



Stony Brook University



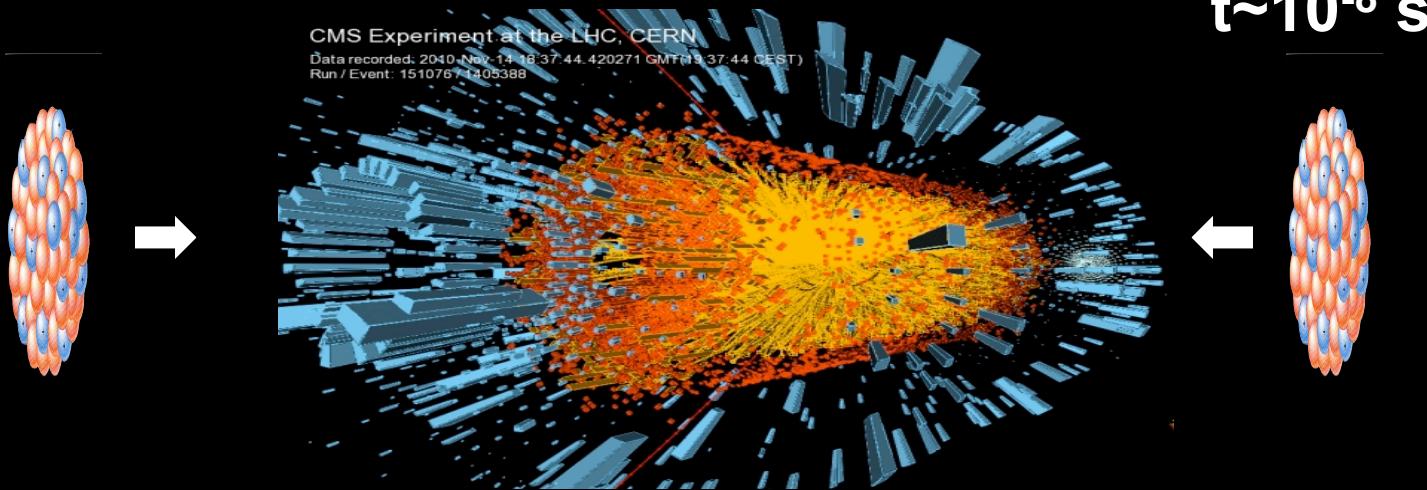
Hydro and flow in nuclear (A+A) collisions

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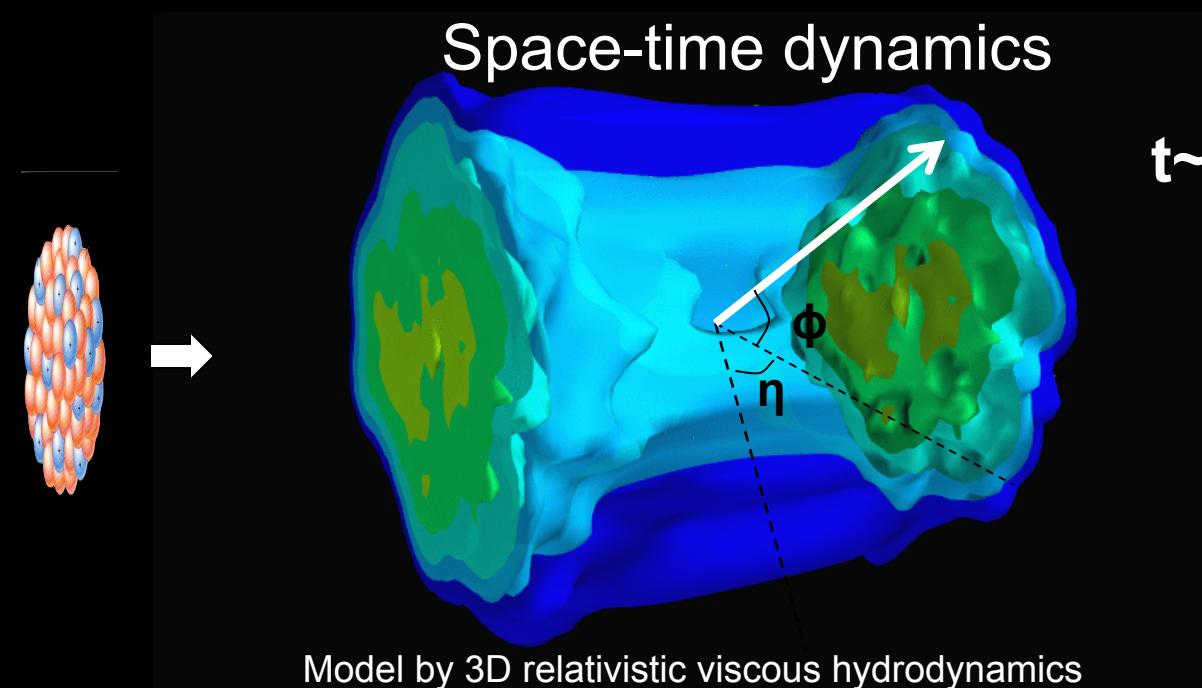
- Space-time dynamics and hydro
- Flow observables and how to measure them
- Flow fluctuations from event to event
- Flow fluctuations within the same event
- Roads toward precision.

Seen by detector



$t \sim 10^{-8} \text{ s}$

Space-time dynamics



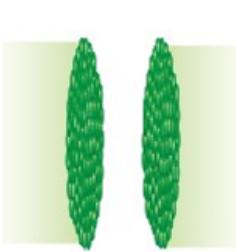
$t \sim 10 \text{ fm}/c = 10^{-22} \text{ s}$

Model by 3D relativistic viscous hydrodynamics

Credit: Bjoern Schenke

Space-time dynamics

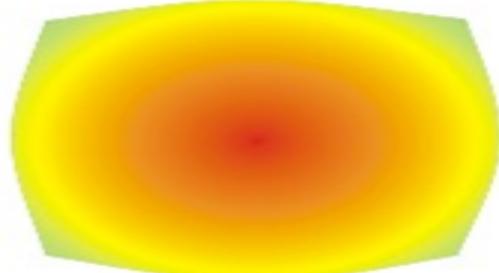
$\tau < 0$



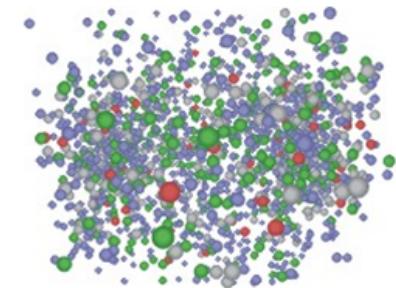
$0 < \tau < 0.5 \text{ fm}/c$



$0.5 < \tau < 6 \text{ fm}/c$



$6 < \tau < 10 \text{ fm}/c$



initial state

pre-equilibrium

QGP & expansion

Phase transition&freeze-out

Glauber
CGC

Field theory
+kinetic

Hydrodynamics

Hydro+kinetic

Use hydro to unfold the space-time dynamics

Basics of hydrodynamics: ideal

Energy-momentum conservation

$$\partial_\mu T^{\mu\nu} = 0$$

Charge conservation

$$\partial_\mu N^\mu = 0$$

System always in local equilibrium: ideal hydrodynamics

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu - Pg^{\mu\nu}$$

$$N^\mu = n u^\mu$$

Six unknown: ϵ , P , u^μ , and n , only five equations-of-motion

Closed by the equation-of-state (EOS) :* $\epsilon=\epsilon(P)$

* zero chemical potential

Hydro-response controlled by QCD EoS.

Basics of hydrodynamics: viscous

Energy-momentum conservation

$$\partial_\mu T^{\mu\nu} = 0$$

Charge conservation

$$\partial_\mu N^\mu = 0$$

Include near-equilibrium corrections: viscous hydrodynamics

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - (P + \Pi) \Delta^{\mu\nu} + \pi^{\mu\nu}$$

Bulk pressure Shear tensor

$$N^\mu = n u^\mu + n^\mu$$

Charge diffusion

Basics of hydrodynamics: 1st order

Energy-momentum conservation

$$\partial_\mu T^{\mu\nu} = 0$$

Charge conservation

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Include near-equilibrium corrections: viscous hydrodynamics

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - (P + \Pi) \Delta^{\mu\nu} + \pi^{\mu\nu}$$

Bulk pressure Shear tensor

$$N^\mu = n u^\mu + n^\mu$$

Charge diffusion

Include 1st-order gradient expansion:

$$\pi^{\mu\nu} = -\eta \sigma^{\mu\nu}$$

η : shear viscosity coefficient

$$\Pi = -\zeta \nabla_\lambda^\perp u^\lambda$$

ζ : bulk viscosity coefficient

Basics of hydrodynamics: 2nd order

Energy-momentum conservation

$$\partial_\mu T^{\mu\nu} = 0$$

Charge conservation

$$\partial_\mu N^\mu = 0$$

Include near-equilibrium corrections: viscous hydrodynamics

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - (P + \Pi) \Delta^{\mu\nu} + \pi^{\mu\nu}$$

Bulk pressure Shear tensor

$$N^\mu = n u^\mu + n^\mu$$

Charge diffusion

Include up to 2nd-order gradient expansion

$$\begin{aligned} \pi^{\mu\nu} = & -\eta \sigma^{\mu\nu} + \eta \tau_\pi \left[\langle D\sigma^{\mu\nu} \rangle + \frac{\nabla_\lambda^\perp u^\lambda}{3} \sigma^{\mu\nu} \right] + \kappa [R^{\langle\mu\nu\rangle} - 2u_\lambda u_\rho R^{\lambda\langle\mu\nu\rangle\rho}] + \lambda_1 \sigma^{\langle\mu}_\lambda \sigma^{\nu\rangle\lambda} \\ & + \lambda_2 \sigma^{\langle\mu}_\lambda \Omega^{\nu\rangle\lambda} + \lambda_3 \Omega^{\langle\mu}_\lambda \Omega^{\nu\rangle\lambda} + \kappa^* 2u_\lambda u_\rho R^{\lambda\langle\mu\nu\rangle\rho} + \eta \tau_\pi^* \frac{\nabla_\lambda^\perp u^\lambda}{3} \sigma^{\mu\nu} + \bar{\lambda}_4 \nabla_\perp^{\langle\mu} \ln \epsilon \nabla_\perp^{\nu\rangle} \ln \epsilon \end{aligned}$$

$$\begin{aligned} \Pi = & -\zeta (\nabla_\lambda^\perp u^\lambda) + \zeta \tau_\Pi D (\nabla_\lambda^\perp u^\lambda) + \xi_1 \sigma^{\mu\nu} \sigma_{\mu\nu} + \xi_2 (\nabla_\lambda^\perp u^\lambda)^2 \\ & + \xi_3 \Omega^{\mu\nu} \Omega_{\mu\nu} + \bar{\xi}_4 \nabla_\mu^\perp \ln \epsilon \nabla_\perp^\mu \ln \epsilon + \xi_5 R + \xi_6 u^\lambda u^\rho R_{\lambda\rho}. \end{aligned}$$

Taken from 1712.05815

Many transport coeff. → probe microscopic theory, QCD

Hydro to decipher the QGP properties?

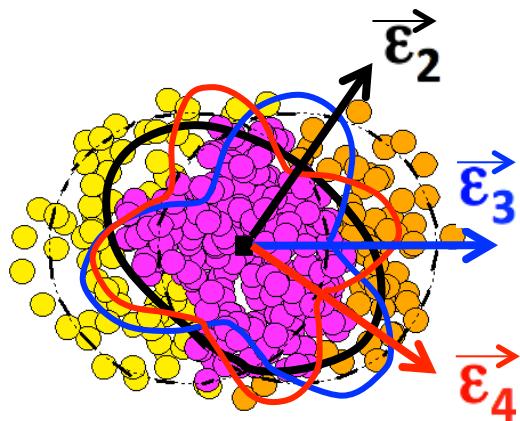
	Gauge/Gravity	Kinetic (BGK)	pQCD	Lattice QCD
$\epsilon(P)$	3 P	Eq. (3.30)	3 P	Eq. (3.125)
η	$\frac{\epsilon+P}{4\pi T}$	$\frac{(\epsilon+P)\tau_R}{5}$	$\frac{3.85(\epsilon+P)}{g^4 \ln(2.765g^{-1})T}$	$0.10(6) \frac{\epsilon+P}{T}$
τ_π	$\frac{2-\ln 2}{2\pi T}$	τ_R	$\frac{5.9\eta}{\epsilon+P}$	
λ_1	$\frac{\eta}{2\pi T}$	$\frac{5}{7}\eta\tau_R$	$\frac{5.2\eta^2}{\epsilon+P}$	
λ_2	$2\eta\tau_\pi - 4\lambda_1$	$-2\eta\tau_R$	$-2\eta\tau_\pi$	
λ_3	0	0	$\frac{30(\epsilon+P)}{8\pi^2 T^2}$	
κ	$\frac{\epsilon+P}{4\pi^2 T^2}$	0	$\frac{5(\epsilon+P)}{8\pi^2 T^2}$	$0.36(15)T^2$
Refs.	[19, 28, 29] [128, 129]	[28, 119, 120]	[121, 123] [130]	[124, 127] [131, 132]

Table 2.1: Compilation of leading-order results for transport coefficients in various calculational approaches, see text for details. table from 1712.05815

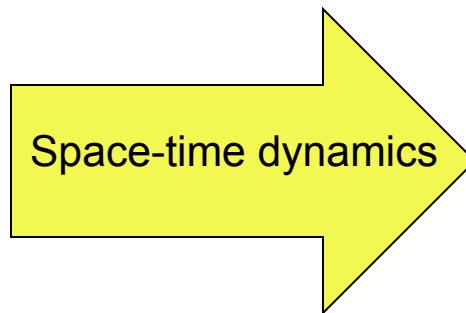
ab.initio calc. for QGP not easy, relies on model/data comparison

Connecting the initial and final state

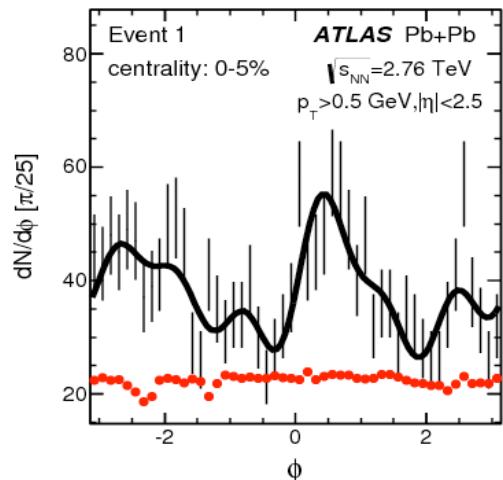
Initial state



Hydro-response



Particle flow

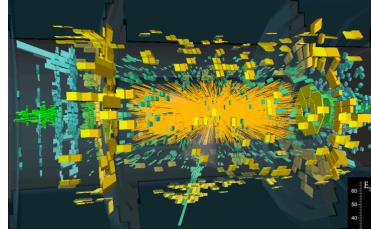


$$\tilde{\epsilon}_n \equiv \epsilon_n e^{in\Phi_n^*} \equiv -\frac{\langle r^n e^{in\phi} \rangle}{\langle r^n \rangle}$$

$$\frac{dN}{d\phi} \propto 1 + 2 \sum_n v_n \cos n(\phi - \Phi_n)$$

- What is the nature of the initial state fluctuation ?
- What is the space-time evolution of the produced matter ?
 - How are (ϵ_n, Φ_n^*) transferred to (v_n, Φ_n) event-by-event?
- What are the properties of the produced matter ?

Hydrodynamic behavior in each event



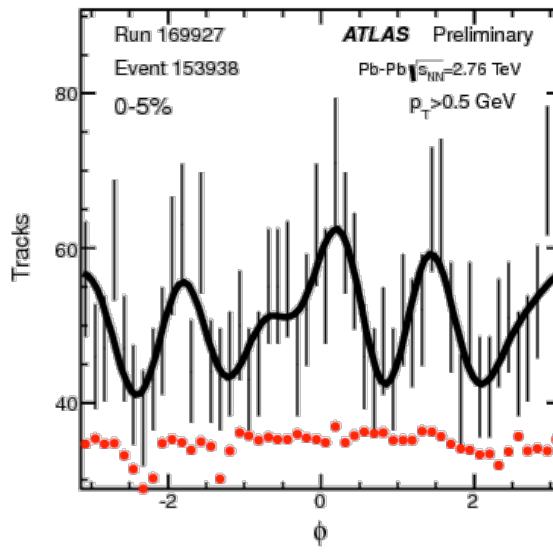
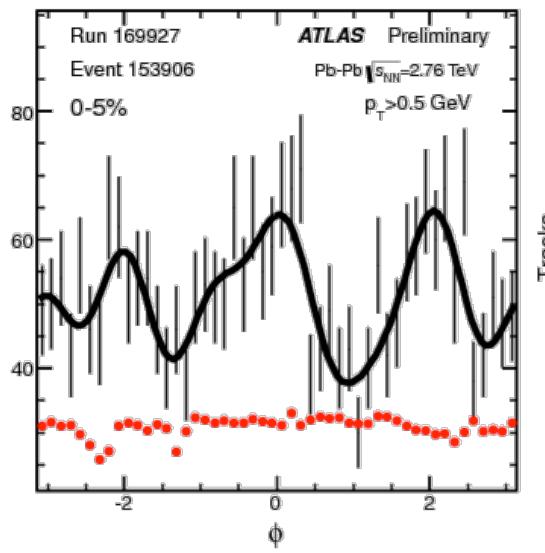
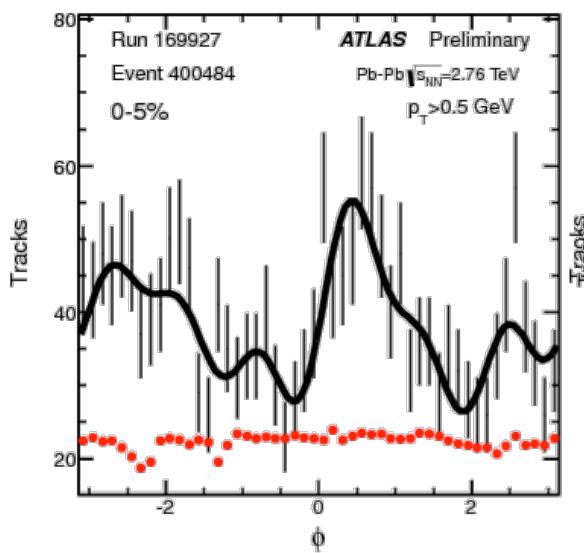
$$\frac{dN}{d\phi} \propto 1 + 2 \sum_n v_n \cos n(\phi - \Phi_n)$$



Event 1

Event 2

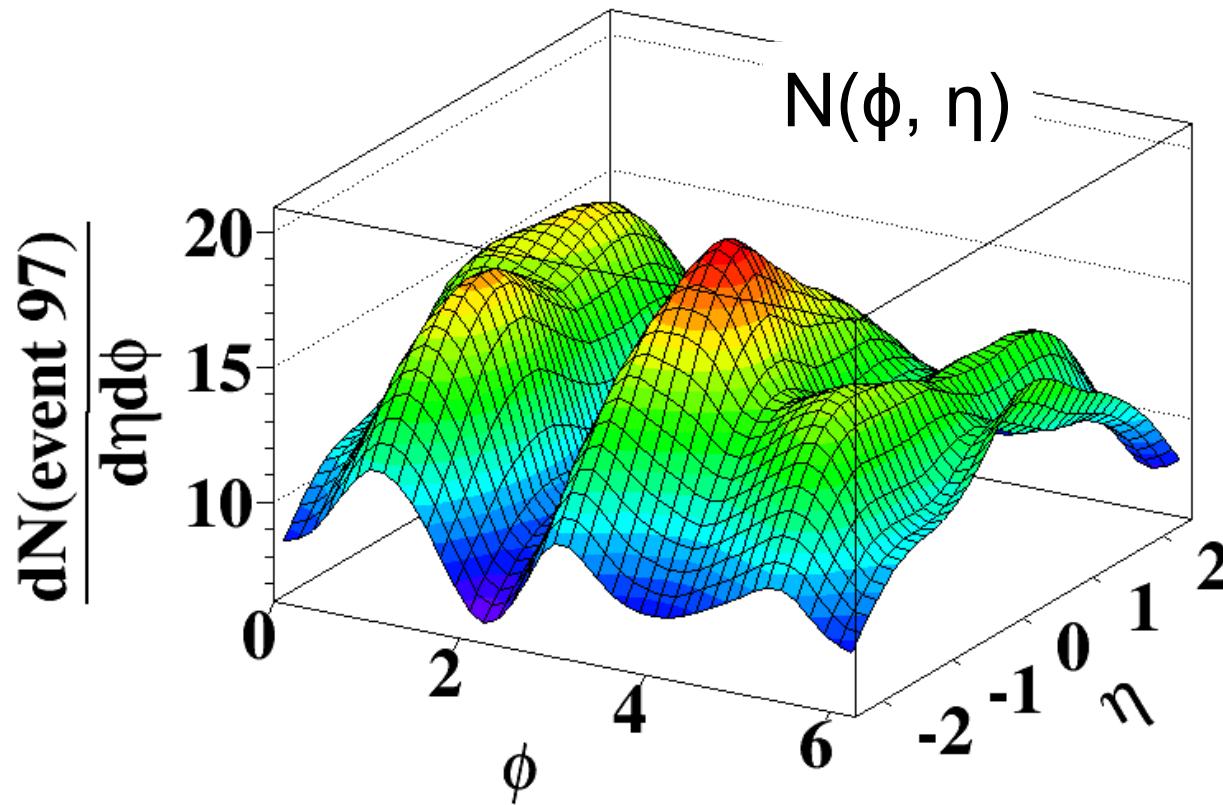
Event 3



- v_n sensitive to initial perturbation and viscosity.
 - Bigger initial fluctuation lead to bigger v_n
 - Small viscosity ensure efficient transfer of initial fluctuation to final state flow.

Richness of flow fluctuations

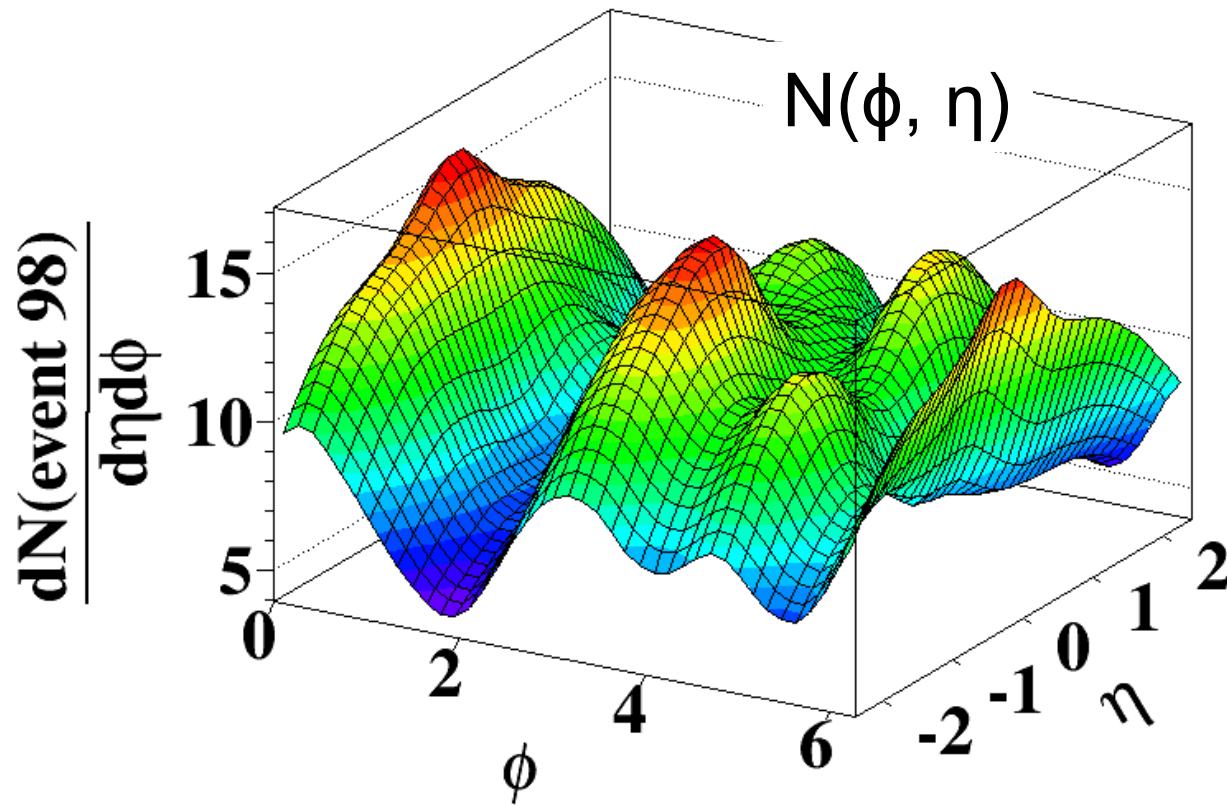
Curtsey of L.Pang and X.N Wang, EbyE 3D hydro+AMPT condition



Fluctuation from event to event

Richness of flow fluctuations

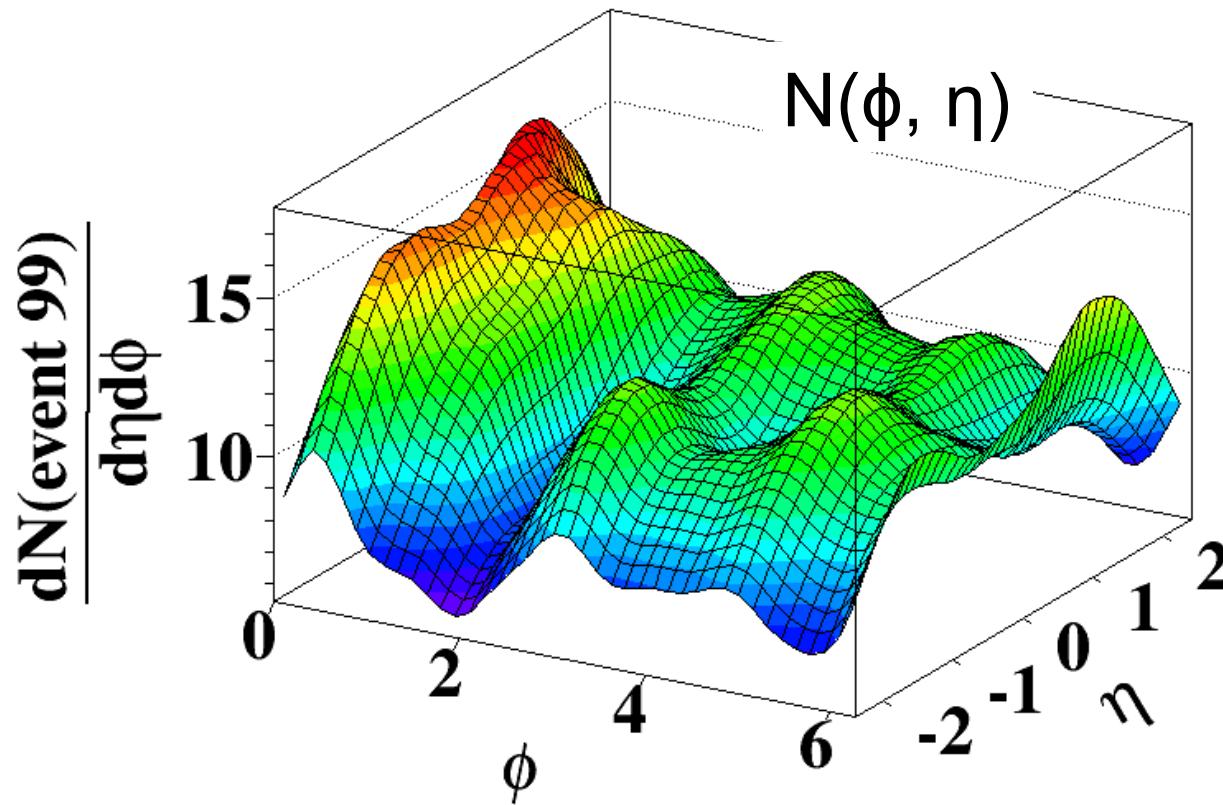
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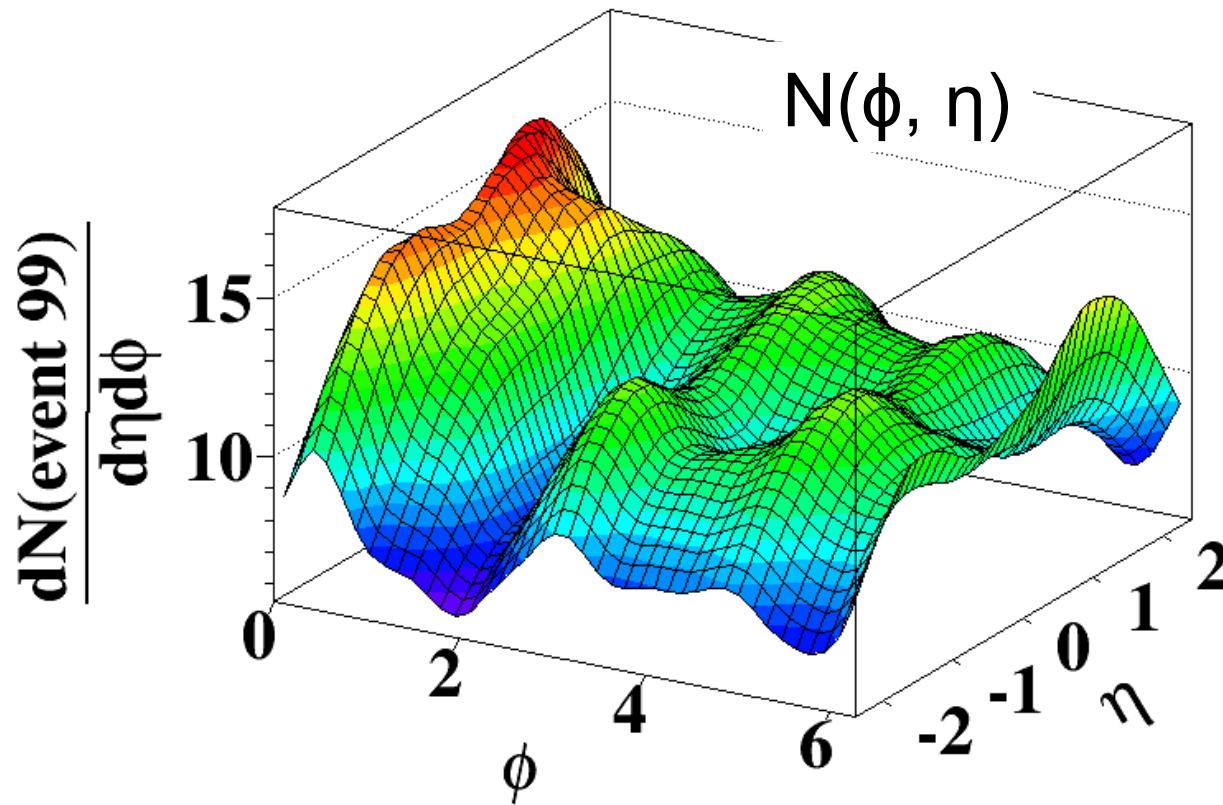
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Fluctuation from event to event

Richness of flow fluctuations

Curtsey of L.Pang and X.N Wang, EbyE 3D hydro+AMPT condition



Fluctuation within a single event

$$\frac{dN}{d\phi} \propto 1 + 2 \sum_n v_n(p_T, \eta, \dots) \cos n(\phi - \Phi_n(p_T, \eta, \dots))$$

Experimental flow observables

- Single particle distribution

$$\frac{dN}{d\phi d\eta dp_T}$$

- Two-particle correlation function

$$\left\langle \frac{dN_1}{d\phi d\eta dp_T} \frac{dN_2}{d\phi d\eta dp_T} \right\rangle$$

- Multi-particle correlation function

$$\left\langle \frac{dN_1}{d\phi d\eta dp_T} \dots \frac{dN_m}{d\phi d\eta dp_T} \right\rangle$$

Experimental flow observables

- Single particle distribution

Flow vector: $V_n = v_n e^{in\Phi_n}$

$$\frac{dN}{d\phi d\eta dp_T} = N(p_T, \eta) \left[1 + 2 \sum_n v_n(p_T, \eta) \cos n(\phi - \Phi_n(p_T, \eta)) \right]$$

$$= N(p_T, \eta) \left[\sum_{n=-\infty}^{\infty} V_n(p_T, \eta) e^{in\phi} \right]$$

Radial flow Anisotropic flow

- Two-particle correlation function

$$\left\langle \frac{dN_1}{d\phi d\eta dp_T} \frac{dN_2}{d\phi d\eta dp_T} \right\rangle$$

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Radial flow Anisotropic flow

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$$\left\langle \frac{dN_1}{d\phi d\eta dp_T} \frac{dN_2}{d\phi d\eta dp_T} \right\rangle \Rightarrow \langle V_n(p_{T1}, \eta_1) V_n^*(p_{T2}, \eta_2) \rangle \quad v_n \text{ from 2PC}$$

- Multi-particle correlation function

$$\left\langle \frac{dN_1}{d\phi d\eta dp_T} \dots \frac{dN_m}{d\phi d\eta dp_T} \right\rangle \Rightarrow \langle V_{n_1} V_{n_2} \dots V_{n_m} \rangle \quad n_1 + n_2 + \dots + n_m = 0$$

$$\langle v_{n_1} v_{n_2} \dots v_{n_m} \cos(n_1 \Phi_{n_1} + n_2 \Phi_{n_2} + \dots + n_m \Phi_{n_m}) \rangle$$

Experimental flow observables

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$$\frac{dN}{d\phi d\eta dp_T} = N(p_T, \eta) \left[1 + 2 \sum_n v_n(p_T, \eta) \cos n(\phi - \Phi_n(p_T, \eta)) \right]$$

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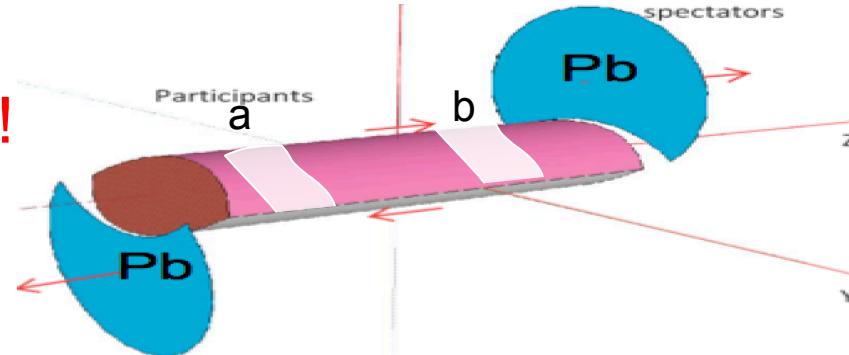
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$$\left\langle \frac{dN_1}{d\phi d\eta dp_T} \dots \frac{dN_m}{d\phi d\eta dp_T} \right\rangle \Rightarrow \langle V_{n_1} V_{n_2} \dots V_{n_m} \rangle \quad n_1 + n_2 + \dots + n_m = 0$$

$$p(v_n, v_m, \dots, \Phi_n, \Phi_m, \dots) = \frac{1}{N_{\text{evts}}} \frac{dN_{\text{evts}}}{dv_n dv_m \dots d\Phi_n d\Phi_m \dots}$$

How to measure flow? $V_n = v_n e^{in\Phi_n}$

By particle correlations!



Determine flow vector in one subevent:

$$\mathbf{q}_n = \frac{\sum_i e^{in\phi_i}}{\sum_i} = v_n e^{in\Phi_n} + \boldsymbol{\delta}$$

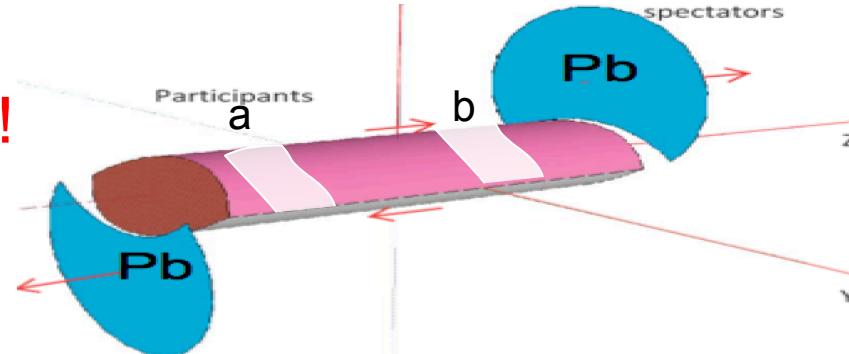
Statistical noise

Noise uncorrelated between two subevents, average over events:

$$\langle \mathbf{q}_n^a \mathbf{q}_n^{b*} \rangle = \left\langle (v_n^a e^{in\Phi_n^a} + \boldsymbol{\delta}^a)(v_n^b e^{-in\Phi_n^b} + \boldsymbol{\delta}^{b*}) \right\rangle = \langle V_n^a V_n^{b*} \rangle$$

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Determine flow vector in one subevent:

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Noise uncorrelated between two subevents, average over events:

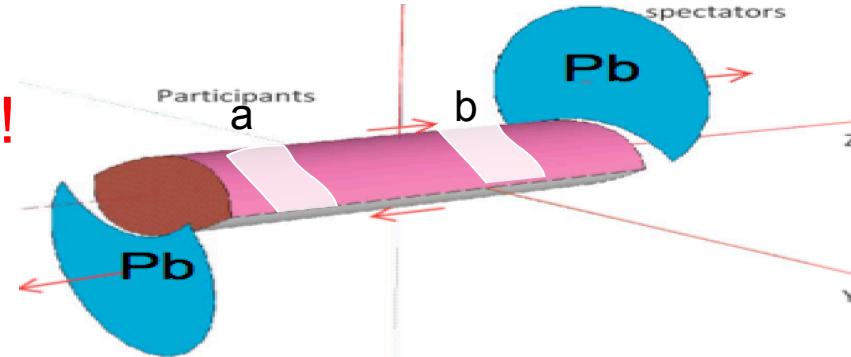
$$\langle \mathbf{q}_n^a \mathbf{q}_n^{b*} \rangle = \left\langle (v_n^a e^{in\Phi_n^a} + \boldsymbol{\delta}^a)(v_n^b e^{-in\Phi_n^b} + \boldsymbol{\delta}^{b*}) \right\rangle = \langle V_n^a V_n^{b*} \rangle$$

We often assume $p(v_n)$ independent of p_T and η , i.e. ignoring intra-event fluctuation $p(V_n) = f(p_T, \eta) p(\bar{v}_n)$

$$\langle V_n^a V_n^{b*} \rangle = f(p_T^a, \eta^a) f(p_T^b, \eta^b) \langle v_n^2 \rangle$$

How to measure flow? $V_n = v_n e^{in\Phi_n}$

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$$\langle V_n^a V_n^{b*} \rangle = f(p_T^a, \eta^a) f(p_T^b, \eta^b) \langle v_n^2 \rangle$$

Event-plane or scalar-product methods, e.g. measure flow in subevent **c** wrt symmetric subevents **a&b**:

$$v_n^{meas} = \frac{\langle \mathbf{q}_n^c \mathbf{q}_n^{a*} \rangle}{\sqrt{\langle \mathbf{q}_n^a \mathbf{q}_n^{b*} \rangle}} = \frac{f(p_T^c, \eta^c) f(p_T^a, \eta^a) \langle \bar{v}_n^2 \rangle}{\sqrt{f(p_T^a, \eta^a) f(p_T^b, \eta^b) \langle \bar{v}_n^2 \rangle}} = f(p_T^c, \eta^c) \sqrt{\langle \bar{v}_n^2 \rangle} = \sqrt{\langle v_n^c v_n^c \rangle}$$

Lessons: 1) We often report RMS value of v_n , 2) relies on factorization assumption!

How to measure EbyE flow fluctuations?

Multi-particle correlations → moments, cumulants

$$C_2 = \langle \delta X^2 \rangle \quad \delta X = X - \langle X \rangle$$

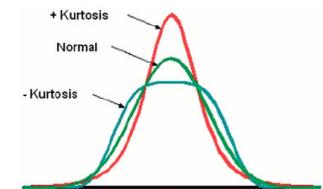
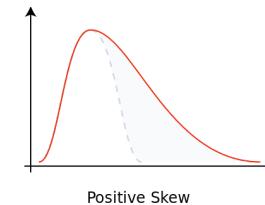
$$C_3 = \langle \delta X^3 \rangle$$

$$C_4 = \langle \delta X^4 \rangle - 3 \langle \delta X^2 \rangle^2$$

$$C_5 = \langle \delta X^5 \rangle - 10 \langle \delta X^3 \rangle \langle \delta X^2 \rangle$$

Quantifies the shape of $p(x)$

C_2 variance, C_3 Skewness, C_4 Kurtosis



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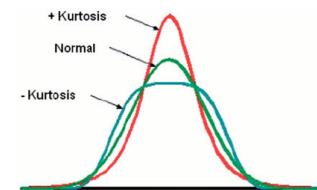
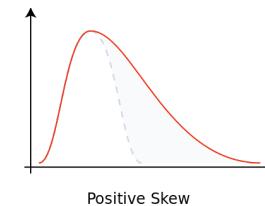
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Replace with harmonics $X = v_n e^{in\Phi_n}$ → cumulants for 2D functions
 Simplification by symmetry → $\langle X \rangle = 0$, $\langle X^n \rangle = 0$, $\langle XX^* \rangle = \langle v_n^2 \rangle \dots$

How to measure EbyE flow fluctuations?

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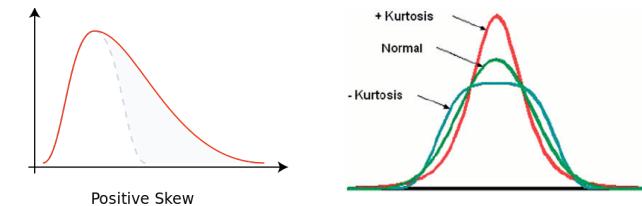
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Cumulant for a single flow harmonic:

N.Borghini, P.Dinh, J.Ollitrault nucl-th/0007063

$$\begin{aligned} c_n\{4\} &= \langle\langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)} \rangle\rangle - \langle\langle e^{in(\phi_1-\phi_3)} \rangle\rangle \langle\langle e^{in(\phi_2-\phi_4)} \rangle\rangle - \langle\langle e^{in(\phi_1-\phi_4)} \rangle\rangle \langle\langle e^{in(\phi_2-\phi_3)} \rangle\rangle - \langle\langle e^{in(\phi_1+\phi_2)} \rangle\rangle \langle\langle e^{in(\phi_3+\phi_4)} \rangle\rangle \\ &= \langle\langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)} \rangle\rangle - 2 \langle\langle e^{in(\phi_1-\phi_2)} \rangle\rangle^2 \quad \text{Vanish by symmetry} \\ &= \langle v_n^4 \rangle - 2 \langle v_n^2 \rangle^2 + \text{non-flow} \end{aligned}$$

↑
Suppress 2PC non-flow

Probe the shape of $p(v_n)$, e.g. non-Gaussianity

How to measure flow fluctuations?

Four-particle **symmetric cumulants**:

A.Bilandzic, C.Christensen, K.Gulbrandsen, A.Hansen, Y.Zhou 1312.3572

$$\begin{aligned}
 \text{sc}_{n,m}\{4\} &= \langle\!\langle e^{in(\phi_1-\phi_2)+m(\phi_3-\phi_4)} \rangle\!\rangle - \langle\!\langle e^{in(\phi_1-\phi_2)} \rangle\!\rangle \langle\!\langle e^{im(\phi_3-\phi_4)} \rangle\!\rangle \\
 &\quad - \cancel{\langle\!\langle e^{i(n\phi_1+m\phi_3)} \rangle\!\rangle} \cancel{\langle\!\langle e^{i(n\phi_2+m\phi_4)} \rangle\!\rangle} - \cancel{\langle\!\langle e^{i(n\phi_1-m\phi_4)} \rangle\!\rangle} \cancel{\langle\!\langle e^{i(-n\phi_2+m\phi_3)} \rangle\!\rangle} \\
 &= \langle\!\langle e^{in(\phi_1-\phi_2)+m(\phi_3-\phi_4)} \rangle\!\rangle - \langle\!\langle e^{in(\phi_1-\phi_2)} \rangle\!\rangle \langle\!\langle e^{im(\phi_3-\phi_4)} \rangle\!\rangle \\
 &= \langle v_n^2 v_m^2 \rangle - \langle v_n^2 \rangle \langle v_m^2 \rangle + \text{non-flow}
 \end{aligned}$$

How to measure flow fluctuations?

Four-particle **symmetric cumulants**:

A.Bilandzic, C.Christensen, K.Gulbrandsen, A.Hansen, Y.Zhou 1312.3572

$$\begin{aligned}
 \text{sc}_{n,m}\{4\} &= \langle\!\langle e^{in(\phi_1-\phi_2)+m(\phi_3-\phi_4)} \rangle\!\rangle - \langle\!\langle e^{in(\phi_1-\phi_2)} \rangle\!\rangle \langle\!\langle e^{im(\phi_3-\phi_4)} \rangle\!\rangle \\
 &\quad - \cancel{\langle\!\langle e^{i(n\phi_1+m\phi_3)} \rangle\!\rangle} \cancel{\langle\!\langle e^{i(n\phi_2+m\phi_4)} \rangle\!\rangle} - \cancel{\langle\!\langle e^{i(n\phi_1-m\phi_4)} \rangle\!\rangle} \cancel{\langle\!\langle e^{i(-n\phi_2+m\phi_3)} \rangle\!\rangle} \\
 &= \langle\!\langle e^{in(\phi_1-\phi_2)+m(\phi_3-\phi_4)} \rangle\!\rangle - \langle\!\langle e^{in(\phi_1-\phi_2)} \rangle\!\rangle \langle\!\langle e^{im(\phi_3-\phi_4)} \rangle\!\rangle \\
 &= \langle v_n^2 v_m^2 \rangle - \langle v_n^2 \rangle \langle v_m^2 \rangle + \text{non-flow}
 \end{aligned}$$

Three-particle **asymmetric cumulants (event-plane correlators)**:

$$\begin{aligned}
 \text{ac}_{n,m}\{3\} &= \langle\!\langle e^{i(n\phi_1+m\phi_2-(n+m)\phi_3)} \rangle\!\rangle - \text{terms involving } \cancel{\langle\!\langle e^{in\phi} \rangle\!\rangle}, \cancel{\langle\!\langle e^{im\phi} \rangle\!\rangle}, \cancel{\langle\!\langle e^{i(n+m)\phi} \rangle\!\rangle} \\
 &= \langle\!\langle e^{i(n\phi_1+m\phi_2-(n+m)\phi_3)} \rangle\!\rangle \\
 &= \langle v_n v_m v_{n+m} \cos(n\Phi_n + m\Phi_m - (n+m)\Phi_{n+m}) \rangle + \text{non-flow}
 \end{aligned}$$

ATLAS 1403.0489

Probe the shape of $p(v_n, v_m)$ and $p(\Phi_n, \Phi_m)$

How to measure flow fluctuations?

Four-particle **symmetric cumulants**:

A.Bilandzic, C.Christensen, K.Gulbrandsen, A.Hansen, Y.Zhou 1312.3572

$$\begin{aligned}
 \text{sc}_{n,m}\{4\} &= \langle\!\langle e^{in(\phi_1-\phi_2)+m(\phi_3-\phi_4)} \rangle\!\rangle - \langle\!\langle e^{in(\phi_1-\phi_2)} \rangle\!\rangle \langle\!\langle e^{im(\phi_3-\phi_4)} \rangle\!\rangle \\
 &\quad - \cancel{\langle\!\langle e^{i(n\phi_1+m\phi_3)} \rangle\!\rangle} \cancel{\langle\!\langle e^{i(n\phi_2+m\phi_4)} \rangle\!\rangle} - \cancel{\langle\!\langle e^{i(n\phi_1-m\phi_4)} \rangle\!\rangle} \cancel{\langle\!\langle e^{i(-n\phi_2+m\phi_3)} \rangle\!\rangle} \\
 &= \langle\!\langle e^{in(\phi_1-\phi_2)+m(\phi_3-\phi_4)} \rangle\!\rangle - \langle\!\langle e^{in(\phi_1-\phi_2)} \rangle\!\rangle \langle\!\langle e^{im(\phi_3-\phi_4)} \rangle\!\rangle \\
 &= \langle v_n^2 v_m^2 \rangle - \langle v_n^2 \rangle \langle v_m^2 \rangle + \text{non-flow}
 \end{aligned}$$

Three-particle **asymmetric cumulants (event-plane correlators)**:

$$\begin{aligned}
 \text{ac}_{n,m}\{3\} &= \langle\!\langle e^{i(n\phi_1+m\phi_2-(n+m)\phi_3)} \rangle\!\rangle - \text{terms involving } \cancel{\langle\!\langle e^{in\phi} \rangle\!\rangle}, \cancel{\langle\!\langle e^{im\phi} \rangle\!\rangle}, \cancel{\langle\!\langle e^{i(n+m)\phi} \rangle\!\rangle} \\
 &= \langle\!\langle e^{i(n\phi_1+m\phi_2-(n+m)\phi_3)} \rangle\!\rangle \\
 &= \langle v_n v_m v_{n+m} \cos(n\Phi_n + m\Phi_m - (n+m)\Phi_{n+m}) \rangle + \text{non-flow}
 \end{aligned}$$

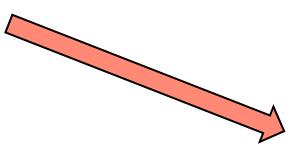
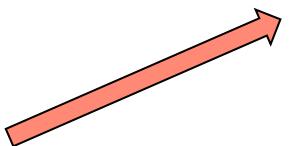
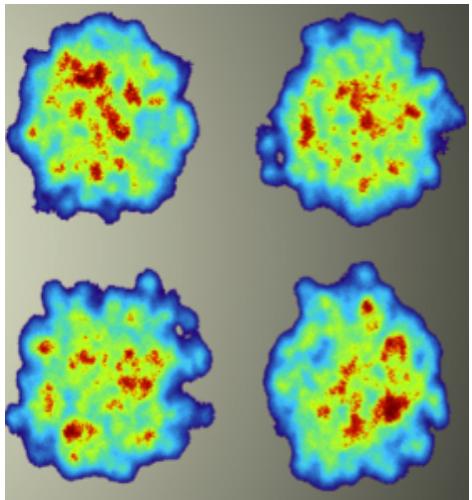
ATLAS 1403.0489

Probe the shape of $p(v_n, v_m)$ and $p(\Phi_n, \Phi_m)$

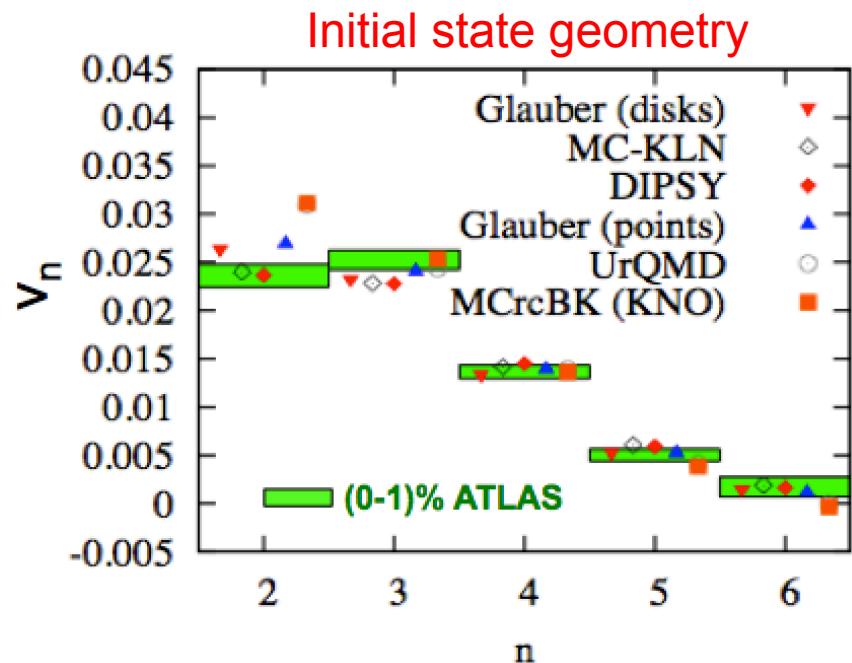
Generalize to more particles, e.g. $\langle \cos 12(\Phi_2 - \Phi_4) \rangle$ is 9-particle correlator

Data/hydro comparison: two-particle correlation²⁸

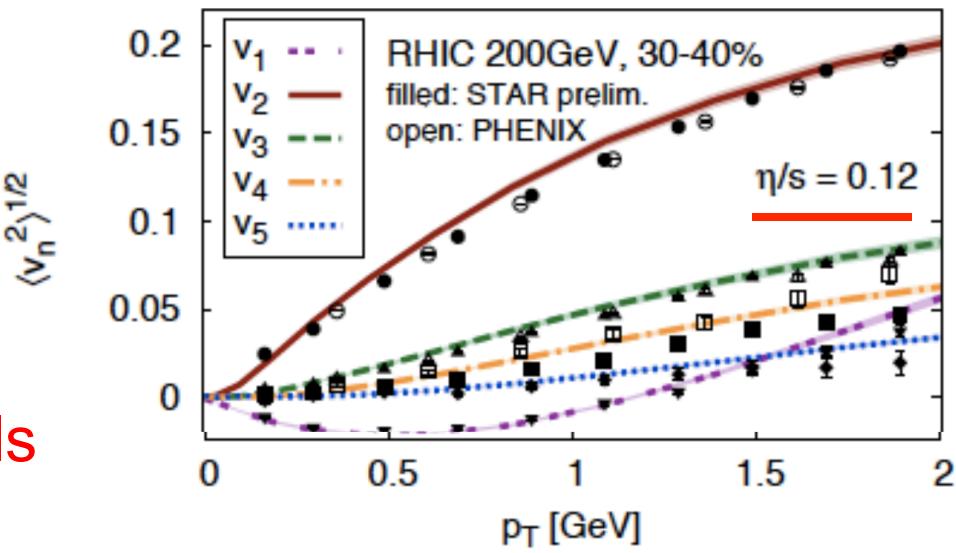
Event-averaged quantities



Constrain initial geometry and shear viscosity via hydro models

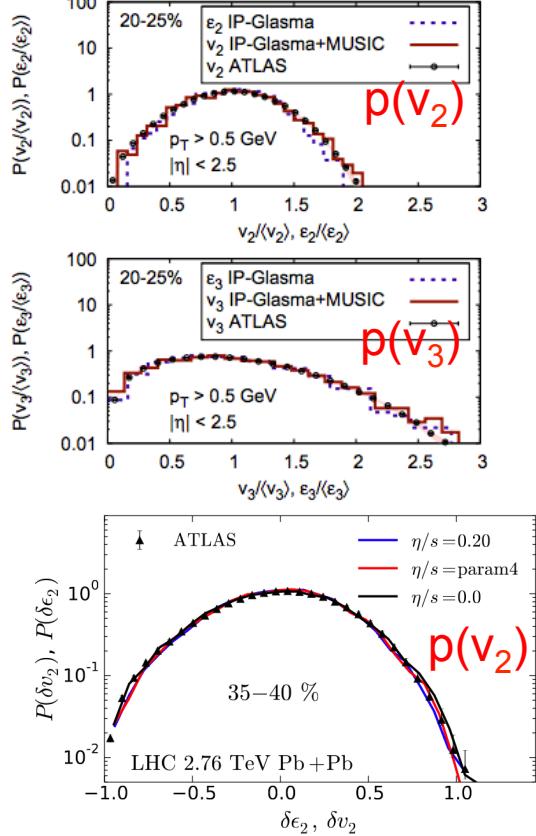


Matter properties

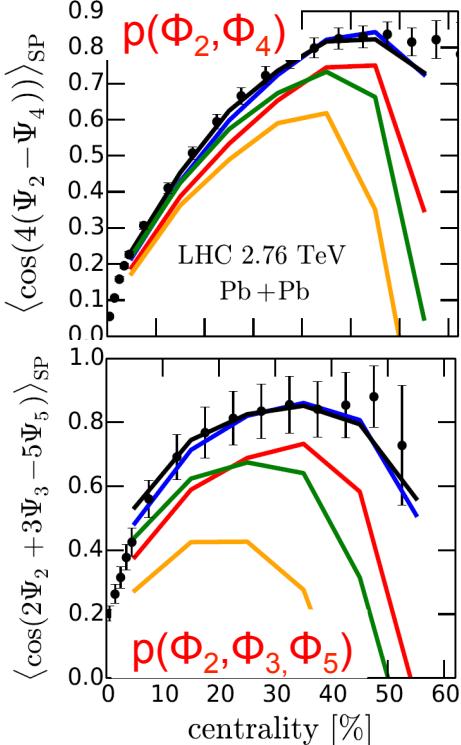


Data/hydro comparison: EbyE flow fluctuations

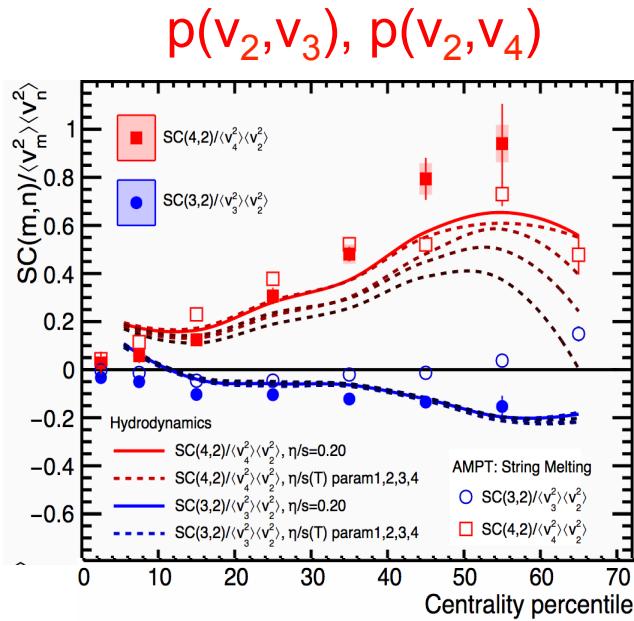
Gale, Jeon, Schenke, Tribedy, Venugopalan 1209.6330



Niemi, Eskola, Paatelainen 1505.02677

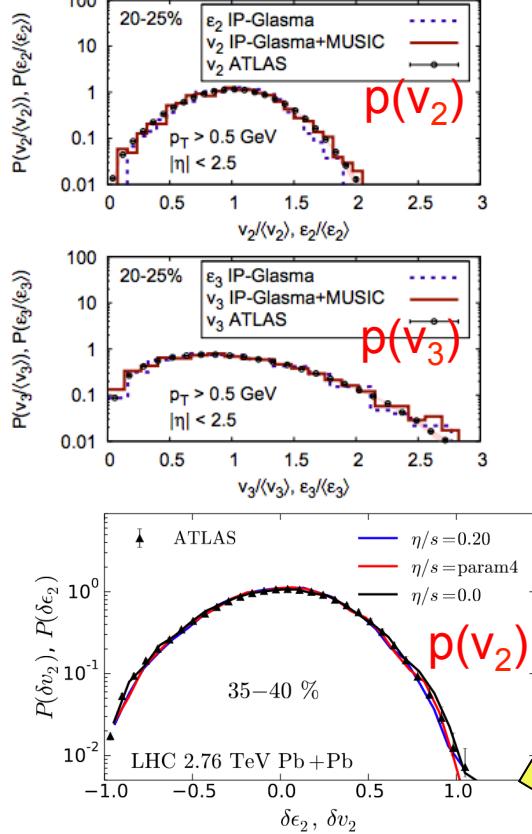


ALICE 1604.07663

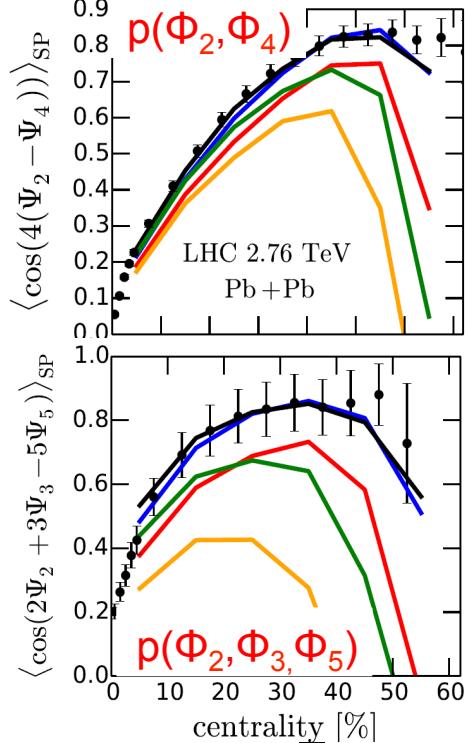


Data/hydro comparison: EbyE flow fluctuations³⁰

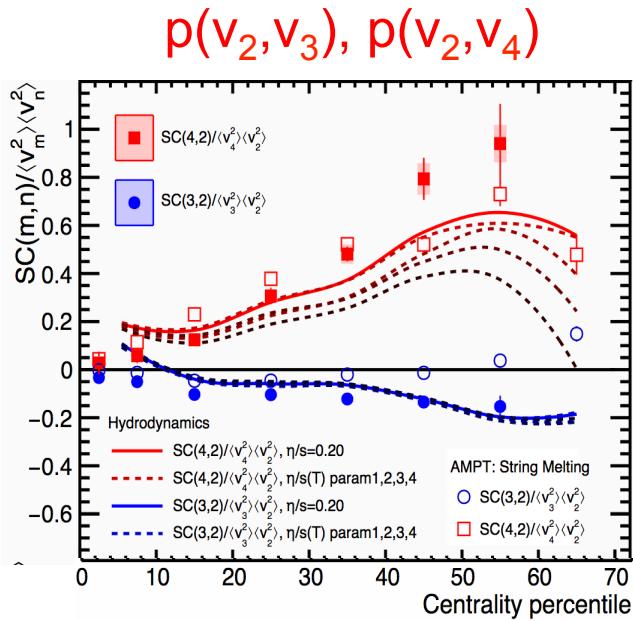
Gale, Jeon, Schenke, Tribedy, Venugopalan 1209.6330



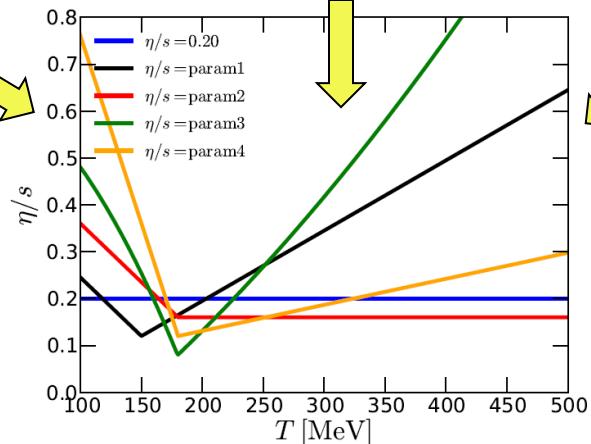
Niemi, Eskola, Paatelainen 1505.02677



ALICE 1604.07663



Probe the hydrodynamic response: $(\epsilon_n, \Phi_n^*) \rightarrow (v_n, \Phi_n)$

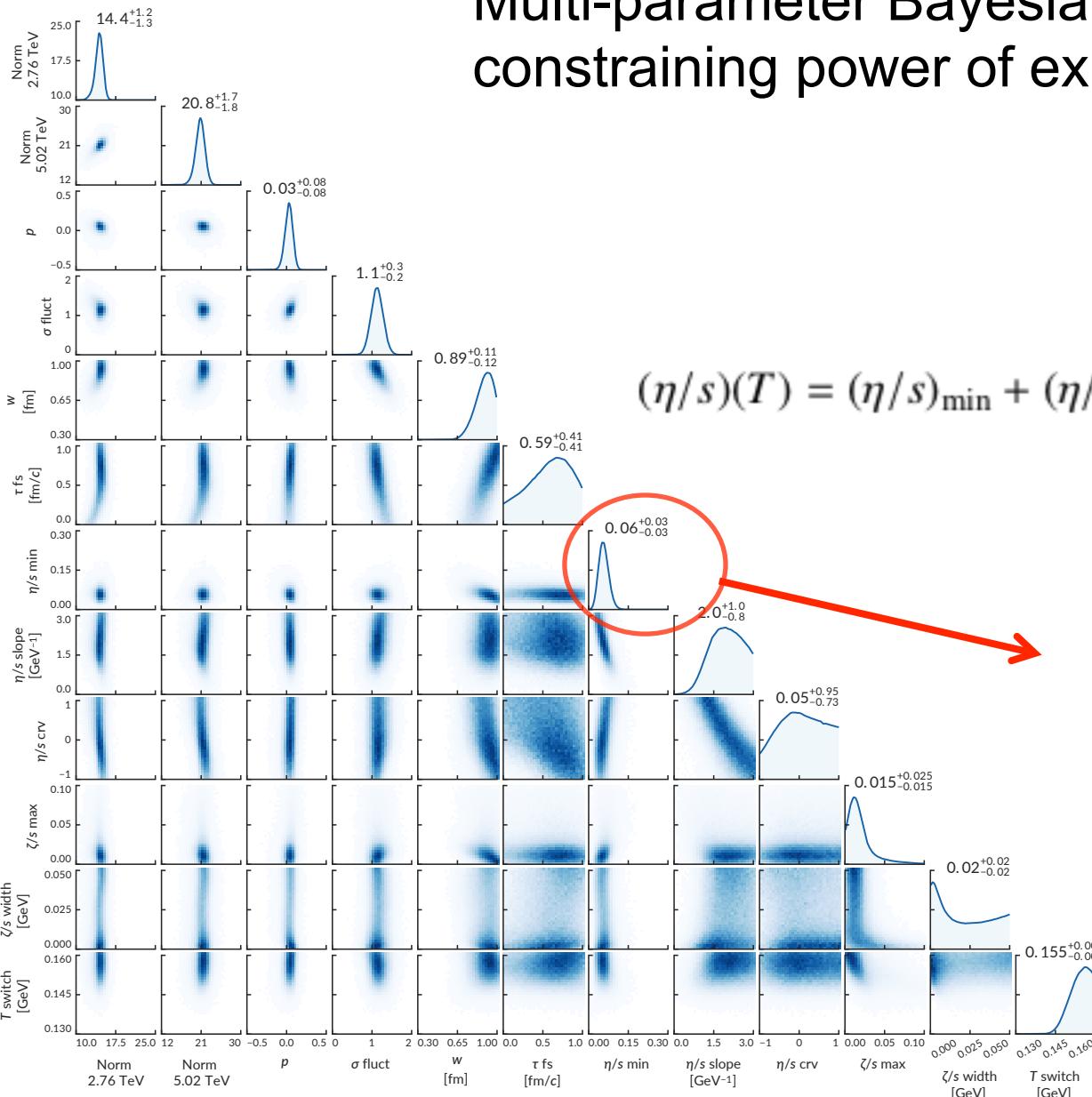


Sensitive to detail of transport coeffis: $\eta/s(T)$

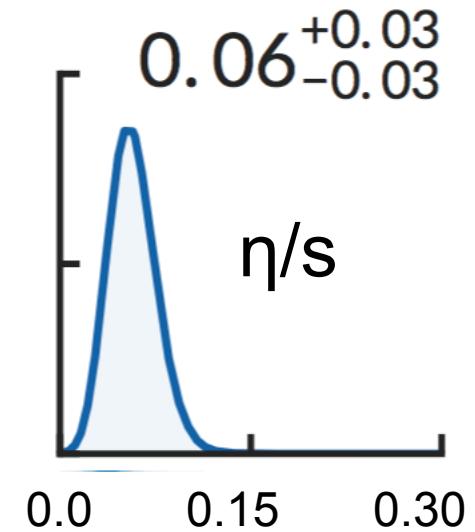
Over-constrain current hydrodynamic models

Maximizing the constraining power

Multi-parameter Bayesian analysis to maximize constraining power of experimental data.



$$(\eta/s)(T) = (\eta/s)_{\text{min}} + (\eta/s)_{\text{slope}} (T - T_c) (T - T_c)^{(\eta/s)_{\text{curvature}}}$$



S.Bass, J. Bernhard, QM2017

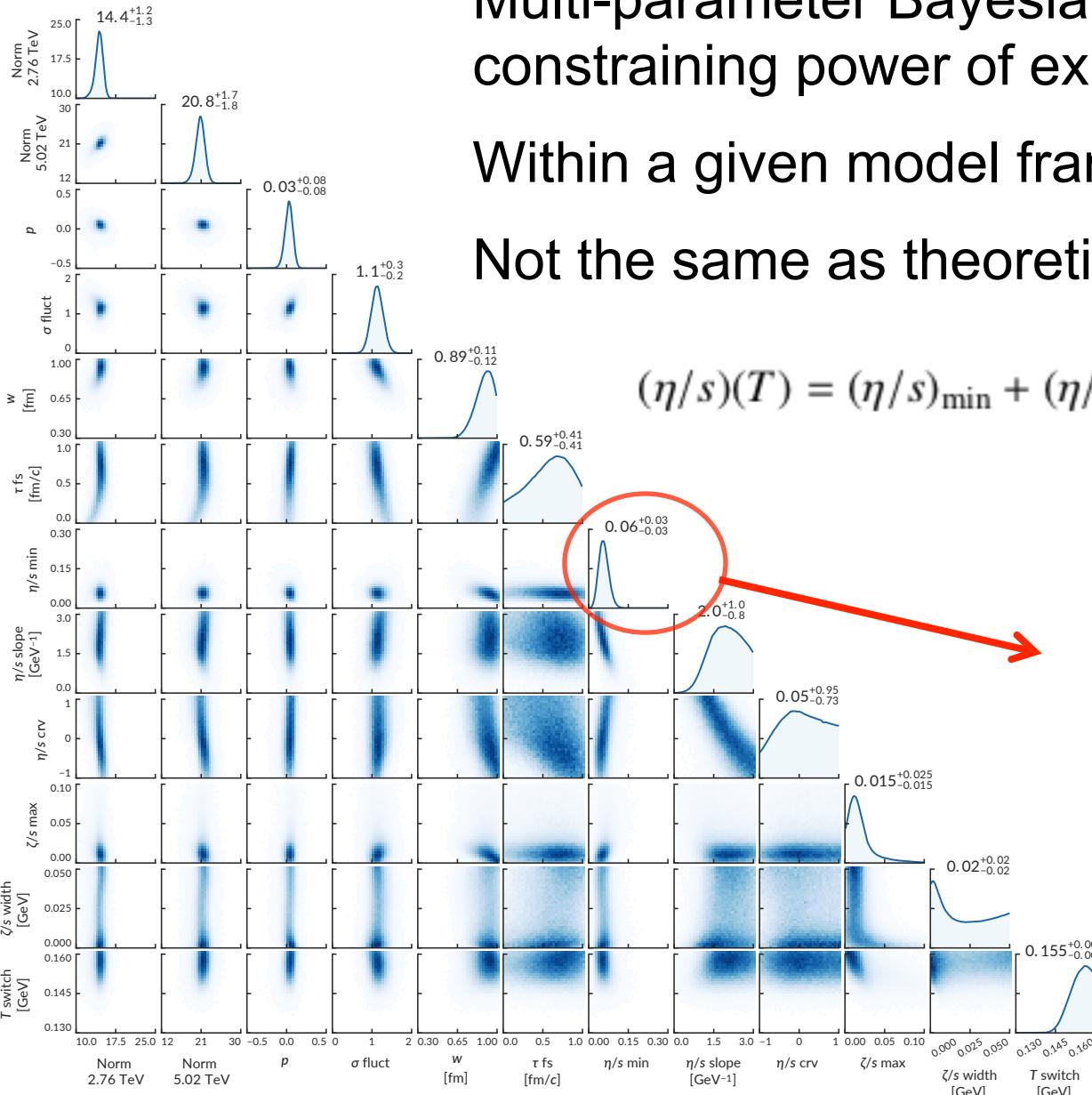
W. Zajc, NPSS2017

Maximizing the constraining power

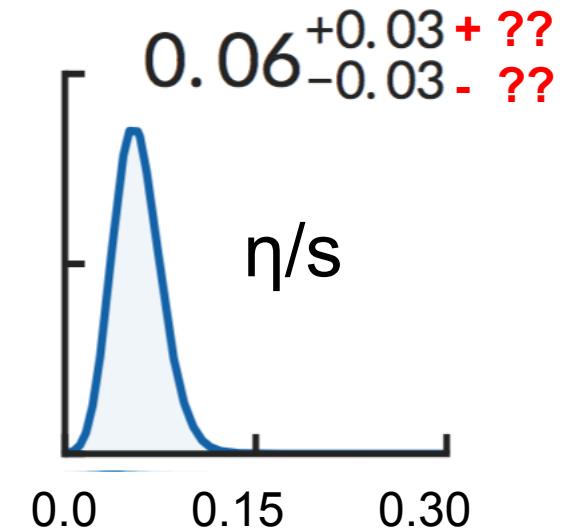
Multi-parameter Bayesian analysis to maximize constraining power of experimental data.

Within a given model framework!

Not the same as theoretical uncertainty.



$$(\eta/s)(T) = (\eta/s)_{\text{min}} + (\eta/s)_{\text{slope}} (T - T_c) (T - T_c)^{(\eta/s)_{\text{curvature}}}$$



S.Bass, J. Bernhard, QM2017

W. Zajc, NPSS2017

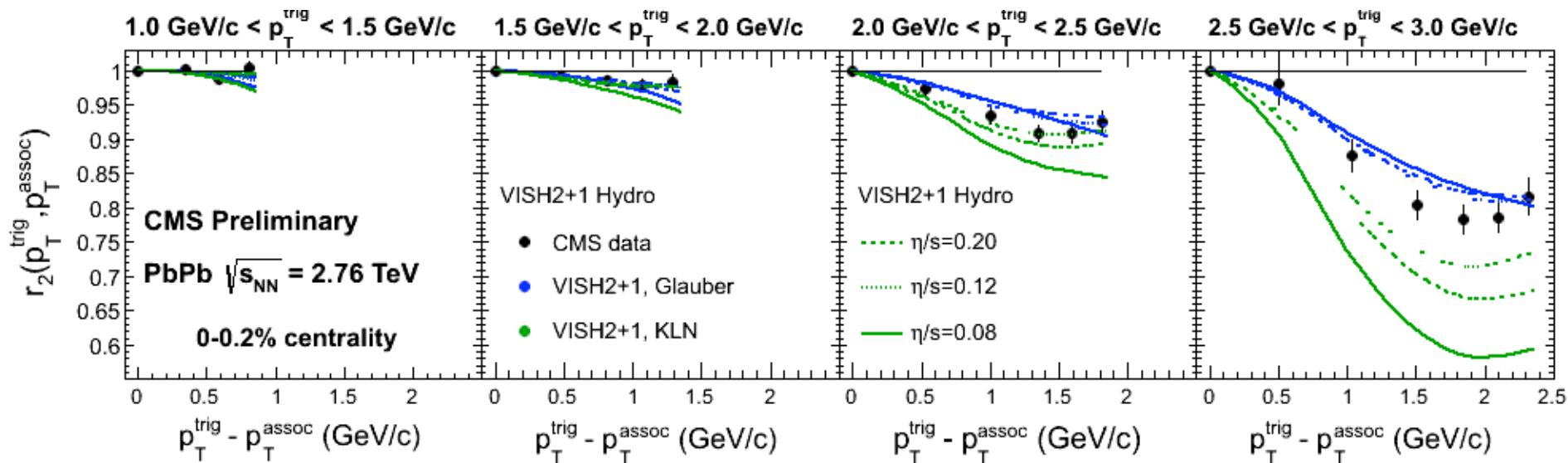
Flow fluctuation in transverse (p_T) direction

- Ollitrault saw v_n angle and amplitude fluctuates in p_T in EbyE hydro

$$\tilde{r}_n(p_{T1}, p_{T2}) := \frac{\langle v_n(p_{T1})v_n(p_{T2}) \cos[n(\Psi_n(p_{T1}) - \Psi_n(p_{T2}))] \rangle}{\langle v_n(p_{T1})v_n(p_{T2}) \rangle}$$

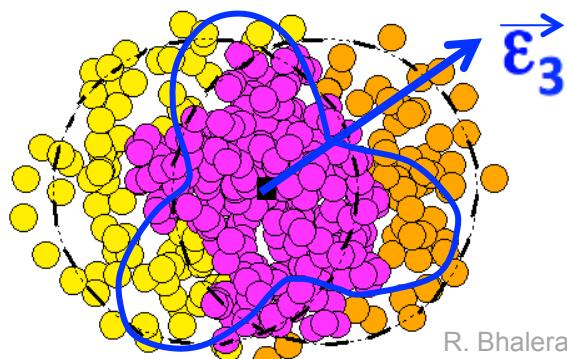
QM2012

- Breaking is largest for v_2 in ultra-central Pb+Pb collisions
 - Also depends strongly on PID

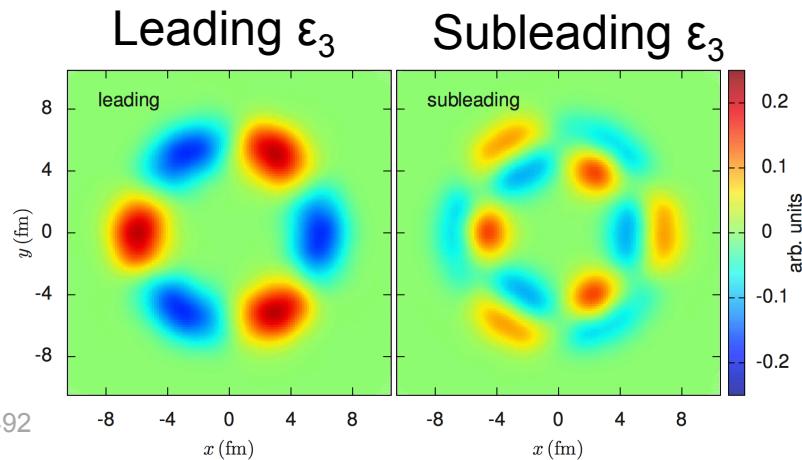
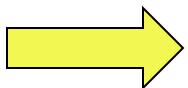


Flow fluctuation in transverse (p_T) direction

Transverse density profile contains multiple eccentricity ε_n mode with different radial length scales. Each ε_n drives its own flow component with its own phase:



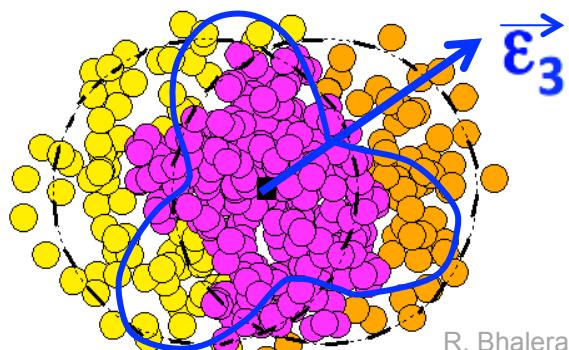
R. Bhalerao, J.Ollitrault, S. Pal, D. Teaney, A.
Mazeliauskas 1410.7739, 1501.03138, 1509.07492



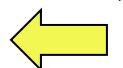
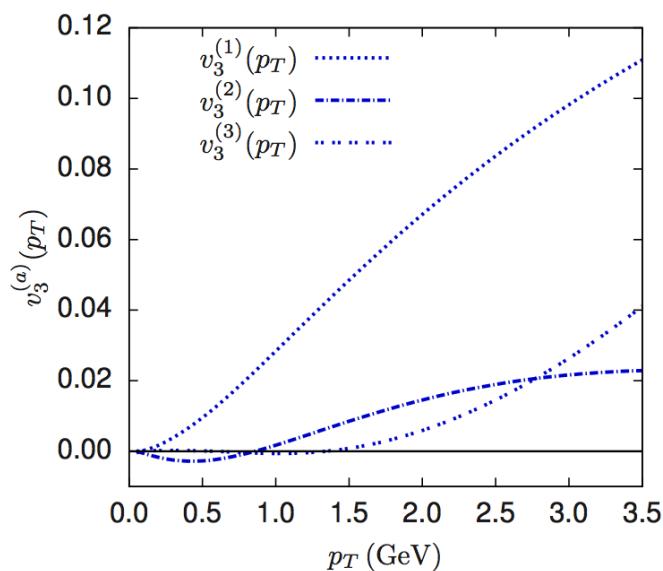
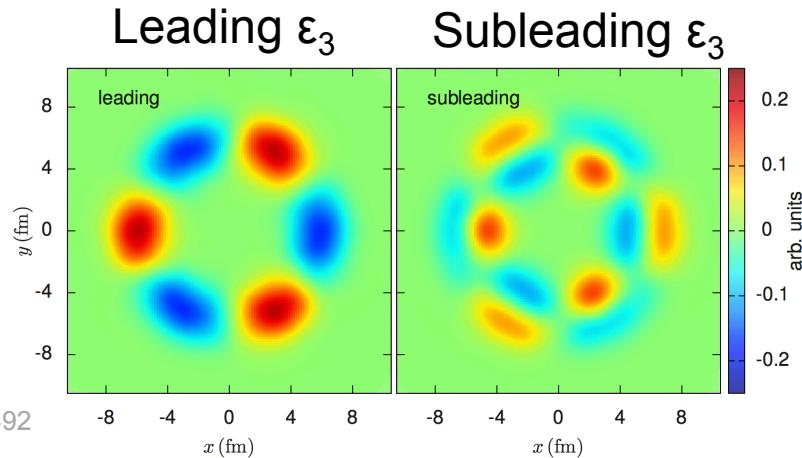
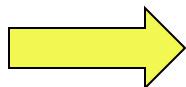
$$\varepsilon_3(r) \propto \varepsilon_{3;1}(r)e^{i3\Phi_3^1} + \varepsilon_{3;2}(r)e^{i3\Phi_3^2}$$

Flow fluctuation in transverse (p_T) direction

Transverse density profile contains multiple eccentricity ε_n mode with different radial length scales. Each ε_n drives its own flow component with its own phase:



R. Bhalerao, J. Ollitrault, S. Pal, D. Teaney, A.
Mazeliauskas 1410.7739, 1501.03138, 1509.07492



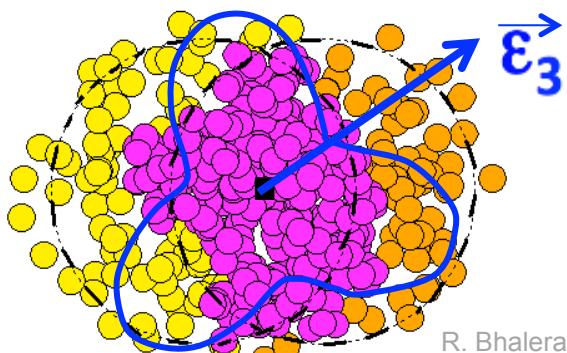
$$\varepsilon_3(r) \propto \varepsilon_{3;1}(r)e^{i3\Phi_3^1} + \varepsilon_{3;2}(r)e^{i3\Phi_3^2}$$



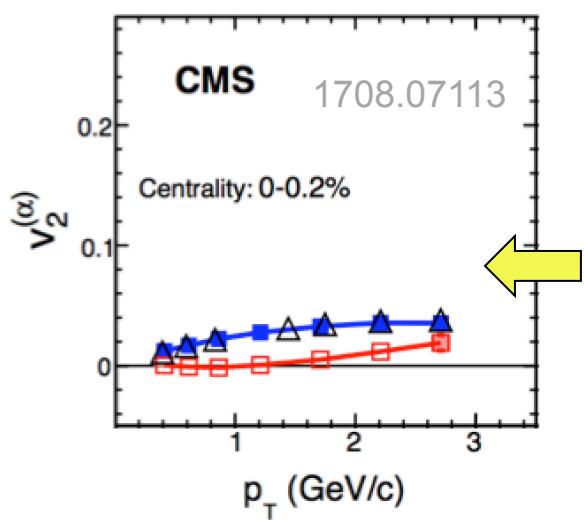
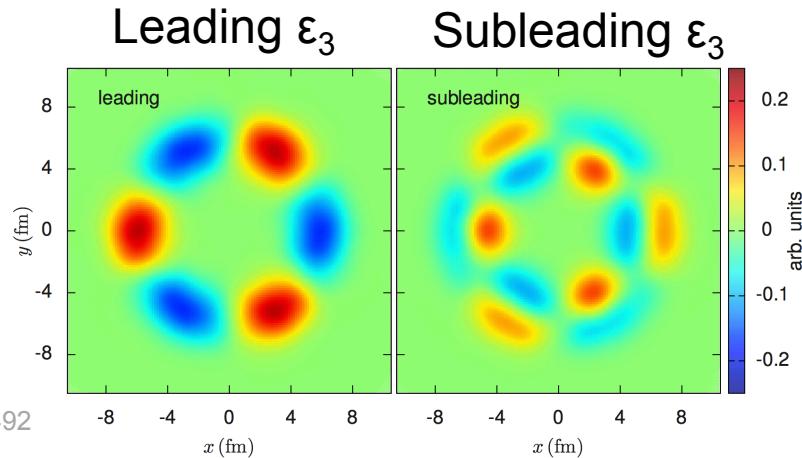
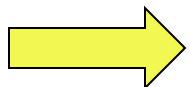
$$v_3(p_T) \propto c_1(p_T)e^{i3\Phi_3^1(p_T)} + c_2(p_T)e^{i3\Phi_3^2(p_T)}$$

Flow fluctuation in transverse (p_T) direction

Transverse density profile contains multiple eccentricity ε_n mode with different radial length scales. Each ε_n drives its own flow component with its own phase:



R. Bhalerao, J. Ollitrault, S. Pal, D. Teaney, A.
Mazeliauskas 1410.7739, 1501.03138, 1509.07492



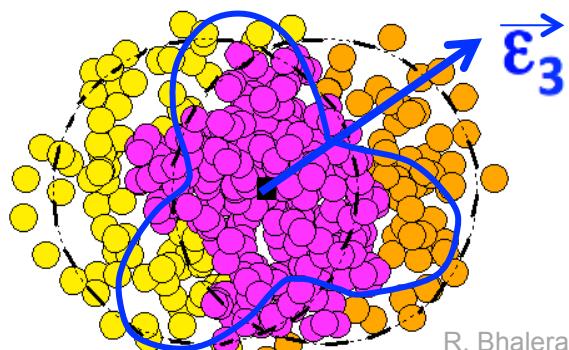
$$\varepsilon_3(r) \propto \varepsilon_{3;1}(r)e^{i3\Phi_3^1} + \varepsilon_{3;2}(r)e^{i3\Phi_3^2}$$



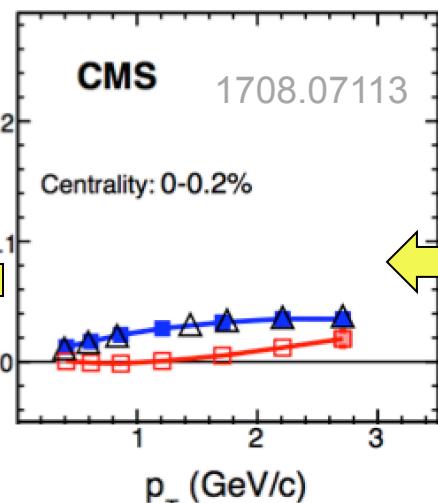
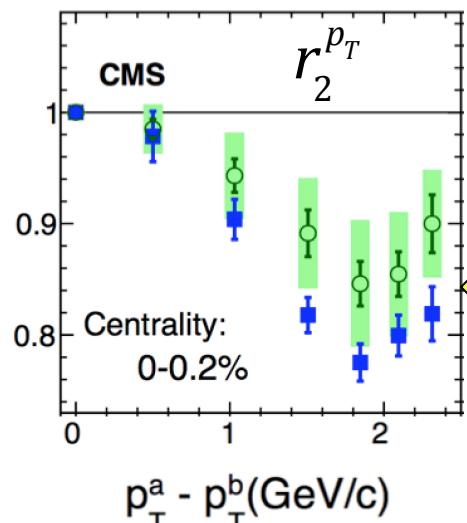
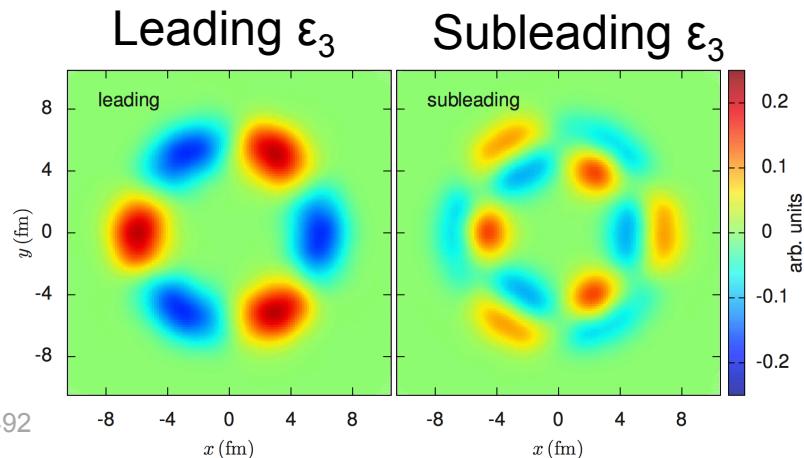
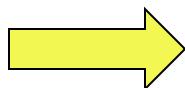
$$v_3(p_T) \propto c_1(p_T)e^{i3\Phi_3^1(p_T)} + c_2(p_T)e^{i3\Phi_3^2(p_T)}$$

Flow fluctuation in transverse (p_T) direction

Transverse density profile contains multiple eccentricity ε_n mode with different radial length scales. Each ε_n drives its own flow component with its own phase:



R. Bhalerao, J. Ollitrault, S. Pal, D. Teaney, A.
Mazeliauskas 1410.7739, 1501.03138, 1509.07492



$$\varepsilon_3(r) \propto \varepsilon_{3;1}(r)e^{i3\Phi_3^1} + \varepsilon_{3;2}(r)e^{i3\Phi_3^2}$$

$$v_3(p_T) \propto c_1(p_T)e^{i3\Phi_3^1(p_T)} + c_2(p_T)e^{i3\Phi_3^2(p_T)}$$

Flow angle and amplitude fluctuates in p_T

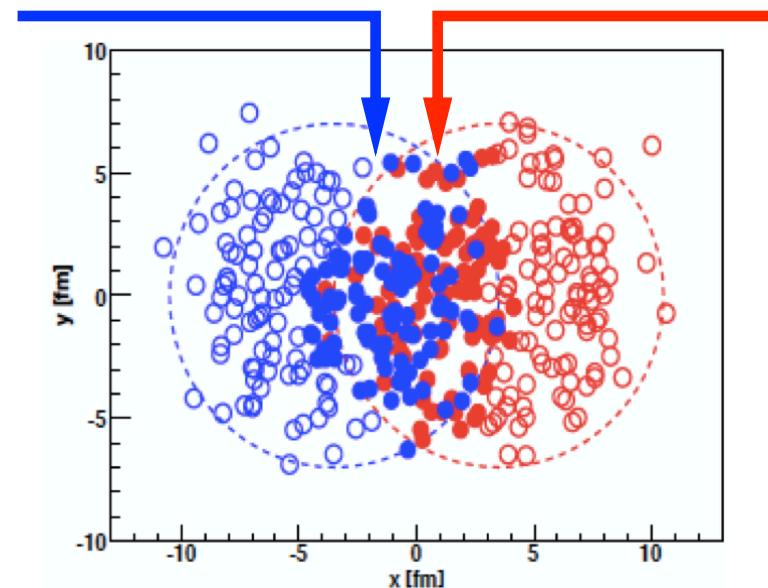
$$V_{n\Delta}(p_T^a, p_T^b) \neq v_n(p_T^a)v_n(p_T^b)$$

Subleading flow responsible for most factorization breaking effect

Flow fluctuation in longitudinal direction

Fluctuation of sources in two nuclei → fluctuation of transverse-shape

$$v_n^F, \Psi_n^F$$



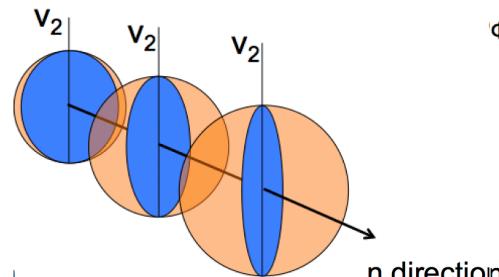
$$v_n^B, \Psi_n^B$$

$$v_n = v_n e^{in\Psi_n}$$

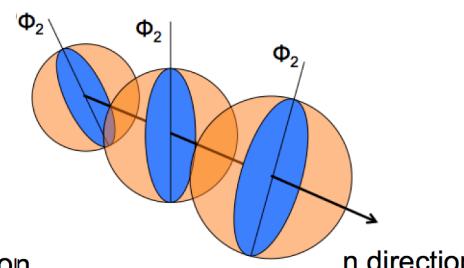
Bozek et.al., arXiv:1011.3354

Consequences:

Asymmetry of a flow magnitude Torque/twist of an event plane



$$v_n(\eta_1) \neq v_n(\eta_2)$$

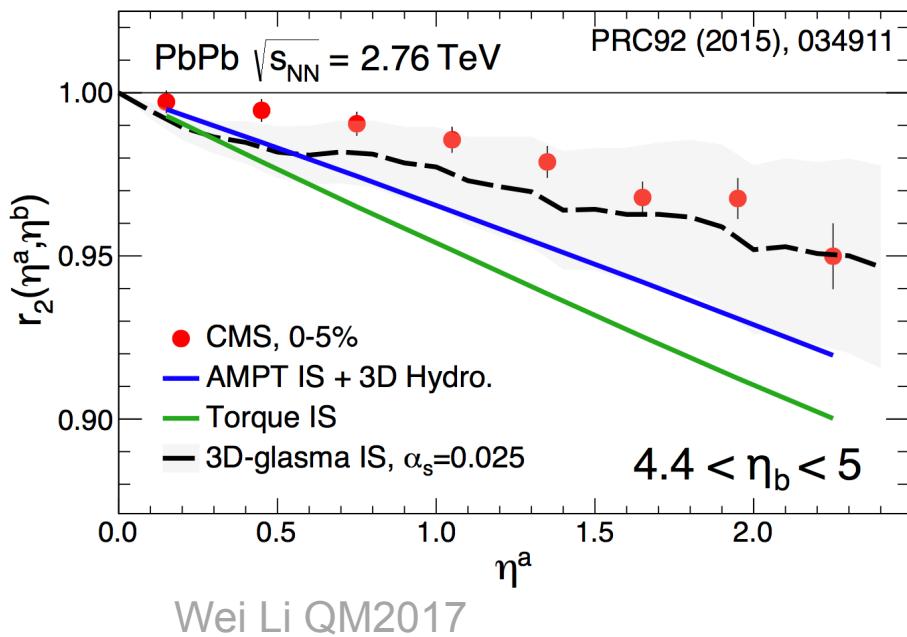


$$\Psi_n(\eta_1) \neq \Psi_n(\eta_2)$$

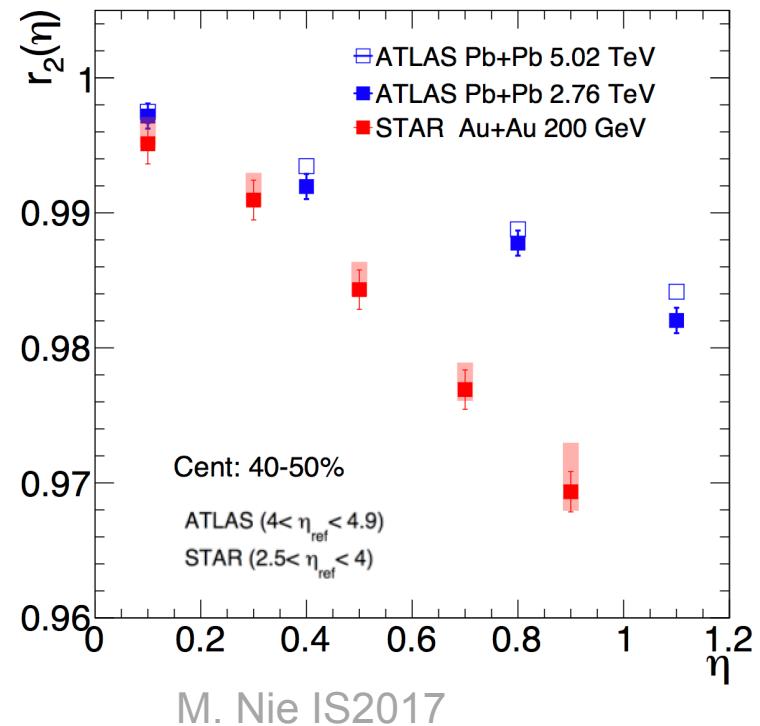
Flow fluctuation in longitudinal direction

Observables: $r_n^\eta = \frac{V_n(-\eta) V_n^*(\eta_{\text{ref}})}{V_n(\eta) V_n^*(\eta_{\text{ref}})} \sim \langle \cos n [\Phi_n(\eta) - \Phi_n(-\eta)] \rangle$

Significant **decorrelation**,
not described by any models



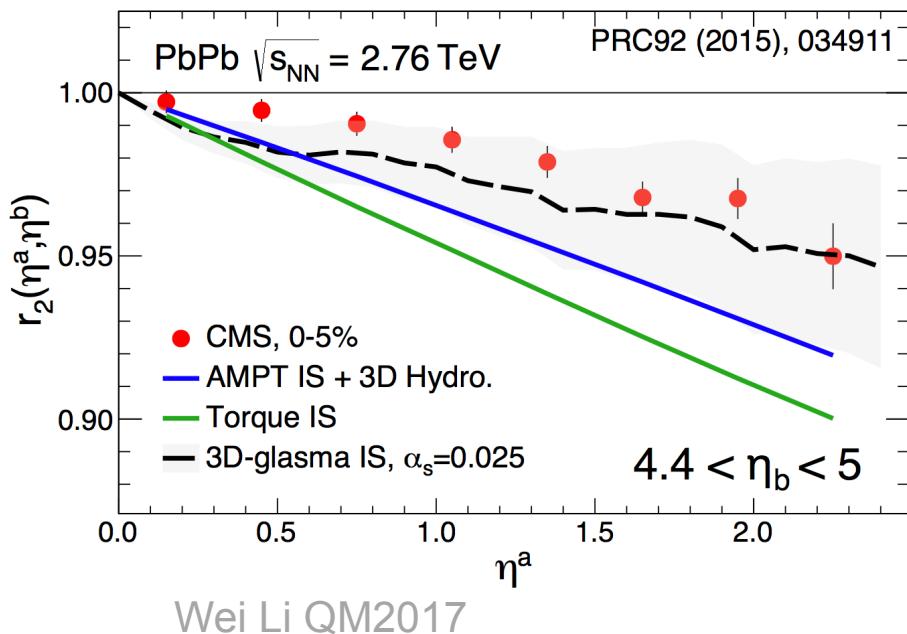
Much stronger at lower \sqrt{s}



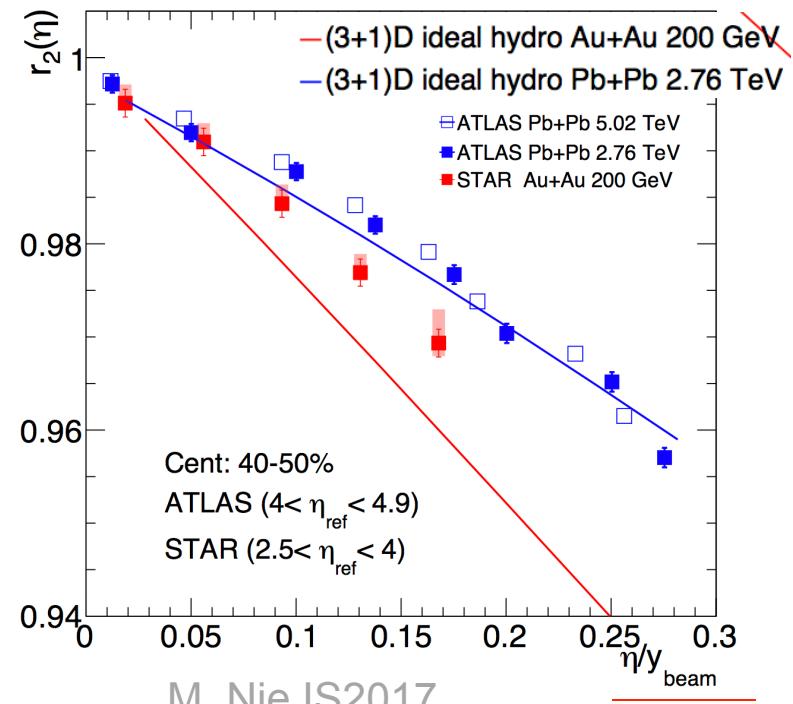
Flow fluctuation in longitudinal direction

Observables: $r_n^\eta = \frac{V_n(-\eta)V_n^*(\eta_{\text{ref}})}{V_n(\eta)V_n^*(\eta_{\text{ref}})} \sim \langle \cos n [\Phi_n(\eta) - \Phi_n(-\eta)] \rangle$

Significant **decorrelation**,
not described by any models



Can't be explained by beam-rapidity scaling, not described by hydro model



Challenge for precision

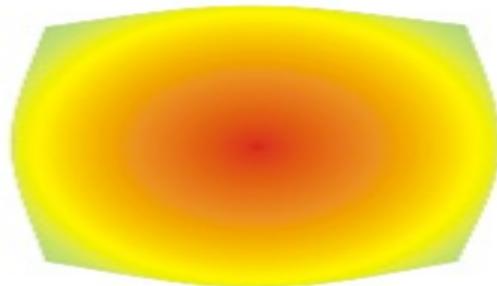
$\tau < 0$



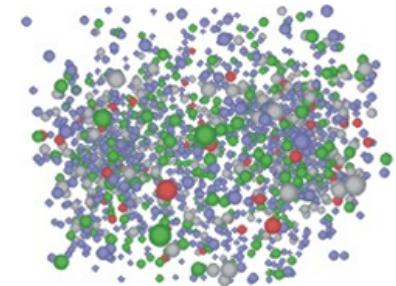
$0 < \tau < 0.5 \text{ fm}/c$



$0.5 < \tau < 6 \text{ fm}/c$



$6 < \tau < 10 \text{ fm}/c$



initial state

pre-equilibrium

QGP & expansion

Phase transition&freeze-out

Glauber, CGC
Sub-nucl. dof
Longi. structure

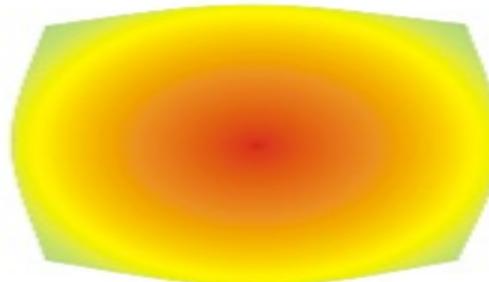
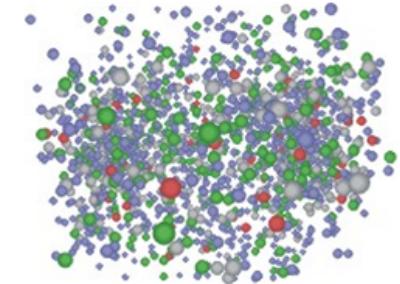
Pre-flow dynamics

Hydrodynamics

Hadronization



Challenge for precision

 $\tau < 0$  $0 < \tau < 0.5 \text{ fm}/c$  $0.5 < \tau < 6 \text{ fm}/c$  $6 < \tau < 10 \text{ fm}/c$ 

initial state

pre-equilibrium

QGP & expansion

Phase transition&freeze-out

Glauber, CGC
Sub-nucl. dof
Longi. structure
External input

Pre-flow dynamics
Non-hydro modes
Kinetic approach

Hydrodynamics
Slow modes
Hydro-fluctuations
Critial-flucuations

Hadronization
Bulk viscosity

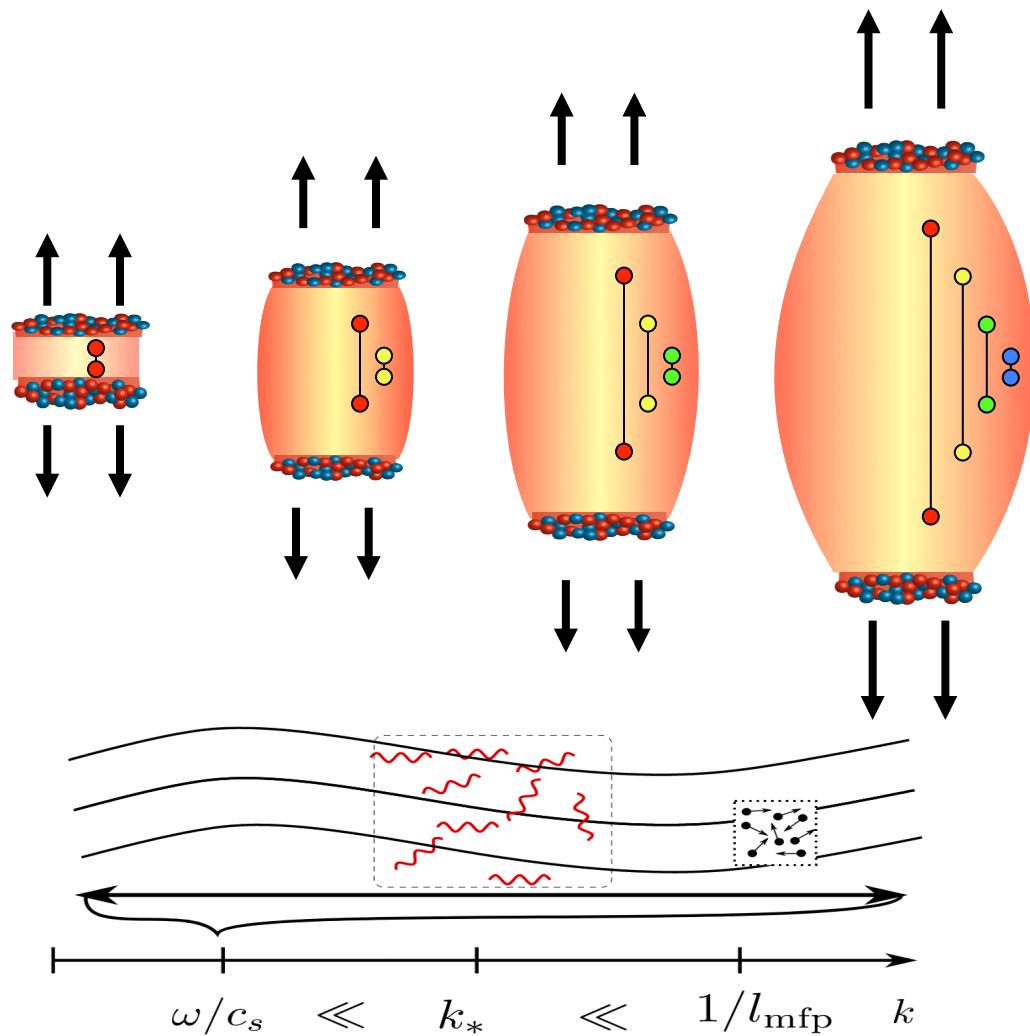
Schenke, Shen, Schlingting,
Venogopalan, Tribedy,Bzdak
et.al 1804.10557, 1710.00881
1605.07158, 1304.3403,
1209.6330

C. Shen, U. Heinz 1504.02160
. Vredevoogd, S. Pratt 0810.4325z
Weller, Romatschke 1701.07145,1704.08699,1712.05815
A.Kurkela, A.Mazeliauskas,J.Paquet,S.Schlichting,
D.Teaney 1805.00961, 1605.04287,

M. Stephanov, Y.Yin K.Rajagopal,.
C.Young, T. Hirano,A.Mazeliauskas,
D.Teaney et.al, 1712.10305,
1606.07742, 1407.1077,1304.3243.
hep-ph/9806219,

S. Ryu, G.Denicol,B. Schenke,S. Jeon,C.
Gale, H. Song, K. Dusling, 1704.04216,
1704.04216, 1502.01675, 0903.3595,
0909.1549, 1109.5181, 1305.1981

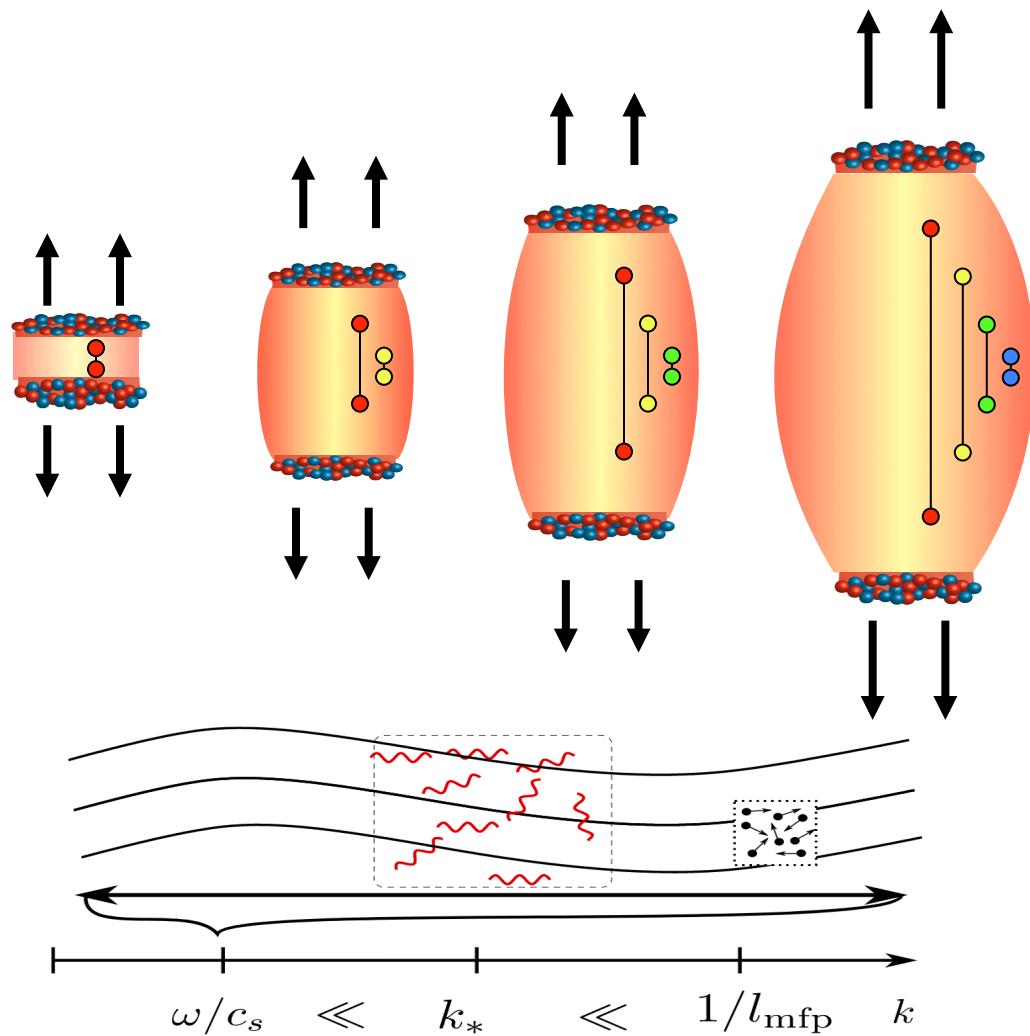
Fluctuations are everywhere



Initial fluctuation
Thermal fluctuation
Critical fluctuation
...

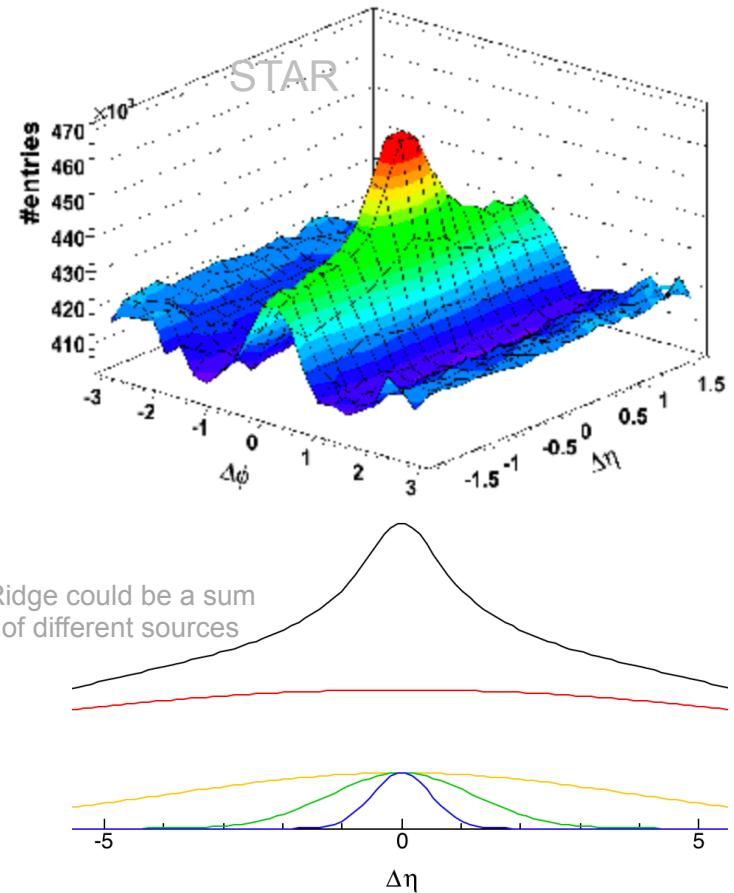
non-hydro modes
Jet quenching,
HBT
Resonance decays,
...

Fluctuations are everywhere



Initial fluctuation
Thermal fluctuation
Critical fluctuation
...

non-hydro modes
Jet quenching,
HBT
Resonance decays,
...



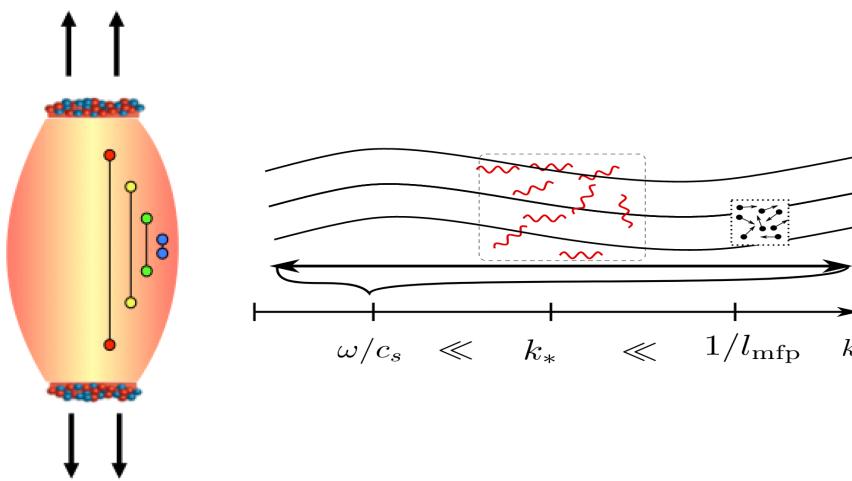
Ridge could be a sum
of different sources

Y. Akamatsu, A. Mazeliauskas, D.Teaney 1606.07742

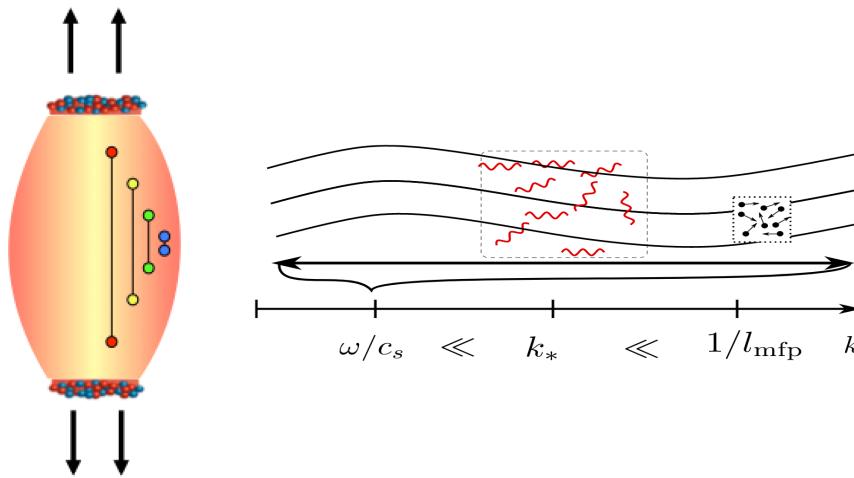
With varying length scale

Disentangle various time-scales
via $\Delta\eta$ correlation, but how?

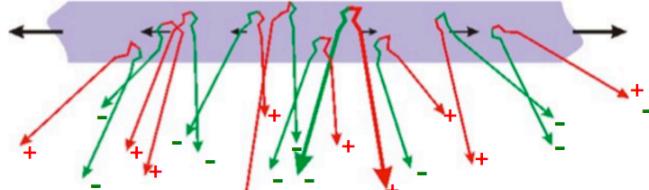
Resolve fluctuations at different time and length scale



Resolve fluctuations at different time and length scale

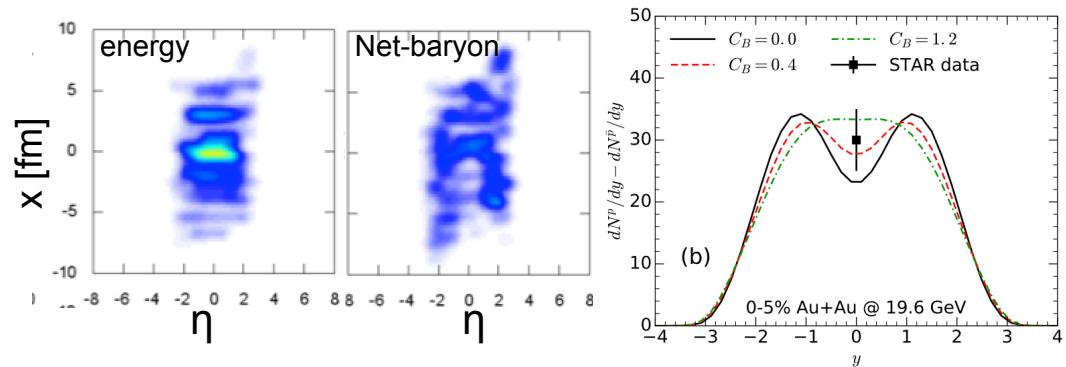


Charge transport



background for CME

Net-baryon transport

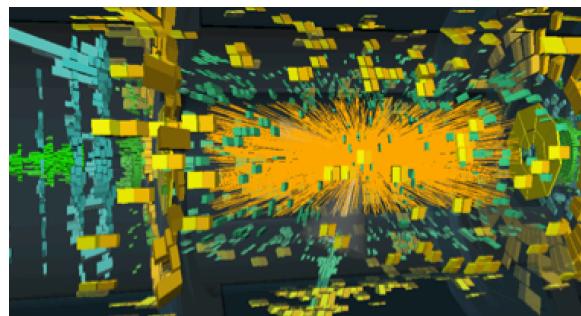
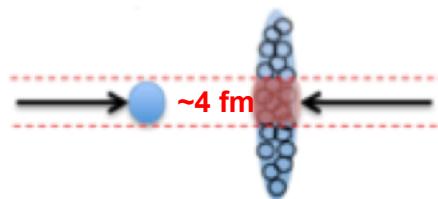
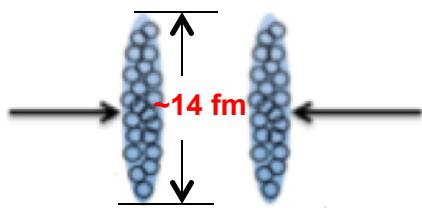


background for critical fluctuation

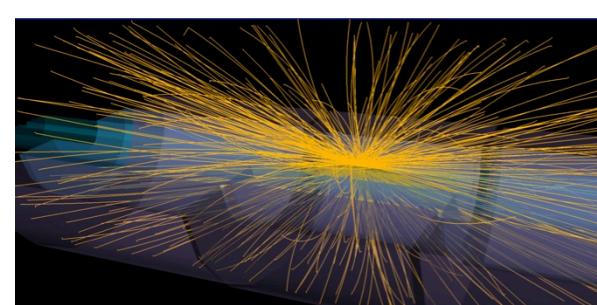
G. Denicol, C.Gale, S.Jeon, A.monnai, B.Schenke C.Shen 1804.10557

Large η coverage from forward upgrade is important

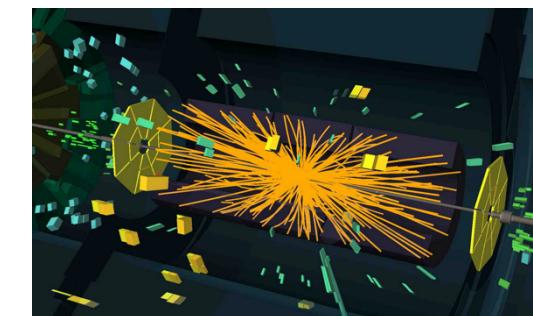
Small systems and early time dynamics



~ 30000 particles*



~ 2000 particles*



~ 600 particles*

What is the smallest droplet of QGP created in these collisions?

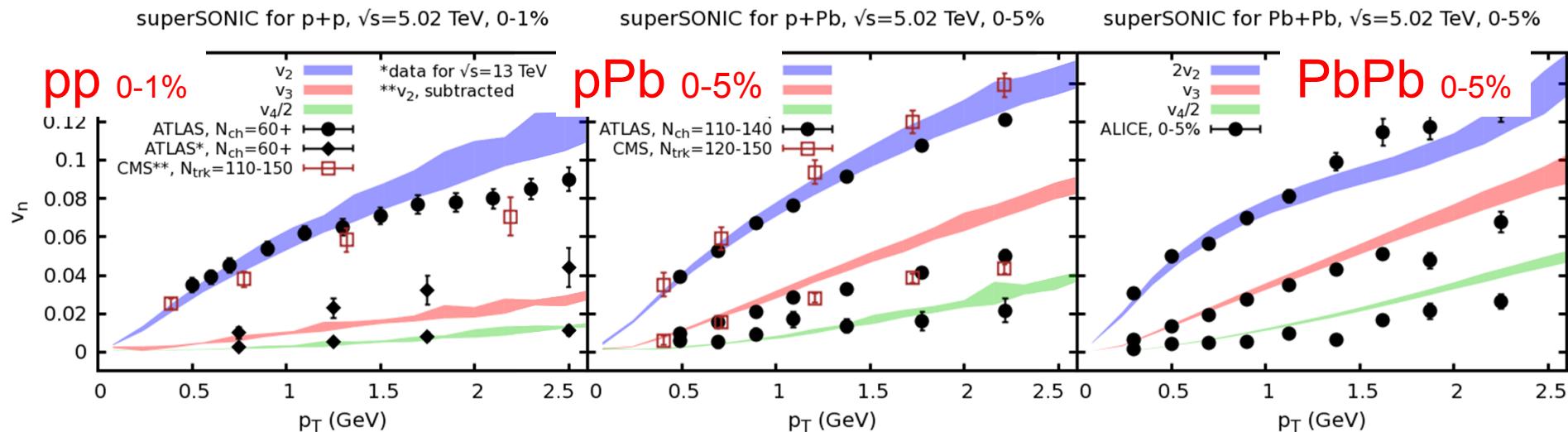
→ Change matter size, life-time and space-time dynamics @ RHIC and LHC

* Rough number in very high-multiplicity events, integrated over full phase space at LHC

Unreasonable success of hydro?

P. Romatschke, R. Weller 1701.07145

Same 2nd-order viscous hydro equations describe all three systems



Actual space-time dynamics & properties should still be different!

- This does not mean $T^{uv}(x,t)_{pp} = T^{uv}(x,t)_{pPb} = T^{uv}(x,t)_{PbPb}$
- The pre-equilibrium effects are not the same

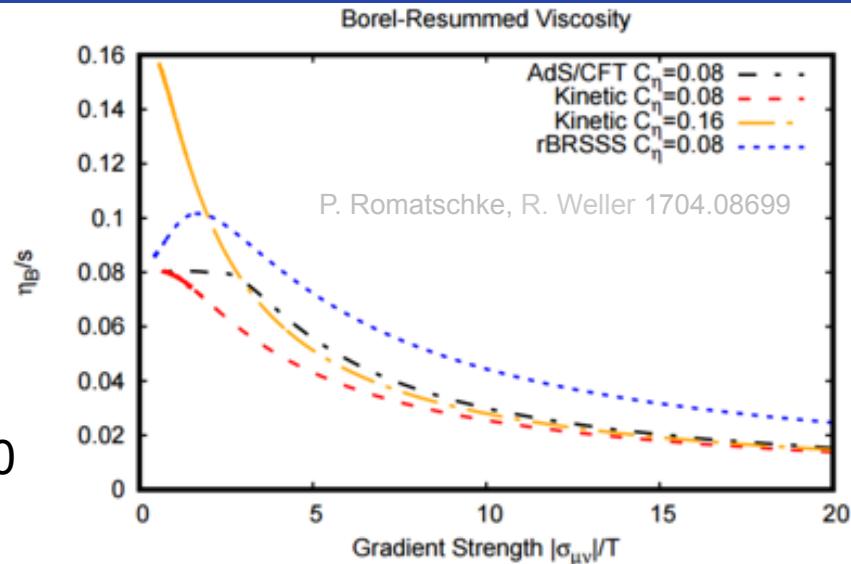
Unreasonable success of hydro?

- In far from equilibrium region, hydro still fit the data, but gives wrong viscosity

$$T_{\text{hydro}}^{\mu\nu} = (\epsilon + P_B)u^\mu u^\nu + P_B g^{\mu\nu} - \eta_B \sigma^{\mu\nu}$$

Small gradients $\eta_B \sim \eta$ Large gradients $\eta_B \rightarrow 0$

Also A.Kurkela, U.Wiedemann, B. Wu 1805.04081



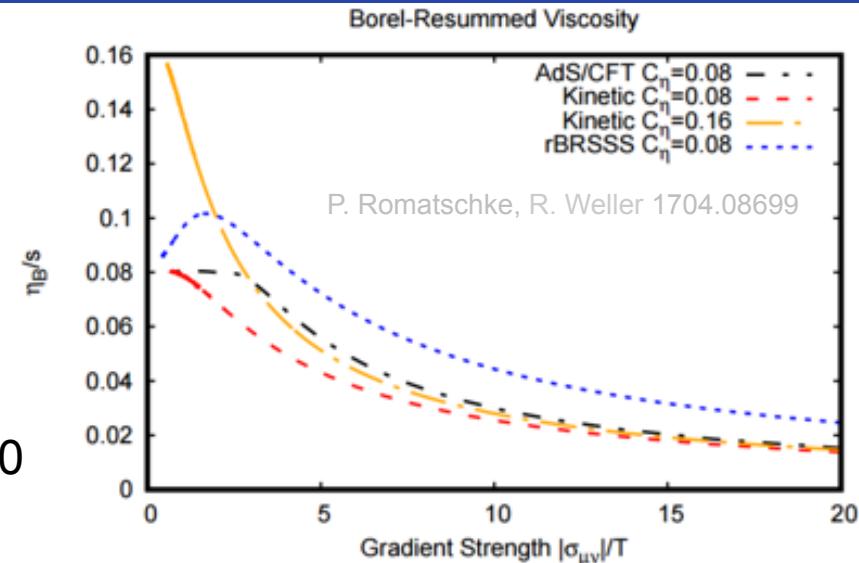
Unreasonable success of hydro?

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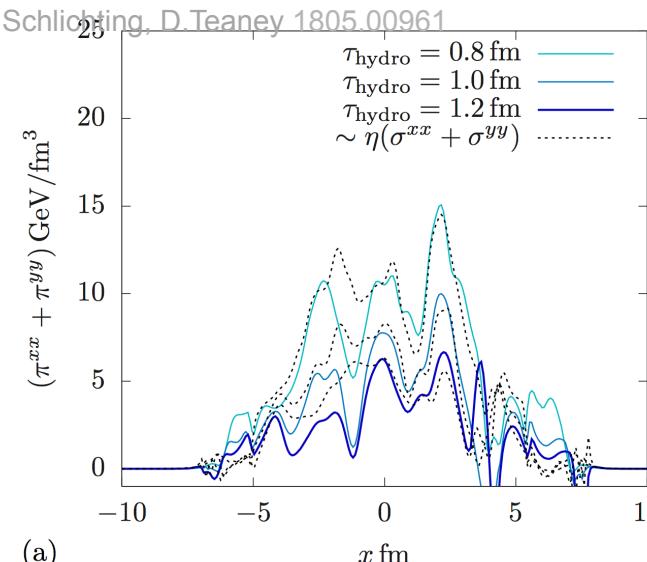
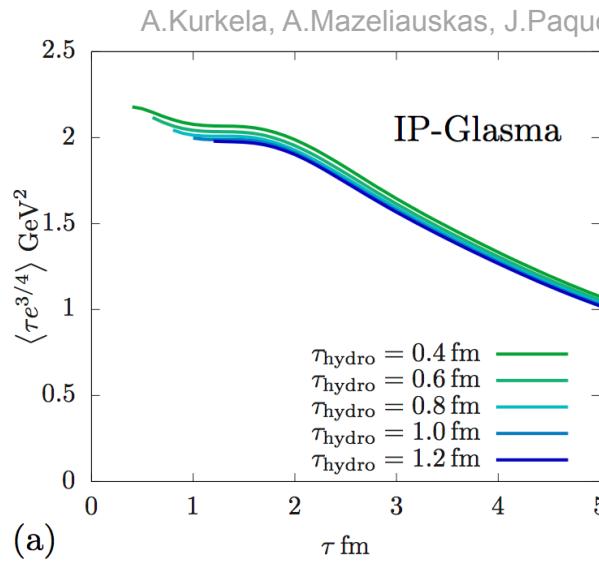
$$T_{\text{hydro}}^{\mu\nu} = (\epsilon + P_B)u^\mu u^\nu + P_B g^{\mu\nu} - \eta_B \sigma^{\mu\nu}$$

Small gradients $\eta_B \sim \eta$ Large gradients $\eta_B \rightarrow 0$

Also A.Kurkela, U.Wiedemann, B. Wu 1805.04081



- Different models for early-time dynamics have similar average hydro-field, but different differential distri., e.g shear tensor $\pi^{\mu\nu}(x)$.

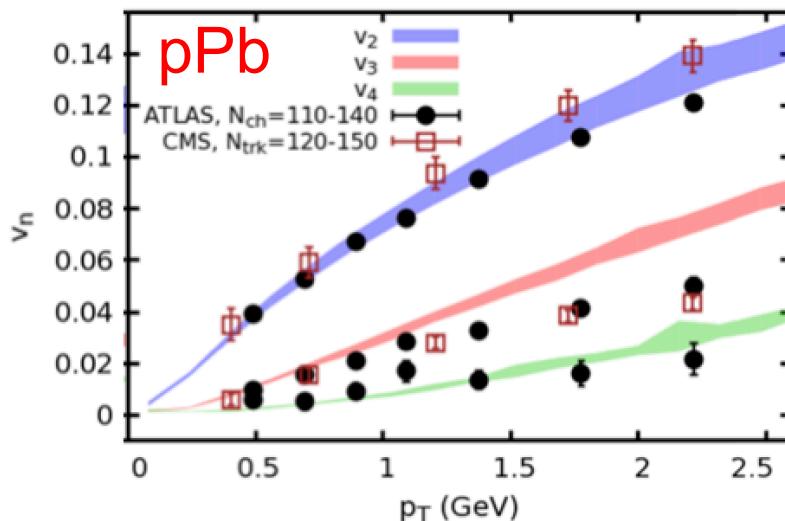


Unreasonable success of hydro?

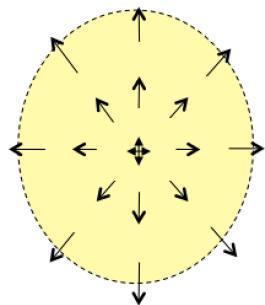
- Different approaches with different setup can describe the same data

P. Romatschke, R. Weller 1701.07145

Hydrodynamics $\text{pPb}, 0\text{-}5\%$

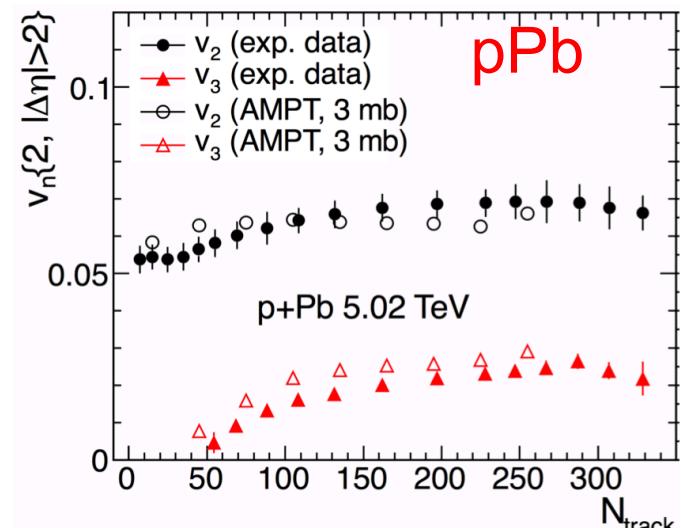


Pressure driven

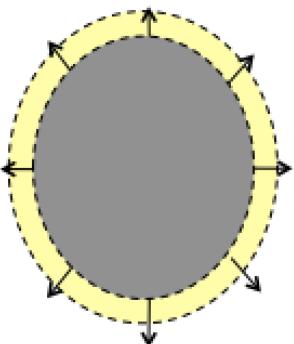


L.He, T.Edmonds, Z.Lin, F.Liu, D. Molnar and F. Wang
1502.05572, G. Ma and A Bzdak 1406.2804

AMPT transport pPb



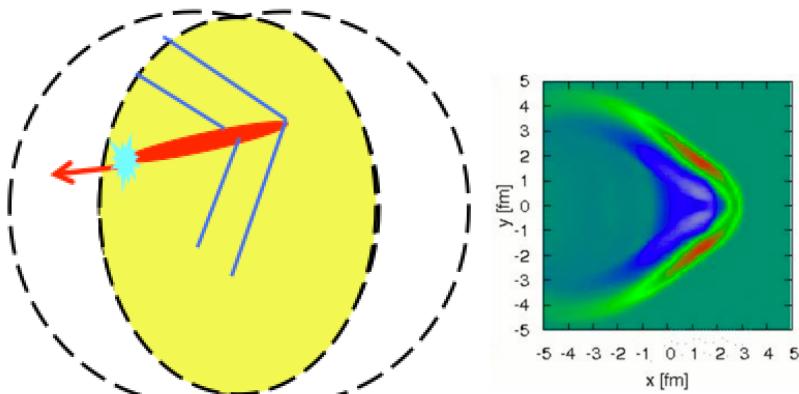
Anisotropic escape
Anisotropic absorption,
surface emission



Actual space-time dynamics are very different

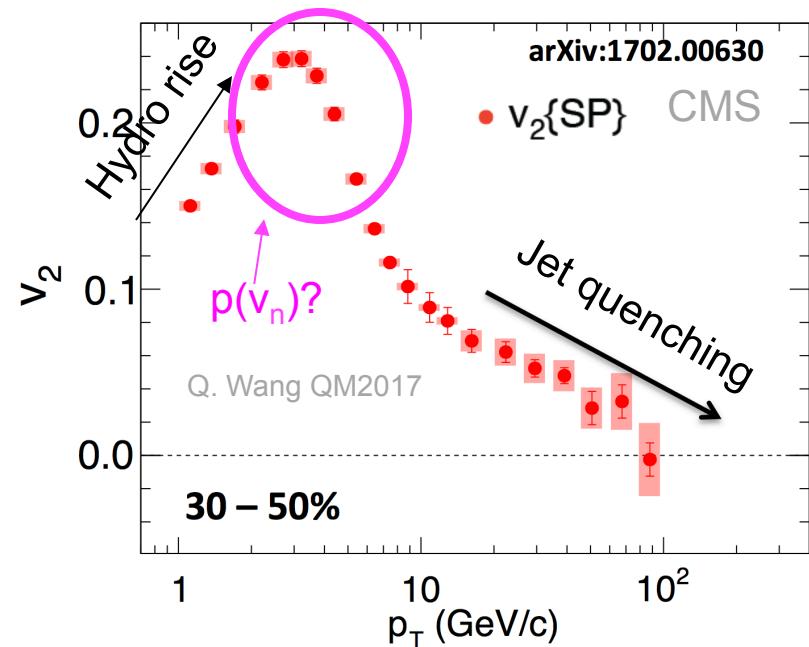
Direct search for non-equilibrium features in v_n ?

Response of the medium to quenched jet, Mach cone?



hard-soft correlations to understand relaxation of non-hydrodynamic perturbations

Nature of flow fluctuations at intermediate p_T ?



Look for systematic failure of gradient expansion of hydrodynamics

Summary

- Flow observables efficiently characterize particle correlations at $\tau=\infty$.
- Hydrodynamic model is a good tool to unfold flow data to extract space-time dynamics of QGP and its properties for $\tau < 10\text{fm}/c$.
- Precision knowledge of HI collisions require more theoretical progress to disentangle contributions from different stages of evolution.
- Some possible directions.
 - Differential measurements of flow fluctuations, mixed correlations
 - Observables to probe fluctuations at different time scales, transport mechanisms for charge and baryon number.
 - Small systems to constrain the pre-equilibrium dynamics.
 - Direct experimental search for effects of non-hydro dynamics.

HOT QUARKS 2018

Scientific Program

- QCD at high temperature/density and lattice QCD
- Initial state effects and Color Glass Condensate
- Relativistic hydrodynamics and collective phenomena
- Correlations and fluctuations
- Jets in the vacuum and in the medium
- Baryons and strangeness
- Heavy-flavour, dileptons and photons
- Application of String Theory and AdS/CFT
- Experimental techniques and future programs

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September 7-14 Texel, The Netherlands

A workshop for young scientists on the physics of ultrarelativistic nucleus-nucleus collisions

Temperature dependence of η/s

Niemi, Eskola, Paatelainen 1505.02677

