Insight into thermal modifications of quarkonia from a comparison of continuum-extrapolated lattice results to perturbative QCD

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Spectral functions of quarkonia hold information on bound states and their in-medium modifications as well as on transport properties. Determining spectral functions is subject of many calculations, in lattice QCD as well as in perturbation theory.
We compare continuum extrapolated lattice results to a perturbatively determined spectral function obtained by interpolating between vacuum asymptotics at high frequencies and resummed thermal effects around the threshold. Modest differences are observed, which may originate from non-perturbative mass shifts and renormalization factors.

Motivation

The spectral functions of heavy quarkonia have been of great interest to the heavy ion community. Many different approaches have been tried to determine the spectral function. On the lattice, the spectral function $\rho(\omega)$ can not be calculated directly but is hidden in the correlator,

$$G(\tau) = \int_0^\infty d\omega K(\omega,\tau)\rho(\omega) \text{ with } K(\omega,\tau) = \frac{\cosh(\omega(\tau - \frac{1}{2T}))}{\sinh(\frac{\omega}{2T})}. \quad (1)$$

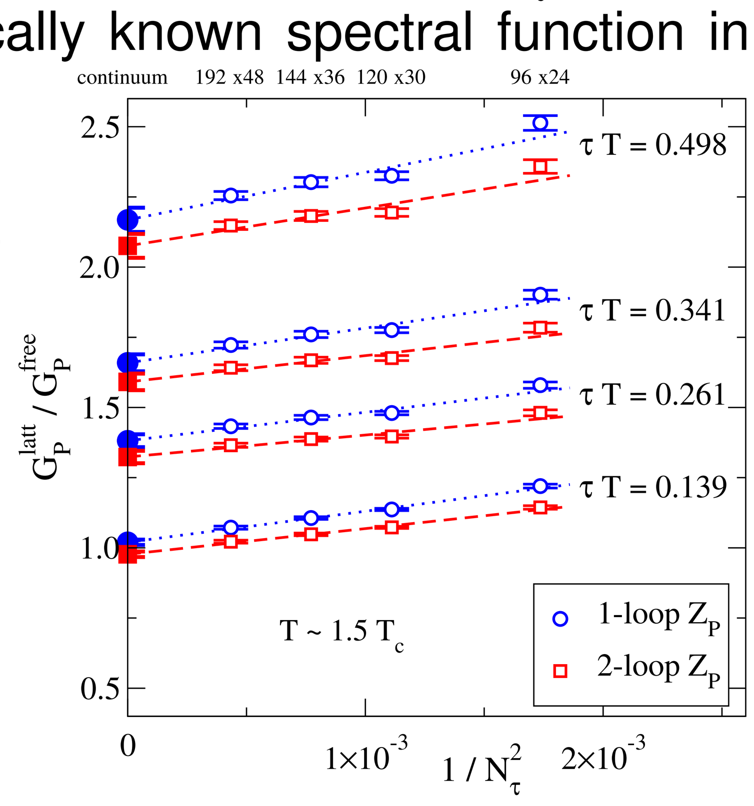
Extracting the spectral function is an ill-posed inversion problem and many different methods have been proposed. We focus on comparing the lattice correlator to a correlator obtained from integrating the perturbative spectral function.

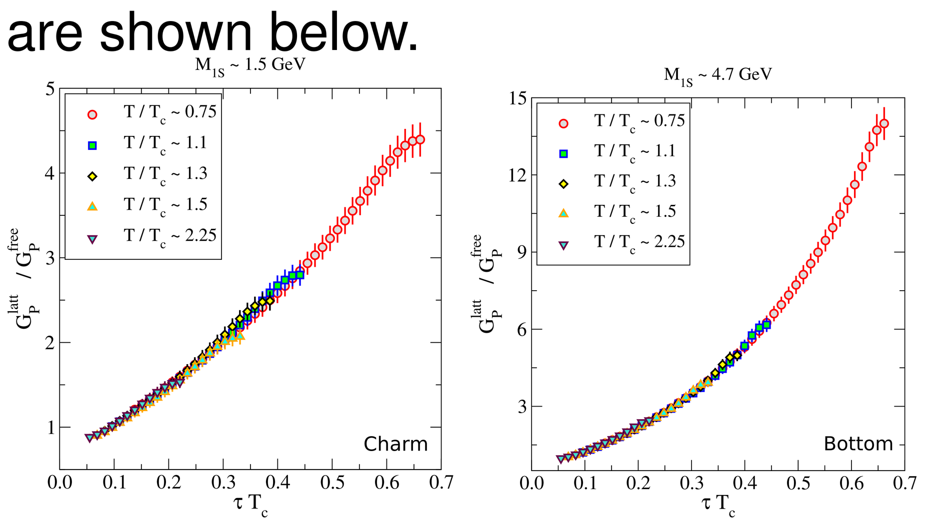
Continuum Correlators

We measured correlation functions in the pseudoscalar and vector channels for 5 temperatures between 0.75 and 2.25T_c with 4 different lattice spacings using Clover-improved Wilson fermions on quenched gauge field configurations. To compare to a perturbation theory, the continuum limit of the lattice correlators needs to be taken. The method is described in [1,2] and consists of the following steps:

Renormalization: In the vector channel, there are different options regarding the renormalization. In addition to perturbative renormalization constants known up to two-loop-order and non-perturbatively determined renormalization constants, we take the continuum limit of renormalization independent ratios with the quark number susceptibility χ_q. In the pseudo-scalar channel only one- and two-loop perturbative renormalization are available. Differences between these renormalizations are used to estimate systematical errors.

Normalization and Continuum Extrapolation: The correlators in the following figures are normalized by the free correlator $G_{free}(\tau T)$, which is calculated using the analytically known spectral function in the non-interacting case. The continuum extrapolation is shown in the figure on the right. The continuum results are shown below.

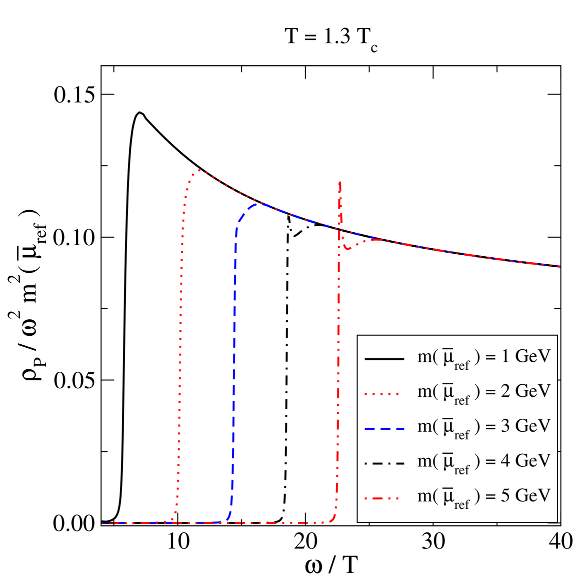




Perturbative Spectral Function

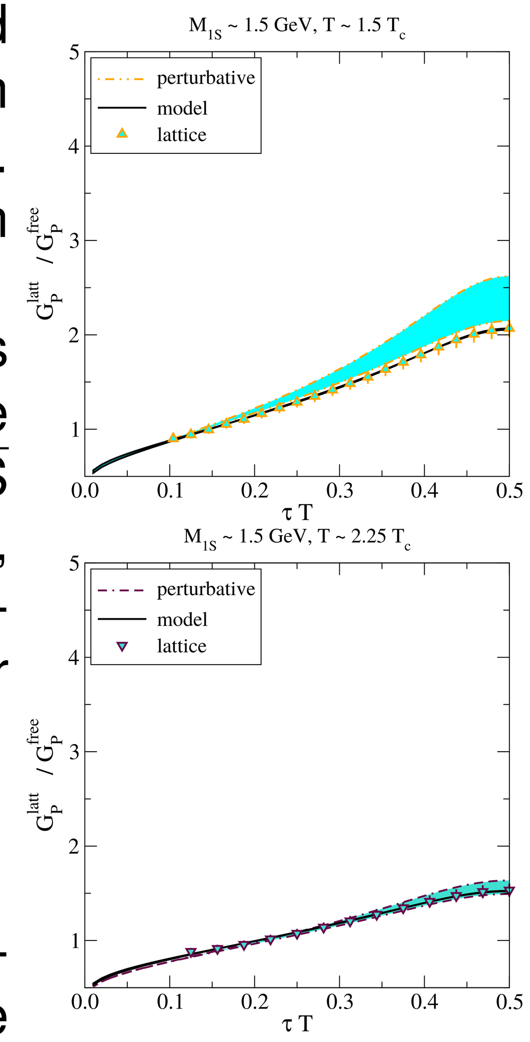
At high frequencies, the spectral function is described by vacuum asymptotics. Around the threshold, thermal contributions arise and are included using a pNRQCD approach with a leading order real-time static potential.

To combine the two frequency regimes, the non-relativistic expression is normalized with a constant to match the vacuum asymptotics. For energies much lower than the threshold, an exponential suppression is introduced since the non-relativistic description mentioned before overestimates the spectral function.



Comparison

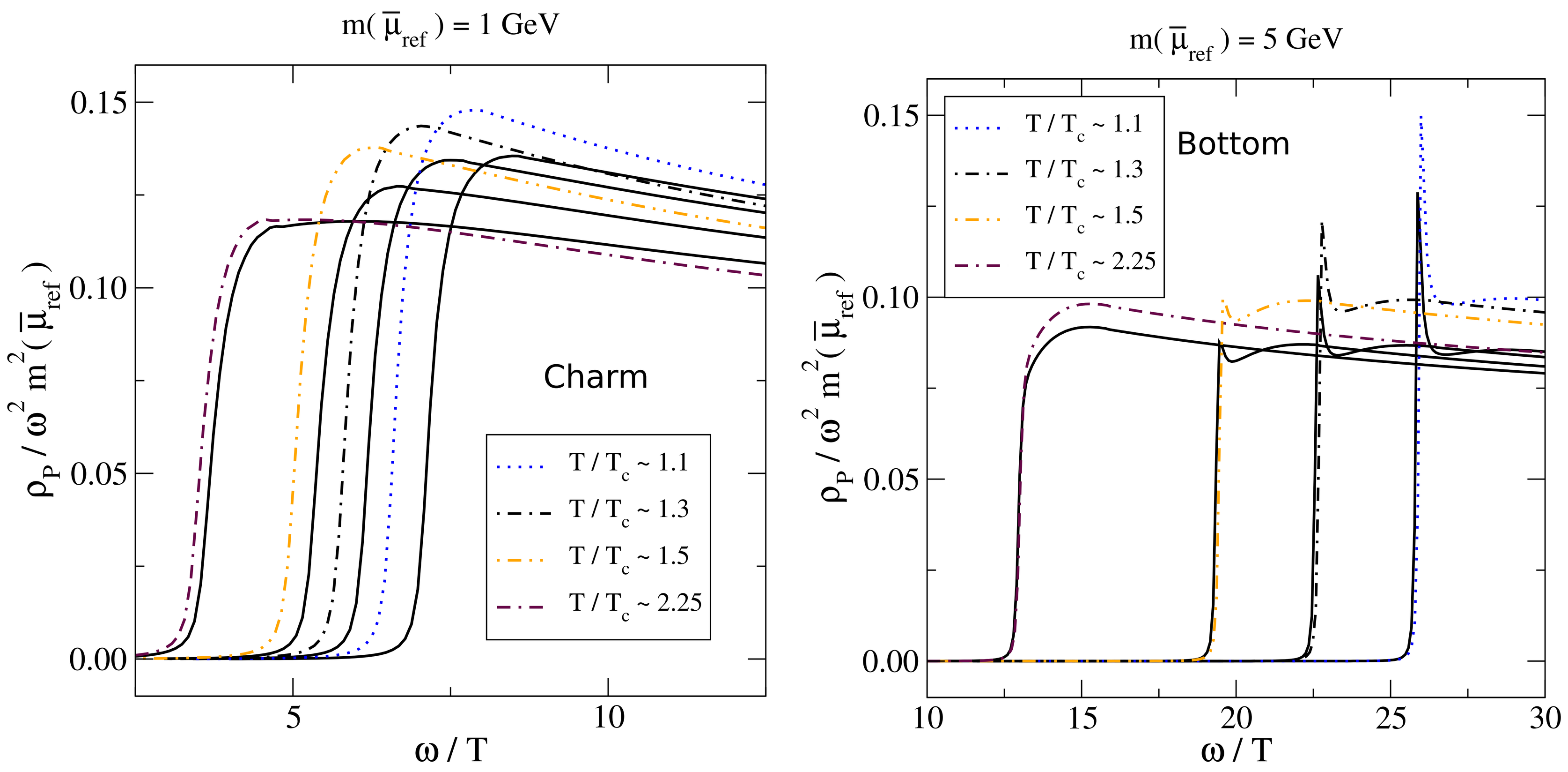
The pictures below provide a first look into the similarities and differences between the perturbation theory (dashed lines) and lattice data (triangle) for charmonium in the pseudo-scalar channel at 1.5 and 2.25T_c. Small differences can be observed, even though the shape of the correlators is similar.



The differences may originate from uncertainties in the renormalization, and on the perturbative side, the relation of the pole mass to the $\overline{\text{MS}}$ mass at a given reference scale is poorly known, which might lead to variations of the threshold location. These two effects are implemented in our model spectral function,

$$\rho^{model}(\omega) = A\rho^{pert}(\omega + B),$$

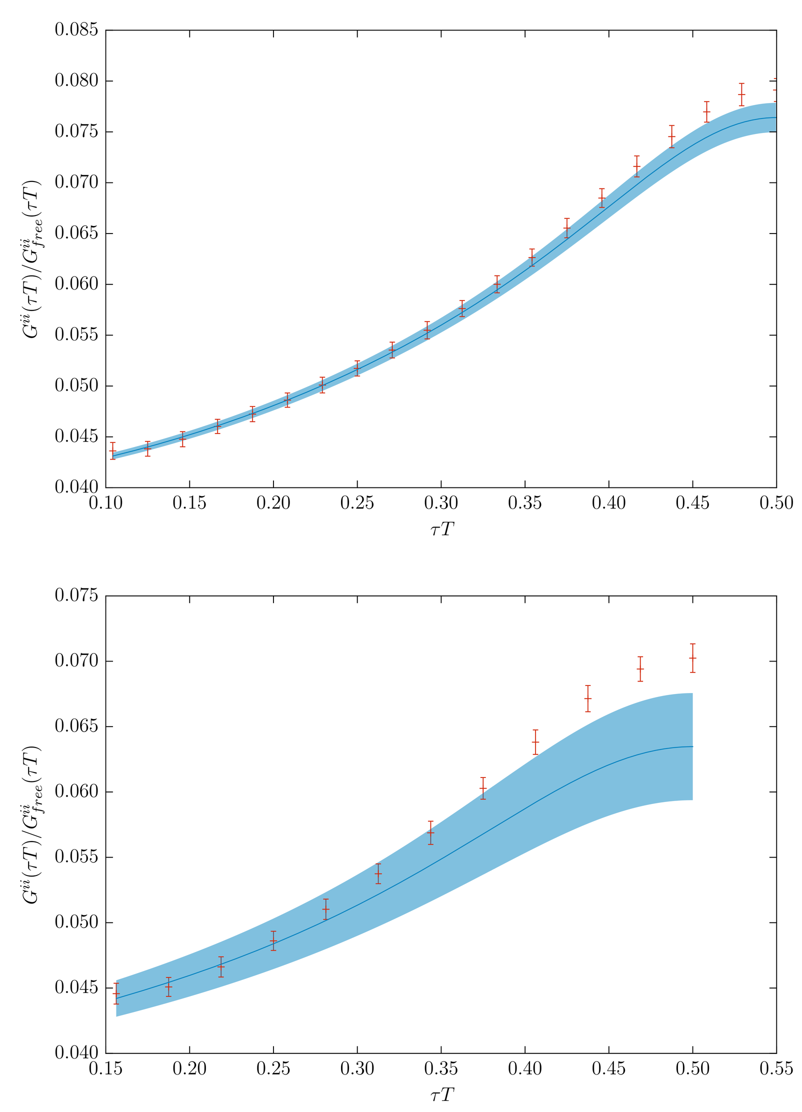
and we fit A and B to the lattice data. The fit results depicted as black lines in the pictures above show good agreement with the lattice data. The normalization factor A is close to 1 and the mass shift B is small.



From this the spectral functions shown above are obtained.

The dashed lines show the original perturbative function and the solid lines show the fit results. For charmonium, no resonance peak is needed, while for bottomonium, a thermally broadened resonance peak gives a good description of the lattice data up to 1.5Tc.

These results are obtained for the pseudo-scalar channel. The knowledge gained from this analysis is used for ongoing studies in the vector channel. A first comparison of the lattice to the perturbative (w/o transport contribution) correlators is shown in the figures on the right for 1.5 and 2.25T_c.



References

[1] H. Sandmeyer et al., *Continuum extrapolation of quarkonium correlators at non-zero temperature.* EPJC 175(2018)07010

[2] Y. Burnier, H.-T. Ding, O. Kaczmarek, A.-L. Kruse, M. Laine, H. Ohno and H. Sandmeyer, *Thermal quarkonium physics in the pseudoscalar channel.* JHEP11(2017)086

[3] See also: Hai-Tao Shu, Talk at QM 2018