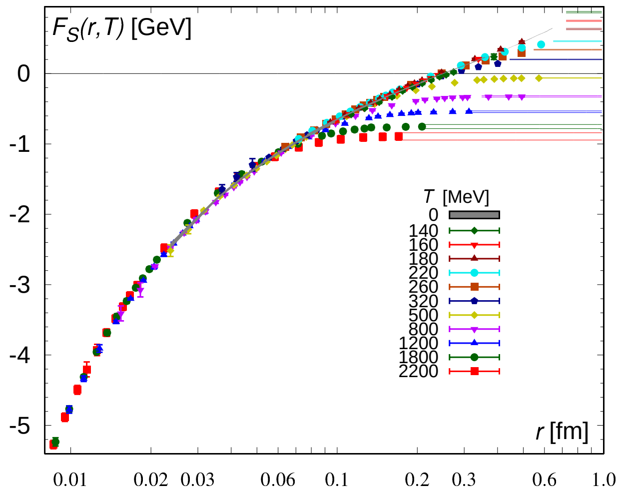
- **Progress on charmonium, bottomonium spectral functions**

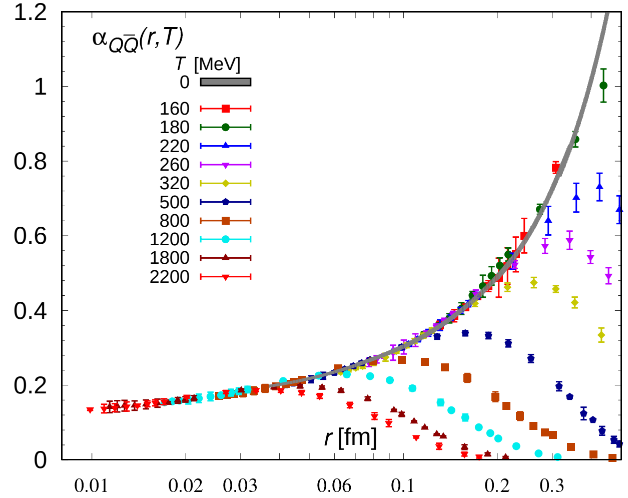
  $\rightarrow$ talks by H.T Shu

- **Progress on open charm spectral functions**

  A. Kelly, A. Rothkopf, J. Skullerud, arXiv:1802.00667 (larger medium effect on D_s, differences between D and D^*)

Screening Properties of the medium at high T (A. Bazavov *et al* arXiv:1804.10600)

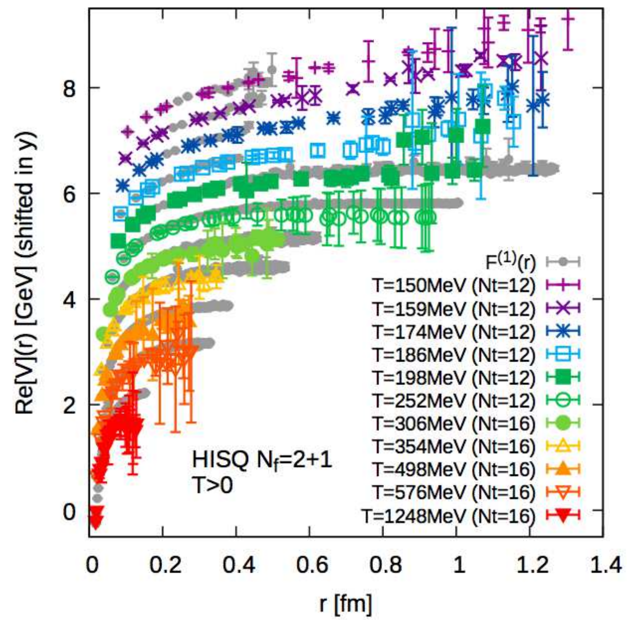




- static $Q\bar{Q}$ free energy in $N_f = 2+1$ QCD. Effective coupling (right): perturbative at short distances, Debye screening at work at intermediate distances and magnetic screening at large distances

Real part of the in-medium potential and comparison with singlet free energies

talk by A. Rothkopf



SUMMARY

- **Precision data on EoS for T at the GeV scale: comparison with HTL-PT and precise estimate of charm contributions**

- **Full convergence from different methods for the pseudo-critical line at small μ_B**

- **Generalized susceptibilities provided up to $O(\mu_B^8)$: information for comparison with experiments and input to various models**
  Are they enough to provide information about the critical point?

- **Progress achieved in the clarification of spectrum modifications across T_c and screening properties of the QGP**