Dissipative effects in ultrarelativistic kinetic theory

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Abstract

The propagation of a harmonic longitudinal wave through a medium of ultrarelativistic particles is considered in the frame of the first- and second-order relativistic hydrodynamics theories. The hydrodynamic equations are solved analytically in the linearised regime. A comparison with the numerical (lattice Boltzmann) solution of the relativistic Anderson-Witting-Boltzmann (AWB) equation validates the expressions for the transport coefficients obtained via the Chapman-Enskog expansion. Some limitations of the first order theory which persist at small relaxation times are discussed. [V. E. Ambruș, Phys. Rev. C 97 (2018) 024914]

Conventions

- Metric signature: $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$.
- Nondimensionalisation with respect to the background state, such that $c = n_0 = P_0 = L = 1$.

Linearised hydrodynamics

- Wave propagates along $z \Rightarrow u^\mu = \gamma(1, 0, 0, \beta)^T$,

$$T^{\mu\nu} = E u^\mu u^\nu + P\Delta^{\mu\nu} + q^\mu u^\nu + u^\mu q^\nu + \Pi^{\mu\nu},$$

where $E = 3P$, $\Delta^{\mu\nu} = \eta^{\mu\nu} + u^\mu u^\nu$ and

$$q^\mu = q \begin{pmatrix} \beta \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad \Pi^{\mu\nu} = \Pi \begin{pmatrix} \beta^2\gamma^2 & 0 & 0 & \beta\gamma^2 \\ 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 \\ \beta\gamma^2 & 0 & 0 & \gamma^2 \end{pmatrix}.$$

- For monocromatic waves ($k = 2\pi/L$) with small amplitudes, a mode decomposition can be performed:

$$\beta = \widetilde{\beta} \sin kz, \qquad \widetilde{\beta} = \sum_\alpha e^{-\alpha t} \beta_\alpha,$$

$$\delta n = \widetilde{\delta n} \cos kz, \qquad \widetilde{\delta n} = \sum_\alpha e^{-\alpha t} \delta n_\alpha, \dots$$

- The conservation equations reduce to:

$$\alpha \delta n_\alpha - k\beta_\alpha = 0,$$
$$3\alpha\delta P_\alpha - 4k\beta_\alpha - kq_\alpha = 0,$$
$$4\alpha\beta_\alpha + \alpha q_\alpha + k\delta P_\alpha + k\Pi_\alpha = 0.$$

- The system is closed by supplying constitutive eqs. for q and Π:

$$\tau_q \partial_t q + q = -\frac{\lambda}{4}(3\partial_z \delta P - 4\partial_z \delta n),$$

$$\tau_\Pi \partial_t \Pi + \Pi = -\frac{\eta}{3}\partial_z (4\beta + q),$$

where the 2nd order hydrodynamics (H2) are not present in the 1st order hydrodynamics (H1) formulation.

- The Chapman-Enskog and Grad methods predict:

$$\eta_{\rm C-E} = \frac{4}{5} P\tau, \qquad \lambda_{\rm C-E} = \frac{4}{3} n\tau,$$

$$\eta_{\rm Grad} = \frac{2}{3} P\tau, \qquad \lambda_{\rm Grad} = \frac{4}{5} n\tau,$$

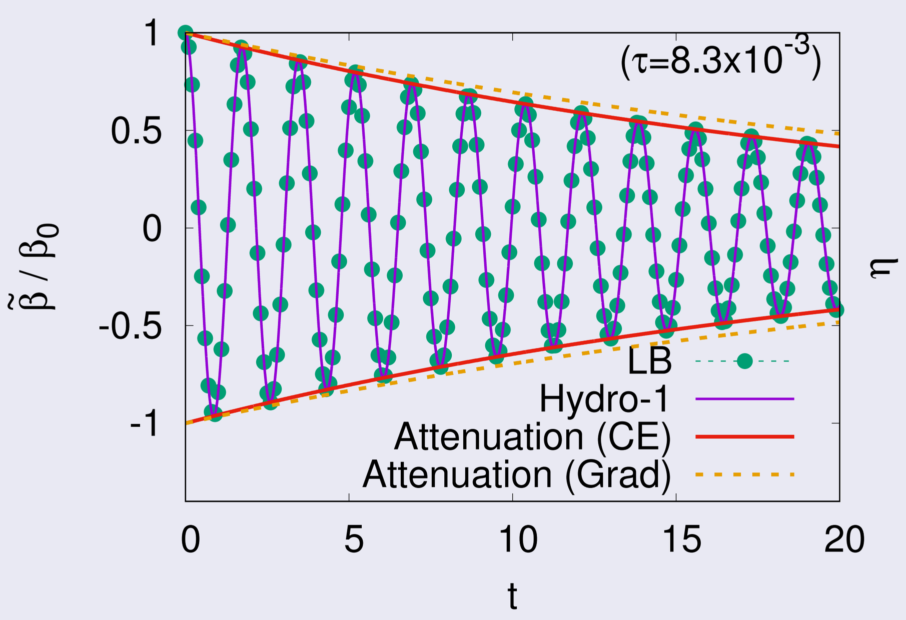
while $\tau_q = \tau_\Pi = \tau$.

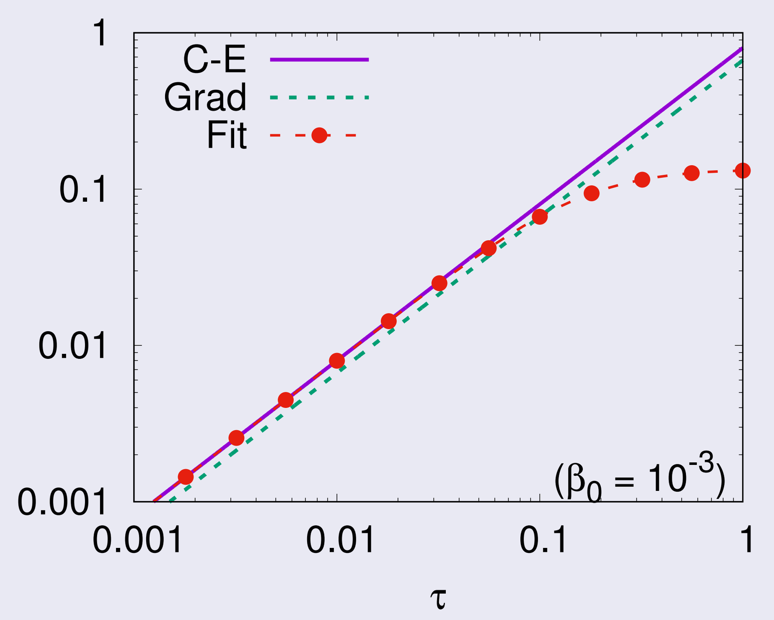
Shear viscosity

- For an adiabatic flow ($\delta n_0 = \delta P_0 = 0$):

$$\widetilde{\beta}_{\rm H2} \simeq \widetilde{\beta}_{\rm H1} = \beta_0 \left(\cos \alpha_o t - \frac{\alpha_d}{\alpha_o} \sin \alpha_o t\right) e^{-\alpha_d t},$$

where $\alpha_d = k^2\eta/6$ and $\alpha_o = \frac{k}{\sqrt{3}}\sqrt{1 - \frac{3\alpha_d^2}{k^2}}$.





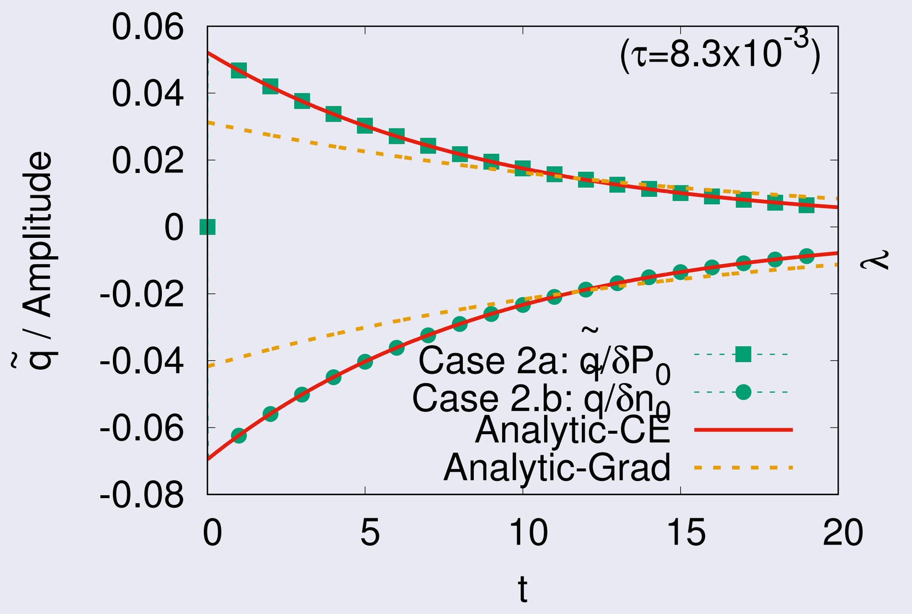
- η is obtained by fitting the numerical data to $\widetilde{\beta}$.

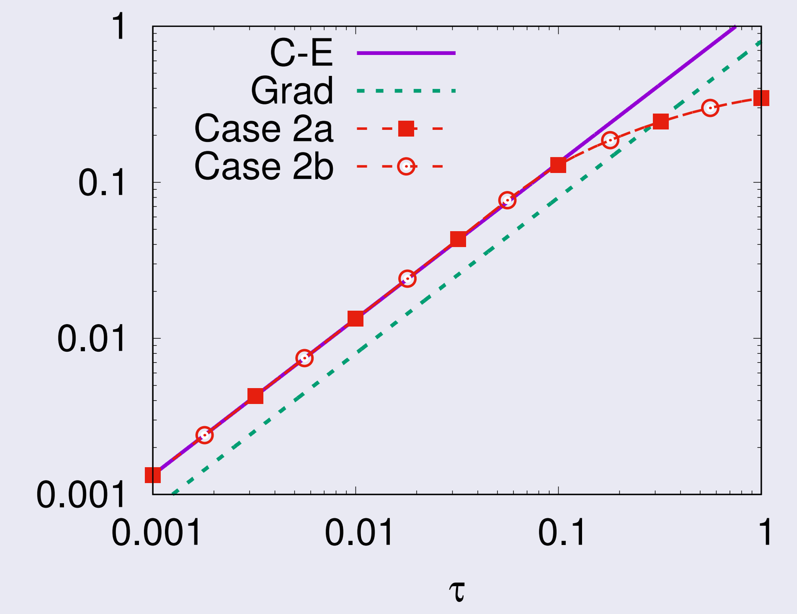
Heat conductivity

- For β_0 and $\delta n_0 = 0$ (Case 2a) or $\delta P_0 = 0$ (Case 2b):

$$\widetilde{q}_{\rm H2} \simeq \widetilde{q}_{\rm H1} = \frac{\alpha_\lambda P_0}{k}\left(\frac{3\delta P_0}{P_0} - \frac{4\delta n_0}{n_0}\right) e^{-\alpha_\lambda t},$$

where $\alpha_\lambda = k^2\lambda/4$.





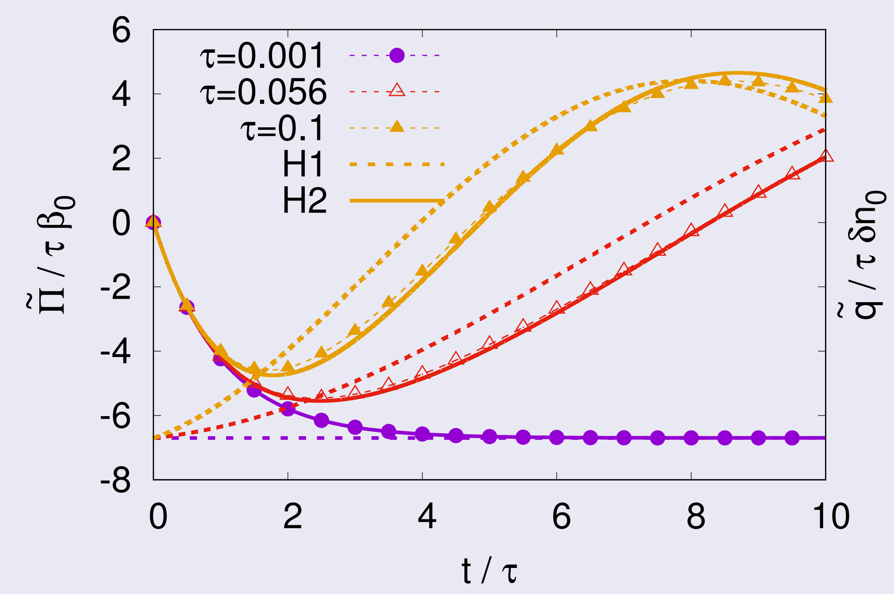
Early time relaxation

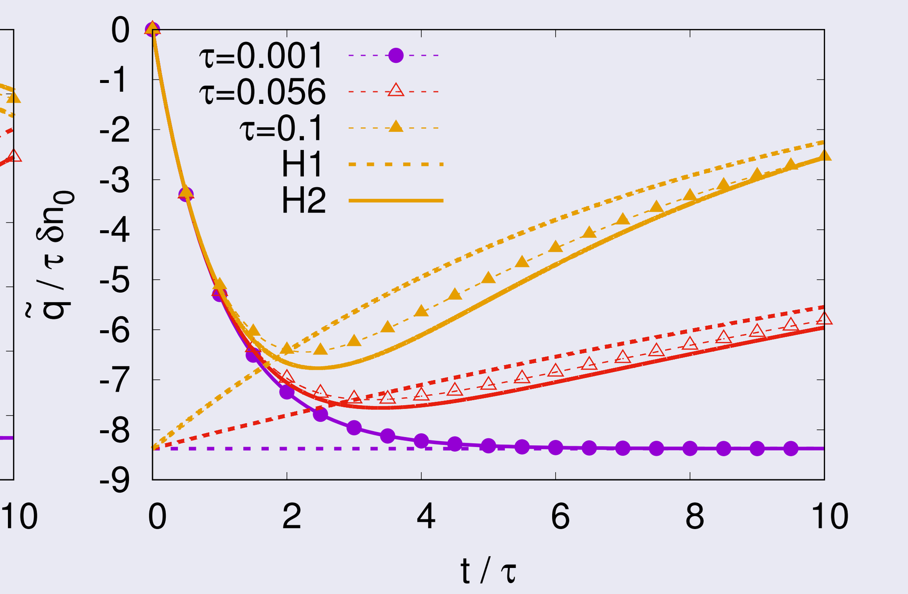
- For $\delta n_0 = \delta P_0 = 0$, $\widetilde{\Pi}$ is:

$$\widetilde{\Pi} \simeq -\frac{8\alpha_d\beta_0}{k}\left[e^{-\alpha_d t}\left(\cos \alpha_o t - \frac{\alpha_d}{\alpha_o}\sin \alpha_o t\right) - e^{-t/\tau_\Pi}\right].$$

- For $\delta P_0 = \beta_0 = 0$, $\widetilde{q}$ is:

$$\widetilde{q} \simeq -\frac{4\alpha_\lambda \delta n_0}{k}\left[e^{-\alpha_\lambda t} - e^{-t/\tau_q}\right].$$



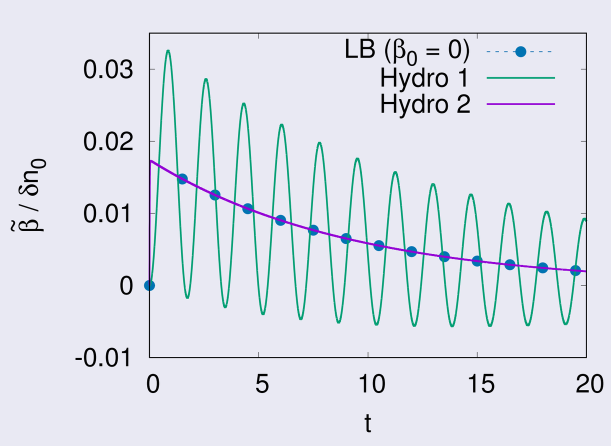


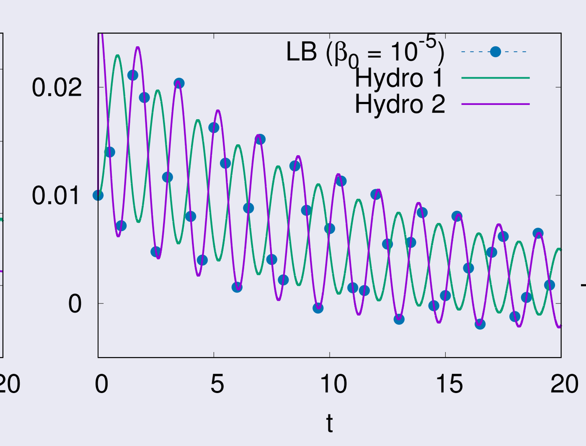
Limitation of the first order theory

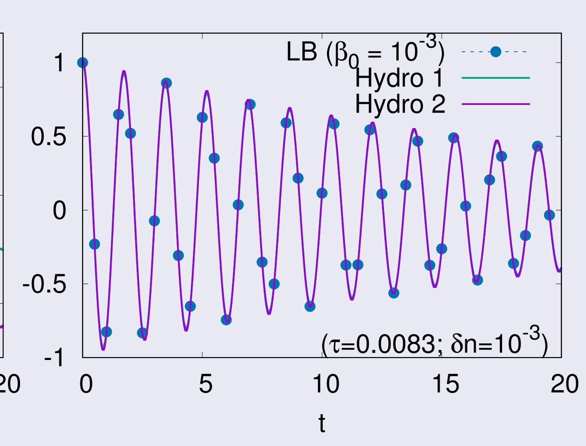
- When $\beta_0 = \delta P_0 = 0$, $\widetilde{\beta}$ in H1 does not approximate the H2 result:

$$\widetilde{\beta}_{\rm H1} = \frac{\alpha_\lambda \delta n_0}{k}\left[e^{-\alpha_\lambda t} - \left(\cos \alpha_d t - \frac{\alpha_d}{\alpha_o}\sin \alpha_d t\right) e^{-\alpha_d t}\right],$$

$$\widetilde{\beta}_{\rm H2} = \frac{\alpha_\lambda \delta n_0}{k}\left[e^{-\alpha_\lambda t} - e^{-t/\tau_q}\right].$$







- In H1, $\widetilde{\beta}$ oscillates with the same magnitude as the purely evanescent H2 sol.

Conclusion

- Comparing numerical (LB) and analytic results confirms the C-E values for η and λ for the AWB equation.
- H2 required to capture early time relaxation of $\widetilde{\Pi}$ and $\widetilde{q}$.
- Insufficient degrees of freedom in H1 formulation results in inaccurate solutions for β even at small τ.
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