Some considerations on the Quark-Gluon Plasma

Giorgio Parisi

Plan of the talk

- An historical remark on the paper with Nicola Cabibbo quark liberation. I have no time to speak also on the complex Langevin equation.
- Some selected topics in off-equilibrium Hamiltonian statistical mechanics. I will concentrate on **slow** approach to equilibrium.
  - Many body localization: an intriguing quantum phenomenon.
  - Energy cascade: what happens if the energy is concentrated on low frequency modes? How does it diffuse from low frequency modes to high frequency modes? Two examples: the Fermi-Pasta-Ulam model and turbulence.
  - Glassy physics: a corrugated landscape.

Quark liberation: Cabibbo Parisi

Two phenomenological models for confinement:

• Flux lines (dual Meisner effect): deconfinement as restoration of the dual gauge symmetry.

• MIT bag model: percolation of bags.

High temperature: free Fermi gas.

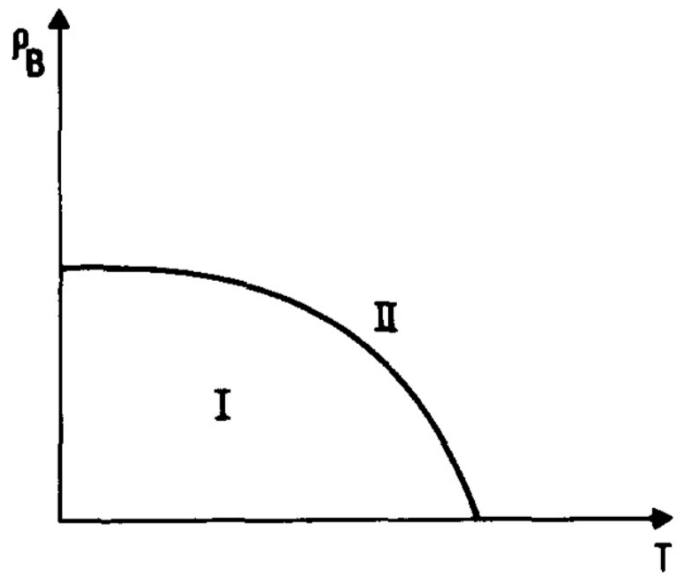
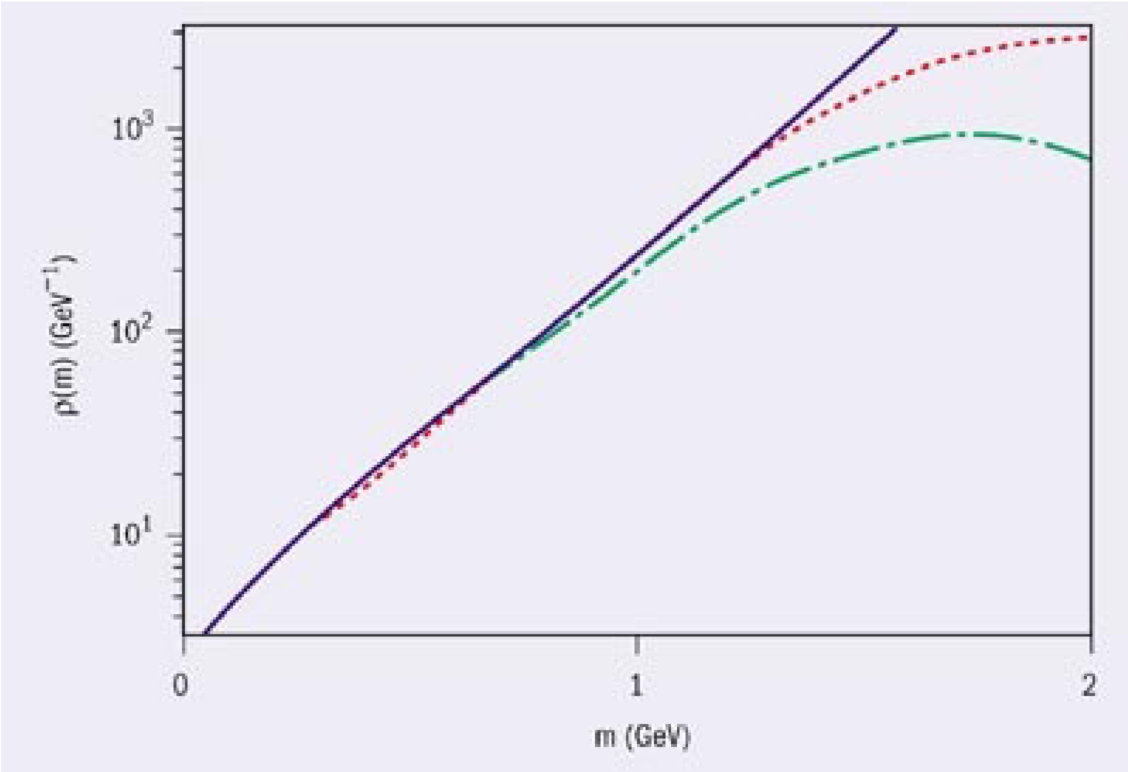


Fig. 1. Schematic phase diagram of hadronic matter. ρ_B is the density of baryonic number. Quarks are confined in phase I and unconfined in phase II.

Which is the value of T_c? $T_H = T_c \approx m_\pi$ in Cabibbo Parisi!

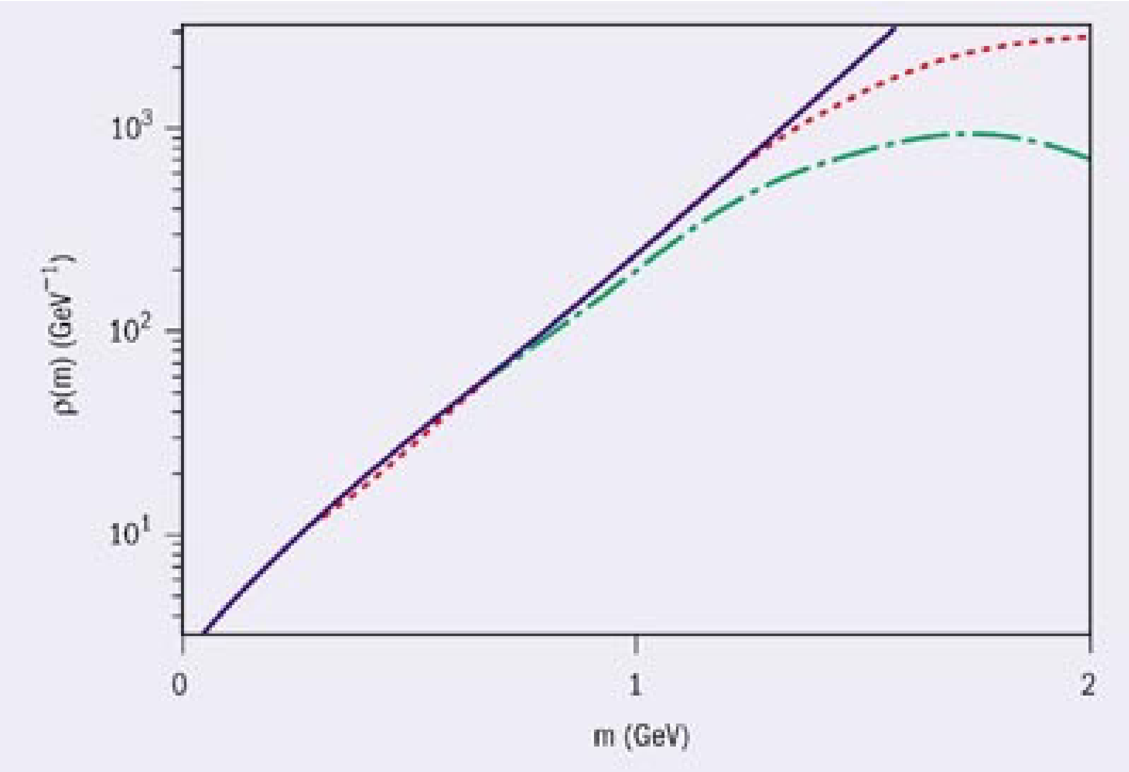
Which is the value of the Hagedorn temperature? Ericson and Rafelski.



The smoothed mass spectrum of hadronic states as a function of mass. Experimental data: long-dashed green line with the 1411 states known in 1967; short-dashed red line with the 4627 states of 1996. The solid blue line represents the exponential fit yielding T_H=158 MeV.

An exponential mass spectrum arises from states that cannot be described by simple two or three quarks bound states. It arises from the excitations of the bag or of the flux tube : i.e multiple states with the same quantum numbers.

Different threshold: pseudoscalar, vector mesons, strangeness, nucleons contribute to an effective exponential spectrum with the **correct** value of the temperature!



Number of quantum states $\approx$ classical phase space

Two quarks with an linear increasing potential: $|x_1 - x_2| < \lambda E$:
the density of states is proportional to E^3

Three quarks with an linear increasing potential: $|x_1 - x_2| < \lambda E$, $|x_1 - x_3| < \lambda E$:
the density of states is proportional to E^6.

Two quarks with a logarithmic potential:
an exponentially increasing density of states.

$$Z = \int d^3x \exp(-\beta\lambda \ln(x))$$

diverges for

$$\beta\lambda \leq 2$$

After the collision of two nuclei thermal equilibrium is reached in a short time.

Why do we believe all systems do reach thermal equilibrium?

Wrong answer: our teacher told us so.

We have a Hamiltonian with $2N$ degrees of freedom.

$$H = H_0 + gH_1$$

If H_0 has N integrals of motion and H_1 is bounded, what happens at $g \neq 0$? (two examples: a slightly anharmonic system or the solar system). What happens to the integrals of motion?

Thermalization and non-trivial integrals of motion are not compatible.

$$H = H_0 + gH_1$$

Poincaré theorem The energy is the only integral of motion that is an analytic function of g.

Perturbation theory is non-convergent: its convergence is ruined by the presence of small denominators.

Naively we expect a microcanonical ensemble at large times.

A newly discovered quantum phenomenon:
Many body localization:

There is no quantum equivalent of the Poincaré theorem.

In some cases one can construct integrals of motion:
small denominators are present but they are not so nasty as in classical theory.

The microcanonical ensemble does not hold.

The worst new:
for large systems, there are local integrals of motion (LIOM) that depends essentially on the behavior of the system near an arbitrary point.

No thermal equilibrium, no Boltzmann statistics: the memory of the initial conditions in a point lasts for an infinite time.

Back to the classic world! KAM theory: a very brief summary.

If we have an Hamiltonian with $2N$ degrees of freedom

$$H = H_0 + gH_1$$

If H_0 has N integrals of motion and H_1 is bounded, for small g often the system behaves like an integral system.

There are values of g where this does not happen. Both kinds of g's have a non-zero measure.

Integrability is most likely at small g and it disappears completely at a g of order 1 (i.e. $g > g^*$).

For these values of g the time evolution does not bring the system to the microcanonical ensamble: the large time limit is not described by the standard statistical mechanics.

Fermi Pasta Ulam models: One dimensional model

$$H_{FPU} = \sum_i \left(\frac{1}{2} \Pi_i^2 + \frac{1}{2} (\phi_i - \phi_{i+1})^2 + \frac{1}{4} g (\phi_i - \phi_{i+1})^4 \right)$$

Fermi Pasta Ulam discovered that the system does not go to equilibrium in the region of not too large g. They started from an energy that is concentrated in the low-frequency modes.

Multiple collisions of low-frequency waves (phonons) **should** produce high-frequency waves.

Later it has been realized that

$$H_{FPU} = H^* + O(g^2)$$

where H^* is integrable and this partially explains the absence of approach to equilibrium.

A modified model

$$H = \sum_i \left(\frac{1}{4}\Pi_i^2 + \frac{1}{2}(\phi_i - \phi_{i+1})^2 + \frac{1}{2}m^2\phi_i^2 + \frac{1}{4}g\phi_i^4 \right)$$

In the continuum limit

$$\frac{\partial^2 \phi}{\partial t^2} = \frac{\partial^2 \phi}{\partial x^2} - m^2\phi - g\phi^3$$

What happens if the energy is concentrated on the low-frequency modes?

Multiple collisions of low-frequency waves (phonons) should produce high-frequency waves.

It is important to consider a momentum (k) dependent energy $E(k)$.

$E(k) = |\tilde{\Pi}(k)|^2 \qquad \tilde{\Pi}(k) = \sum_k \exp(ipj)\Pi_j$

At equilibrium $\langle \Pi_i \Pi_j \rangle = 0$ hence $E(k) = const$: ultraviolet catastrophe

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For small g and times not to large than 10^9 natural units:
$E(k) \propto \exp(-\beta(t)|k|)$: an effective quantum behaviour.

Also if $\phi(x,t)$ is analytic for complex x at $t = 0$, at later times $\phi(x,t)$ develops poles. If the nearest poles are at

$$\mathrm{Im}\ x = b(t)$$

we get

$$E(k) \propto \exp(-2b(t)|k|)$$

KAM theory and the infinite volume limit

We know that for most of the $g < g^*$ we do not reach equilibrium.

How g^* depends on the volume?

One can argue that in the infinite volume limit $g^* \to 0$ for a random starting configuration.
At $g = 0$ in the infinite volume limit at large times $\phi(x, t)$ as a Gaussian distribution.

For small g, $g\phi^4$ may be arbitrarily large so that $g\phi^4$ may be arbitrarily large.

The characteristic time increases very fast when g becomes small.

3-D turbulence

The velocity field $v_\mu(x,t)$ is divergence free (incompressible fluid)

$$\sum_\mu \partial_\mu v_\mu = 0$$

.

Passive matter $\rho(x,t)$ is transported by the velocity: it satisfies the continuity equation.

$$\dot\rho = \sum_\mu \partial_\mu \left(v_\mu \; \rho\right)$$

The zero viscosity equation are the limit $\eta \to 0$ of Navier-Stokes equations

$$\dot v_\nu = \sum_\mu \partial_\mu \left(v_\mu \; v_\nu\right) - \eta \Delta v_\nu + \partial_\nu p$$

$p(x,t)$ is a function that enforces the zero divergence condition.

Energy is formally conserved when $\eta \to 0$ (zero viscosity).

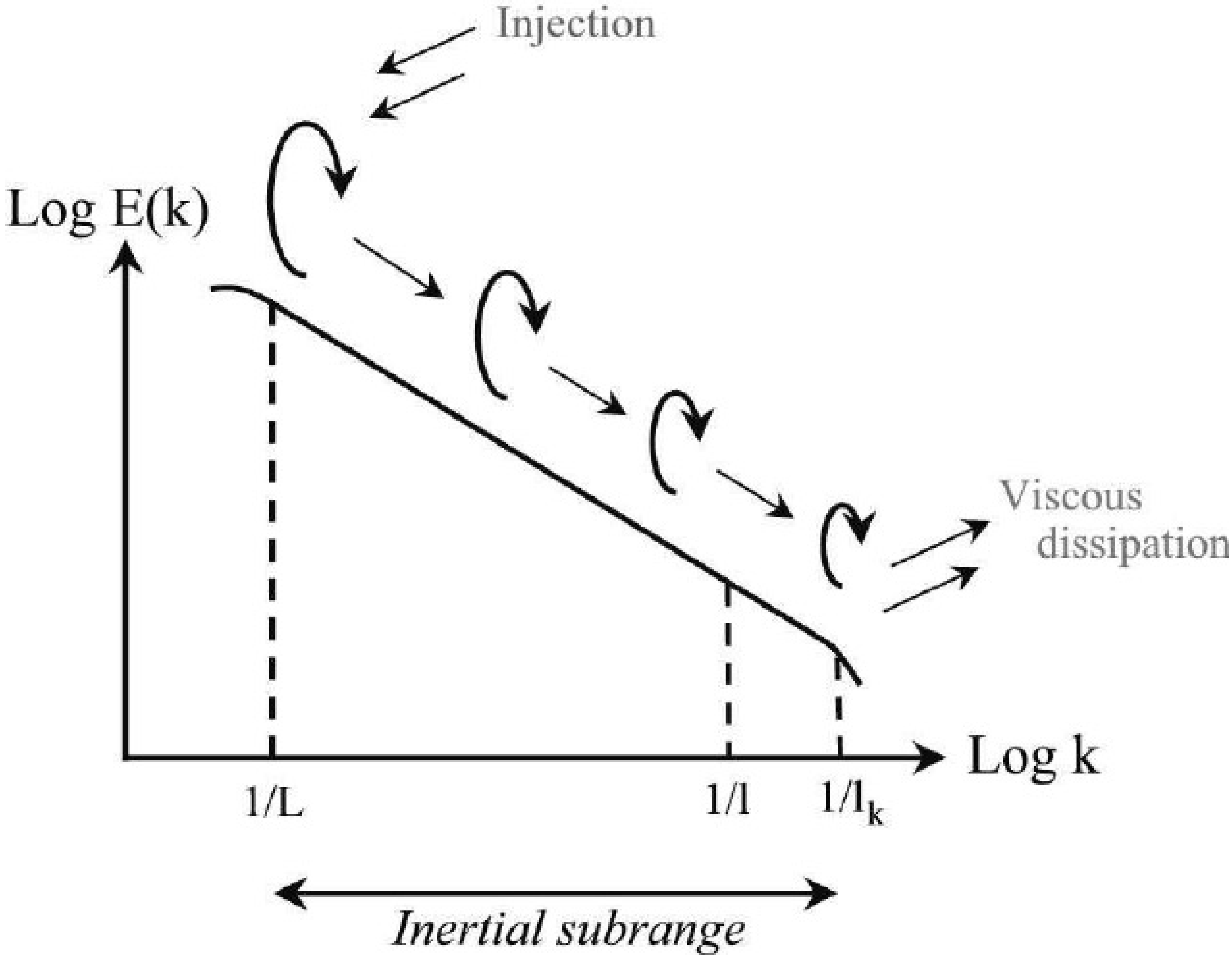
$E = \int dx \sum_\mu v_\mu^2$

We can define $E(k)$. $E(k) \propto \exp(-2b(t)|k|)$ and $b(t) = 0$ for $t > t^*$

For $t > t^*$

$$E(k) \propto |k|^{-\zeta}$$

with $\zeta \approx \frac{5}{3}$ (Kolmogorov theory)


Injection
Log E(k)
Viscous
dissipation
Log k
1/L
1/l
$1/l_k$
Inertial subrange

The ultraviolet catastrophe is present in 3D turbulence.

At a time of order 10, the energy starts to be transferred from low k to infinite k.

At a later time the energy content of low modes decays quite fast (exponentially?).

Glassy dynamics

When cooling a system, some very slow degrees of freedom may go out of equilibrium.

The interesting case is when this degree of freedom correspond to changes in a large space region (of size ξ).

Many different local minima of the free energy. The minima differ one from the other on a region of size ξ. The barriers for going from one minimum to the others are of order $\exp(\Delta(\xi))$, $\Delta(\xi)$ being an increasing function of ξ. Slow dynamics is dominated by **tunnelling**.

In glasses one measure times to approach equilibrium that are very high (10^{18} the natural scale).

Glassy dynamics

- Two very different speeds of approach to equilibrium
  - Inside a minimum: faster
  - Hopping from one minimum to an other: slower.
- Two different temperatures:
  - Inside a minimum: lower
  - Hopping from one minimum to an other: higher.
- When one changes external conditions, the system remains in the same free energy minimum up to the point where it becomes unstable.

  You move at the bottom of a deep canyon and you do not jump to the nearby canyon. Canyons may merge, biforcate or disappear into a jump.

Conclusions

- There are mechanisms that may induce a slow, very slow, infinitely slow approach to equilibrium.

- The situation may strongly depend on the initial conditions (small momenta or large momenta are out of equilibrium)

- It would be interesting to understand if some of them may at action in the case of quark matter.

- How to parametrize phenomenologically **incomplete** thermalization in order to put constraints on this effect and eventually to measure it?