



Student Programme

The Modern View of Quark-Gluon Plasma

presented at

Quark Matter 2018 Student Day

Venice, Italy
May 13th, 2018

W.A. Zajc
Columbia University



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The Modern View of Quark-Gluon Plasma

Thanks to (JN)³ :
Jamie Nagle,
Jorge Noronha,
Jaki Noronha-Hostler
for many useful
discussions

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Outline

- Original expectations
- Exact solutions
- Why “almost” doesn’t work
- Measures of coupling
- The emerging modern view

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for many useful discussions

A “Strong” Hint (prior to QCD)

- Hagedorn ~1968: *Is there an ultimate temperature?*
- The very rapid increase of hadron levels with mass
- ~ equivalent to an *exponential* level density

$$\rho(m) \equiv \frac{dn}{dm} \sim m^a e^{m/T_H}$$

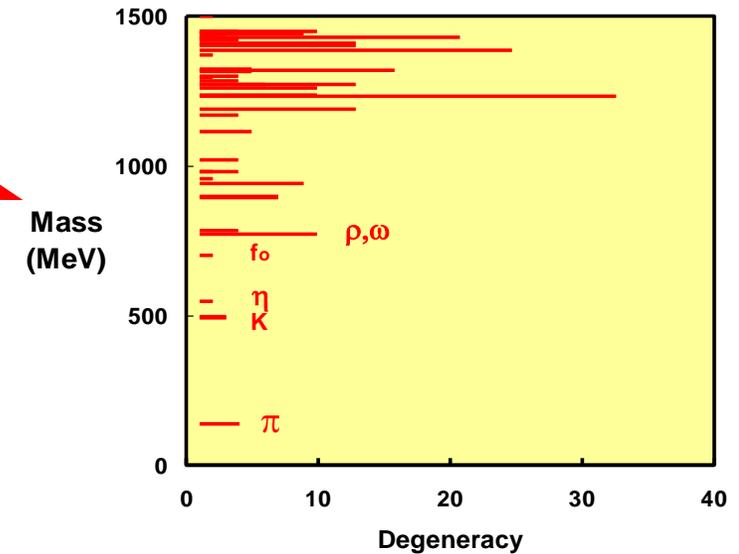
$$\Rightarrow Z = \int \rho(m) e^{-m/T} dm$$

$$\sim \int m^a e^{m/T_H} e^{-m/T} dm$$

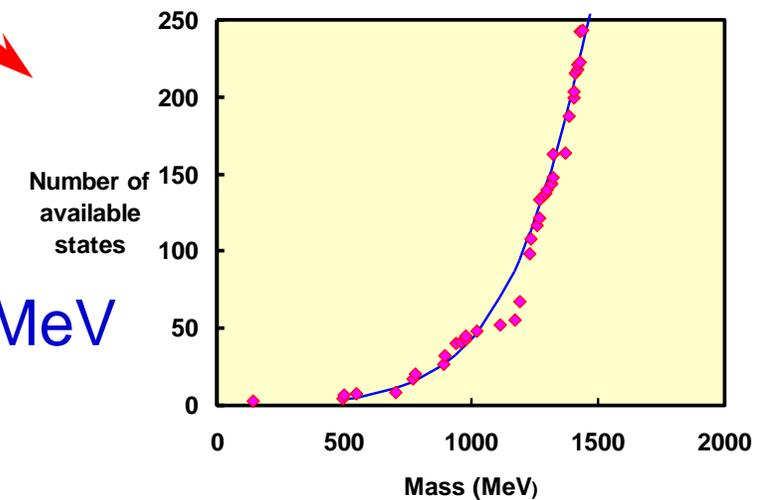
$$\rightarrow \infty \quad \text{when } T > T_H (!)$$

- Which implies an “Ultimate Temperature” (!) $T_H \sim 170 \text{ MeV}$

Hadron 'level' diagram



Density of States vs Energy





Puzzles from pre-History

- Huang and Weinberg (1970):
 - ▶ *Ultimate Temperature and the Early Universe*, Phys. Rev. Lett. **25**, 896 (1970)
 - ▶ Difficulties in constructing a consistent theory of the early universe with a limiting temperature
 - ▶ Its own fine-tuning problem(s)
 - “A curious tentative view of cosmic history emerges from these considerations... at earlier times ($T \sim T_0$), ρ was, once again, dominated by non-relativistic baryons!”

• “...a veil, obscuring our view of the very beginning.”

Steven Weinberg, *The First Three Minutes* (1977)

VOLUME 25, NUMBER 13 PHYSICAL REVIEW LETTERS 28 SEPTEMBER 1970

ULTIMATE TEMPERATURE AND THE EARLY UNIVERSE*

Kerson Huang and Steven Weinberg
 Laboratory for Nuclear Science and Physics Department, Massachusetts Institute of Technology,
 Cambridge, Massachusetts 02139
 (Received 5 August 1970)

The early history of the universe is discussed in the context of an exponentially rising density of particle states.

There are now plausible theoretical models¹ for the thermal history of the universe back to the time of helium synthesis, when the temperature was 0.1 to 1 MeV. Our present theoretical apparatus is really inadequate to deal with much earlier times, say when $T \geq 100$ MeV, and in lieu of any better ideas it is usual to treat the matter of the very early universe as consisting of a number of species of essentially free particles. But how many species?

At one extreme, it might be assumed that the number of particle species stays fixed (perhaps just quarks, antiquarks, leptons, antileptons, photons, and gravitons). In this case, the temperature T will vary with the cosmic scale factor² $R(t)$ according to the relation $T \propto 1/R$. The present universe should then contain various relics of the early inferno: There would be a 1°K blackbody gravitational radiation,³ if $1/R$ stayed roughly constant between the times that the gravitons and the photons decoupled from the rest of the universe; also, according to Zeldovich,⁴ the leftover quarks should be about as common as gold atoms. The gravitational radiation would not have been seen, but the quarks would have been, unless, of course, quarks do not exist.

At the other extreme, one might assume that the number of species of particles with mass between m and $m + dm$ increases as $m \rightarrow \infty$ as fast as possible:

$$N(m)dm \sim Am^{-k} e^{b_0 m} dm, \quad (1)$$

If $N(m)$ increased any faster, the partition function would not converge. With the k in (1), the partition function converges only if the temperature⁵ is less than $1/b_0$. The quantity $T_0 = 1/b_0$ is thus a maximum temperature for any system in thermal equilibrium.

Support for this latter sort of model comes from two quite different directions:

(1) The transverse momentum distribution of secondaries in very high energy collisions is observed to be roughly $\exp(-|p_\perp|/100 \text{ MeV})$. Hagdorn⁶ interprets this distribution in terms of a statistical model with $T_0 = 100$ MeV and

$$B = \frac{1}{2}.$$

(2) If particles fall on families of parallel linearly rising Regge trajectories, their masses take discrete values m_1, m_2, \dots , where

$$\alpha' m_n^2 + \alpha_0 = n. \quad (2)$$

Here $\alpha' = 1 \text{ GeV}^{-2}$ is the universal Regge slope and α_0 is a number, of order unity, characterizing the family. The extension of the Veneziano model⁷ to multiparticle reactions requires⁸ that the number of particle states with mass m_n equals the degeneracy of the eigenvalues λ of the operator

$$N = \sum_{\mu=1}^D \sum_{\nu=1}^D k_{\nu\mu} a_{\nu\mu}, \quad (3)$$

where $k_{\nu\mu}$ and $a_{\nu\mu}$ are an infinite set of multiplication and creation operators. For $n \rightarrow \infty$, this number is⁸

$$P_{\alpha_0} = 2^{-1/2} (D/24)^{D+5/4} e^{-D\alpha_0/4} \times \exp[2\pi(\frac{1}{2} D \alpha_0)^{1/2}]. \quad (4)$$

Equations (2) and (4) lead to an asymptotic level density of form (1), with

$$\beta_0 = 2\pi(D\alpha')^{1/2}, \quad B = \frac{1}{2}(D+1). \quad (5)$$

The value of D is not certain—originally Fubini and Veneziano⁷ had $D=4$, but Lovelace⁹ argues that D is larger, possibly $D=5$.

Table I summarizes the values of T_0 and B for these various models. Lovelace⁹ has emphasized the striking agreement between the values of T_0 derived in such different ways. We now see that

Table I. Possible values of the parameters in the level-density formula (1).

Model	$T_0 = 1/b_0$	B
(1) Hagdorn ⁶	~100 MeV	$\frac{1}{2}$
(2) Veneziano ⁷ (with $\alpha' = 1 \text{ GeV}^{-2}$)		
$D=4$	100 MeV	$\frac{3}{2}$
$D=5$	174 MeV	3
$D=6$	189 MeV	$\frac{7}{2}$
$D=7$	187 MeV	4

*Ref. 6. [†]Ref. 9.



1973

- 1973 = Birth of **QCD**

- Gross, Politzer, Wilczek

PHYSICAL REVIEW D

VOLUME 8, NUMBER 10

15 NOVEMBER 1973

Asymptotically Free Gauge Theories. I*

David J. Gross[†]

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and Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08540

Frank Wilczek

Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08540
(Received 23 July 1973)

Asymptotically free gauge theories of the strong interactions are constructed and analyzed. The reasons for doing this are recounted, including a review of renormalization-group techniques and their application to scaling phenomena. The renormalization-group equations are derived for Yang-Mills theories. The parameters that enter into the equations are calculated to lowest order and it is shown that these theories are asymptotically free. More specifically the effective coupling constant, which determines the ultraviolet behavior of the theory, vanishes for large spacelike momenta. Fermions are incorporated and the construction of realistic models is discussed. We propose that the strong interactions be mediated by a "color" gauge group which commutes with $SU(3) \times SU(3)$. The problem of symmetry breaking is discussed. It appears likely that this would have a dynamical origin. It is suggested that the gauge symmetry might not be broken and that the severe infrared singularities prevent the occurrence of noncolor singlet physical states. The deep-inelastic structure functions, as well as the electron-positron total annihilation cross section are analyzed. Scaling obtains up to calculable logarithmic corrections, and the naive light-cone or parton-model results follow. The problems of incorporating scalar mesons and breaking the symmetry by the Higgs mechanism are explained in detail.

VOLUME 30, NUMBER 26

PHYSICAL REVIEW LETTERS

25 JUNE 1973

¹⁴Y. Nambu and G. Jona-Lasino, Phys. Rev. **122**, 345 (1961); S. Coleman and E. Weinberg, Phys. Rev. D **7**, 1888 (1973).

¹⁵K. Symanzik (to be published) has recently suggested that one consider a $\lambda\phi^4$ theory with a negative λ to achieve UV stability at $\lambda=0$. However, one can show, using the renormalization-group equations, that in such theory the ground-state energy is unbounded from below (S. Coleman, private communication).

¹⁶W. A. Bardeen, H. Fritzsch, and M. Gell-Mann, CERN Report No. CERN-TH-1538, 1972 (to be published).

¹⁷H. Georgi and S. L. Glashow, Phys. Rev. Lett. **28**, 1494 (1972); S. Weinberg, Phys. Rev. D **5**, 1962 (1972).

¹⁸For a review of this program, see S. L. Adler, in Proceedings of the Sixteenth International Conference on High Energy Physics, National Accelerator Laboratory, Batavia, Illinois, 1972 (to be published).

Reliable Perturbative Results for Strong Interactions?*

H. David Politzer

Jefferson Physical Laboratories, Harvard University, Cambridge, Massachusetts 02138
(Received 3 May 1973)

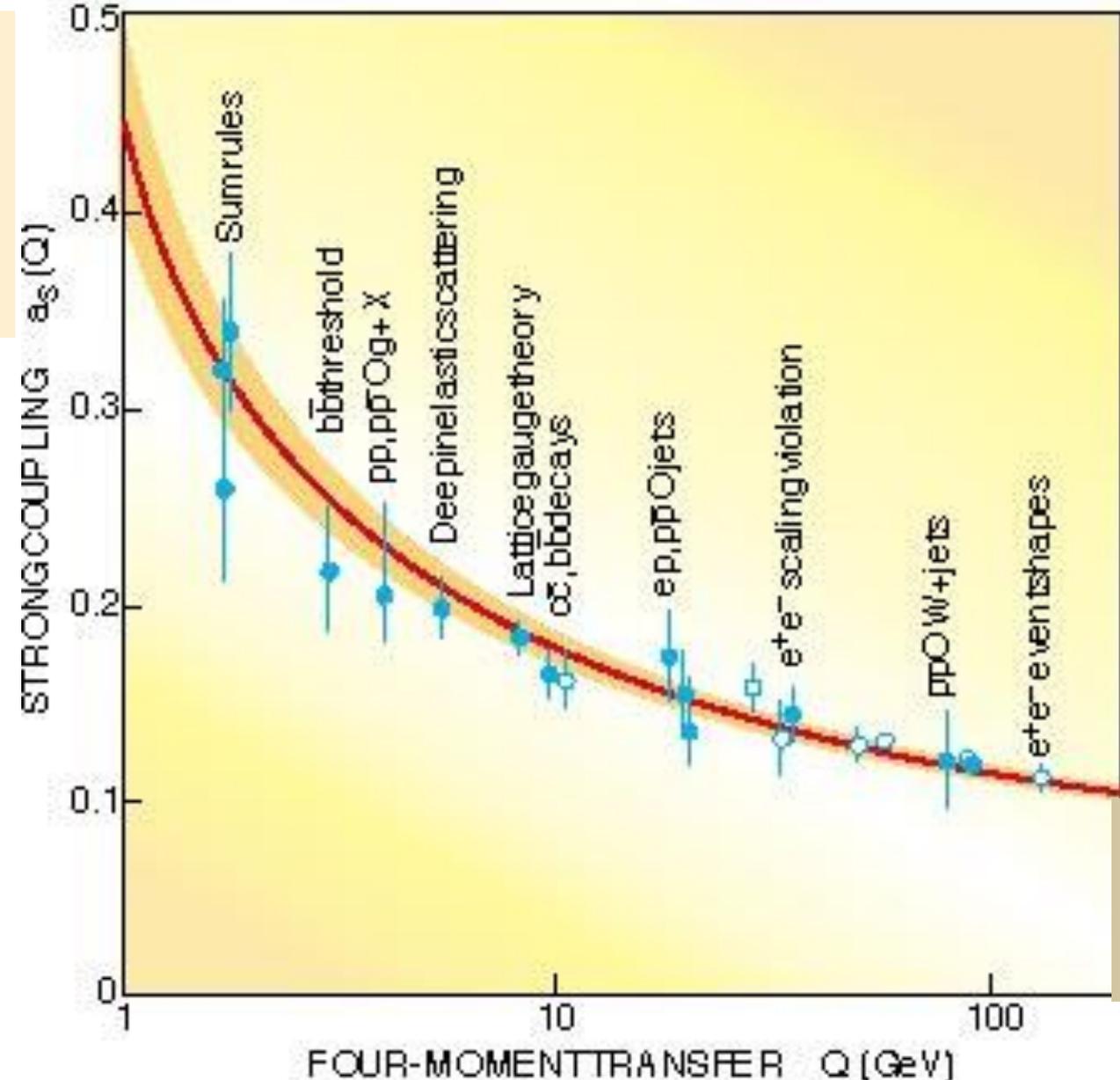
An explicit calculation shows perturbation theory to be arbitrarily good for the deep Euclidean Green's functions of any Yang-Mills theory and of many Yang-Mills theories with fermions. Under the hypothesis that spontaneous symmetry breakdown is of dynamical origin, these symmetric Green's functions are the asymptotic forms of the physically significant spontaneously broken solution, whose coupling could be strong.

Renormalization-group techniques hold great promise for studying short-distance and strong-coupling problems in field theory.^{1,2} Symanzik²

goes to zero, compensating for the fact that there are more and more of them. But the large- β^2 divergence represents a real breakdown of

QCD's Running Coupling "Constant"

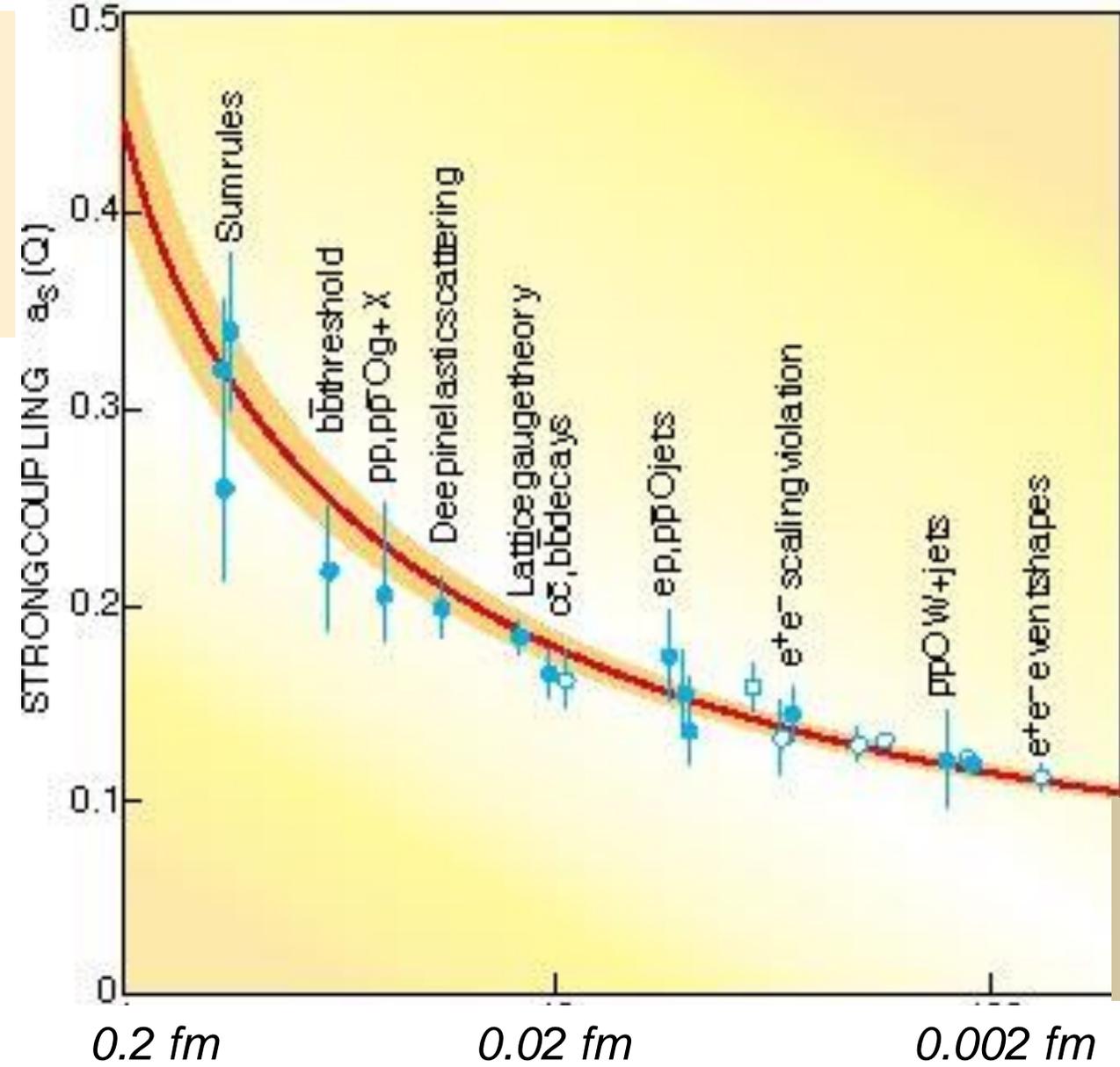
Strong coupling → non-perturbative confinement "infrared slavery"



← Weak coupling perturbative "asymptotic freedom"

QCD's Running Coupling "Constant"

Strong coupling → non-perturbative confinement "infrared slavery"



← Weak coupling perturbative "asymptotic freedom"



Q. What Sets the Scale ?

- Simple answer – Quantum Mechanics:
 - ▶ $\hbar c = 200 \text{ MeV-fm} = 0.2 \text{ GeV-fm}$
 - ▶ So $\hbar / (1 \text{ fm}) = 200 \text{ MeV} \sim \text{light hadron masses}$
- Deeper answer – vacuum condensate that establishes confinement scale $\Lambda \sim 200 \text{ MeV}$

$$\alpha_s(Q^2) = \frac{12\pi}{(33 - 2N_F) \log\left(\frac{Q^2}{\Lambda^2}\right)} \sim \frac{1}{\log\left(\frac{Q^2}{\Lambda^2}\right)} \quad \Lambda \approx \frac{\hbar c}{r_o} \approx 0.2 \text{ GeV}$$



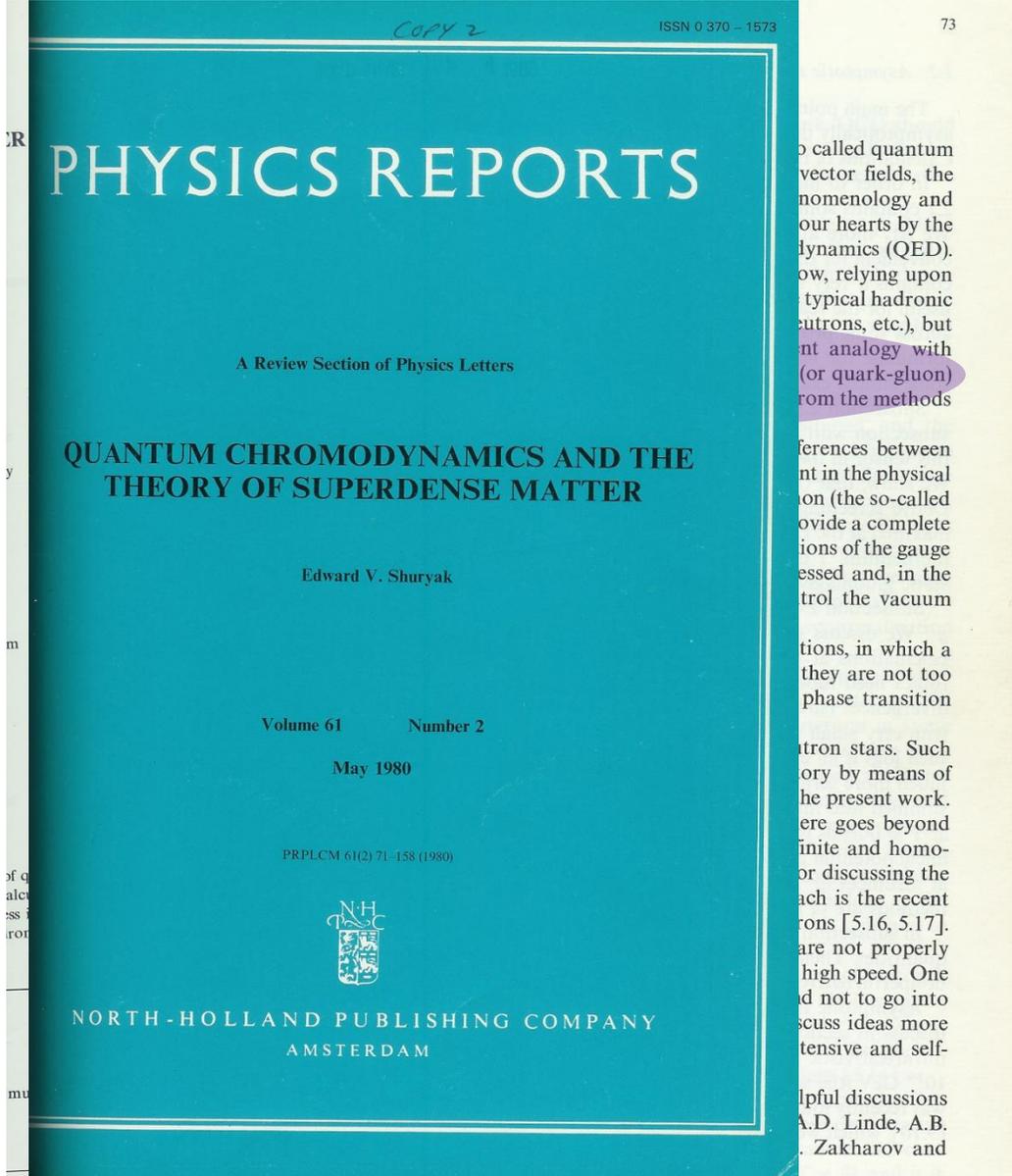
Naming It

- It was quickly realized that asymptotic freedom would lead to deconfined quarks and gluons at high densities and/or pressures:
 - ▶ *Superdense Matter: Neutrons or Asymptotically Free Quarks?*, J.C. Collins and M.J. Perry, Phys. Rev. Lett. 34, 1353 (1975).
 - “We suggest that matter at such high densities is a **quark soup**.”
 - ▶ *Hot Quark Soup*, L. Susskind, submitted to Phys. Rev. D (1978...) available as <http://slac.stanford.edu/pubs/slacpubs/2000/slac-pub-2070.pdf> .
 - “At high temperatures a transition to a **plasma-like phase** occurs.”



Naming It

- Shuryak publishes first “review” of thermal QCD- and coins a phrase:

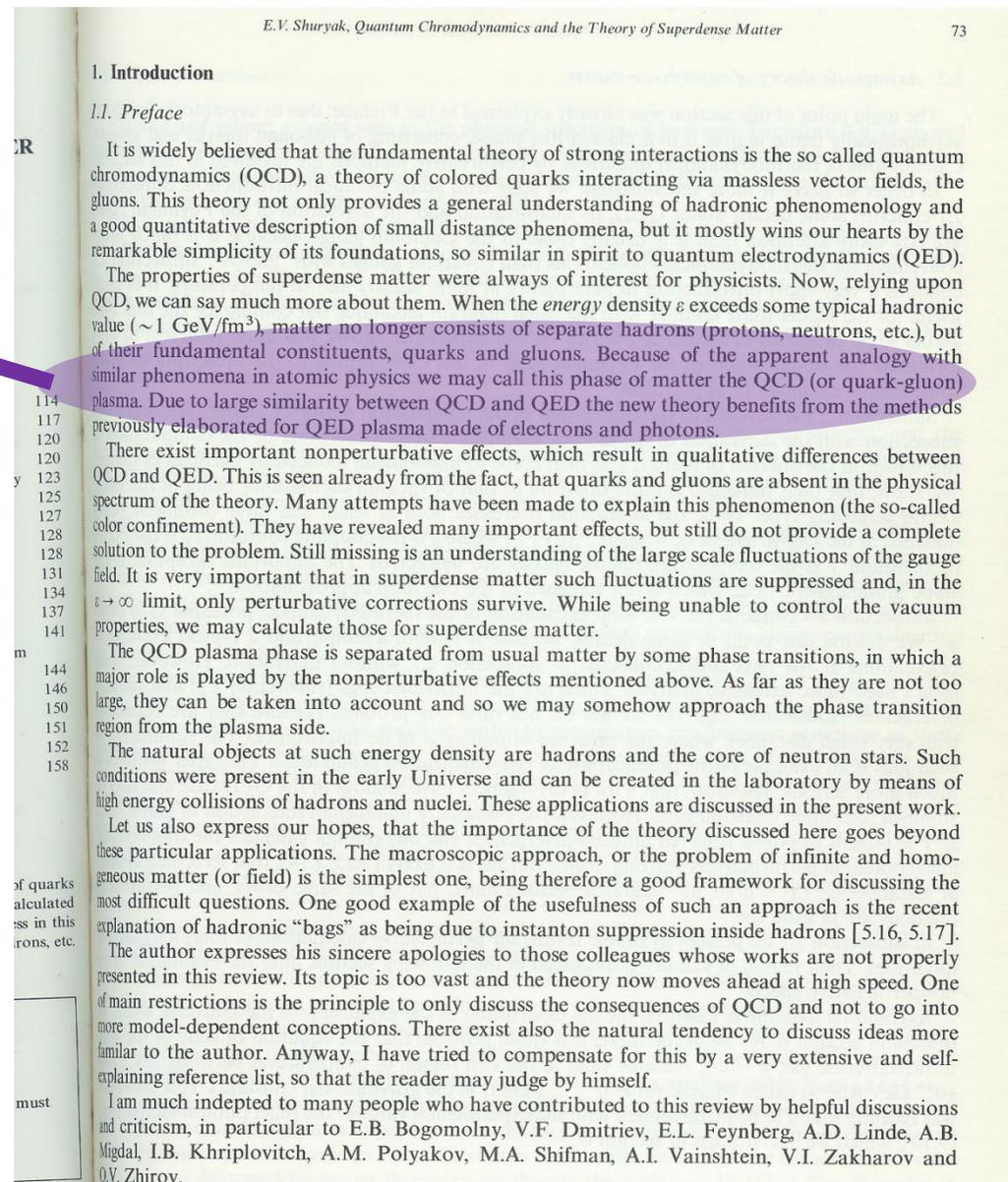


Naming It

- Shuryak publishes first “review” of thermal QCD- and coins a phrase:

“Because of the apparent analogy with similar phenomena in atomic physics, we may call this phase of matter the QCD (or quark-gluon) plasma.”

QGP



“Plasma” Leads To Prejudice

Student Programme



Dictionary

Enter a word, e.g. "pie"



plas·ma

/ˈplazmə/ 

noun

1. the colorless fluid part of blood, lymph, or milk, in which corpuscles or fat globules are suspended.
2. **an ionized gas** consisting of positive ions and free electrons in proportions resulting in more or less no overall electric charge, **typically at low pressures** (as in the upper atmosphere and in fluorescent lamps) or at very high temperatures (as in stars and nuclear fusion reactors).



Translations, word origin, and more definitions



“Plasma” Leads To Prejudice

A screenshot of the Wikipedia article for "Plasma (physics)". The browser address bar shows "en.wikipedia.org/wiki/Plasma_(physics)#Ultracold_plasma". The article title is "Plasma (physics)" with a subtitle "From Wikipedia, the free encyclopedia". There are tabs for "Article" and "Talk", and buttons for "Read", "Edit", and "View history". A search bar is visible on the right.

plasma is a state of matter similar to gas in which a certain portion of the particles are ionized... **plasma** is a state of matter similar to gas in which a certain portion of the particles are ionized.

 A screenshot of the Wikipedia article for "Plasma (physics)" showing the table of contents and two images. The table of contents is as follows:

- 1 Common plasmas
- 2 Plasma properties and parameters
 - 2.1 Definition of a plasma
 - 2.2 Ranges of plasma parameters
 - 2.3 Degree of ionization
 - 2.4 Temperatures
 - 2.4.1 Thermal vs. non-thermal plasmas
 - 2.5 Potentials
 - 2.6 Magnetization
 - 2.7 Comparison of plasma and gas phases
- 3 Complex plasma phenomena
 - 3.1 Filamentation
 - 3.2 Shocks or double layers
 - 3.3 Electric fields and circuits
 - 3.4 Cellular structure
 - 3.5 Critical ionization velocity
 - 3.6 Ultracold plasma
 - 3.7 Non-neutral plasma
 - 3.8 Dusty plasma and grain plasma
- 4 Mathematical descriptions
 - 4.1 Fluid model
 - 4.2 Kinetic model
- 5 Artificial plasmas
 - 5.1 Generation of artificial plasma

 Two images are shown:

- The top image is a "Plasma globe" showing complex filamentation patterns. The caption reads: "Plasma globe, illustrating some of the more complex phenomena of a plasma, including filamentation. The colors are a result of relaxation of electrons in excited states to lower energy states after they have recombined with ions. These processes emit light in a spectrum characteristic of the gas being excited."
- The bottom image is a close-up of the Sun, showing its surface and solar flares.

Q. What Temperature Is Required for Deconfinement?

I.e., How to compute location of transition from

A gas of hadrons at temperature T

to

A gas of deconfined quarks and gluons at T ?

A1: The only scale in QCD is $\Lambda_{\text{QCD}} \rightarrow T_c \sim \Lambda_{\text{QCD}}$

A2: Thermodynamics of phase equilibria

- ▶ Compute the pressure P in each phase
- ▶ The phase with the higher pressure wins

• (See back-up for refresher on phase equilibria):



Needed Statistical Mechanics

- For massless non-interacting *bosons* :

- ▶ Number density

$$n(T) = \frac{1.202}{\pi^2} T^3 \approx \left(\frac{T}{2}\right)^3$$

- ▶ Energy density

$$\varepsilon(T) = \frac{\pi^2}{30} T^4$$

- ▶ Pressure

$$P(T) = \frac{1}{3} \varepsilon(T) = \frac{\pi^2}{90} T^4$$

☞ *Per degree of freedom*

⇒ Next steps: Counting degrees of freedom

Exercise: Convince yourself this handy pocket formal is correct (in natural units). Use it to compute the density of microwave background photons (T=2.7 K)

Counting Degrees of Freedom

- Hadronic phase

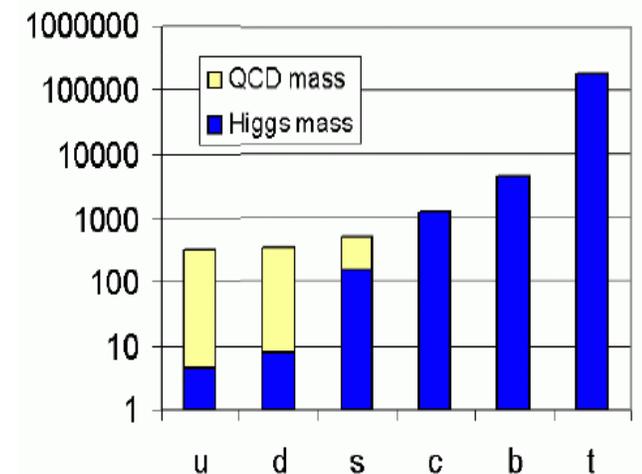
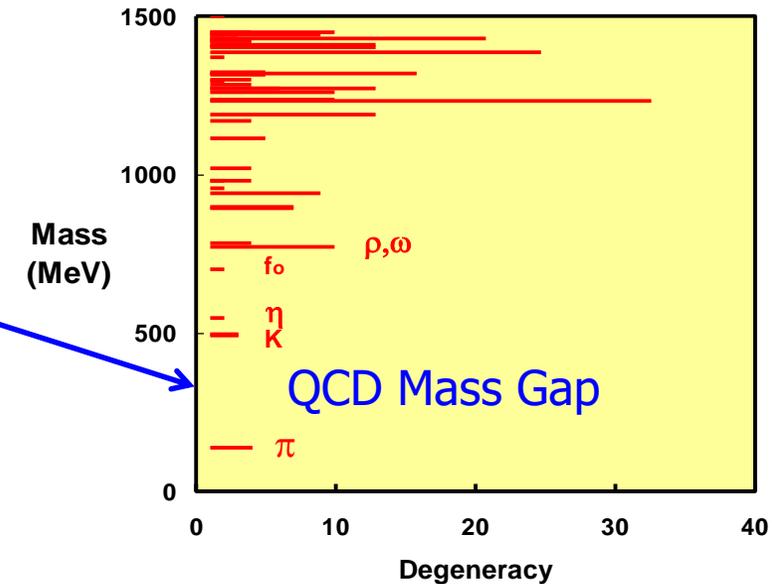
- ▶ Assume relevant $T < 500 \text{ MeV}$
- ▶ $\text{ndf} = 3$ (π^-, π^0, π^+)

- Quark-gluon phase

- ▶ Gluons: $\text{ndf} = 2_s \times 8_c = 16$
 - ▶ Quarks: $\text{ndf} = (7/8) \times 2_s \times 2_f \times 2_a \times 3_c = \underline{21}$
 - ▶ Total $\text{ndf} = 37$
- $47.5 (3_f)$

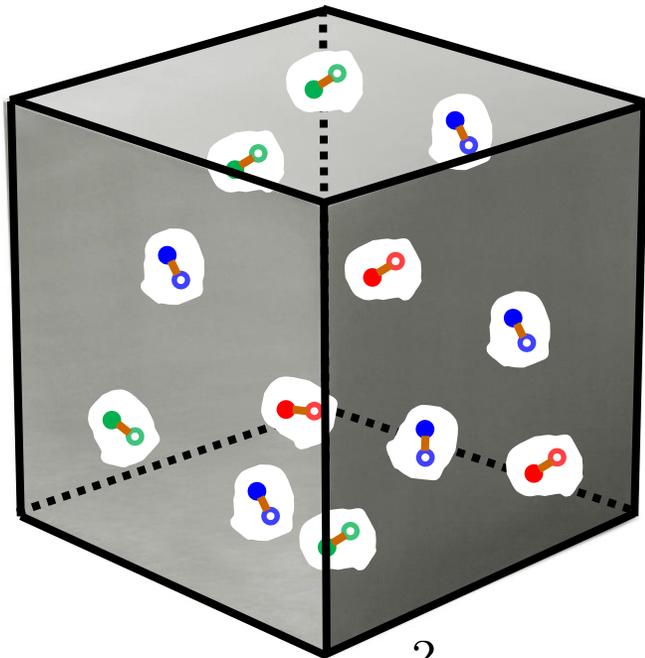
- Bottom line: $\text{ndf}_{\text{QGP}} \sim 10 \times \text{ndf}_{\text{Hadrons}}$

Hadron 'level' diagram

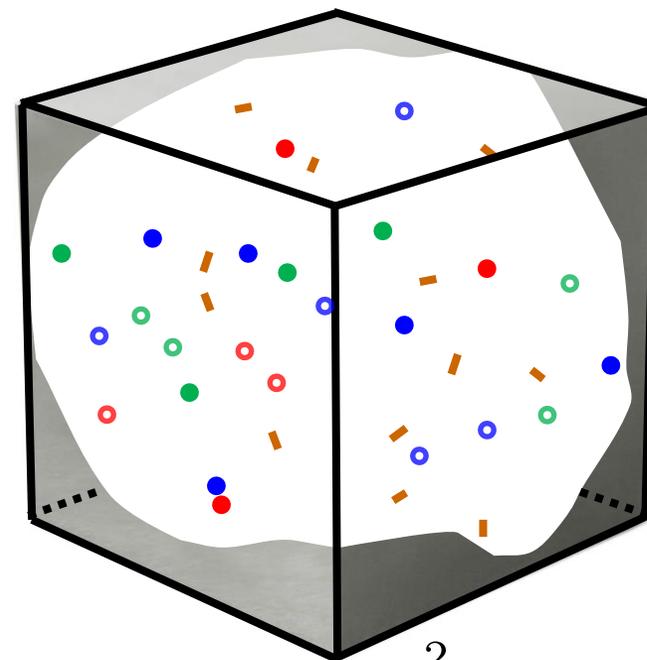


It's a High Pressure Game

- Compare pressure of pion gas to that of a bag of q's and g's



$$P_{\pi}(T) = 3 \frac{\pi^2}{90} T^4$$

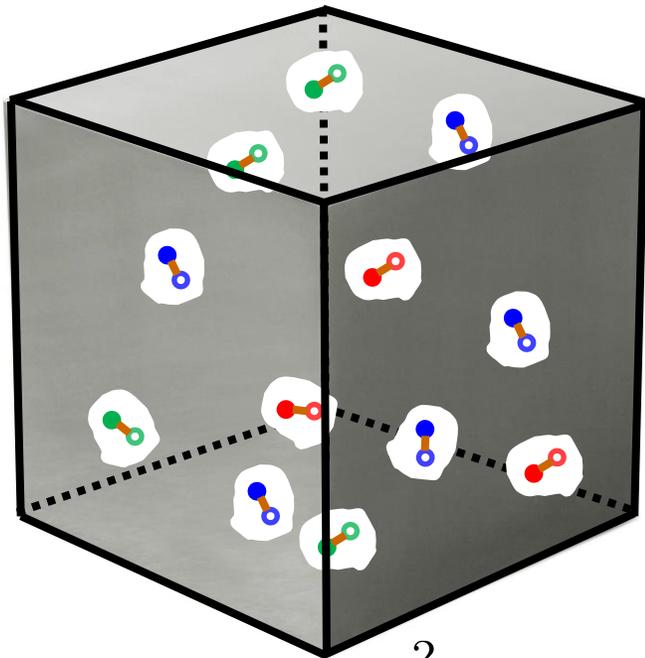


$$P_{QGP}(T) = g \frac{\pi^2}{90} T^4 - B$$

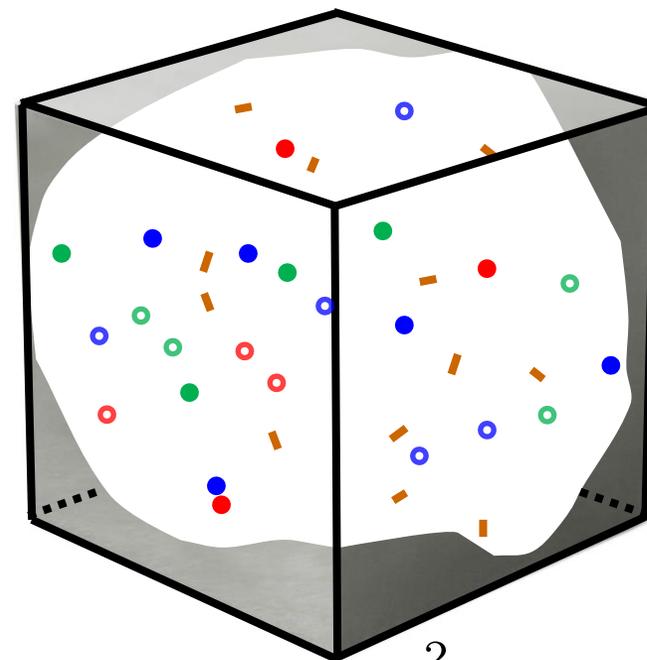
$$g \sim 37-47.5$$

It's a High Pressure Game

- Compare pressure of pion gas to that of a bag of q's and g's



$$P_{\pi}(T) = 3 \frac{\pi^2}{90} T^4$$



$$P_{QGP}(T) = g \frac{\pi^2}{90} T^4 - B$$

$$g \sim 37-47.5$$

- Nature selects system with higher pressure...



Simple Model Result

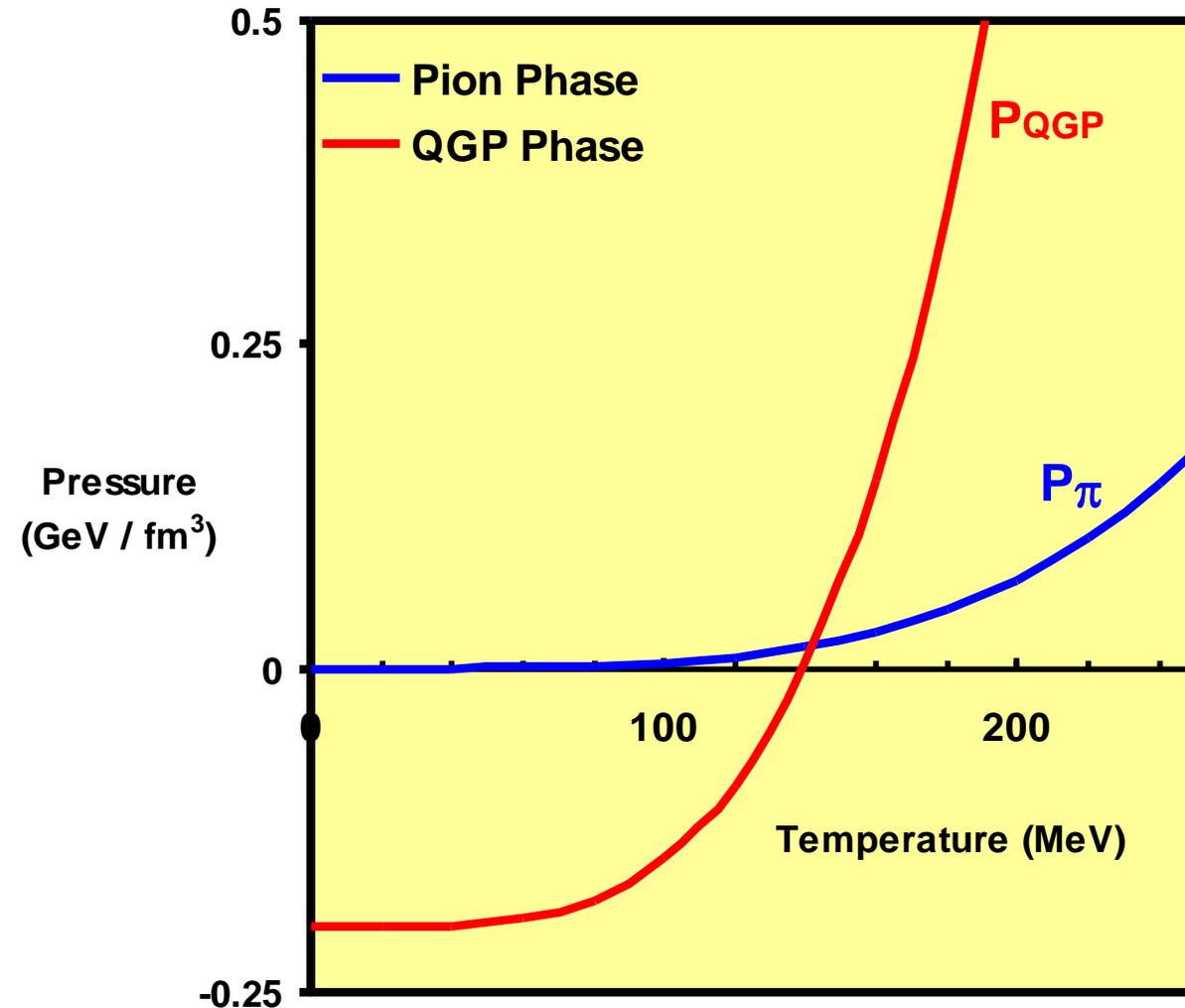
- For $T > (0.68-0.72) B^{1/4}$,

$$P_{\text{QGP}} > P_{\pi}$$

→ a phase transition to deconfined state

- For $B^{1/4} \sim 200 \text{ MeV}$,
 $T_c \sim 140 \text{ MeV}$,

with modest latent heat = $4B$





Damage Control

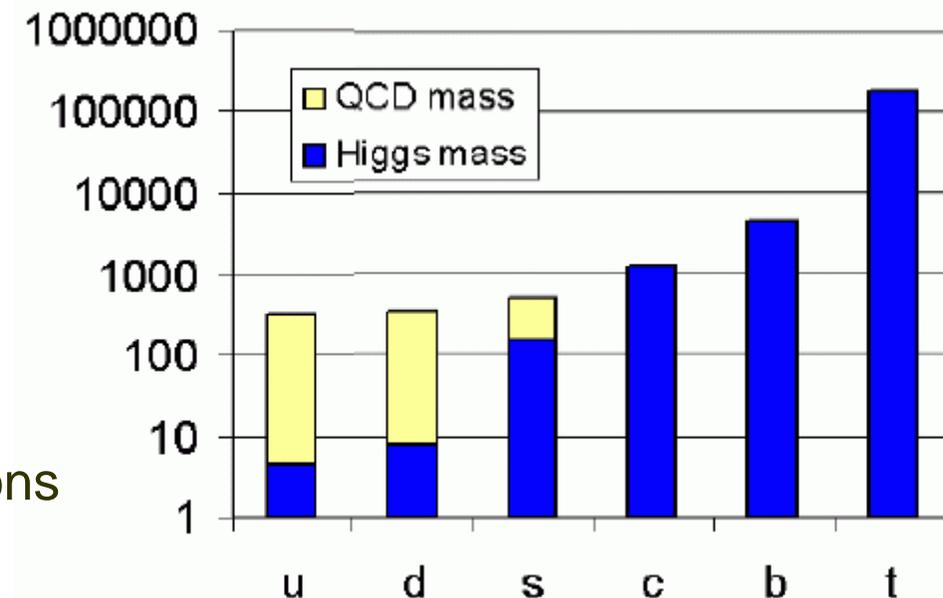
- In reality, this is not such a great estimate:

- ▶ **Hadron side**

- Pions hardly massless relative to 140 MeV ☺
 - Ignores exponential growth at higher T
 - Ignores (strong!) interactions

- ▶ **QGP side**

- Strange quark neither massless nor massive
 - Bag constant stand-in for QCD vacuum fluctuations
 - Ignores (strong!) interactions



- To do better

- ▶ Program some of this in *Mathematica*
or

- ▶ Calculate ~ 1 TeraFlops x 100 days $\sim 10^{19}$ Flops

Lattice QCD

- “Solve” the theory on a discrete space-time lattice
- Requires massive (parallel) computing

Below slide from Derek Teaney

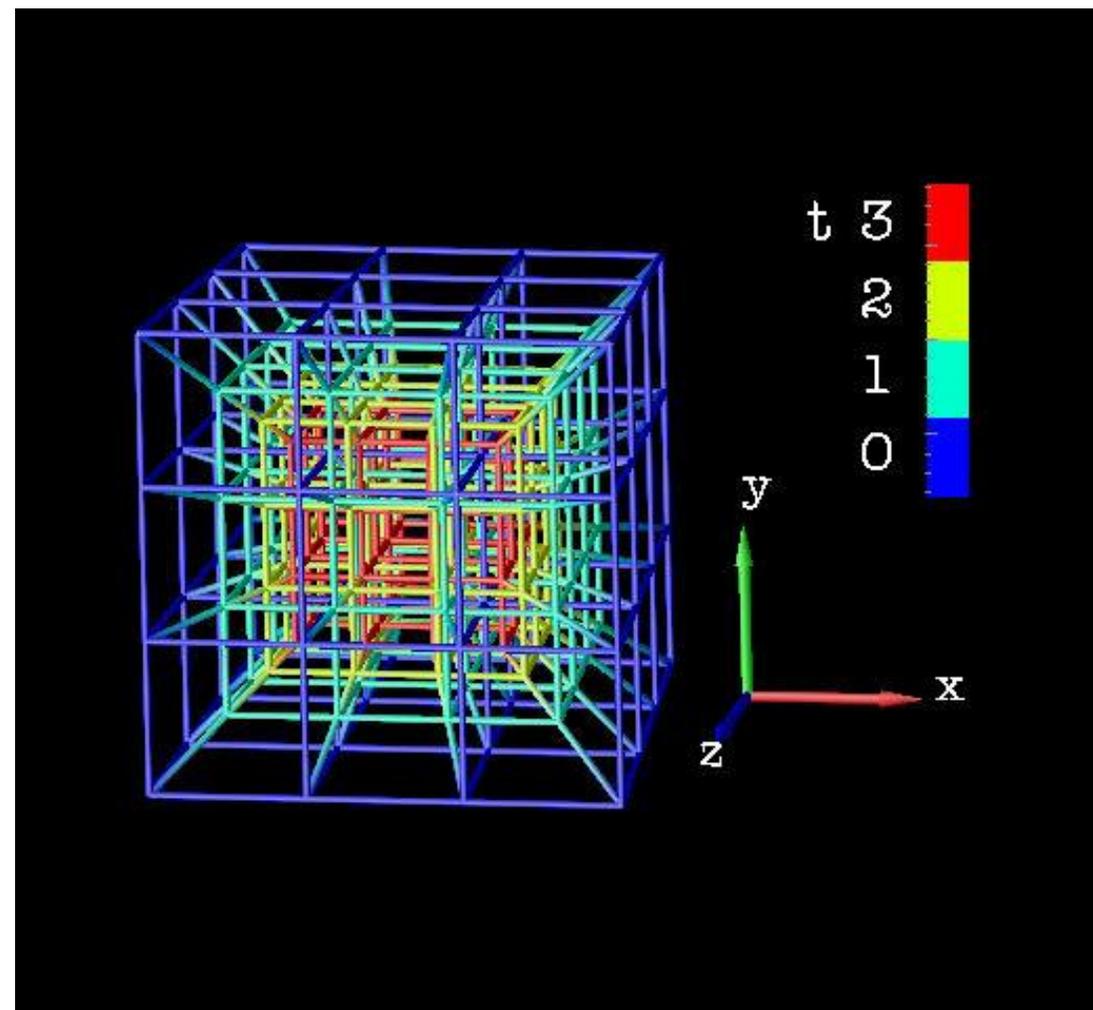
Lattice QCD and the QCD equation of state:



Compute the equation of state by sampling fields with the statistical weight:

$$Z \sim \int [DA] e^{-S_{QCD}[A]}$$

The largest single computational project in human history!

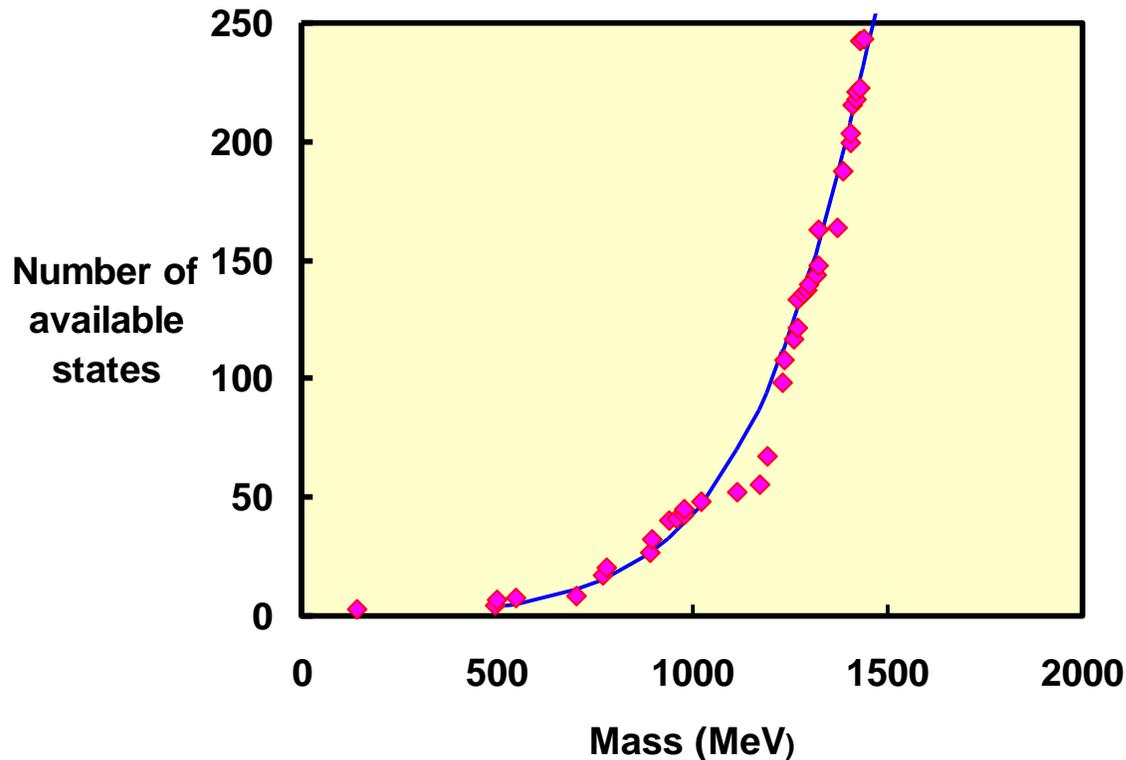




Selected Lattice QCD Results (I)

- The density of states really is exponential:

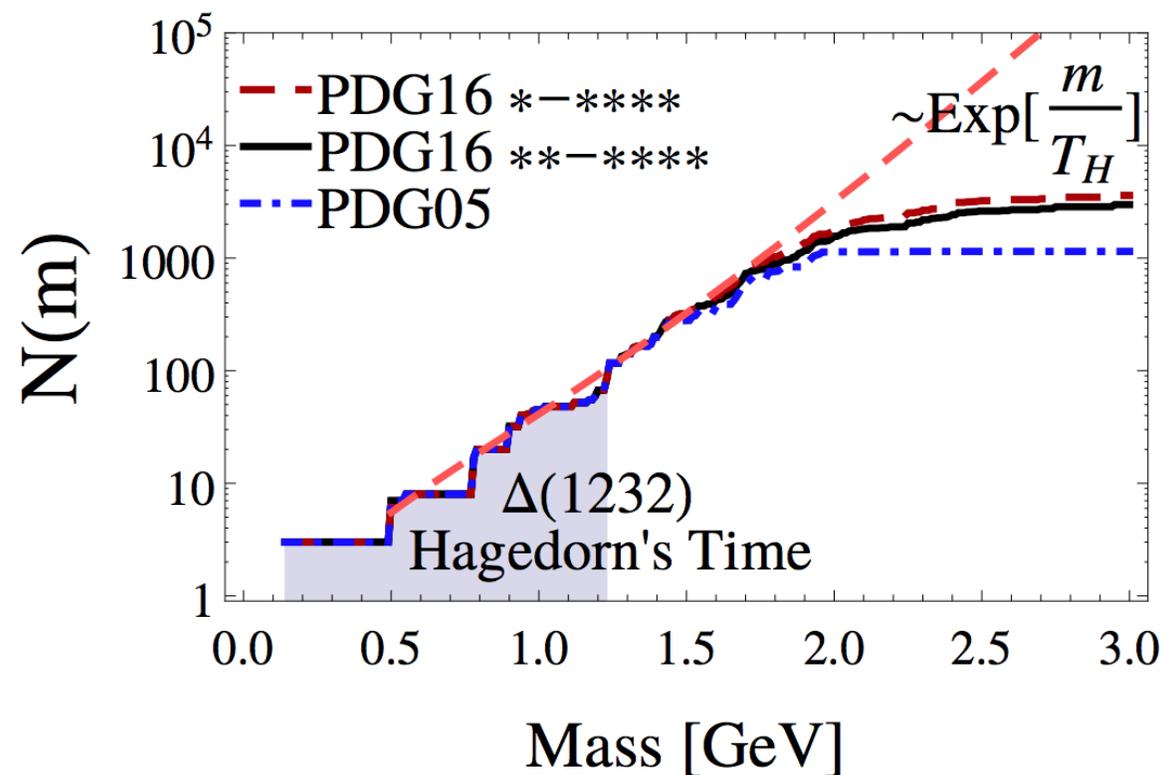
Density of States vs Energy



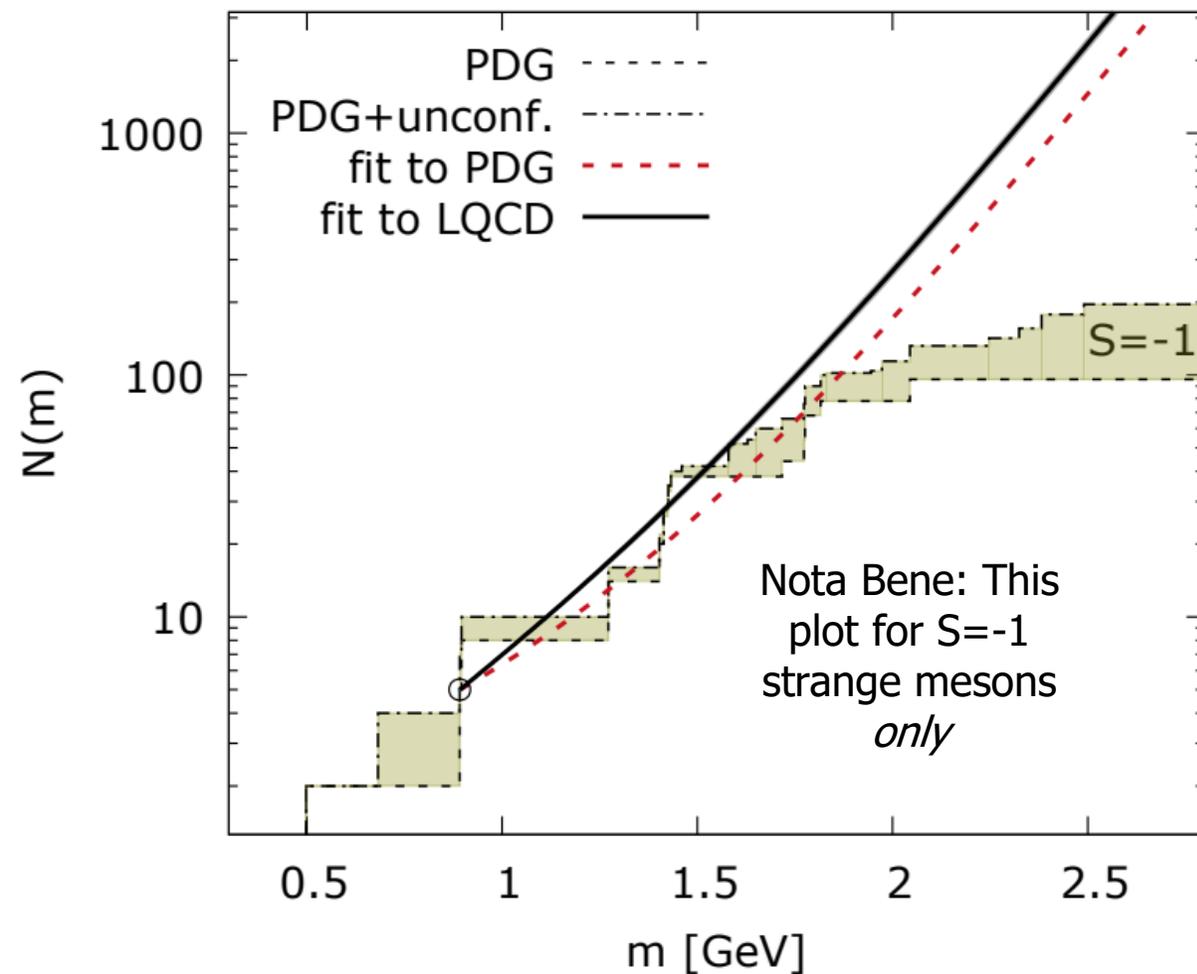
(hand-compilation by WAZ c. 1995)

Selected Lattice QCD Results (I)

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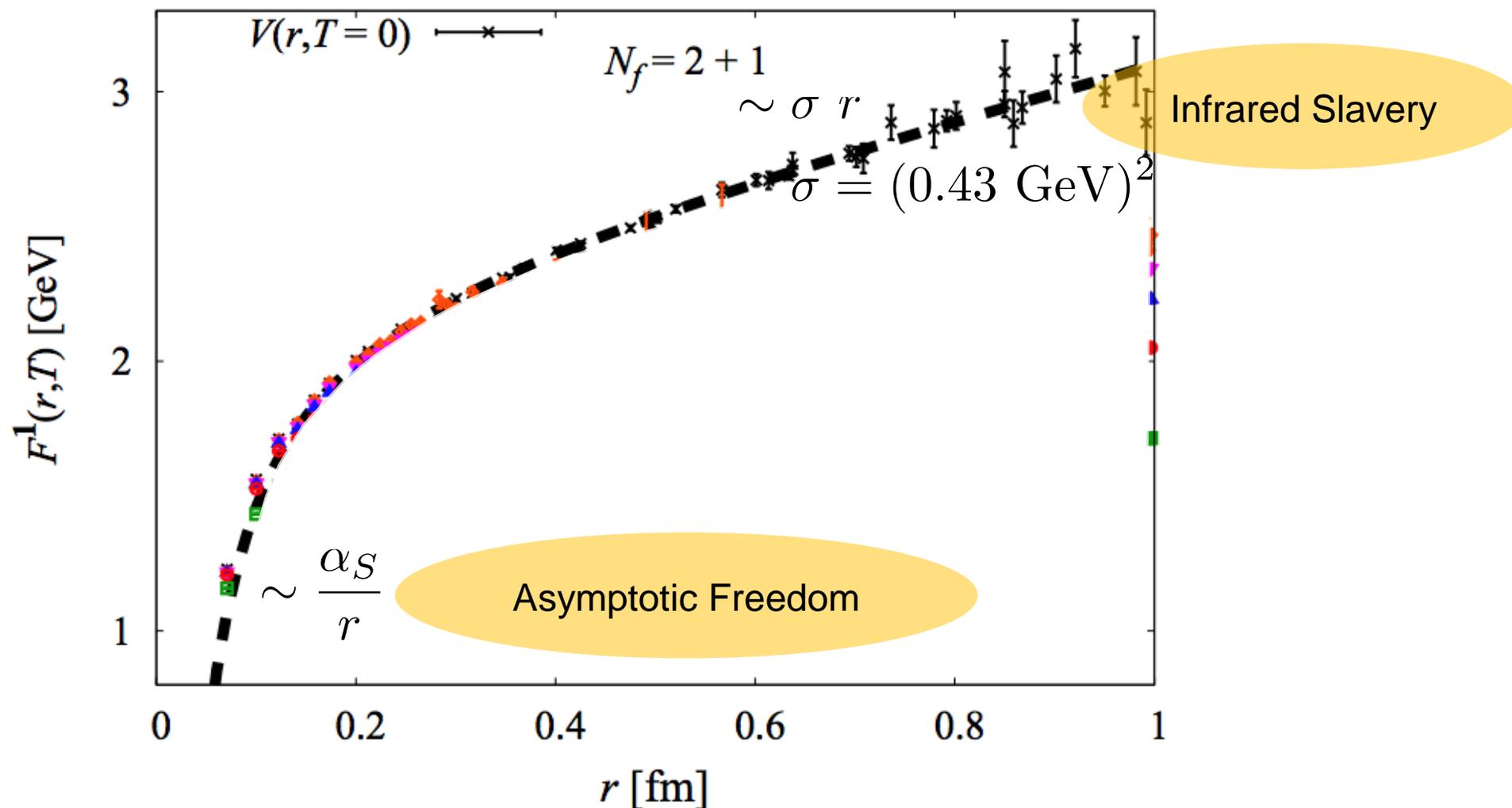
J. Noronha-Hostler, [arXiv:1612.07765](https://arxiv.org/abs/1612.07765)



P.M. Lo *et al.*, [arXiv: 1507.06398](https://arxiv.org/abs/1507.06398)

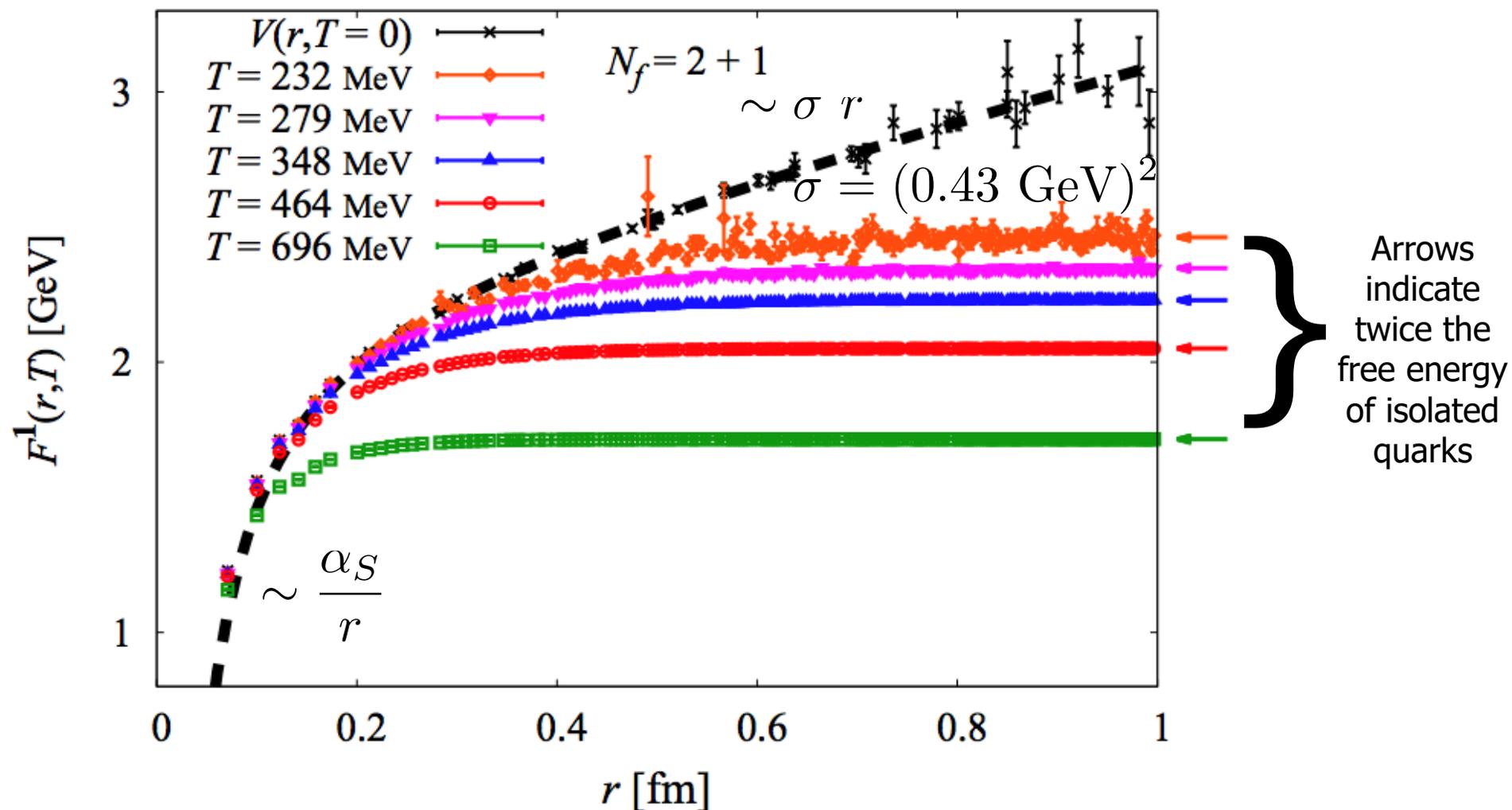
Selected Lattice QCD Results (II)

- The quark-antiquark potential is strongly modified at $T > T_H$.



Selected Lattice QCD Results (II)

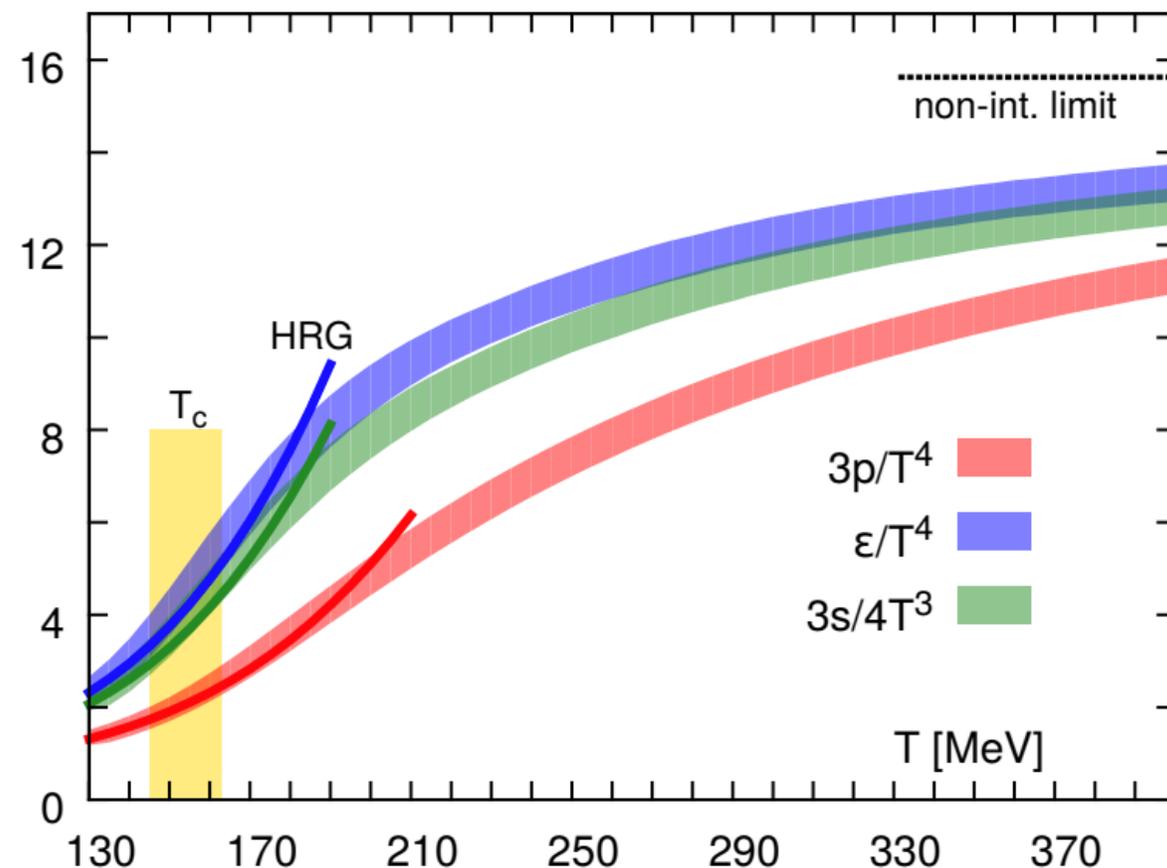
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Selected Lattice QCD Results (III)

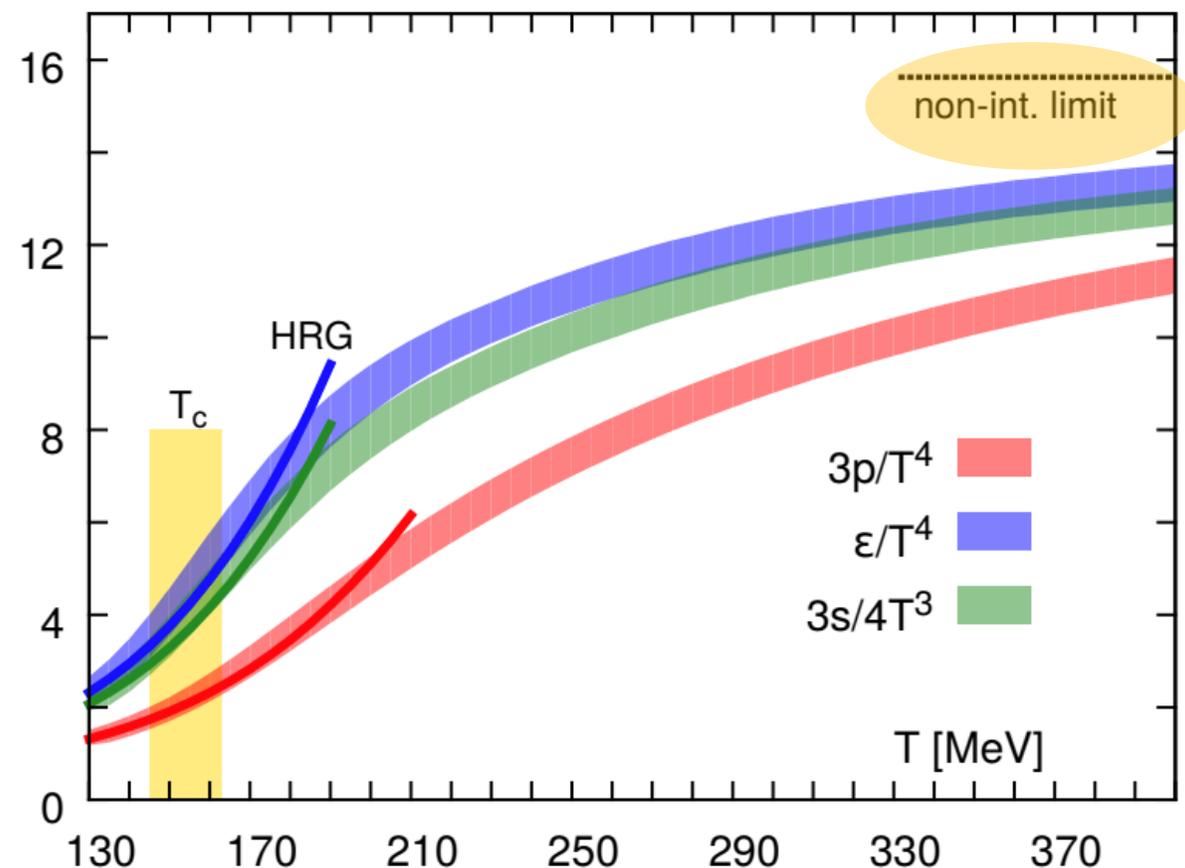
- The pressure p , energy density ε and entropy density s all increase dramatically for $T > \sim 140$ MeV
- Smooth crossover (like ionization in forming regular plasma)
- At low temperature, LQCD results nicely consistent with a Hadron Resonance Gas (HRG) calculation containing exponential increase of states
- At high temperature, ε/T^4 approaches estimates from counting free quarks and gluons





Selected Lattice QCD Results (III)

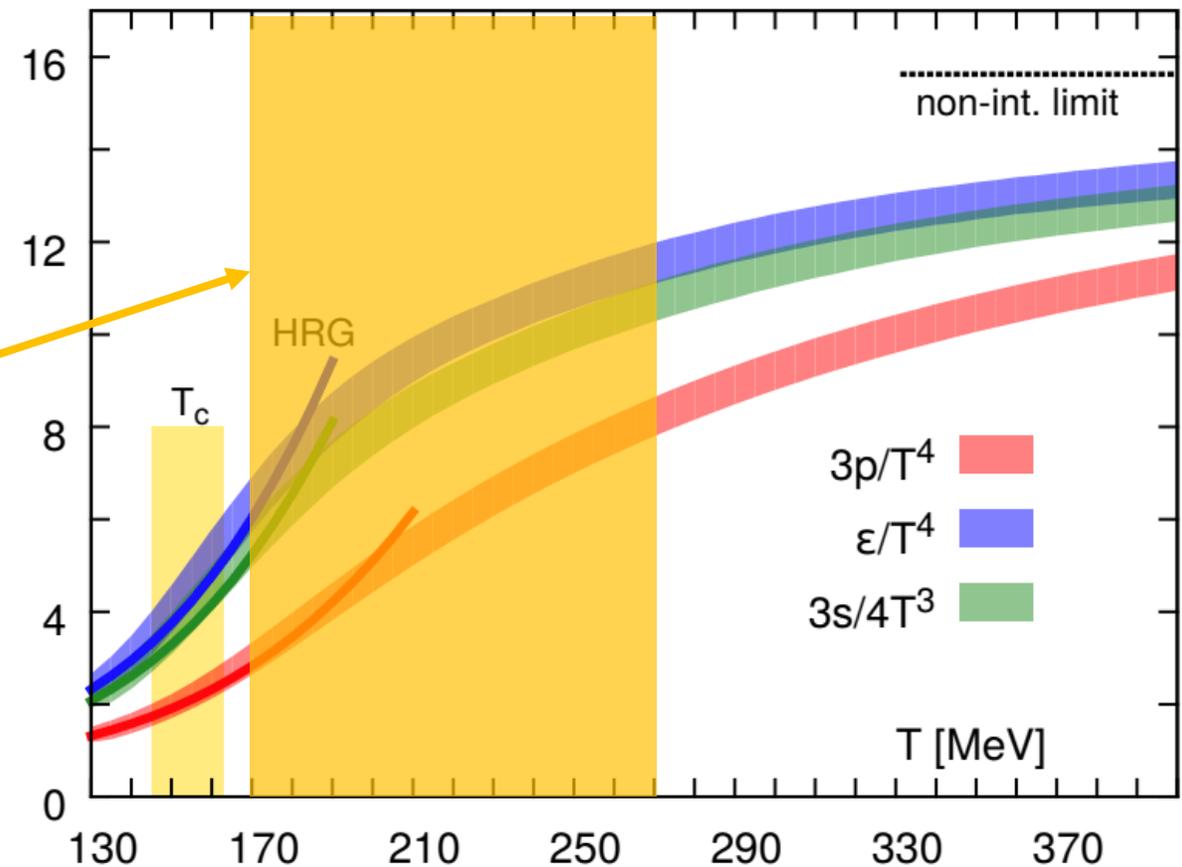
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Resisting Temptation

- First results from RHIC in 2000-2002 especially
 - ▶ thermal spectra indicative of temperatures ~ 140 MeV (or higher)
 - ▶ Strong suppression of high p_T particles
- Compelling evidence for matter in this regime
- Very tempting to identify this with free quarks and gluons!



Theoretical Guidance

- 1983: “an extended **quark-gluon plasma** within which the quarks are deconfined and move independently”

PHASE DIAGRAM OF NUCLEAR MATTER

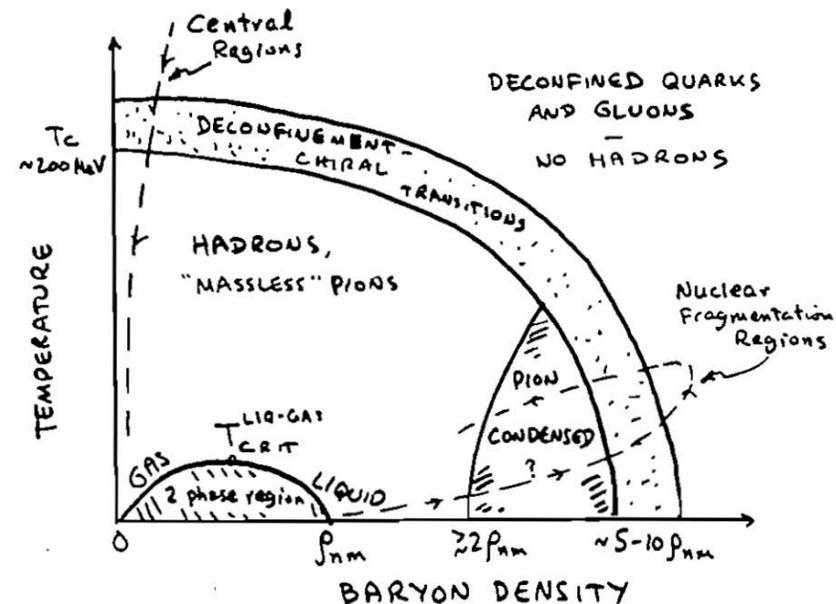


Fig. II.9-A. Expected phases of nuclear matter at various temperatures and baryon (or nucleon) densities, showing the “hadronic phase” including a gas-liquid phase transition region, and the transition region to deconfined quarks and gluons. The dashed lines illustrate trajectories in this phase diagram that can be explored in ultra-relativistic heavy ion collisions.

Theoretical Guidance

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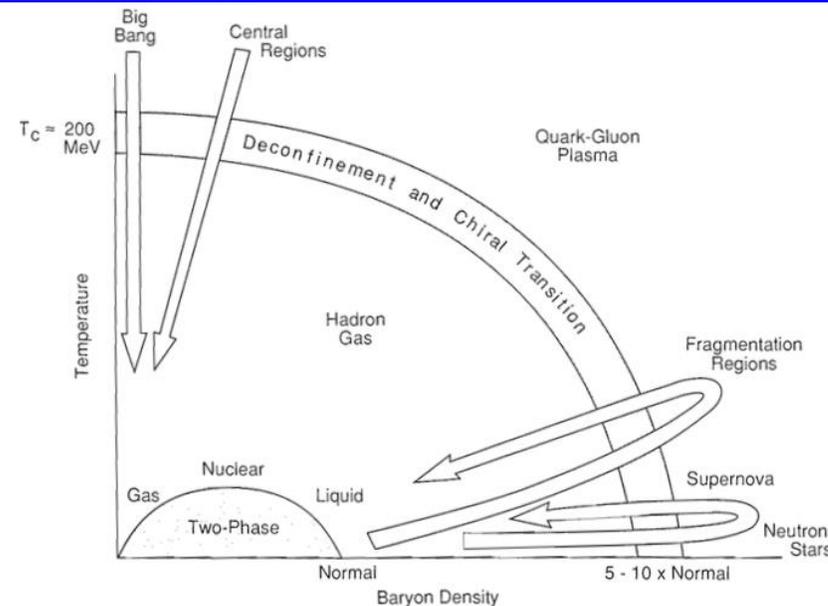


Figure 24: Expected phases of nuclear matter at various temperatures and baryon (or nucleon) densities, showing the “hadronic phase,” including a gas-liquid phase-transition region, and the transition region to deconfined quarks and gluons. The dashed lines illustrate trajectories in this phase diagram that can be explored in ultrarelativistic heavy-ion collisions.

- 1989: “quark-gluon plasma, in which hadrons dissolve into a plasma of quarks and gluons, which are then free to move over a large volume.”

Theoretical Guidance



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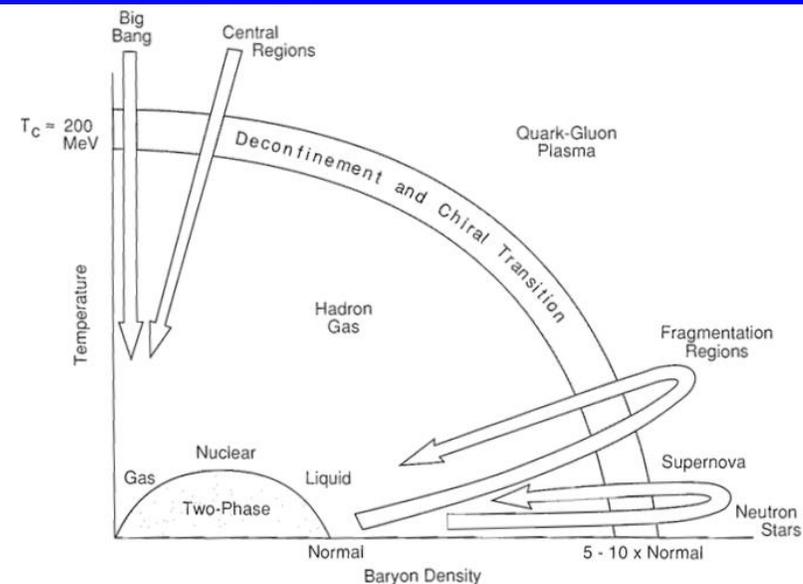


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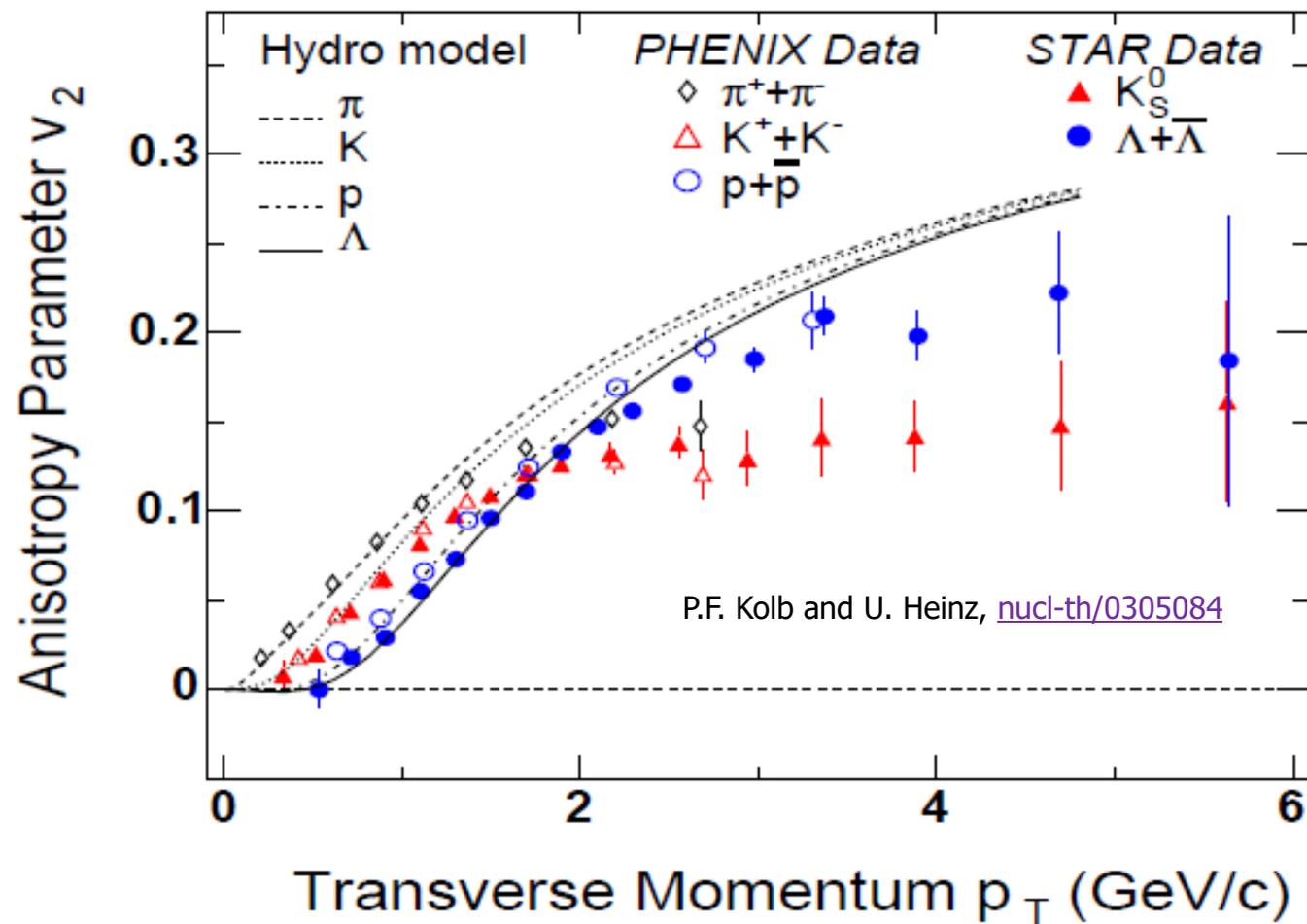
- 1989: “quark-gluon plasma, in which hadrons dissolve into a plasma of quarks and gluons, which are then free to move over a large volume.”

- 2000: “Quarks and gluons would then freely roam within the volume of the fireball created by the collision.”



Lead Us Not Into Temptation...

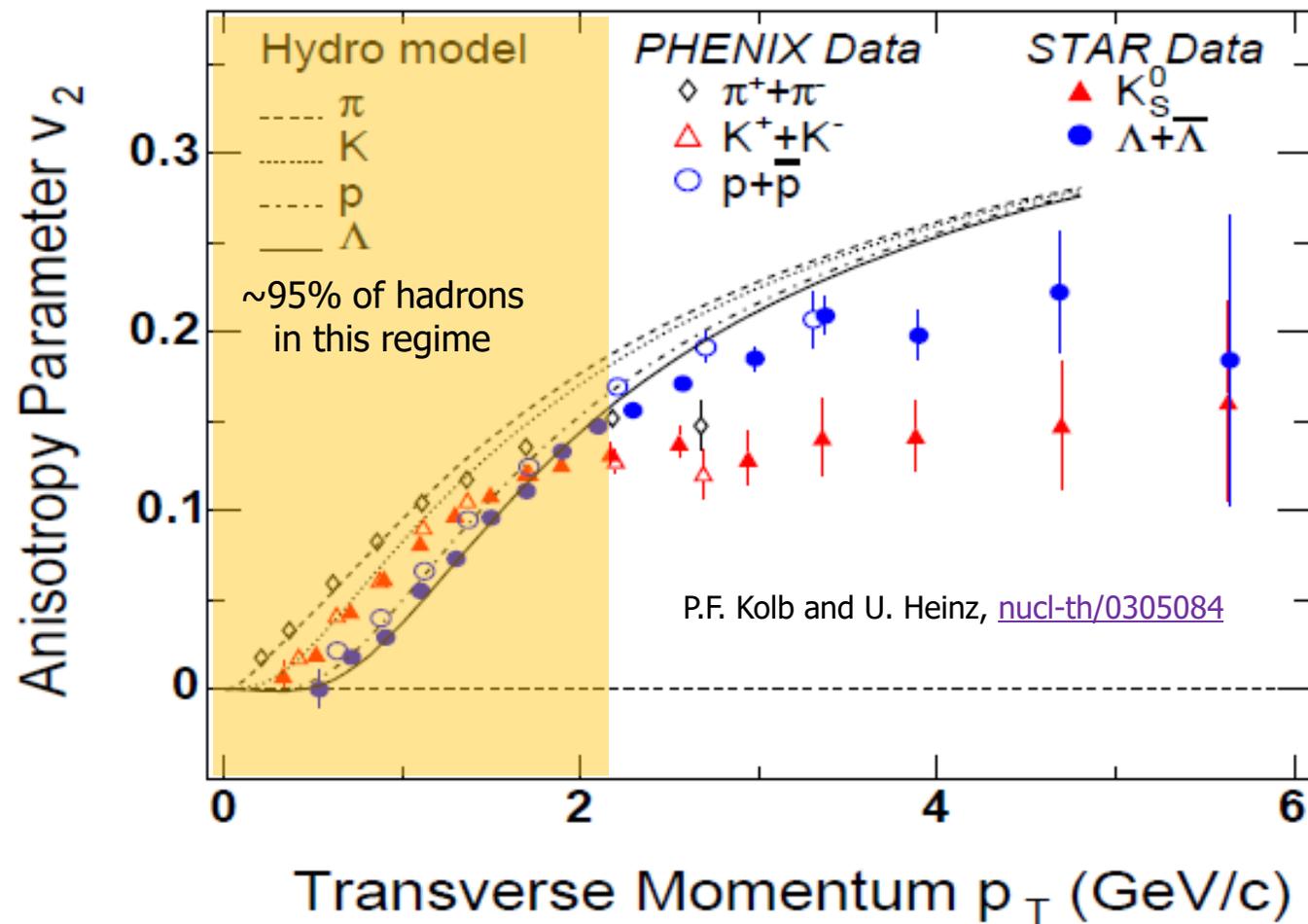
- Those putatively “free” quarks and gluons exhibited flow patterns consistent with *ideal* hydrodynamics
- Ideal hydro \rightarrow viscosity set to zero
- Zero viscosity limit \rightarrow infinite cross sections between our “free” quarks and gluons (!)





Lead Us Not Into Temptation...

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A Funny Thing About Viscosity

- **Strong** interactions imply **small** viscosity:

- ▶ Viscosity $\eta \sim$ Transverse momentum diffusion

$$\sim n \langle p \rangle \lambda \quad , \quad n = \text{number density}$$

$$, \quad \langle p \rangle = \text{mean momentum}$$

$$, \quad \lambda = \text{mean free path}$$

$$= 1 / n \sigma ,$$

$$\sigma = \text{cross section}$$

- ▶ So

$$\eta \sim n \langle p \rangle \lambda \sim \langle p \rangle / \sigma$$

- ▶ **Large** inter-particle cross section \rightarrow **small** viscosity



A Perturbative Calculation for Viscosity

- Lattice suggests deconfined quarks and gluons above T_c .
- A harbinger of asymptotic freedom?
- \rightarrow Try perturbation in g , where $\alpha_S = \frac{g^2}{4\pi}$
 - P. Arnold, G. Moore, Y. Yaffe, [hep-ph/0010177](https://arxiv.org/abs/hep-ph/0010177)

▶ Leading-order result:

$$\eta \sim \#_1 \frac{T^3}{g^4 \log(\frac{\#_2}{g})} \quad \text{where } \#_1 \sim 90 \quad ; \quad \#_2 \sim 2.95$$

- ▶ Small perturbative cross-sections (< mb) produce a *large* viscosity



A Perturbative Calculation for Viscosity

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Aside: This result is often quoted as

“parametrically, $\eta \sim \frac{T^3}{g^4 \log(\frac{1}{g})}$ ”

This is confusing,
well, more accurately, *wrong*.

Thinking Like a Physicist



- Viscosity is a dimensionful quantity
→ we must ask “*Small or large compared to what?*”
- Answer: *Entropy density s*
- Motivation:
 - ▶ Relativistic $F = ma$ results in terms containing $\frac{\eta}{\epsilon + P} \sim \frac{\text{drag force}}{\text{inertial mass}}$
 - ▶ Thermodynamics (at zero chemical potential) $\rightarrow \epsilon + P = T s$
 - ▶ So $\frac{\eta}{\epsilon + P} = \left(\frac{\eta}{s}\right) \frac{1}{T} \sim (\text{dimensionless number}) \times (\text{natural thermal time})$



How Small Can We Make η/s ?

- Recall

- ▶ Viscosity $\eta \sim$ Transverse momentum diffusion $\sim n \langle p \rangle \lambda$
- ▶ Entropy density $s \sim 4 n$ (massless non-interacting quanta)

- Then
$$\frac{\eta}{s} \sim \frac{n \langle p \rangle \lambda}{4n} \sim \frac{1}{4} \langle p \rangle \lambda$$

- But uncertainty principle $\rightarrow \langle p \rangle \lambda \sim \hbar$

- Which in turn implies
$$\frac{\eta}{s} > \sim \frac{1}{4} \hbar$$



Lost Knowledge (1985)

• Miklos Gyulassy and Pawel Danielewicz:

- ▶ *Dissipative Phenomena In Quark-Gluon Plasmas*
P. Danielewicz, M. Gyulassy
Phys.Rev. D31, 53,1985.



noted restrictions on smallest allowed

• Most restrictive:

- $\lambda > \hbar / \langle p \rangle \Rightarrow \eta > \sim n / 3$
- But recall $s = 4 n$ for the quanta they were considering
- $\Rightarrow \eta/s > \hbar / (4 \times 3) \sim \hbar / (4 \pi)$

$$\frac{\eta}{s} > \sim \frac{1}{4\pi} \hbar$$

Before estimating λ_i via Eq. (3.2) we note several physical constraints on λ_i . First, the uncertainty principle implies that quanta transporting typical momenta $\langle p \rangle$ cannot be localized to distances smaller than $\langle p \rangle^{-1}$. Hence, it is meaningless to speak about mean free paths smaller than $\langle p \rangle^{-1}$. Requiring $\lambda_i \geq \langle p \rangle_i^{-1}$ leads to the lower bound

$$\eta \geq \frac{1}{3} n, \quad (3.3)$$

where $n = \sum n_i$ is the total density of quanta. What seems amazing about (3.3) is that it is independent of dynamical details. There is a finite viscosity regardless of how large is the free-space cross section between the quanta. See Refs. 21 and 22 for examples illustrating how the thermalization rate of many-body systems is limited by the uncertainty principle.



Simultaneously in 2003-4

- An estimate (bound?) on viscosity appeared from string theory's AdS/CFT correspondence:
 - ▶ *A Viscosity Bound Conjecture*,
P. Kovtun, D.T. Son, A.O. Starinets, [hep-th/0405231](https://arxiv.org/abs/hep-th/0405231)

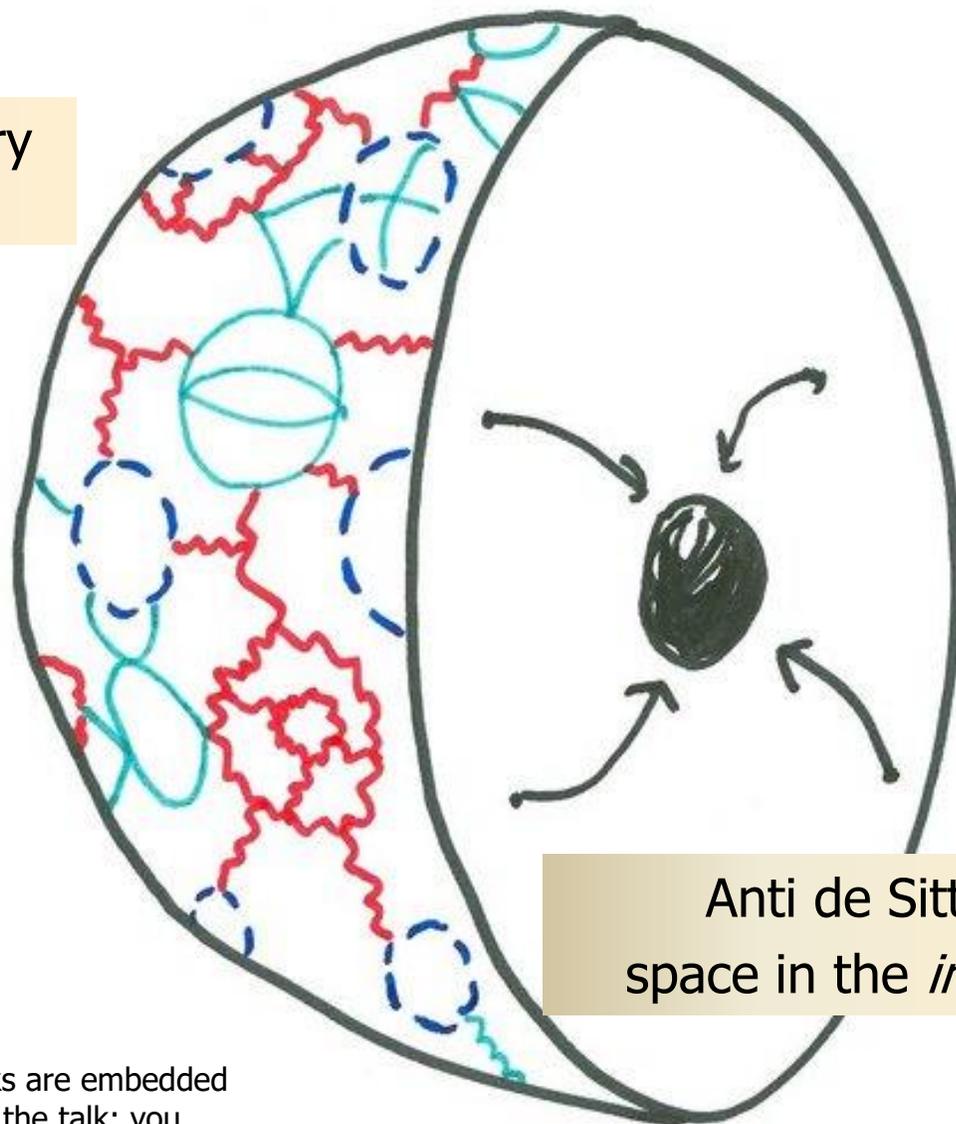
$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi}$$

⇒ Very important to quantify *departures* from ideal hydrodynamics

AdS / CFT in a Picture

Conformal Field Theory
on the *boundary*

quantum
field
theory
on
surface



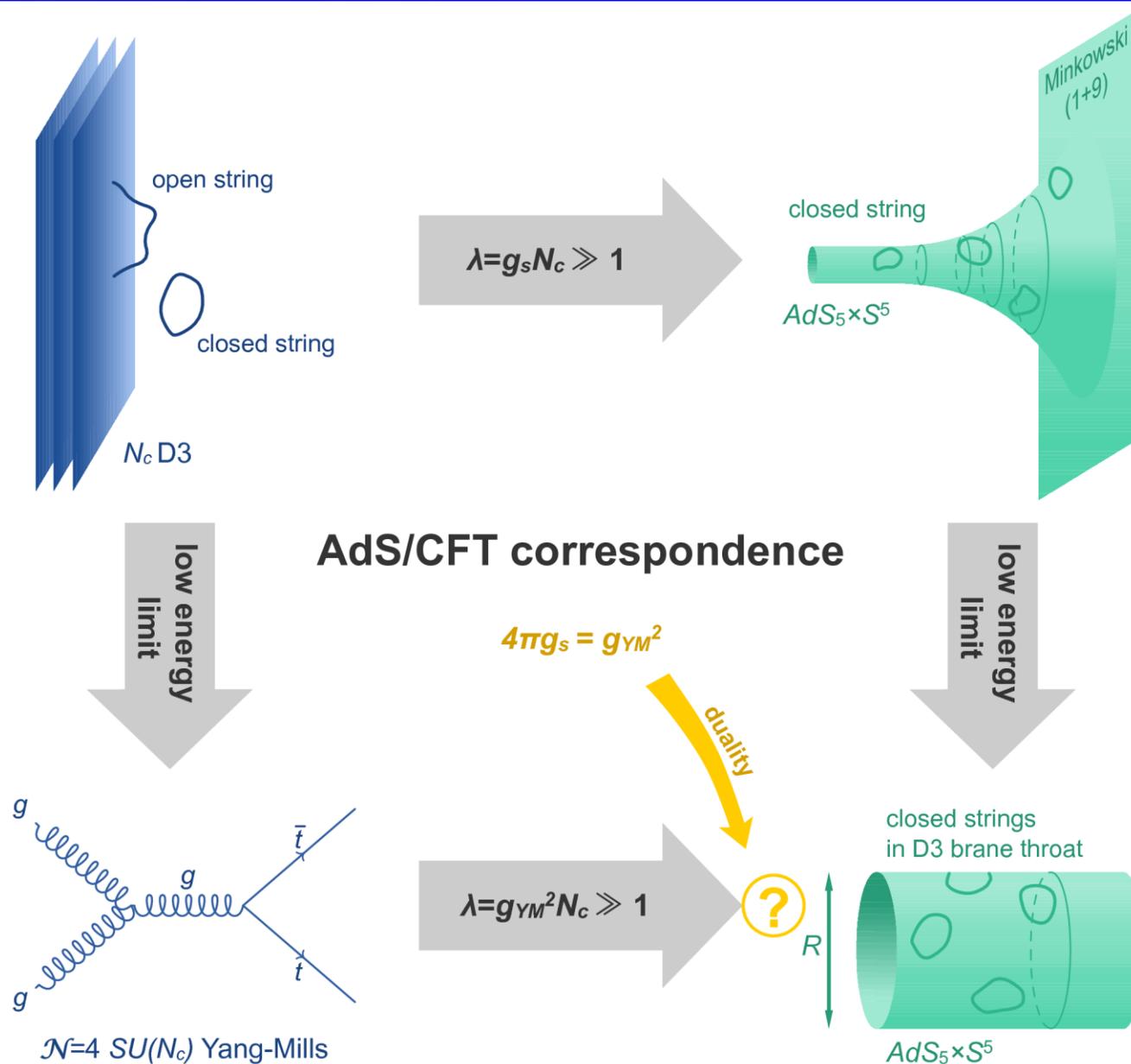
Anti de Sitter
space in the *interior*

gravity
theory
with
black hole
inside
ball

☞ A hard
(strongly-coupled)
gauge
theory
calculation
is **dual** to
an easy
semi-
classical
gravitational
calculation.

Note: None of these figures are mine; links are embedded with each on in the PowerPoint version of the talk; you can find all of them simply by searching on "AdS/CFT"

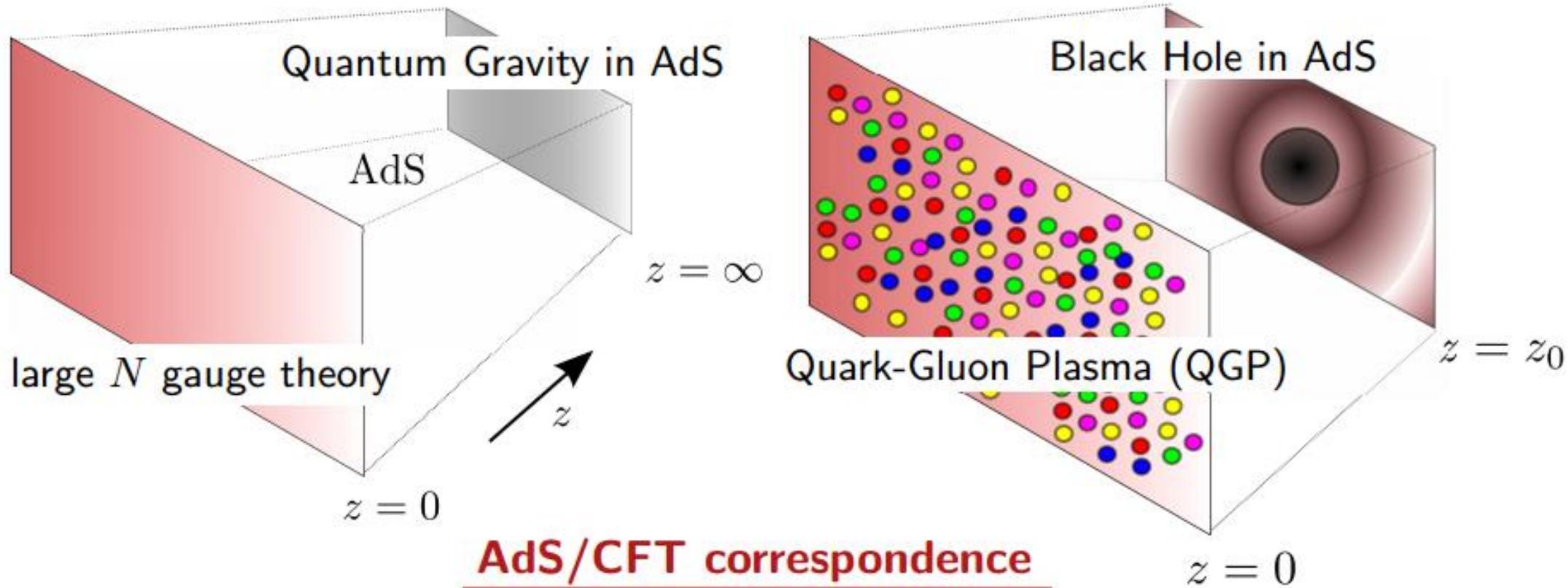
AdS / CFT in a Picture



A hard (strongly-coupled) gauge theory calculation is **dual** to an easy semi-classical gravitational calculation.

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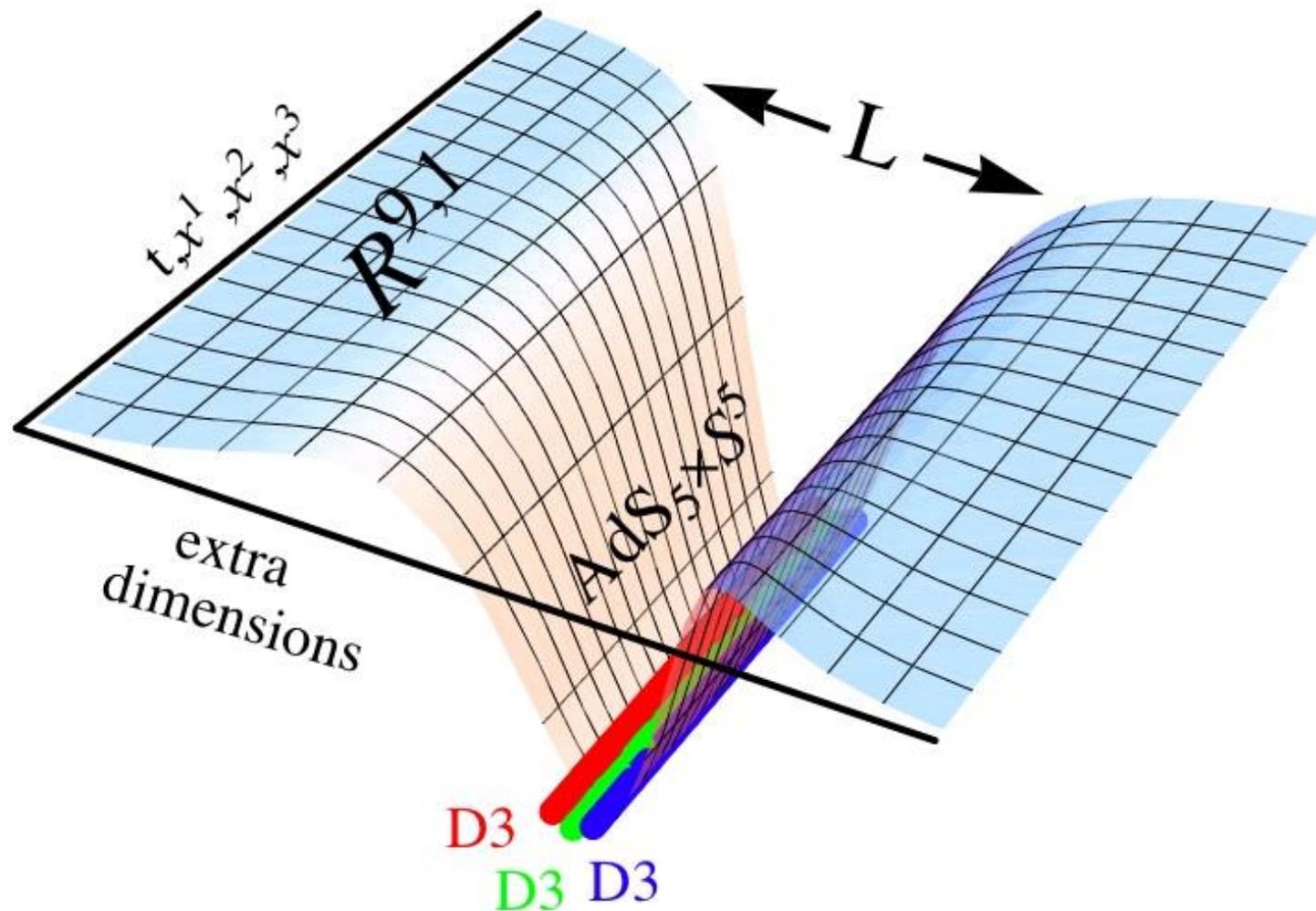
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AdS / CFT in a Picture



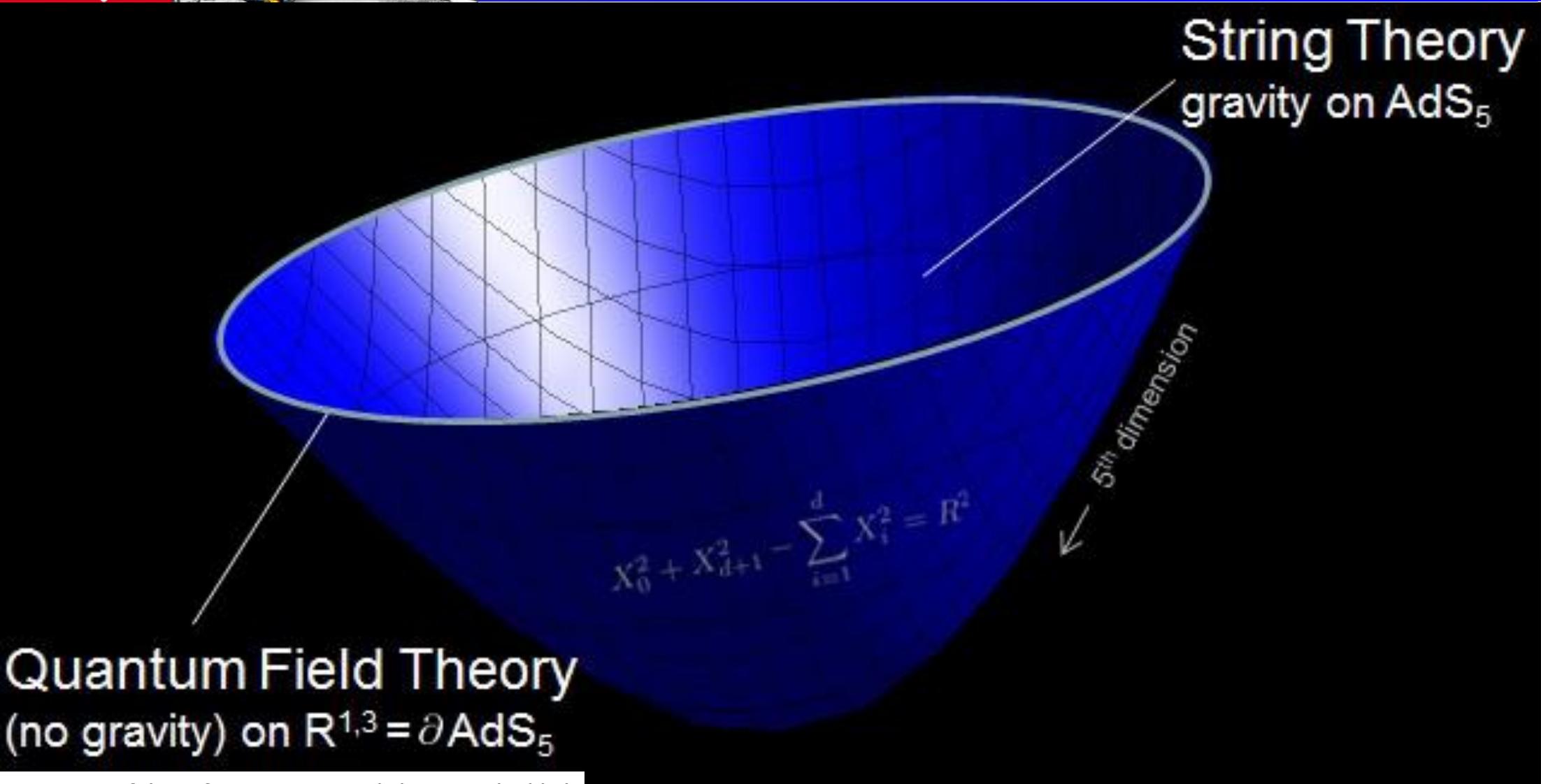
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strong coupling

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AdS / CFT in a Picture

Student Programme

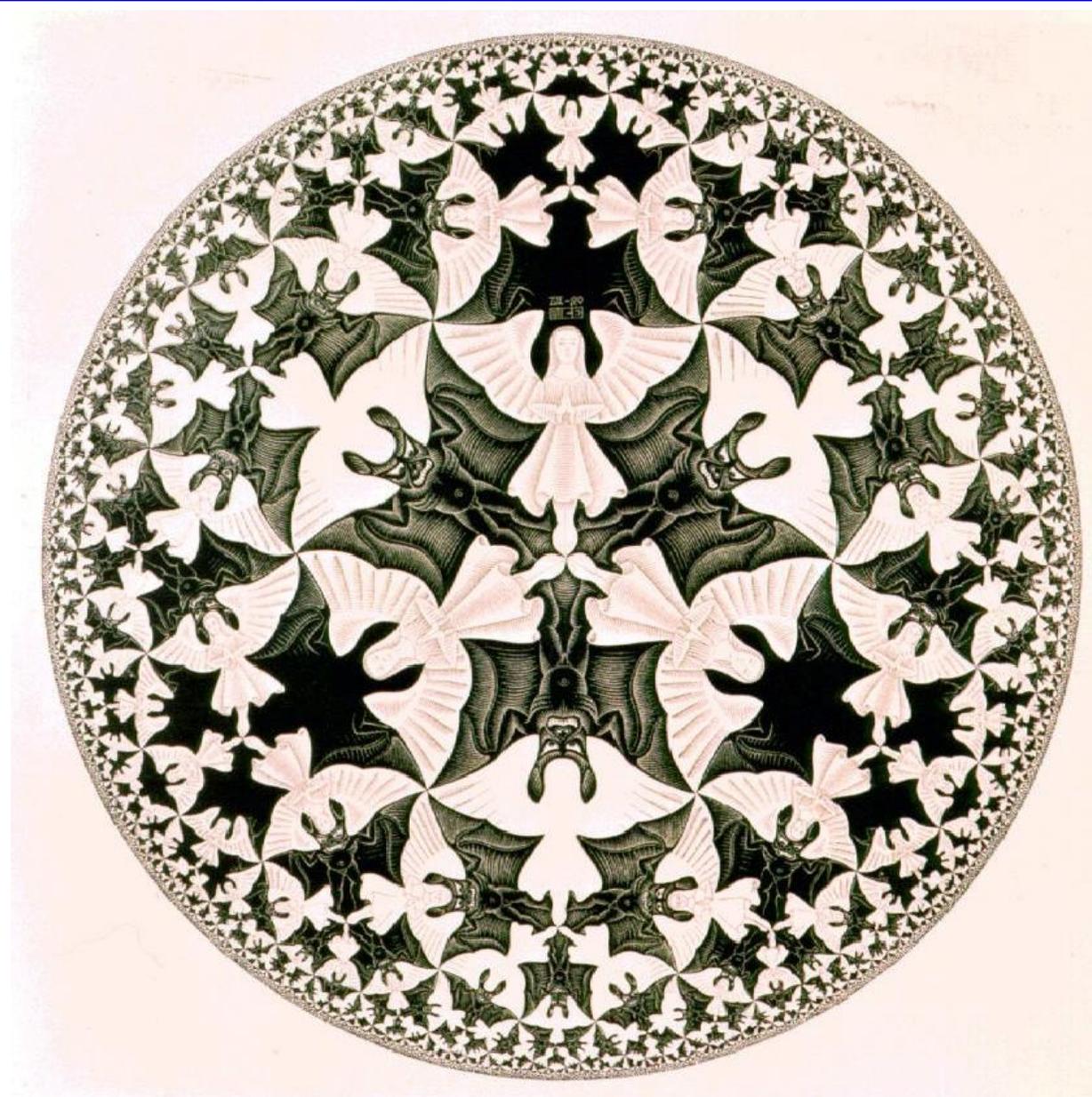


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AdS / CFT in a Picture

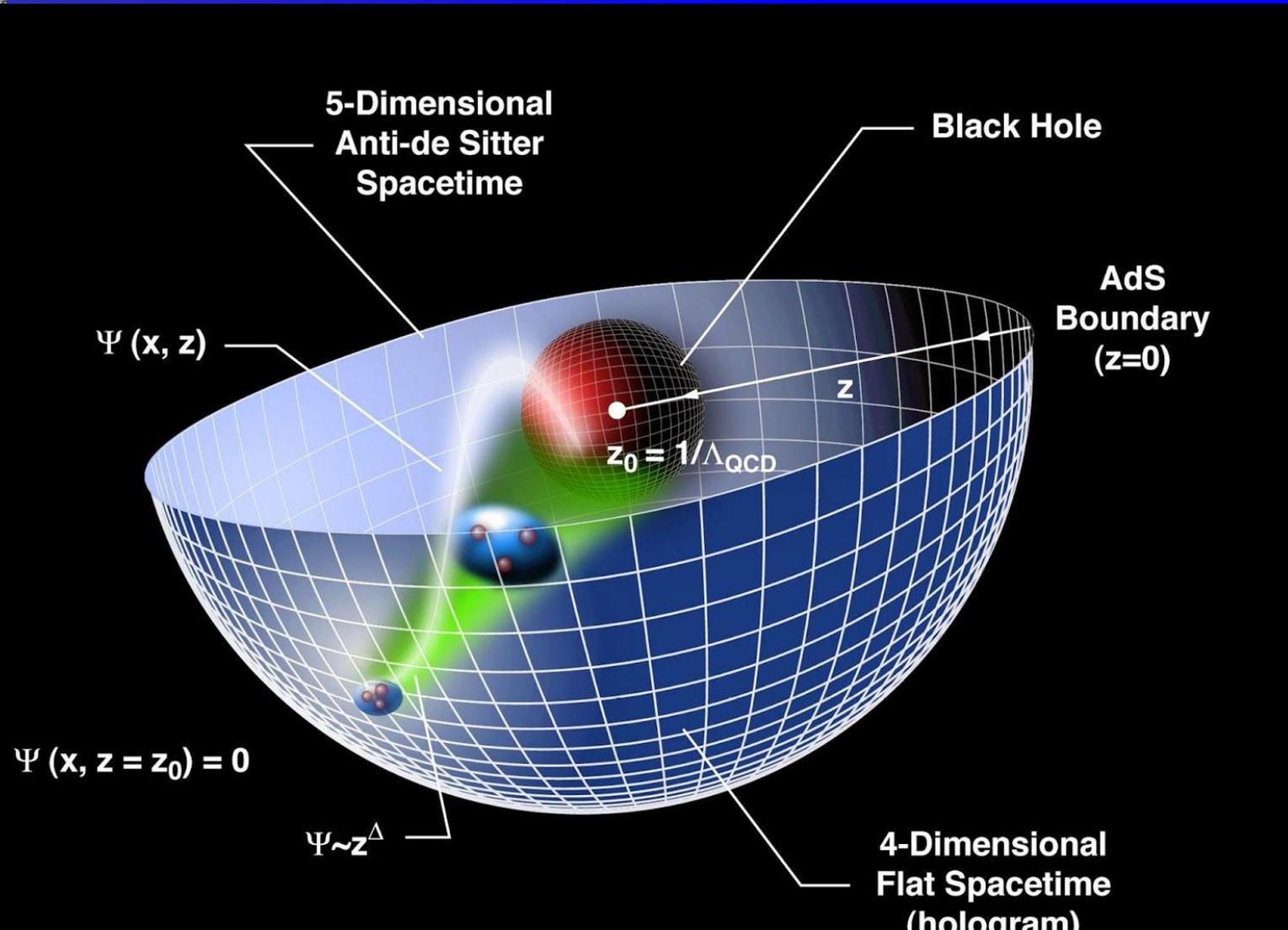


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AdS / CFT in a Picture

Student Programme



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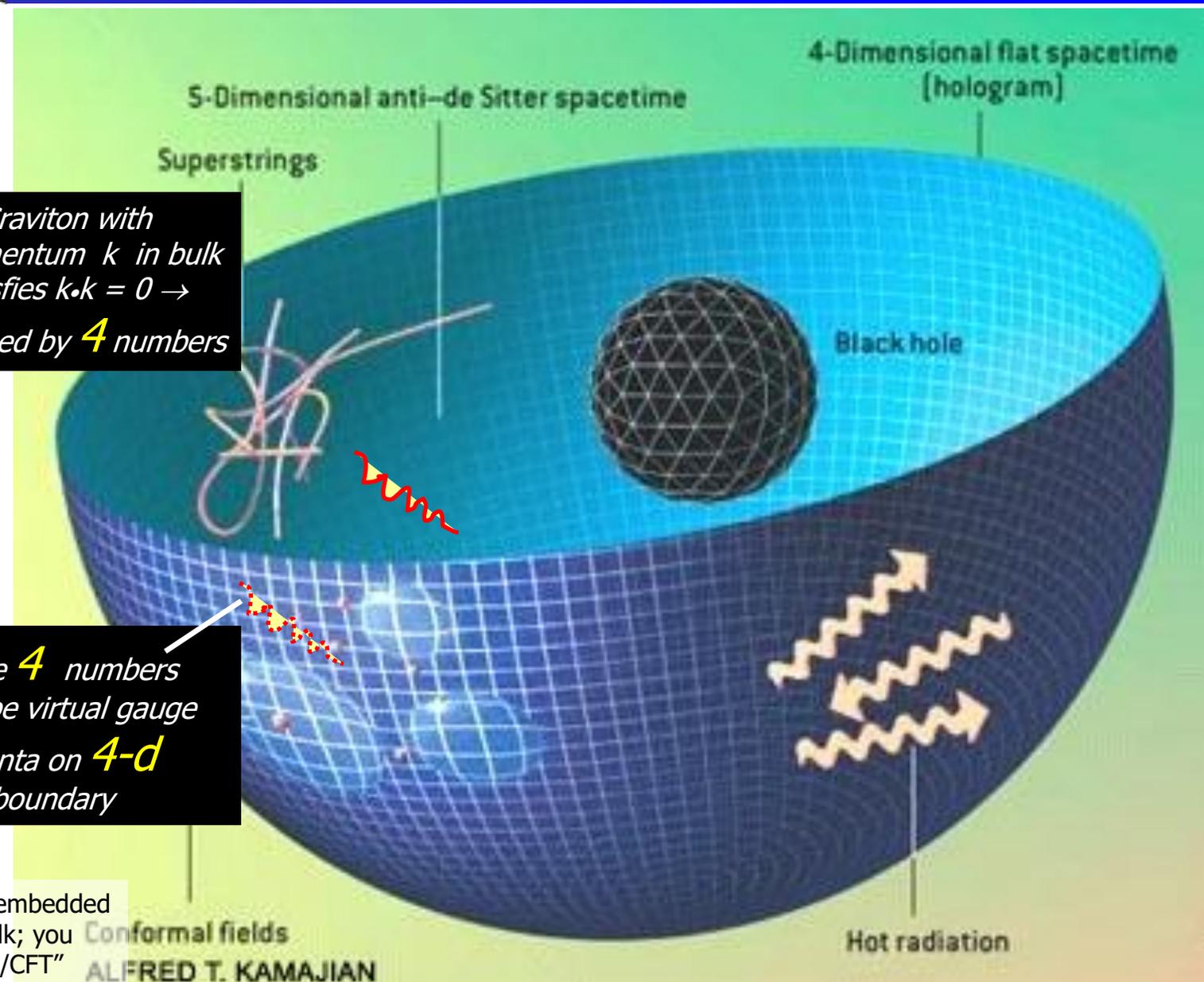
AdS / CFT in a Picture

Student Programme



Graviton with 5-momentum k in bulk satisfies $k \cdot k = 0 \rightarrow$ described by **4** numbers

Those **4** numbers describe virtual gauge quanta on **4-d** boundary



☞ A hard (strongly-coupled) gauge theory calculation is **dual** to an easy semi-classical gravitational calculation.

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ALFRED T. KAMAJIAN



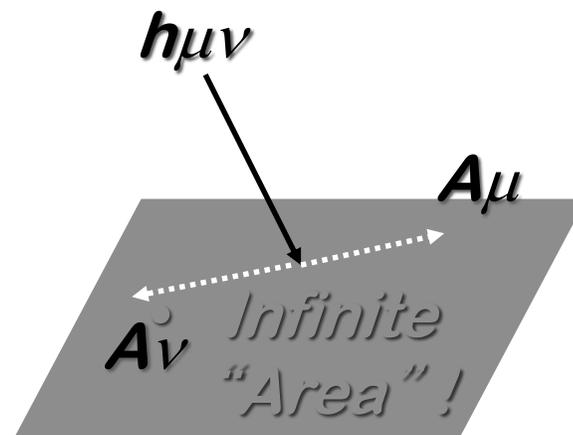
The (Assumed) Connection

- **Exploit** Maldacena's "D-dimensional strongly coupled gauge theory \Leftrightarrow (D+1)-dimensional stringy gravity"
- **Thermalize** with massive black brane
- **Calculate** viscosity $\eta = \text{"Area"}/16\pi G$

- **Normalize** by entropy (density) $s = \text{"Area"} / 4G$

- **Dividing out** the infinite "areas" :

$$\frac{\eta}{s} = \left(\frac{\hbar}{k}\right) \frac{1}{4\pi}$$



- **Conjectured** to be a lower bound "for all relativistic quantum field theories at finite temperature and zero chemical potential".
- **See** "Viscosity in strongly interacting quantum field theories from black hole physics", P. Kovtun, D.T. Son, A.O. Starinets, Phys.Rev.Lett.94:111601, 2005, [hep-th/0405231](https://arxiv.org/abs/hep-th/0405231)

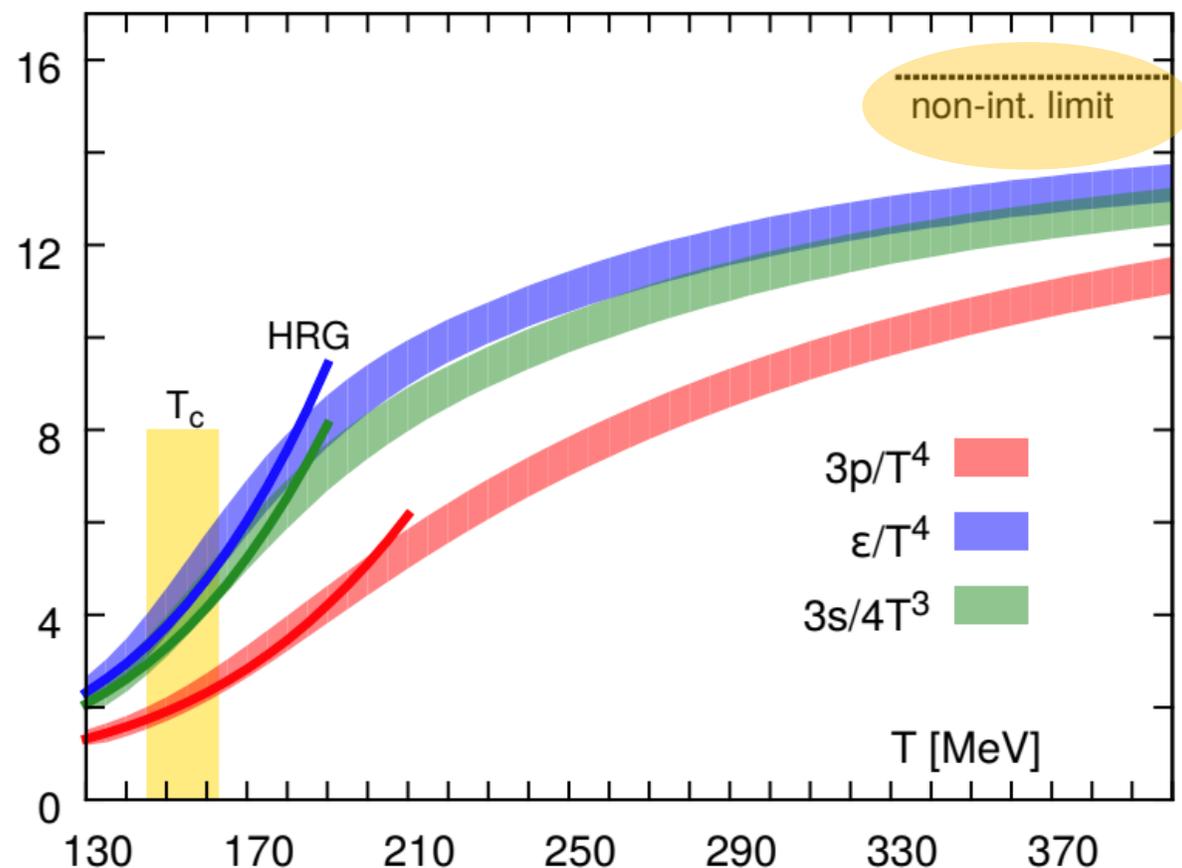
Implications

- The bulk behavior of “strongly-coupled QGP” (sQGP) depends only on $\alpha_s(T)$ and \hbar .
 - ▶ A true “quantum liquid”.
- The observed very low value of $\eta/s \sim 0.1-0.2 \Rightarrow$ no quasiparticles!
 - ▶ Urs Wiedemann, Quark Matter 2012:
“This plasma is unique in that it does not carry quasi-particle excitations”
 - ▶ National Academy Decadal Survey:
“Does the QGP have a particulate description at any length scale?”



Another Lesson From AdS/CFT

- The "approach" observed to the non-interacting limit may be misleading.
- The Super Yang-Mills plasma on the boundary in AdS/CFT is in some sense has infinitely strong coupling
- But...

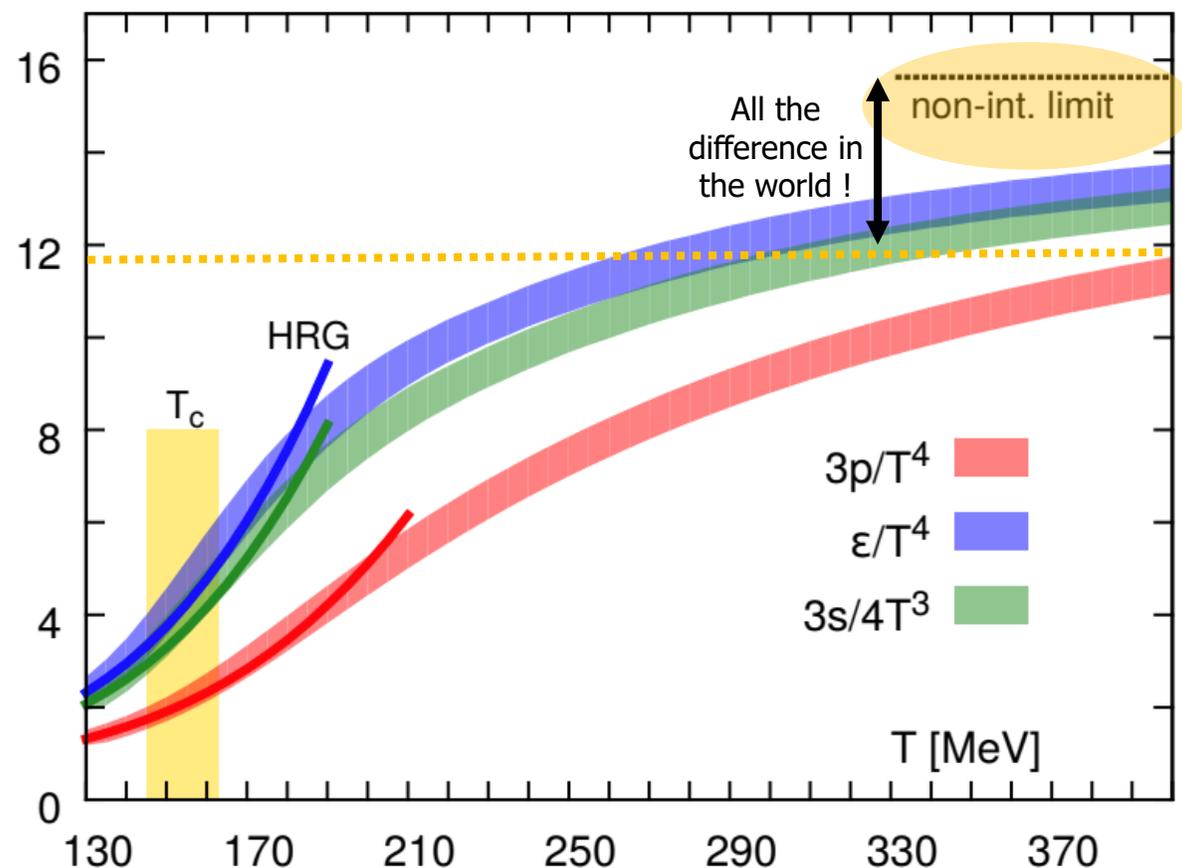




Another Lesson From AdS/CFT

- The "approach" observed to the non-interacting limit may be misleading.
- The Super Yang-Mills plasma on the boundary in AdS/CFT is in some sense has infinitely strong coupling
- But

$$s(g = \infty) = \frac{3}{4} s(g = 0) \quad (!)$$





Asymptotic Freedom Redux

- The colloquial is not the physical:

- QCD potential
$$V(r) \sim \frac{a_s(r)}{r} \sim \frac{a_0}{\log(rL)r}$$

- But: QCD *number density*
$$n \sim T^3 \sim \frac{1}{\langle r \rangle^3} \Rightarrow \frac{1}{\langle r \rangle} \sim T$$

- So – in a *thermal* system, $\langle V(r) \rangle \sim \frac{a_0 T}{\log(T/L)}$
 the **slow** $\log(T/\Lambda)$ **decrease** in α_s
 is overwhelmed by the fast **increase** in $1/r \sim T$

→ asymptotic freedom is asymptotic indeed !

- The absence of asymptotic freedom known long enough to be forgotten several times:

- ▶ 1982: Gordon Baym, proceedings of Quark Matter '82:

- A hint of trouble can be seen from the first order result for the entropy density ($N_f = 3$)

which turns negative for $\alpha_s > 1.1$

$$s(T) = \frac{19\pi^2}{9} \left\{ 1 - \frac{54}{19\pi} \alpha_s(T) + \dots \right\} T^4$$

- ▶ 1992: Berndt Mueller, Proc. of NATO Advanced Study Institute

- For plasma conditions realistically obtainable in the nuclear collisions ($T \sim 250$ MeV, $g = \sqrt{4\pi\alpha_s} = 2$) the effective gluon mass $m_g^* \sim 300$ MeV. We must conclude, therefore, that the notion of almost free gluons (and quarks) in the high temperature phase of QCD is quite far from the truth. Certainly one has $m_g^* \ll T$ when $g \ll 1$, but this condition is never really satisfied in QCD, because $g \sim 1/2$ even at the Planck scale (10^{19} GeV), and $g < 1$ only at energies above 100 GeV.

- ▶ 2002: Ulrich Heinz, Proceedings of PANIC conference:

- Perturbative mechanisms seem unable to explain the phenomenologically required very short thermalization time scale, pointing to strong non-perturbative dynamics in the QGP even at or above $2T_c$ The quark-hadron phase transition is arguably the most strongly coupled regime of QCD.



Recent Developments

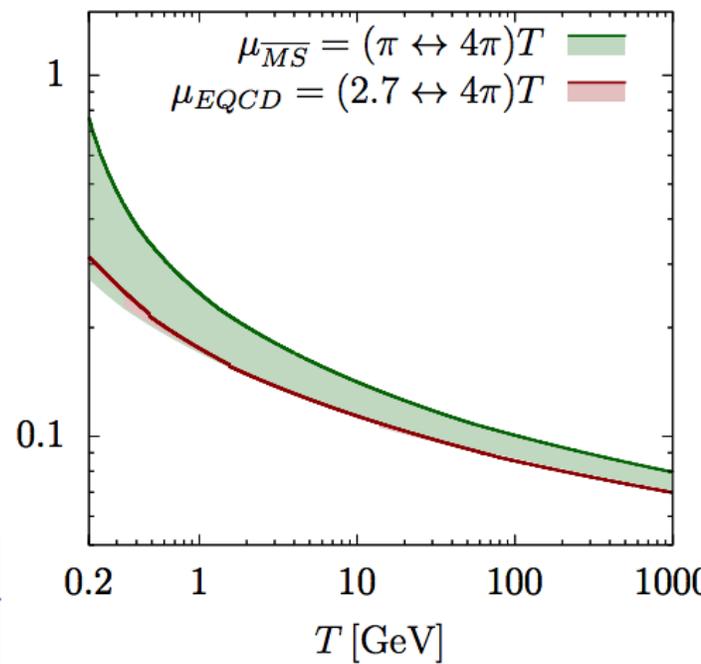
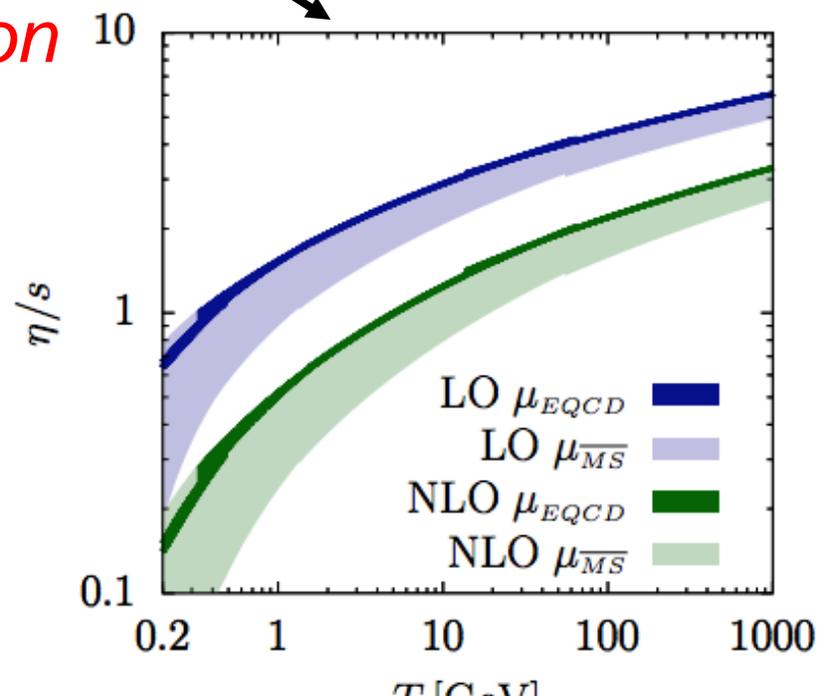
- New results on perturbative calculation of η/s :

- Investigate sensitivity to subtraction scheme, renormalization scale

- Find NLO corrections are *large*

- “*The perturbative expansion is problematic even for $T \sim 100$ GeV...*”

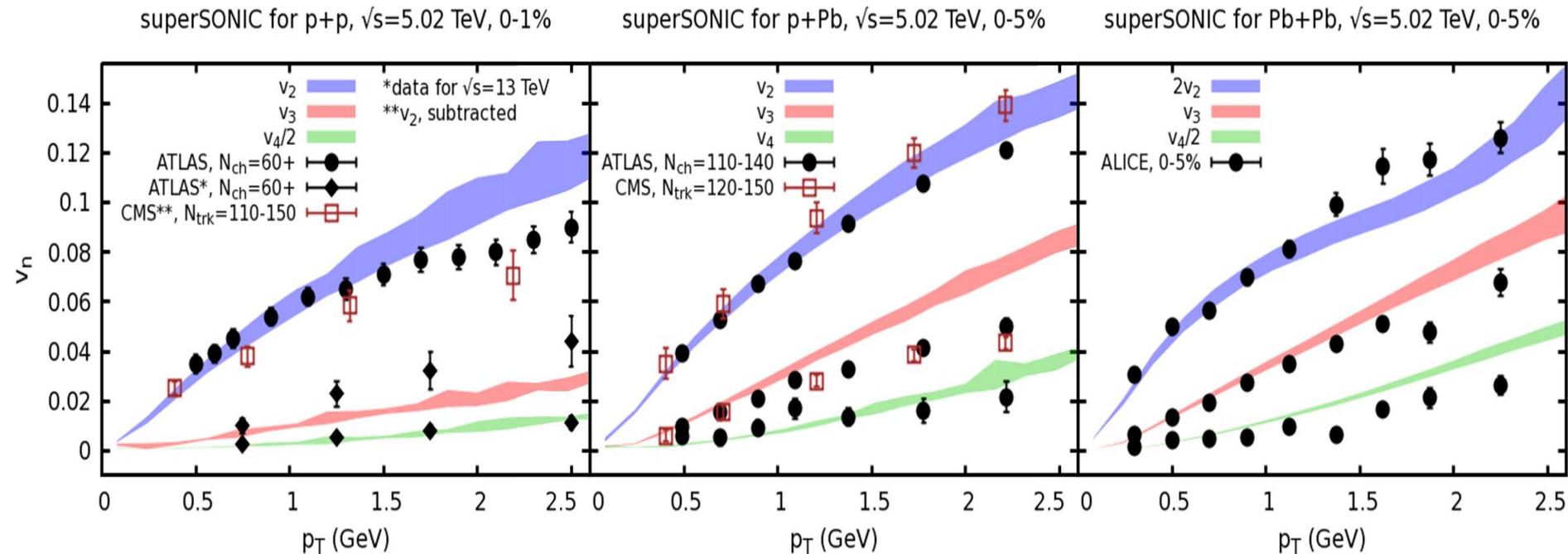
QCD Shear Viscosity at (Almost) NLO,
J. Ghiglieri, G. Moore and D. Teaney,
[arXiv:1802:09535](https://arxiv.org/abs/1802.09535)





Hydrodynamics in Small Systems

- *Single* hydro description provides “reasonable” *simultaneous* description of $v_2(p_T)$, $v_3(p_T)$ and $v_4(p_T)$ in p+p, p+Pb, and Pb+Pb:



R.D. Weller and P. Romatschke,
[arXiv:1701:07145](https://arxiv.org/abs/1701.07145)

- “One fluid to rule them all...”
- N.B.: *Geometry initialized at constituent quark level*

Recent review of collective behavior in small systems: J.L. Nagle and WAZ, [arXiv:1801.03477](https://arxiv.org/abs/1801.03477)

Recent Developments



- Hydrodynamic behavior $\not\Rightarrow$ thermalization
 - ▶ That is, applicability of hydrodynamics does not *require* thermalization
 - and therefore
 - ▶ Should not be assumed to be *evidence* for thermalization

P. Romatschke, [arXiv:1609.02820](https://arxiv.org/abs/1609.02820)

Reviews:

W. Florkowski, M.P. Heller, and M. Spalinski, [arXiv:1707.02282](https://arxiv.org/abs/1707.02282)

P. Romatschke and U. Romatschke, [arXiv:1712.05815](https://arxiv.org/abs/1712.05815)



Recent Developments

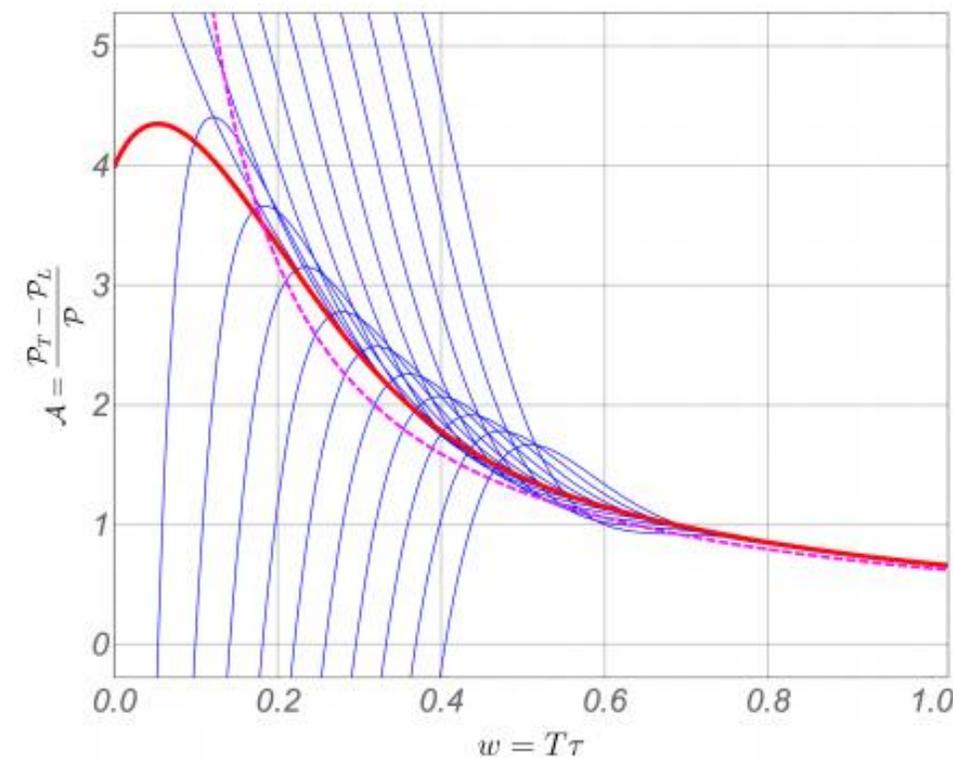
- Hydrodynamic behavior $\not\Rightarrow$ thermalization
 - ▶ That is, applicability of hydrodynamics does not *require* thermalization
 - and therefore
 - ▶ Should not be assumed to be *evidence* for thermalization
- Instead – strongly-coupled systems exhibit *relaxation to universal attractor* for $\tau > 0.7 / T$

P. Romatschke, [arXiv:1609.02820](https://arxiv.org/abs/1609.02820)

Reviews:

W. Florkowski, M.P. Heller, and M. Spalinski, [arXiv:1707.02282](https://arxiv.org/abs/1707.02282)

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Open Questions

- How does non-zero baryon chemical potential affect the fluid properties of QGP?
- Are there experimentally accessible regimes where the plasma become weakly coupled?
- Can we use jets to probe the microscopic structure of QGP?
- How can we understand the success of parton transport models that seemingly violate quantum mechanical limits yet reproduce flow-like features in small systems?
- Are there experimental observables sensitive to the nonhydro modes? Can they be used to determine the associated relaxation parameters?
- What is the smallest drop of QGP describable by relativistic viscous hydrodynamics?

- Good News...



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- Good News: *There is much work to be done!*



Student Programme

Thank You!



Venezia
Quark Matter

Student Programme

Back-up



A Question

- Why does the system select the *higher* pressure ?
- After all, systems tend to ‘select’ the lowest energy...
- Possible answers:
 - ▶ To get the right answer
 - ▶ Higher pressure pushes harder on lower pressure...
 - ▶ It's more chaotic
 - ▶ **2nd Law of Thermodynamics**
 - Basic result in phase equilibria at fixed volume and temperature
 - $dS = - d\Phi/T$, $\Phi = \text{Grand Potential} = - P V$
 - Details provided in back-up slides

Statistical Mechanics I



- Density of states: $dN = \frac{d^3 r d^3 p}{h^3}$
 - ▶ (Incredibly ubiquitous and useful)

- Boson occupation factor:

$$f_B(\mathbf{p}) = \frac{1}{e^{(E(\mathbf{p})-\mu)/T} - 1}$$

- Fermion occupation factor:

$$f_F(\mathbf{p}) = \frac{1}{e^{(E(\mathbf{p})-\mu)/T} + 1}$$

- Then

$$N = \int \frac{1}{e^{(E(\mathbf{p})-\mu)/T} \pm 1} \frac{d^3 r d^3 p}{h^3} = \frac{V}{h^3} \int \frac{1}{e^{(E(\mathbf{p})-\mu)/T} \pm 1} \frac{d^3 p}{h^3}$$

$$U = \int \frac{E(\mathbf{p})}{e^{(E(\mathbf{p})-\mu)/T} \pm 1} \frac{d^3 r d^3 p}{h^3} = \frac{V}{h^3} \int \frac{E(\mathbf{p})}{e^{(E(\mathbf{p})-\mu)/T} \pm 1} \frac{d^3 p}{h^3}$$



Statistical Mechanics II

- Huge simplification for **(non-interacting) massless** quanta at **zero** chemical potential μ .

- Mathematics: $\int \frac{s^a ds}{e^s - 1} = \Gamma(a+1)\zeta(a+1)$

$$\int \frac{s^a ds}{e^s + 1} = \left(1 - \frac{1}{2^a}\right)\Gamma(a+1)\zeta(a+1)$$

Exercise: "Prove" this

Exercise: "Prove" this

- Physics: $n_B(T) \equiv \frac{N_B}{V} = \frac{\xi(3)}{\pi^2} T^3$, $n_F(T) = \frac{3}{4} n_B(T)$

$$\varepsilon_B(T) \equiv \frac{U_B}{V} = \frac{3\xi(4)}{\pi^2} T^4$$
 , $\varepsilon_F(T) = \frac{7}{8} \varepsilon_B(T)$

Exercise: Derive these expressions using relations above.

- Mathematics: $\xi(2) = \frac{\pi^2}{6}$, $\xi(4) = \frac{\pi^4}{90}$

Exercise: "Prove" this. (Either you know the tricks or this will be very challenging...)



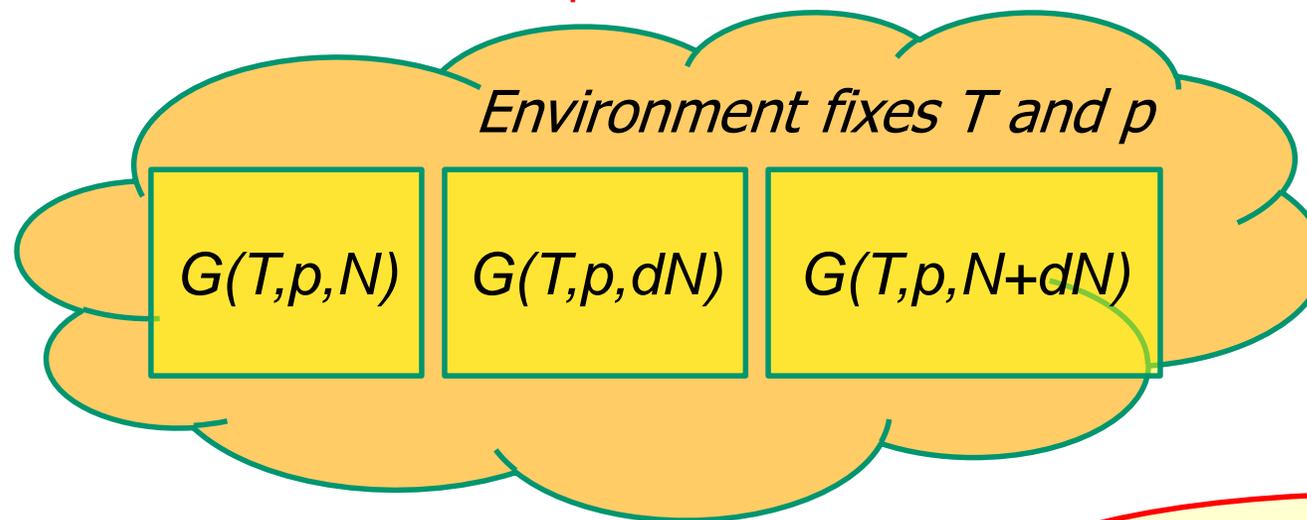
Thermodynamics I

- $U \equiv$ Internal energy of a system
 - ▶ $dU = dQ + dW$ (1st Law, energy conservation)
 - ▶ $dU = T dS - p dV + \mu dN \Rightarrow U(S, V, N)$
- Enthalpy $H \equiv U + pV$
 - ▶ $dH = T dS + V dp + \mu dN \Rightarrow H(S, p, N)$
- Free Energy $F \equiv U - TS$
 - ▶ $dF = -S dT - p dV + \mu dN \Rightarrow F(S, T, N)$
- Gibbs Free Energy $G \equiv F + pV$
 - ▶ $dG = -S dT + V dp + \mu dN \Rightarrow G(T, p, N)$
- Grand Potential $\Phi \equiv F - \mu N$
 - ▶ $d\Phi = -S dT + V dp - N d\mu \Rightarrow \Phi(T, p, \mu)$

Always remember this form, derive the rest

Thermodynamics II

- Hiding in the Legendre Transformation formalism is some very useful physics:
- Gibbs Free Energy $G \equiv F + pV = U - TS + pV$
 - ▶ $dG = -S dT + V dp + \mu dN \Rightarrow G(T,p,N)$
 - ▶ So $\mu = (\partial G/\partial N)_{T,p}$, which implies $G \sim N$



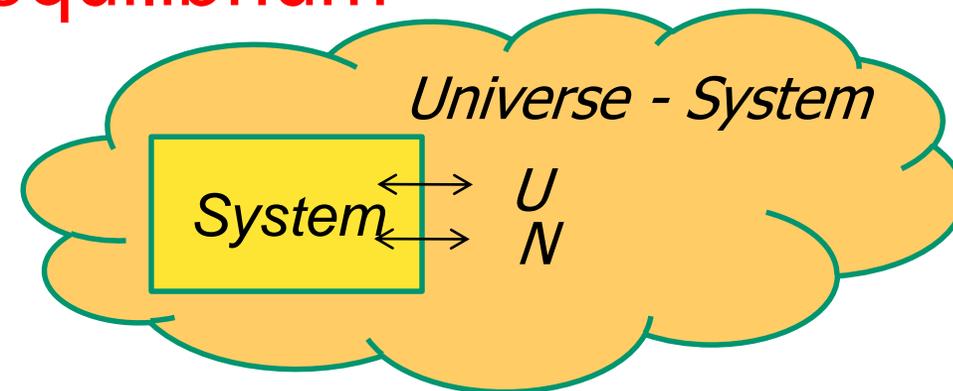
- So $G = \mu N = U - TS - pV \Rightarrow U = TS - pV + \mu N$

Thermodynamics III



- Now use $U = TS - pV + \mu N$ together with definition of **grand potential** $\Phi \equiv F - \mu N$:
- $\Phi = \Phi (T, p, \mu) \equiv F - \mu N = U - TS - \mu N = -pV$
- **Consider system at fixed (T, p, μ) in equilibrium with rest of universe:**

- $dS_{\text{TOT}} = dS_S + dS_{U-S}$



- $T dS_{U-S} = dU_{U-S} + p dV_{U-S} - \mu dN_{U-S}$
 $= dU_{U-S} + \quad \quad \quad - \mu dN_{U-S}$ (Since V_{U-S} fixed)

- Then $dS_{\text{TOT}} = \frac{dS_S + (dU_{U-S} - \mu dN_{U-S})}{T}$
 $= \frac{(T dS_S - dU_S + \mu dN_S)}{T} = -d\Phi/T$

Exercise: Work through this argument



Alternative History

- So the “perfect fluid” observed at RHIC with

$$\left(\frac{\eta}{s} \right)_{RHIC} \sim 0.1 \hbar$$

was immediately recognized as confirming the 1985 uncertainty principle estimate of Danielewicz and Gyulassy

- Except that's not what happened...



Instead ...

- In 2003-4 a new ~~estimate~~ (bound?) appeared from the AdS/CFT correspondence in string theory (!):

▶ *A Viscosity Bound Conjecture*,
 P. Kovtun, D.T. Son, A.O. Starinets,
hep-th/0405231

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi} \sim 0.08\hbar$$

in a rigorous calculation with no (apparent) appeal to the uncertainty principle.



Theoretical Discovery 2003-4

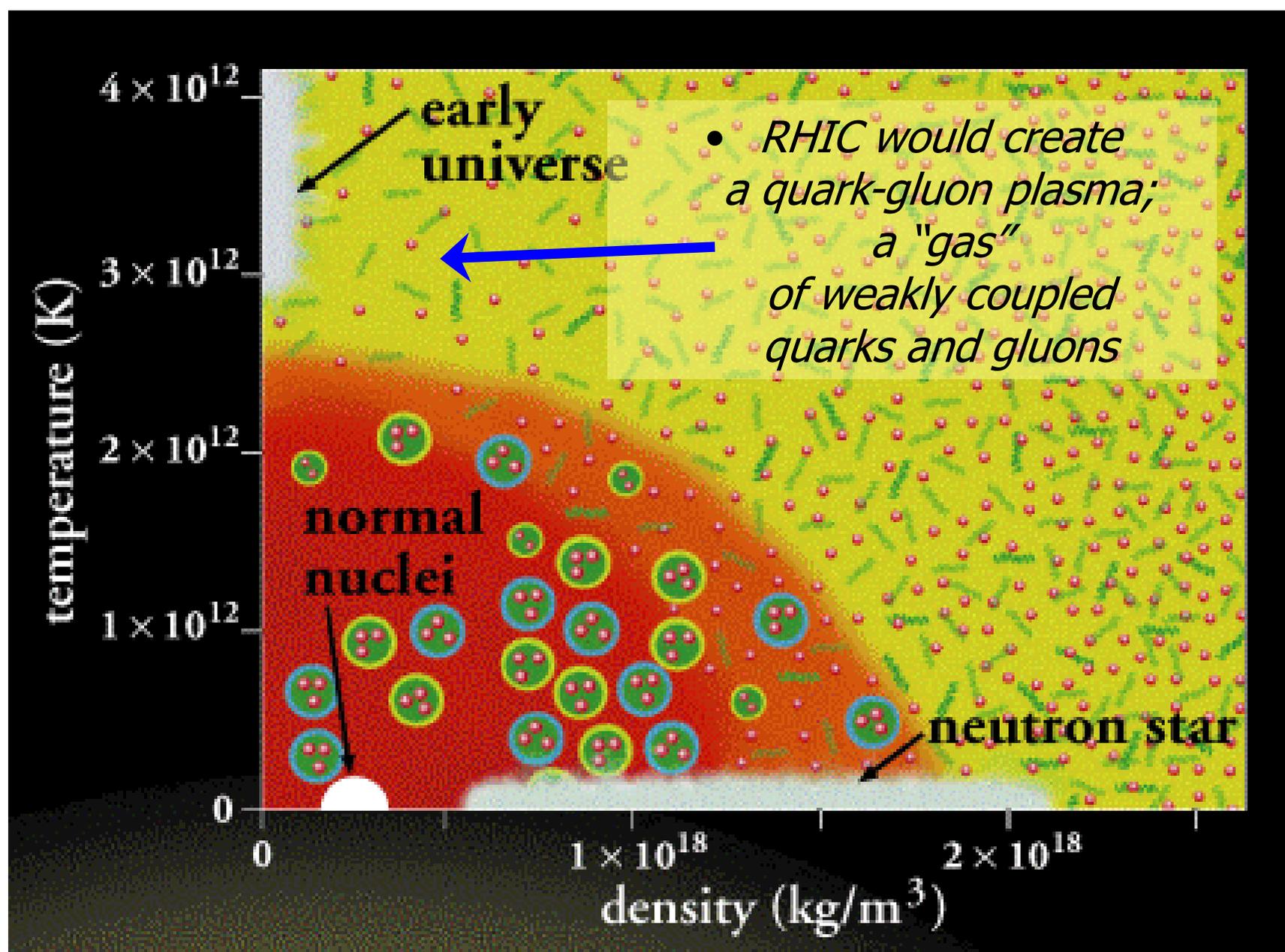
- An estimate (bound?) on viscosity appeared from string theory's AdS/CFT correspondence:
 - ▶ *A Viscosity Bound Conjecture*,
P. Kovtun, D.T. Son, A.O. Starinets,
[hep-th/0405231](https://arxiv.org/abs/hep-th/0405231) (1300+ citations!)

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi} \sim 0.08\hbar$$

- ⇒ *Fundamental* measure of strong coupling
- ⇒ Cleanest result from gauge/gravity duality
- ⇒ A measure of “quantum liquidity”

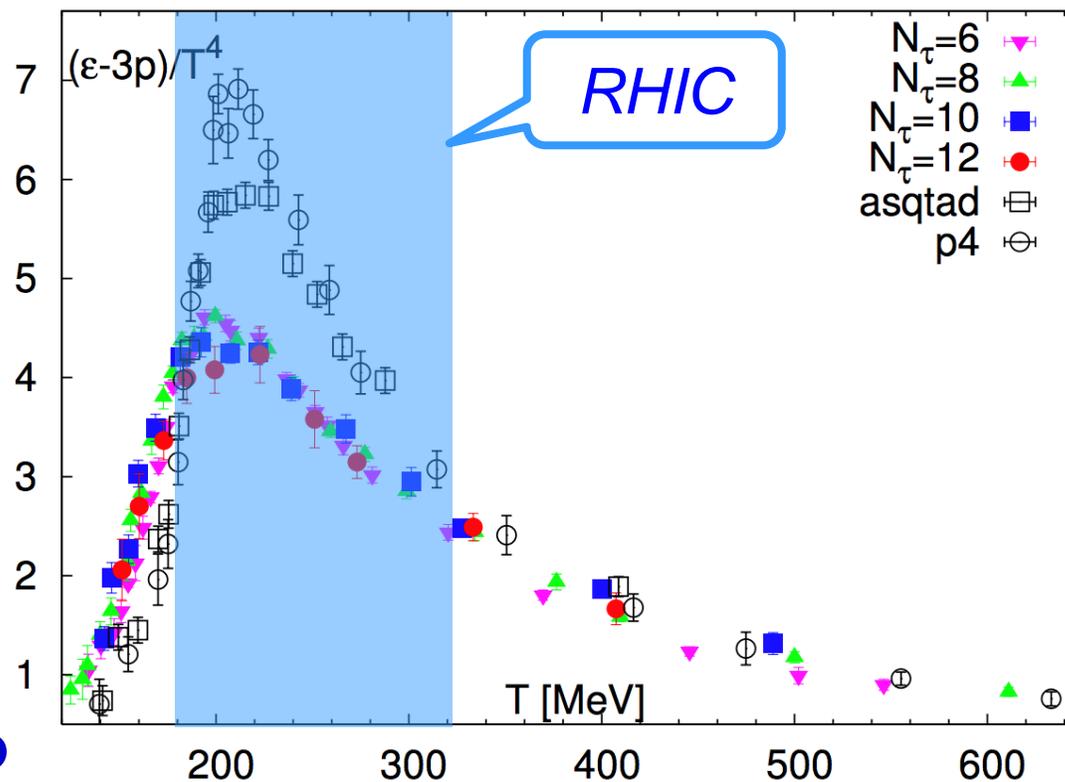
Expectations circa 2000

Student Programme



The Key Technique

- Working at the *strongest possible coupling*
- Tune temperature to produce ~"most liquid" QGP
- Cross sections as large as allowed by unitarity (?)



- *The Equation of State in 2+1 QCD*, A. Bazavov *et al.*, Phys. Rev. **D90** 094503 (2014), [arXiv:1407.6387](https://arxiv.org/abs/1407.6387)