Magnetohydrodynamics with chiral anomaly [1711.08450]

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Abstract

We study the relativistic hydrodynamics with chiral anomaly and dynamical electromagnetic fields, called the chiral magnetohydrodynamics (MHD), which plays an important role in quantifying anomaly-induced effects in heavy-ion collisions. We formulate the chiral MHD as a low-energy effective theory based on a derivative expansion. The modification of ordinary MHD due to chiral anomaly can be obtained from the 2nd law of thermodynamics and is tied to the chiral magnetic effect with the universal coefficient. When the chirality imbalance exceeds a critical value, a new type of collective gapless excitation emerges, as a result of the interplay among magnetic field, flow velocity, and chiral anomaly; we call it the chiral magnetohelical mode (CMHM). These modes carry definite magnetic and fluid helicities and either grow exponentially or dissipate in time, depending on the relative sign between their helicity and axial charge density. The presence of exponentially growing CMHM indicates a hydrodynamic instability.

Introduction

The chiral magnetic effect

$$\boldsymbol{j}_{\mathrm{CME}} = C_{\mathrm{A}} \mu_{\mathrm{A}} \boldsymbol{B}$$

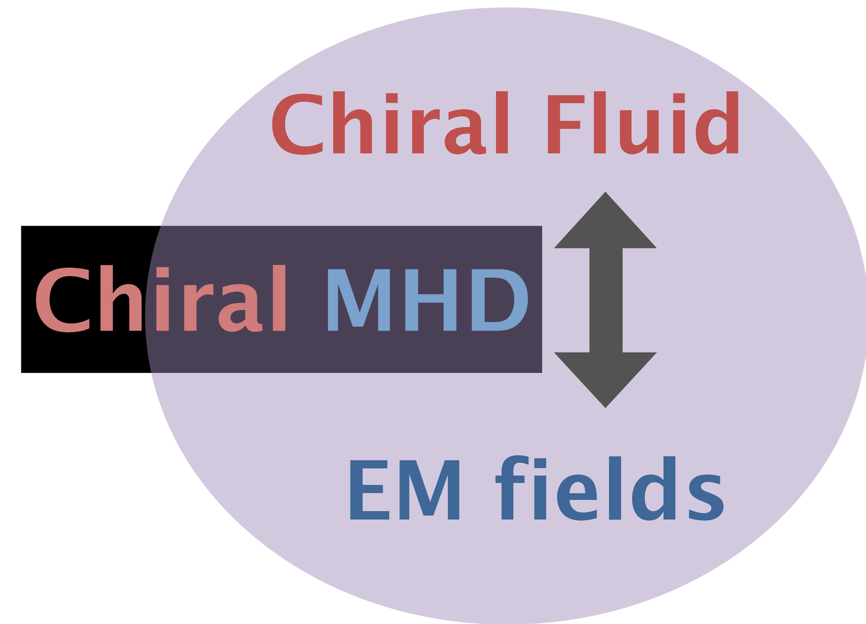
- Macroscopic transport
- Dissipationless (no heat production)
- Transport coefficient is **universal**

Where does it happen?

- Heavy-ion collisions/Early Universe
  Dirac/Weyl semimetals

What is chiral MHD?

Chiral MHD is a low-energy effective theory describing the coupled system of chiral fluid electromagnetic fields



MHD = Magnetohydrodynamics

We formulate the chiral MHD as a low-energy hydrodynamic effective theory based on a derivative expansion

Formulation of chiral MHD

Hydrodynamic (slow) variables

$$\{e(\boldsymbol{x}), u^\mu(\boldsymbol{x}), n_{\mathrm{A}}(\boldsymbol{x}), B^\mu(\boldsymbol{x})\}$$

$$E^\mu \equiv F^{\mu\nu} u_\nu, \quad B^\mu \equiv \tilde{F}^{\mu\nu} u_\nu$$

EOM of chiral MHD

$$\partial_\mu T^{\mu\nu}_{\mathrm{tot}} = 0 \qquad \partial_\mu \tilde{F}^{\mu\nu} = 0 \qquad \partial_\mu J^\mu_{\mathrm{A}} = -C_{\mathrm{A}} E_\mu B^\mu$$

$$T^{\mu\nu}_{\mathrm{tot}} = T^{\mu\nu}_{\mathrm{fluid}} + T^{\mu\nu}_{\mathrm{EM}} \qquad T^{\mu\nu}_{\mathrm{EM}} = -F^\mu_{\ \alpha} F^{\nu\alpha} + \frac{1}{4}\eta^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta}$$

Constitutive relations

$$T^{\mu\nu} = \epsilon\, u^\mu u^\nu - p\,(g^{\mu\nu} - u^\mu u^\nu) - H^\mu B^\nu + T^{\mu\nu}_{(1)}$$

$$\tilde{F}^{\mu\nu} = B^\mu u^\nu - B^\nu u^\mu + \tilde{F}^{\mu\nu}_{(1)}$$

$$J^\mu_{\mathrm{A}} = n_{\mathrm{A}}\, u^\mu + J^\mu_{\mathrm{A}(1)}$$

where the subscript (1) indicates the first-order corrections, which are fixed by the 2nd law of thermodynamics as

$$T^{\mu\nu}_{(1)} = 2\eta \nabla^{<\mu} u^{\nu>} + \zeta \Delta^{\mu\nu} \nabla \cdot u \qquad J^\mu_{\mathrm{A}(1)} = D_{\mathrm{A}} \nabla^\mu \bar{\mu}_{\mathrm{A}}$$

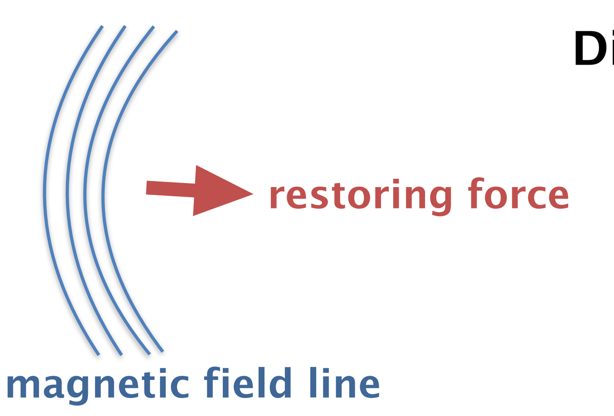
$$E^\mu_{(1)} = \frac{1}{\sigma\beta} \epsilon^{\mu\nu\alpha\beta} u_\nu \partial_\alpha (\beta H_\beta) \boxed{- \frac{C_{\mathrm{A}}\mu_{\mathrm{A}}}{\sigma} B^\mu}$$

CME current

β : inverse temperature $\qquad$ σ : electric conductivity

A hydrodynamic instability in chiral MHD

Alfven wave



Dispersion relation

$$\omega = \pm v_A k_{||}$$

momentum along **B**

$$v_A^2 = \frac{\boldsymbol{B}^2}{e + p + \boldsymbol{B}^2}$$

Alfven velocity

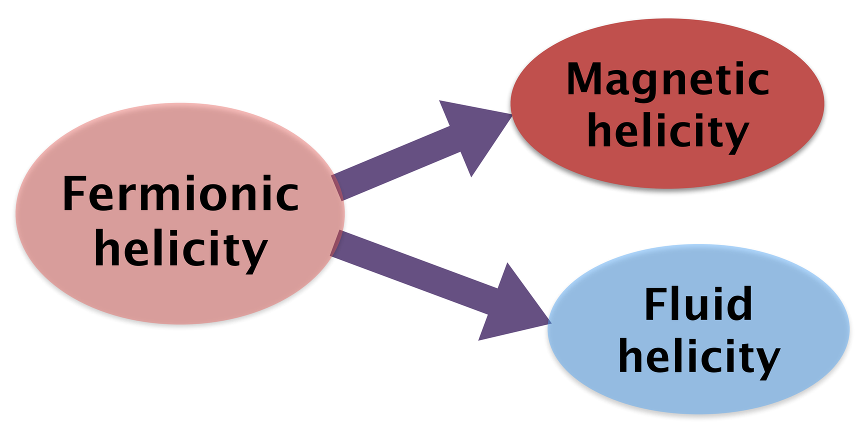
B fields are "frozen in" to the fluid

Dissipative & chiral modification

$$\omega = \pm v_A k_{||} - \frac{i}{2}\left[\bar{\eta} + \lambda\right] k^2 \boxed{+ \frac{i}{2} s \epsilon_B k_{||}}$$

Instability

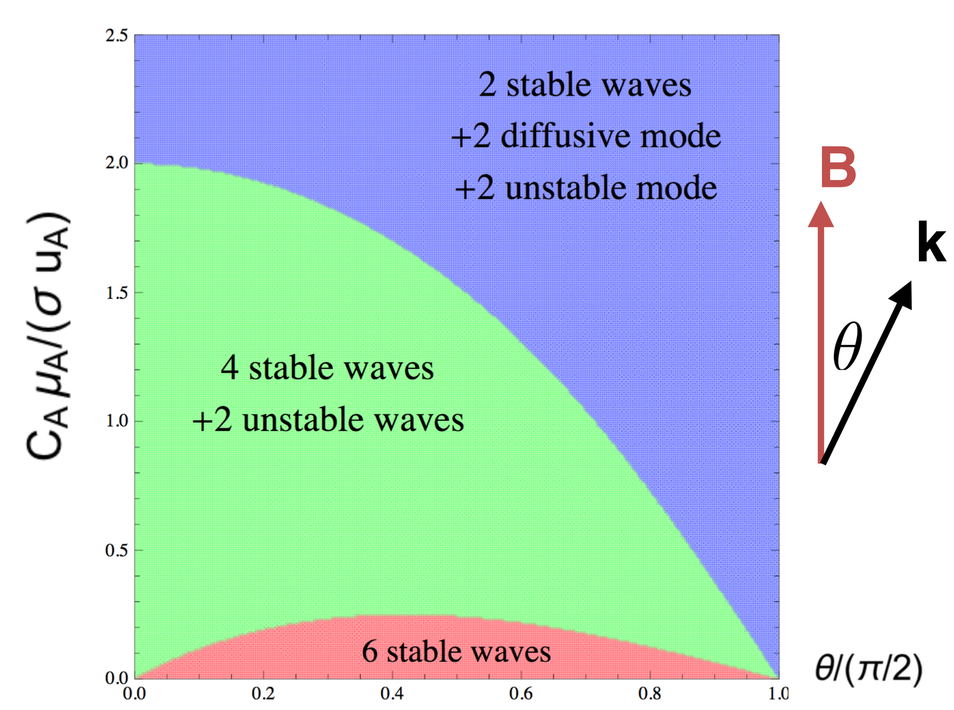
$$\epsilon_B \equiv \frac{C_{\mathrm{A}}\mu_{\mathrm{A}}}{\sigma} \qquad s = \pm 1 \text{ : handedness of the mode}$$

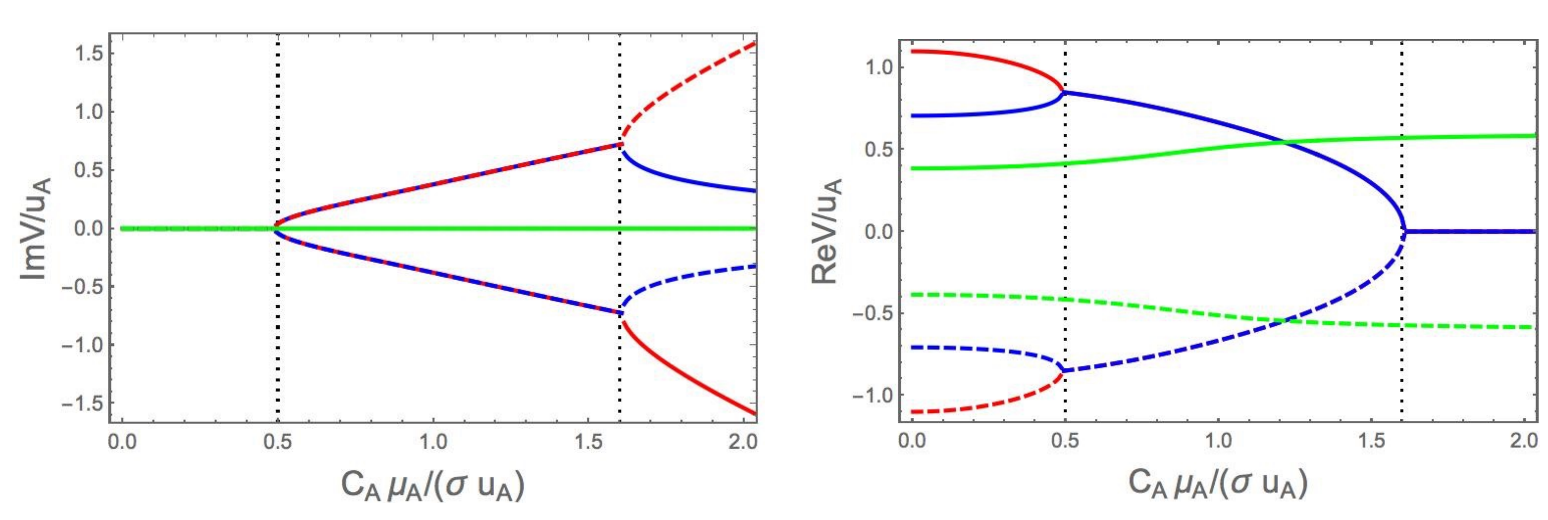


- **Cases helicity transfer**
- **Generation of large-scale helical magnetic field & fluid flow**

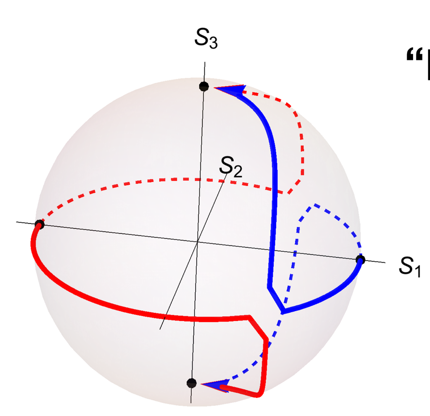
Detail properties of waves

Phases of collective modes





Real and imaginary part of the velocity for $\theta = \pi/4$



"Poincare sphere"

Poles: circularly polarized (R&L)

Equator: linear polarization

Trajectory of the Stokes vector as a function of ϵ_{A}

$$\boldsymbol{s} = \frac{\left(b_1^2 - b_2^2, 2\mathrm{Re}\left[b_1 b_2^*\right], 2\,\mathrm{Im}\left[b_1 b_2^*\right]\right)}{b_1^2 + b_2^2}$$