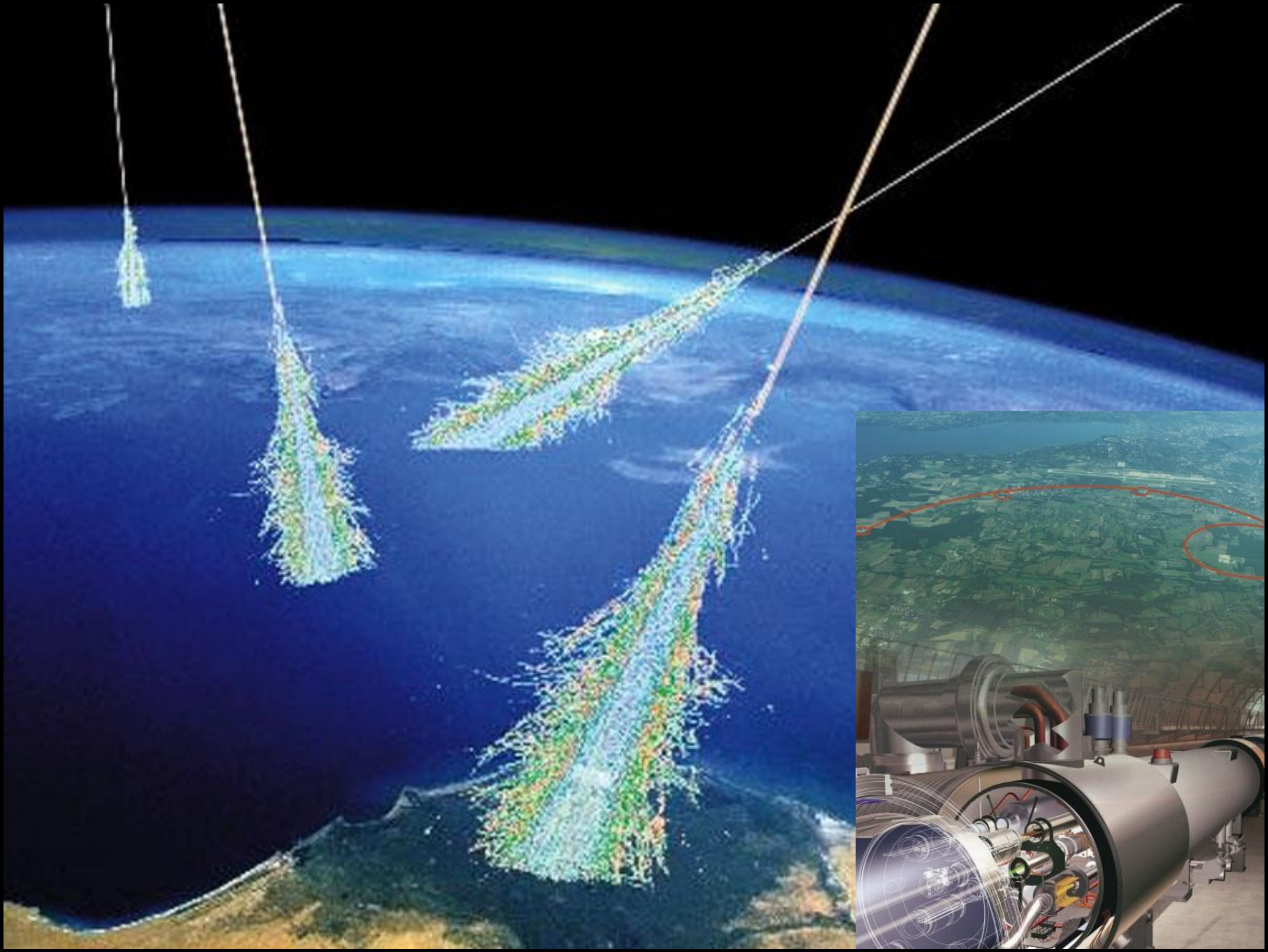
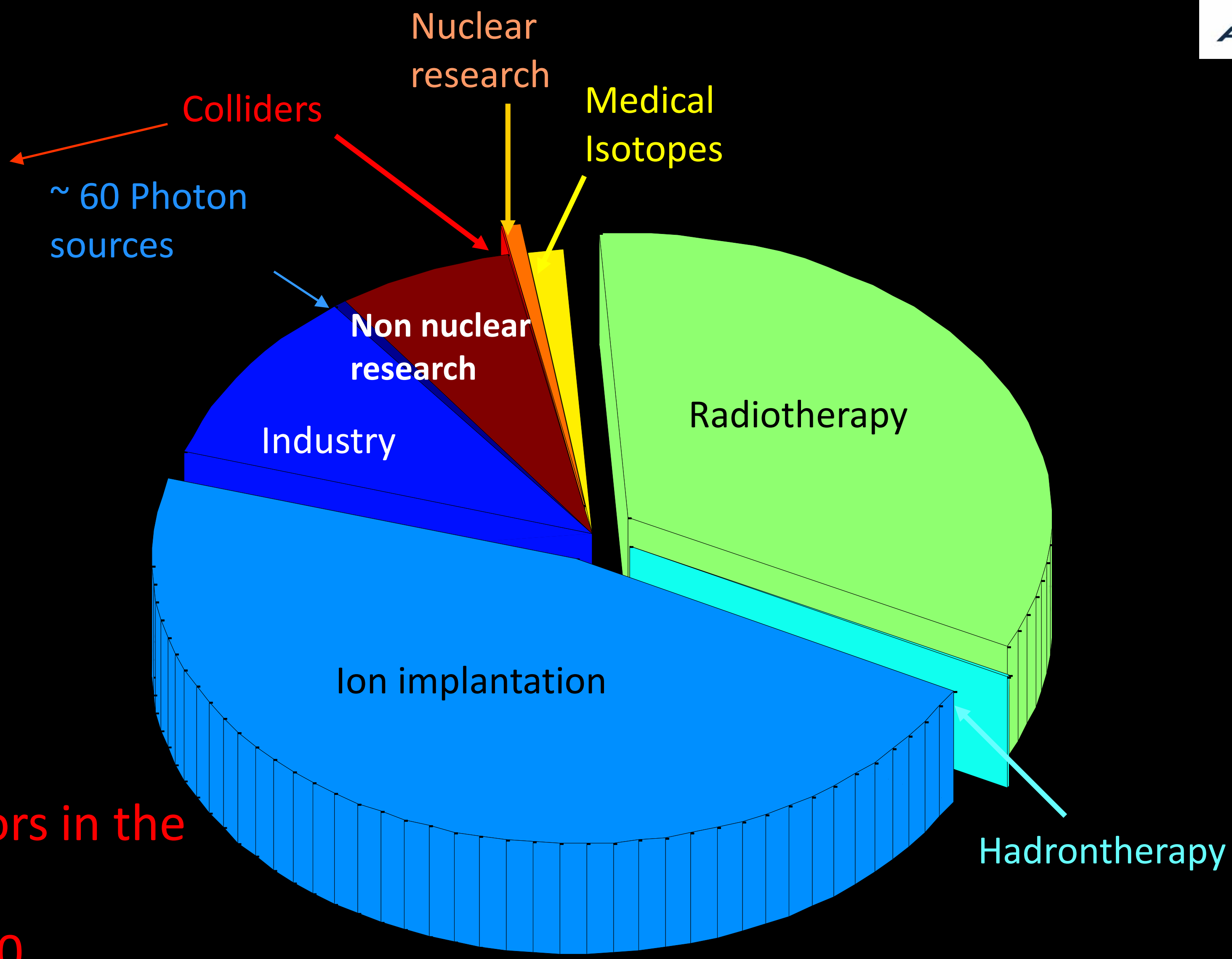
Introduction to Accelerators



1 CERN
1 Italy
2 Russia
1 China
1 USA
1 Japan



Accelerators in the world:
over 35000
(15000 in 2000)

Livingston's chart of accelerators – ~ One century of history

BASICS OF ACCELERATORS AND OF THE ART OF INVENTIVENESS 5

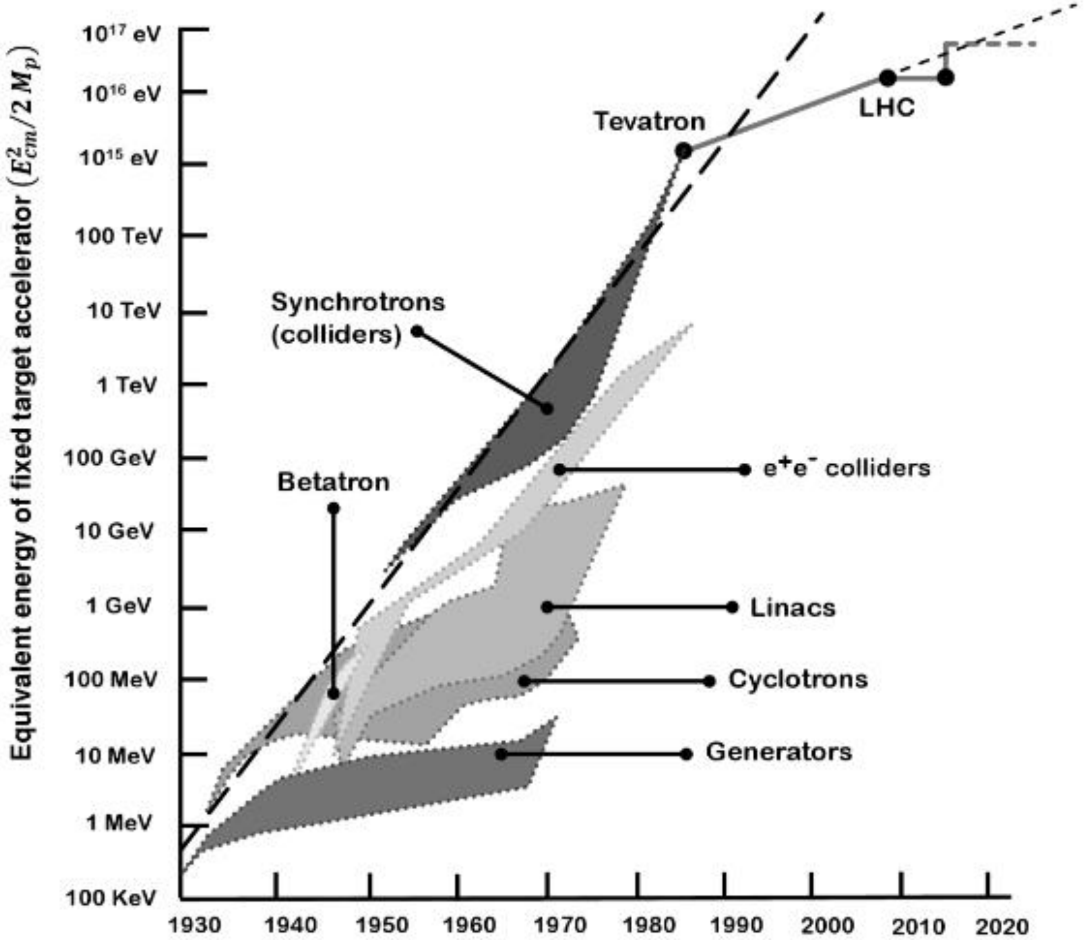
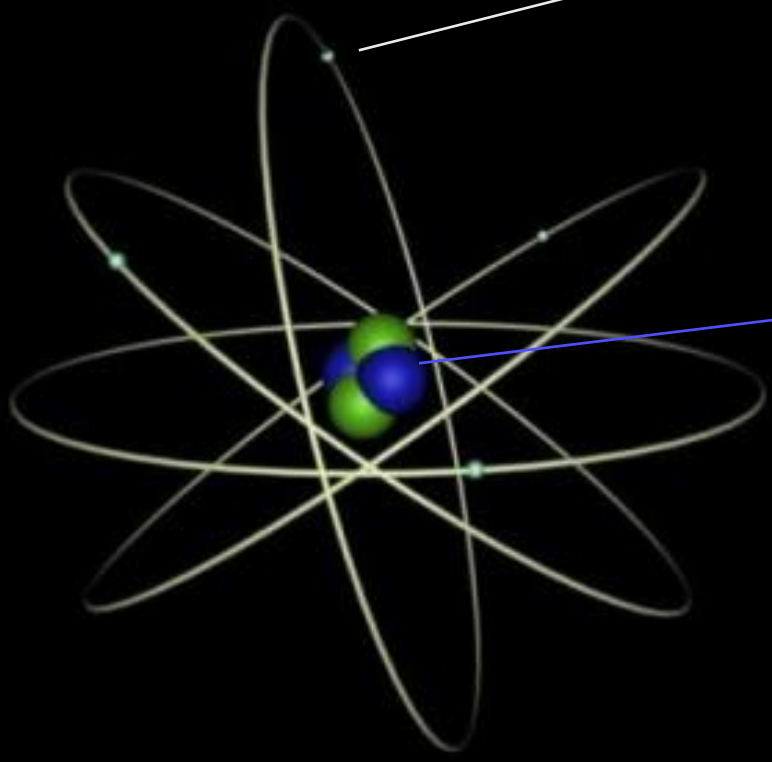


FIGURE 1.6
Livingston plot of evolution of accelerators.

Nearly nine decades of continued growth in the energy reach of accelerators
Driven by continuous innovation in acceleration techniques
Many new acceleration techniques developed to keep pushing the energy frontier

Particle in the accelerators =
= charged particles

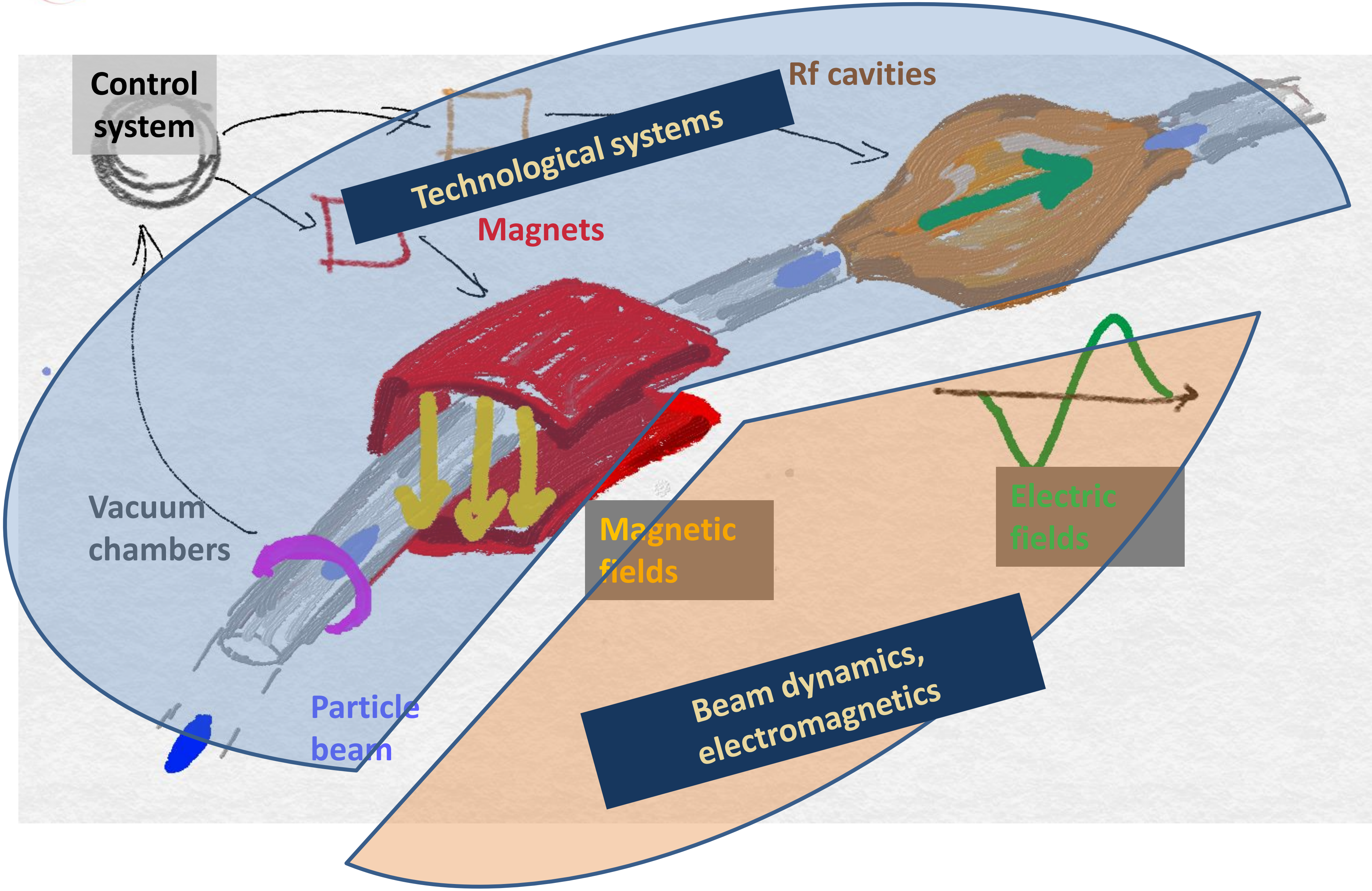


Electrons
Mass = $9.1 * 10^{-31}$ kg

Protons
Mass = $1,7 \times 10^{-27}$ kg

and Ions

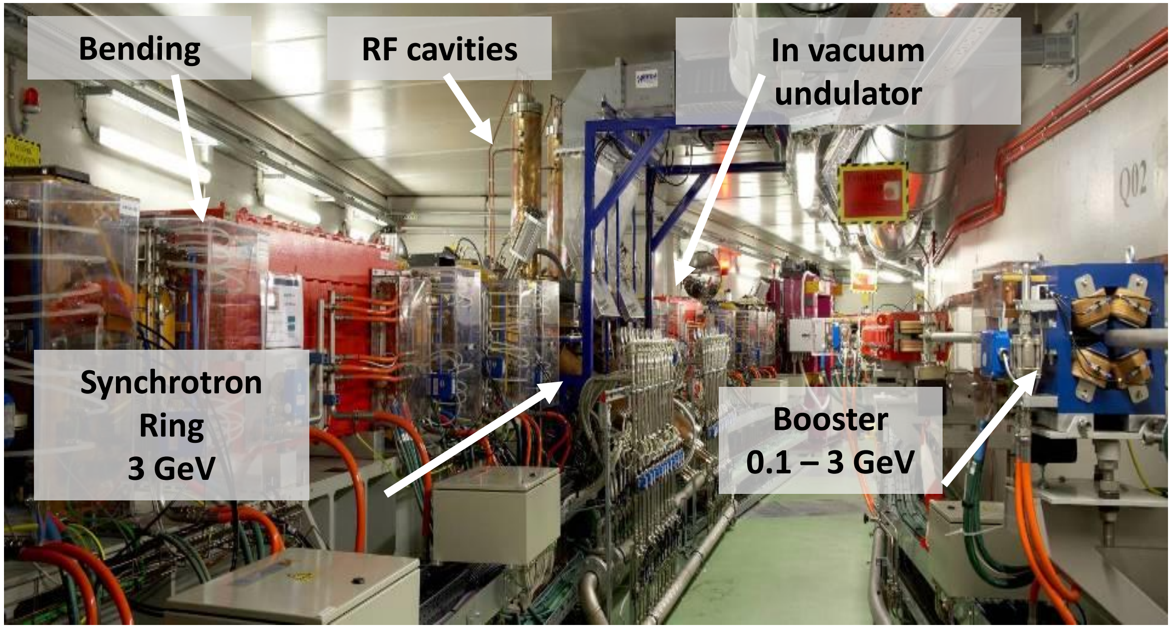
ATOM


Control system
Technological systems
Rf cavities
Magnets
Vacuum chambers
Particle beam
Magnetic fields
Electric fields
Beam dynamics, electromagnetics

Main systems of an accelerator

- Sources
- Rf cavities
- Magnets
- Vacuum system
- Beam diagnostics
- Control system

Example of a synchrotron (ALBA)



ALBA accelerator expertise in

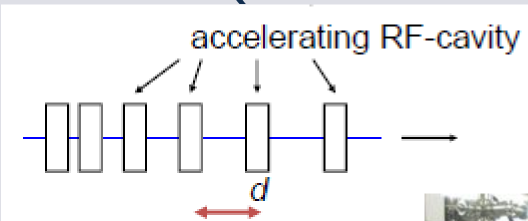
- Accelerator design / simulation / construction / operation at highest international standards
- Magnet design / realization; magnetic measurement lab used also by other institutions and companies
- RF systems – cavities, IOTs, Klystrons, LLRF;
- Vacuum systems – vacuum chamber design and optimization; realization; maintenance
- e- beam diagnostics and instrumentations systems – BPMs, current monitors, SR monitors, streak cameras
- Conventional infrastructure characteristics for high stability at mechanical and electromagnetic level

Main accelerators types and their major utilization

LINACs Radiotherapy, FELs, neutron sources, colliders, injectors	**Cyclotrons** Isotope production, proton therapy, nuclear physics, neutron sources
Synchrotrons colliders, photon sources, injectors	**Others: betatrons, van der Graaf, electrostatic, FFAG, ERL,…**

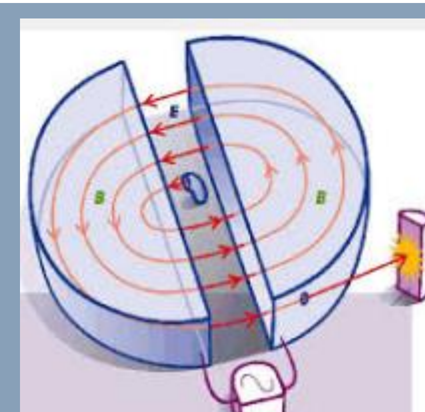
Main accelerators types and their major utilization

Linac (Linear Accelerator)



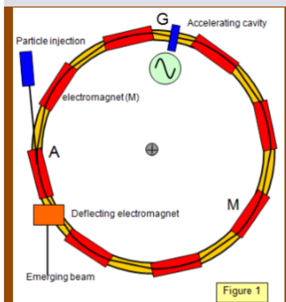
$d = \beta\lambda/2$

Cyclotron



$$\omega_o = \frac{qB}{m} = \omega_{RF} = \text{costante}$$

Synchrotron



Main accelerators applications and their typical parameters

Colliders (LHC, SuperKEKB,..) e+, e-, p $\bar{p}$, ions *Energy, Luminosity*	**Photon Sources (ESRF, LCLS, ALBA,…)** electrons *Energy, Brilliance*
HPPA (SNS, GSI, ESS,..) Protons *Energy, Power*	**Medical applications (radio and hadrontherapy)** e-, p, C ions, *Energy, Dose*

Basics on beam dynamics

Relativity

For the most part, we will use SI units, except

- *Energy: eV (keV, MeV, etc) [1 eV = 1.6x10^{-19} J]*
- *Mass: eV/c^2 [proton = 1.67x10^{-27} kg = 938 MeV/c^2]*
- *Momentum: eV/c [proton @ β=.9 = 1.94 GeV/c]*

$$\beta \equiv \frac{v}{c} = \frac{pc}{E}$$

$$\gamma \equiv \frac{1}{\sqrt{1-\beta^2}}$$

$$\text{momentum } p = \gamma m v$$

$$\text{total energy } E = \gamma m c^2$$

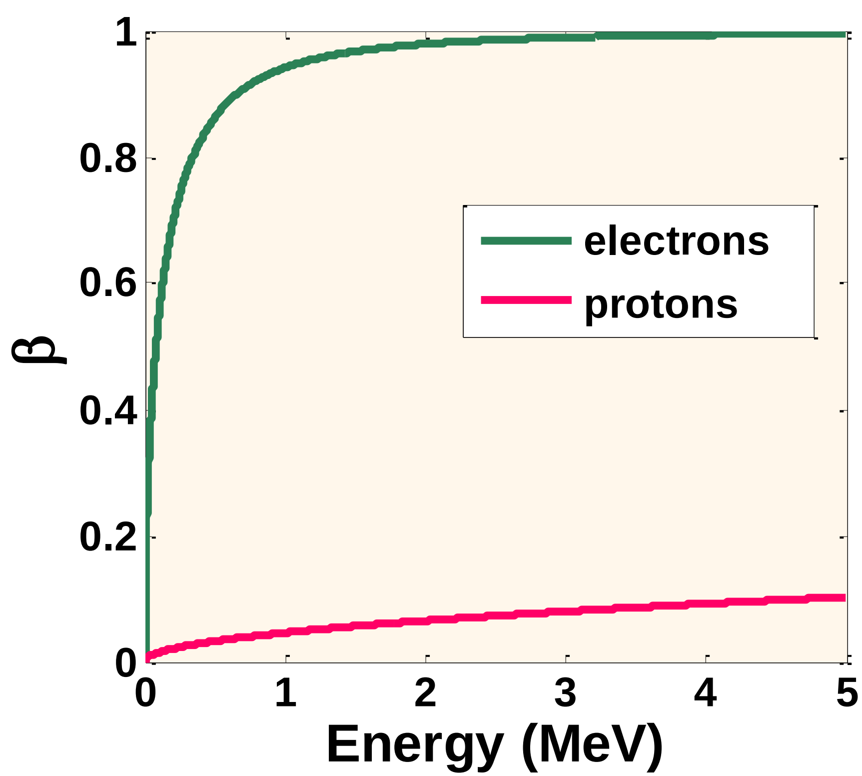
$$\text{kinetic energy } K = E - mc^2$$

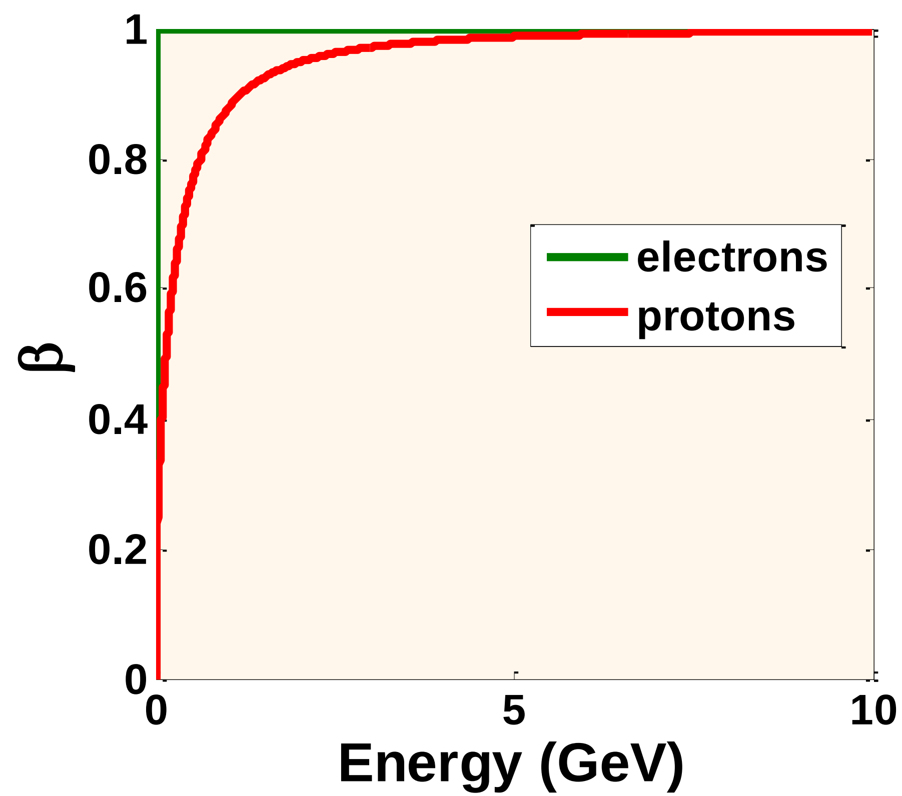
$$E = \sqrt{\left(mc^2\right)^2 + \left(pc\right)^2}$$

Important:
When we speak of beam particle energy in an accelerator we refer to Kinetic Energy! (unless specified)

Particle velocity as a function of kinetic energy

$$\beta = \frac{v}{c} \qquad \beta = 1 \longrightarrow \text{Particle at light velocity } c$$





Electrons are ultrarelativistic at few MeV
Protons at few GeV (mass ~2000 times electron mass)

Lorentz's Force

Particle dynamics are governed by the Lorentz force law

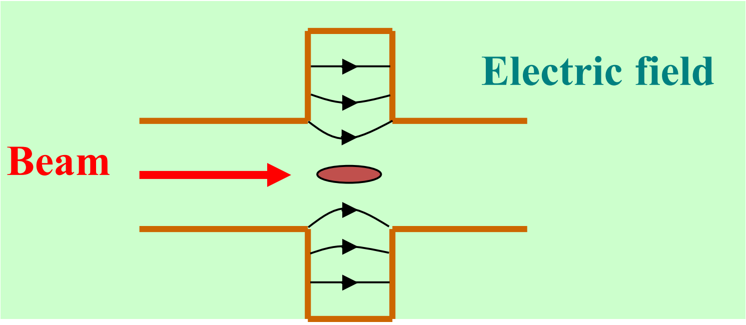
$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

$$\vec{F} = m\frac{d\vec{v}}{dt} \quad \text{for } v << c$$

$$\vec{F} = \frac{d\vec{p}}{dt} \quad \text{for any } v$$

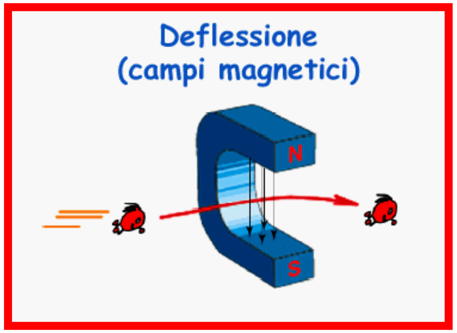
Acceleration

$$\vec{E} = \text{electric field}$$



Bending and focusing

$$\vec{B} = \text{magnetic field}$$



Rigidity:
Relation between radius and momentum given a certain magnetic field

$$B\rho = \frac{p}{q}$$

How hard (or easy) is a particle to deflect?

- Often expressed in [Tm] (easy to calculate B)
- Be careful when q≠e!!

$$B\rho[Tm] \approx 3.33 \frac{p[\frac{GeV}{c}]}{q[e]}$$

Electrons and protons (now in MeV)

$$E_o = 0.511\ MeV \quad or\ 938.27\ MeV$$
$$E_{tot} = E_{kin} + E_o$$
$$\gamma = \frac{E_{tot}}{E_o}, \beta = \sqrt{1 - \frac{1}{\gamma^2}}$$
$$p = \beta E_{tot}$$

$$B\rho = 3.33\ 10^{-3} p$$

Energia [MeV]	*Rigidità Bρ [Tm]*	
	p	*e⁻*
1	0,14	0,005
10	0,44	0,035
100	1,45	0,34
1.000	5,66	3,34
10.000	36,35	33,36
100.000	336	336
1.000.000	3335	3335

Ions

Kinetic energy per nucleon: E_{kin}

Total charge = Qe

$$E_e = 0.511\ MeV \qquad E_p = 938.27\ MeV \qquad E_n = 939.57 MeV$$

$$N_e, Z, N \Rightarrow A = Atomic\ mass\ (Z + N)$$

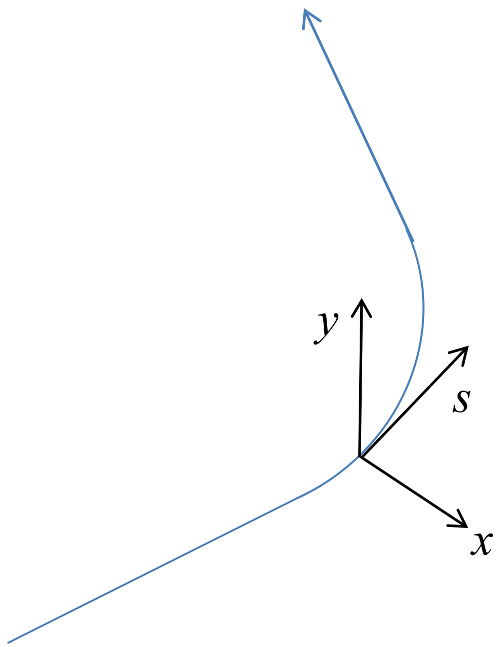
$$E_o = ZE_p + NE_p + N_e E_e - A * 0.8$$

$$E_{tot} = A\ E_{kin} + E_o$$

$$\gamma = \frac{E_{tot}}{E_o}, \beta = \sqrt{1 - \frac{1}{\gamma^2}}$$

$$p = \beta E_{tot}$$

$$B\rho = 3.33\ 10^{-3} p/\mathrm{Q}$$

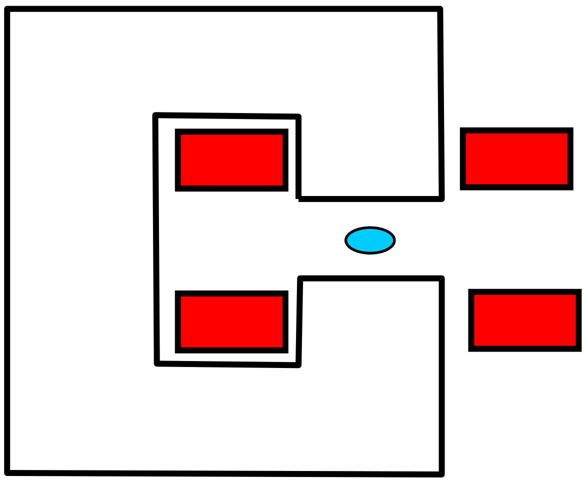


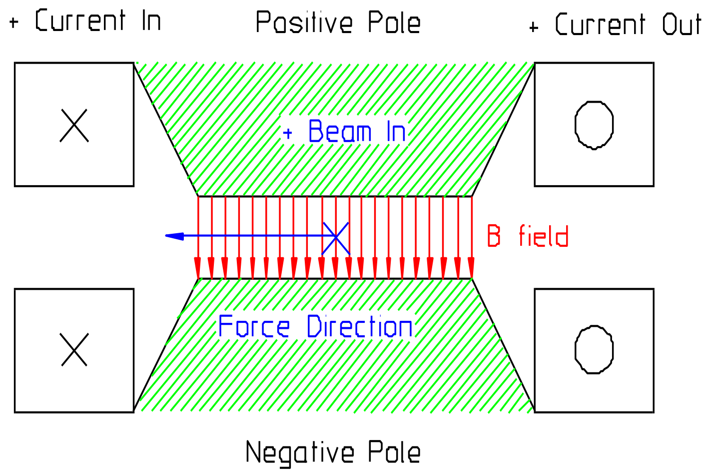
Reference system

x : horizontal
y : vertical
s : longitudinal along the trajectory

Dipole magnets

A dipole magnet has two magnetic poles which are flat and parallel to each other. The poles are equipotential surfaces.
The magnetic field is perpendicular to the poles, is uniform and is used to steer the beam around accelerator.

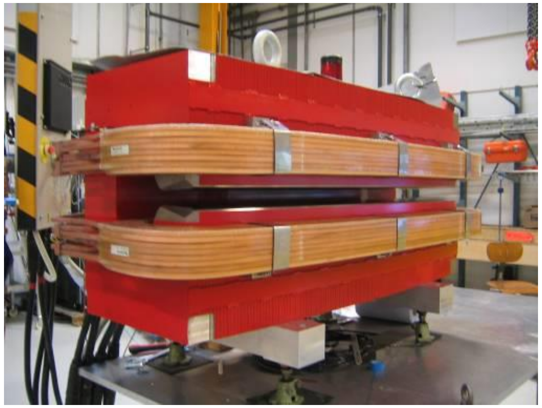




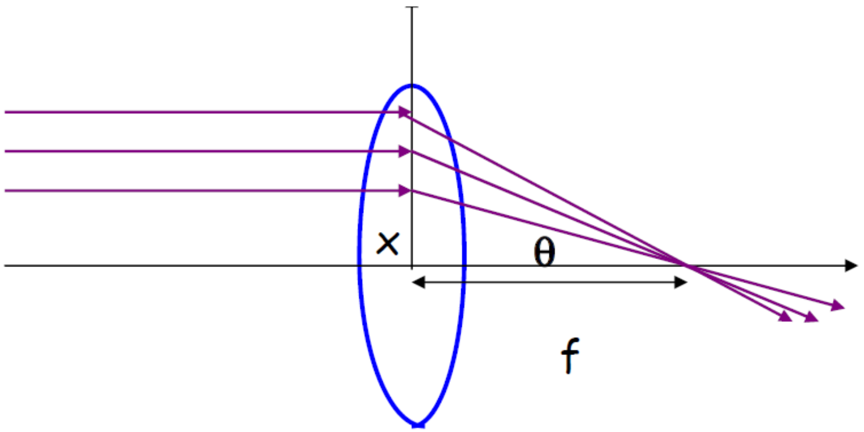
$$B\,\rho = 3.3356\,E\,[GeV]$$

F = q **· (v x B)**

$$\theta = \frac{B\,L}{3.3356\,E\,[GeV]}$$



In addition we need something to focus the beam, because without focusing a beam will diverge: a magnetic lens



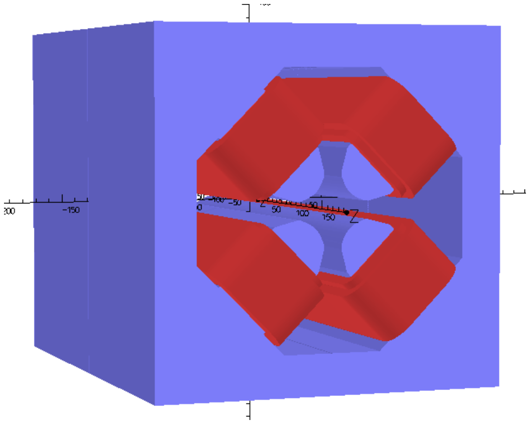
Optical analogy

$$\tan\theta = \frac{x}{f}$$

$$\theta = \frac{x}{f} \quad \text{for small x}$$

The farther off axis, the stronger the focusing effect: we use quadrupoles

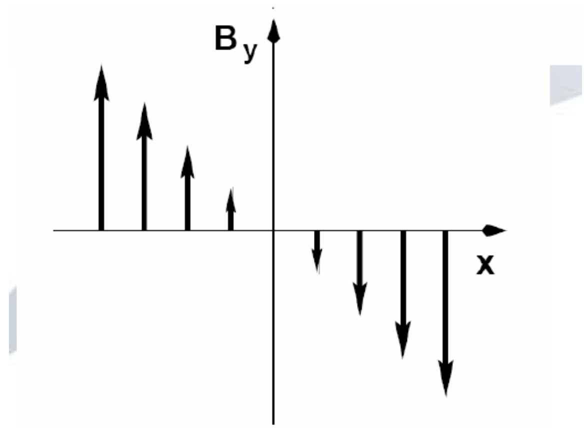
Quadrupole magnets



The Quadrupole Magnet has four poles. The field varies *linearly* with the the distance from the magnet center. It focuses the beam along one plane while defocusing the beam along the orthogonal plane.

The field of the quadrupole is proportional to the distance from the centre (x or y).

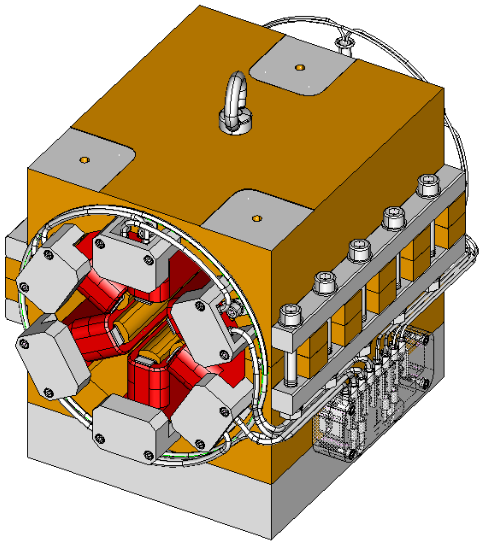
The ideal pole profile of a quadrupole is a hyperbolic one.

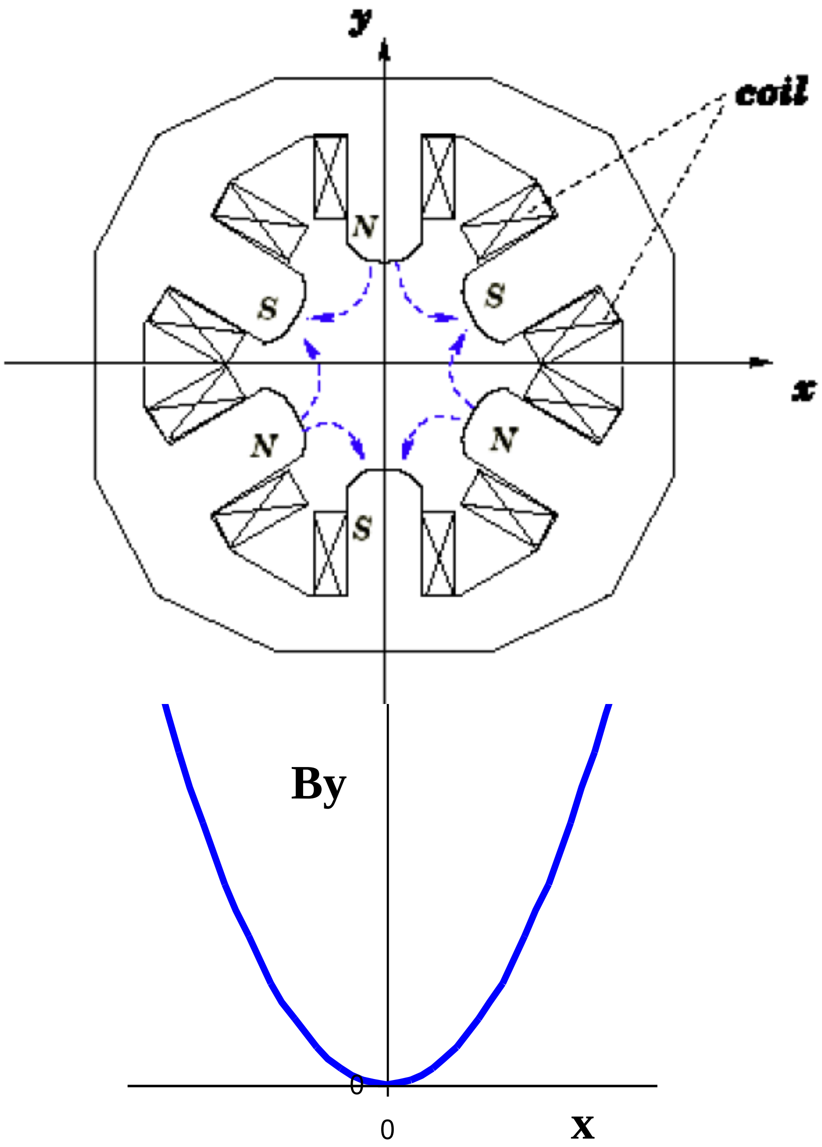


Sextupole magnet

Lens have chromaticity, and so do quadrupoles: to correct for it we use sextupoles

The Sextupole Magnet has six poles. The field varies *quadratically* with the distance from the magnet center.



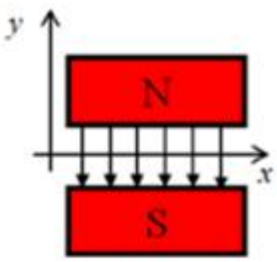


Properties of Typical Magnets

Dipoles: used for guiding the particle trajectories

$B_x = 0$

$B_y = B_o = \text{proportional to } B\rho$ $\quad\quad B_o/B\rho = 1/\rho \; [m^{-1}]$

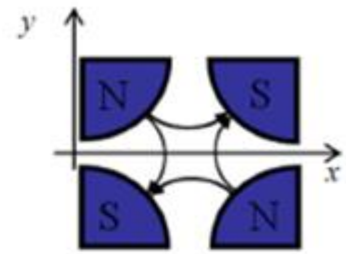


Quadrupoles: used to focus the particle trajectories

$B_x = G\,y$

$B_y = -G\,x$ $\quad\quad k = G/B\rho = (1/B\rho)\, dB_y/dx \; [m^{-2}]$

$G = \text{proportional to } B\rho$

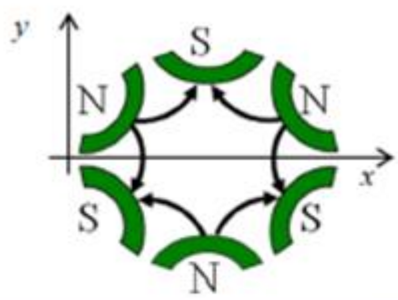


Sextupoles: used to correct chromatism and non linear terms

$B_x = 2\,S\,x\,y$

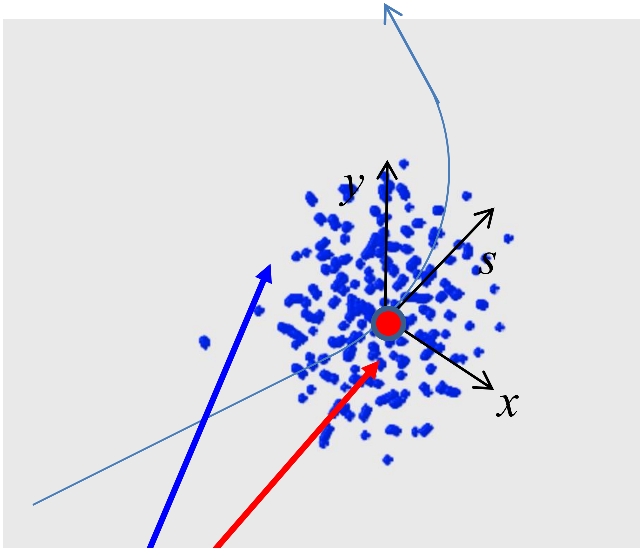
$B_y = S\,(x^2 - y^2)$ $\quad\quad k^2 = S/B\rho = (1/B\rho)\, d^2B/dx^2 \; [m^{-3}]$

$S = \text{proportional to } B\rho$



Fields $\quad\quad$ **Normalized fields (with magnetic rigidity)**

Single particle dynamics in magnetic fields



Reference system

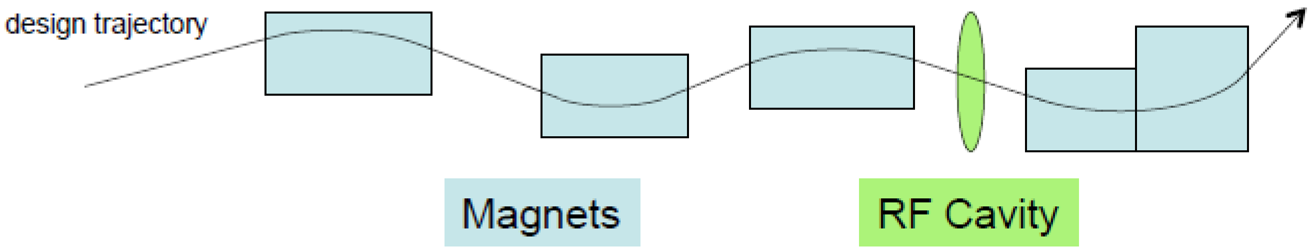
x : horizontal
y : vertical
s : longitudinal along the trajectory

Reference particle: the particle which does not exist, but dominates all the others.

- It has always the nominal energy
- It travels on the nominal trajectory inside dipoles
- It is exactly on time, always and everywhere
- It travels on axis on quadrupoles and sextupoles (where magnetic field is zero)

Real particles: all particles which travel in the accelerator trying to maintain their trajectories and energies the closest to the reference one – These are the interesting ones

Simplified Particle Motion



Design trajectory

- Particle motion will be expanded around a **design trajectory** or **orbit**
- This orbit can be over linacs, transfer lines, rings

Separation of fields: Lorentz force

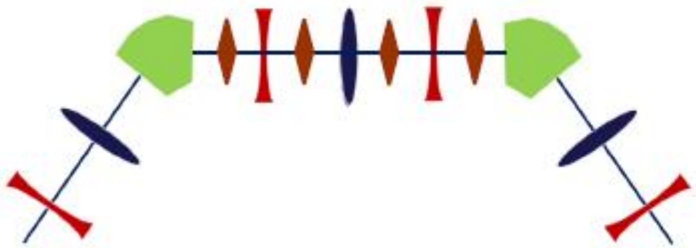
- Magnetic fields from static or slowly-changing magnets transverse to design trajectory
- Electric fields from high-frequency RF cavities in direction of design trajectory
- Relativistic charged particle velocities

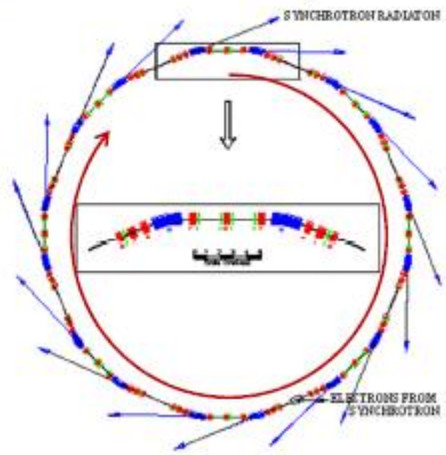
The accelerator from the particle point of view is a sequence of

- Drifts – No external fields – Particles go straight
- Magnetic fields – Particles are bent according to the magnetic rigidity
- Electric fields – Particles go straight, gain or loose energy

Lattice in an accelerator

Lattice: Sequence of magnets interleaved with drifts (used for diagnostics, vacuum pumping, Injection, extraction, etc)





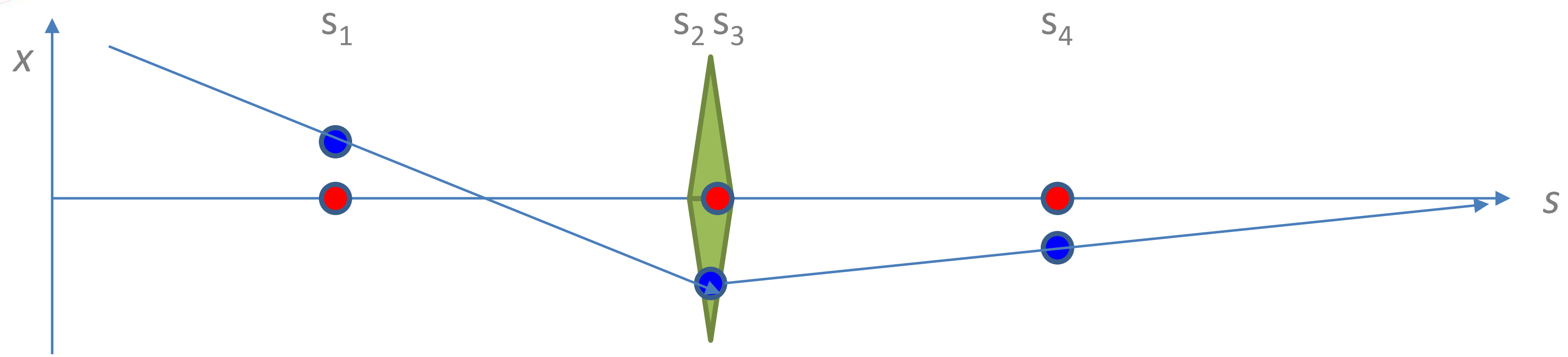
The first step in calculating a lattice is to consider only the linear components of it (quadrupoles and dipoles). Non-linear effects and chromatic aberration corrections will be evaluated later.

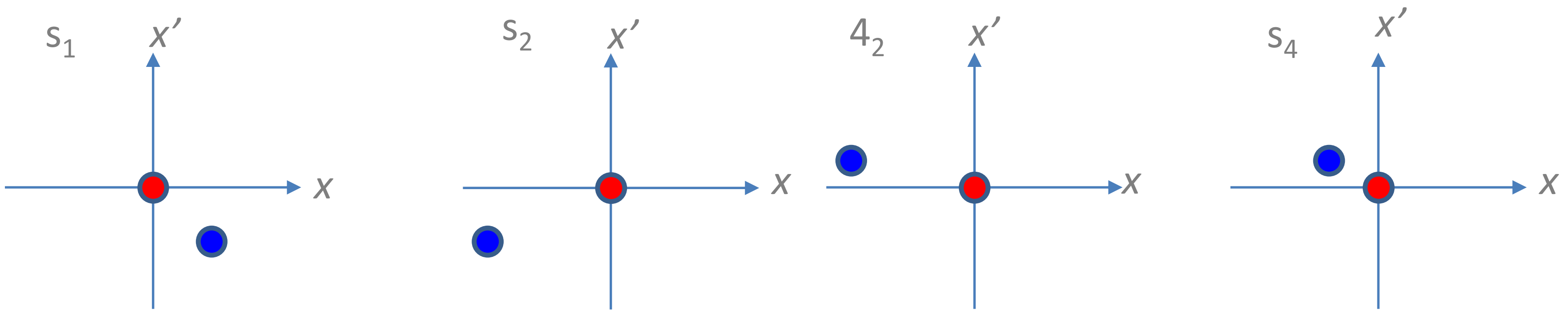
The trajectory of the reference particle along the optics is calculated.

All the other beam particles are represented in a frame moving along the reference trajectory, and where the reference particle is always in the center.

Coordinate systems used to describe the motion correspond to the **difference (in position or slope or energy or time)** between the particle and the reference one. They are usually locally Cartesian or cylindrical (typically the one that allows the easiest field representation)

What equations of motion describe





PHASE SPACE

$$\frac{dx}{ds} \text{ and } \frac{dx'}{ds} == x' \text{ and } x''$$

Linear transverse motion
(in presence of dipoles and quadrupoles)

Lorentz force +
Linear magnetic fields +
Derivative along s

$$x'' + \left(\frac{1}{\rho^2} - k\right) x = 0 \quad \text{H}$$

$$y'' + ky = 0 \quad \text{V}$$

$$k = \frac{g}{p/e} = \frac{1}{B\rho}\frac{dB_y}{dx}$$

Solution of equations of motion (harmonic oscillator):

$a_1 = x_0 \quad a_2 = \frac{x'_0}{\sqrt{K}}$

Horizontal : $K = \frac{1}{\rho^2} - k$

Vertical : $K = k$

Here x represents both x or y

$$x(s) = x_0 \cos(\sqrt{K}s) + x'_0 \frac{1}{\sqrt{K}} \sin(\sqrt{K}s)$$

$$x'(s) = -x_0 \sqrt{K} \sin(\sqrt{K}s) + x'_0 \cos(\sqrt{K}s)$$

Matrix formalism

We can write the equations in matrix formalism: coordinates at point s_1 can be obtained knowing the coordinates at s_0

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s_1} = M \begin{pmatrix} x \\ x' \end{pmatrix}_{s_0}$$

$$M = \begin{pmatrix} \cos(\sqrt{K}L) & \frac{1}{\sqrt{K}}\sin(\sqrt{K}L) \\ -\sqrt{K}\sin(\sqrt{K}L) & \cos(\sqrt{K}L) \end{pmatrix}$$

Example: Drift
Length: L
$K = 0$

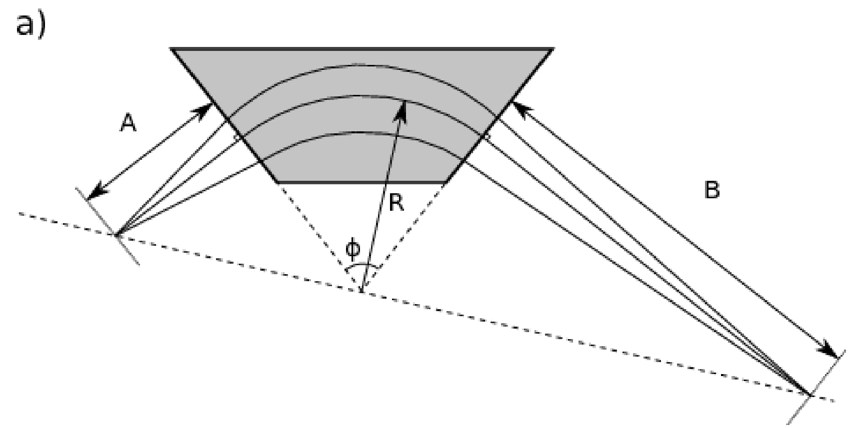
$$M_{Drift} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$$

$$x_1 = x_0 + Lx'_0$$
$$x'_1 = x'_0$$

Focusing quadrupole:
Length L, $K > 0$

$$M_Q = \begin{pmatrix} \cos(\sqrt{K}L) & \frac{1}{\sqrt{K}}\sin(\sqrt{K}L) \\ -\sqrt{K}\sin(\sqrt{K}L) & \cos(\sqrt{K}L) \end{pmatrix}$$

Dipole



Sector magnet:
Nominal particle trajectory is perpendicular to dipole entrance
Horizontal plane: $K = {}^{1}/_{\rho^2} - k$
Vertical plane: $K = k$

If $k = 0, L = \rho\theta$

$$M_H = \begin{pmatrix} cos\theta & \rho sin\vartheta \\ -\frac{1}{\rho} sin\theta & cos\theta \end{pmatrix}$$

ρ = bending radius
θ = bending angle

$$M_V = \begin{pmatrix} 1 & \rho\vartheta \\ 0 & 1 \end{pmatrix}$$

Matrix of a lattice

System of lattice elements: Drifts (M_D), quads (M_Q), bendings (or dipoles) (M_B)

Starting with $\begin{pmatrix} x_0 \\ x'_0 \end{pmatrix}$ The final position and divergence of the particle will be $\begin{pmatrix} x_1 \\ x'_1 \end{pmatrix}$

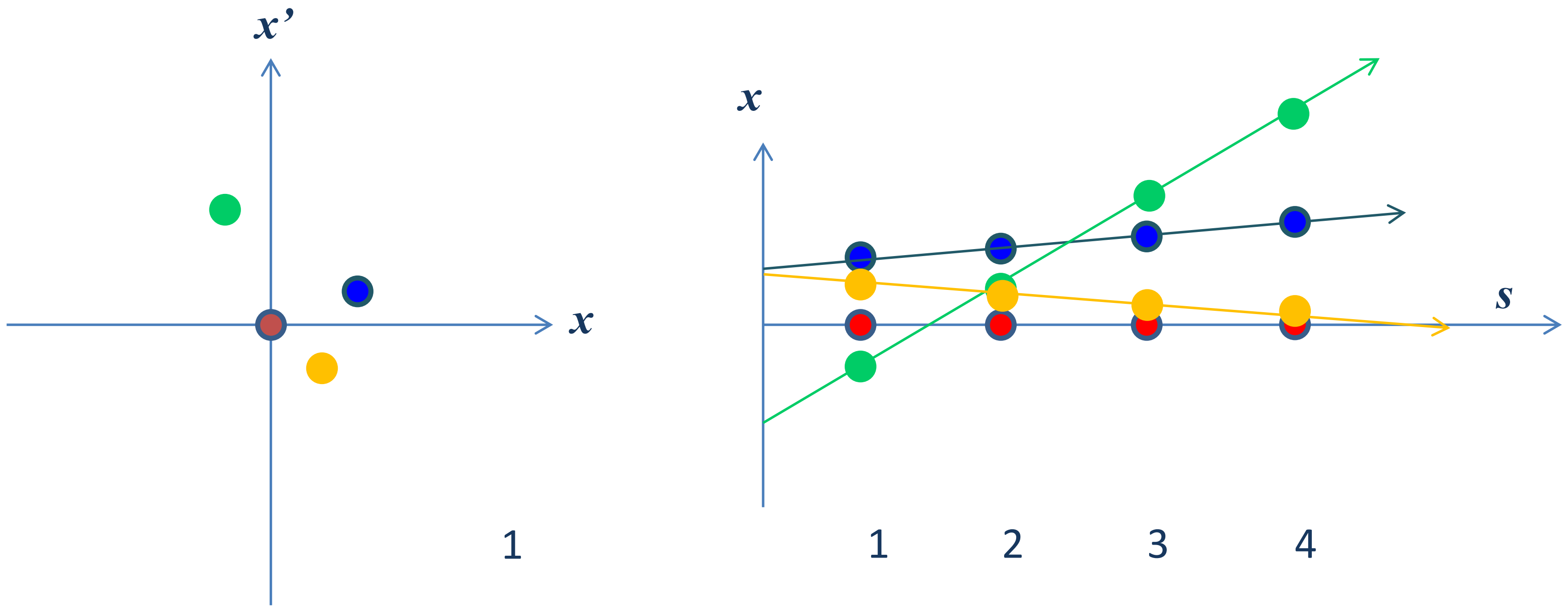
$$\begin{pmatrix} x_1 \\ x'_1 \end{pmatrix} = M_{Dn} \cdot M_{Qn} \cdot M_{Dn-1} \cdots \cdot M_{B1} \cdot M_{D2} \cdot M_{Q1} \cdot M_{D1} \cdot \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix}$$

Or simpler

$$\begin{pmatrix} x_1 \\ x'_1 \end{pmatrix} = M(s_1, s_0) \cdot \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix}$$

The mathematical representation of an accelerator lattice is a sequence of matrices

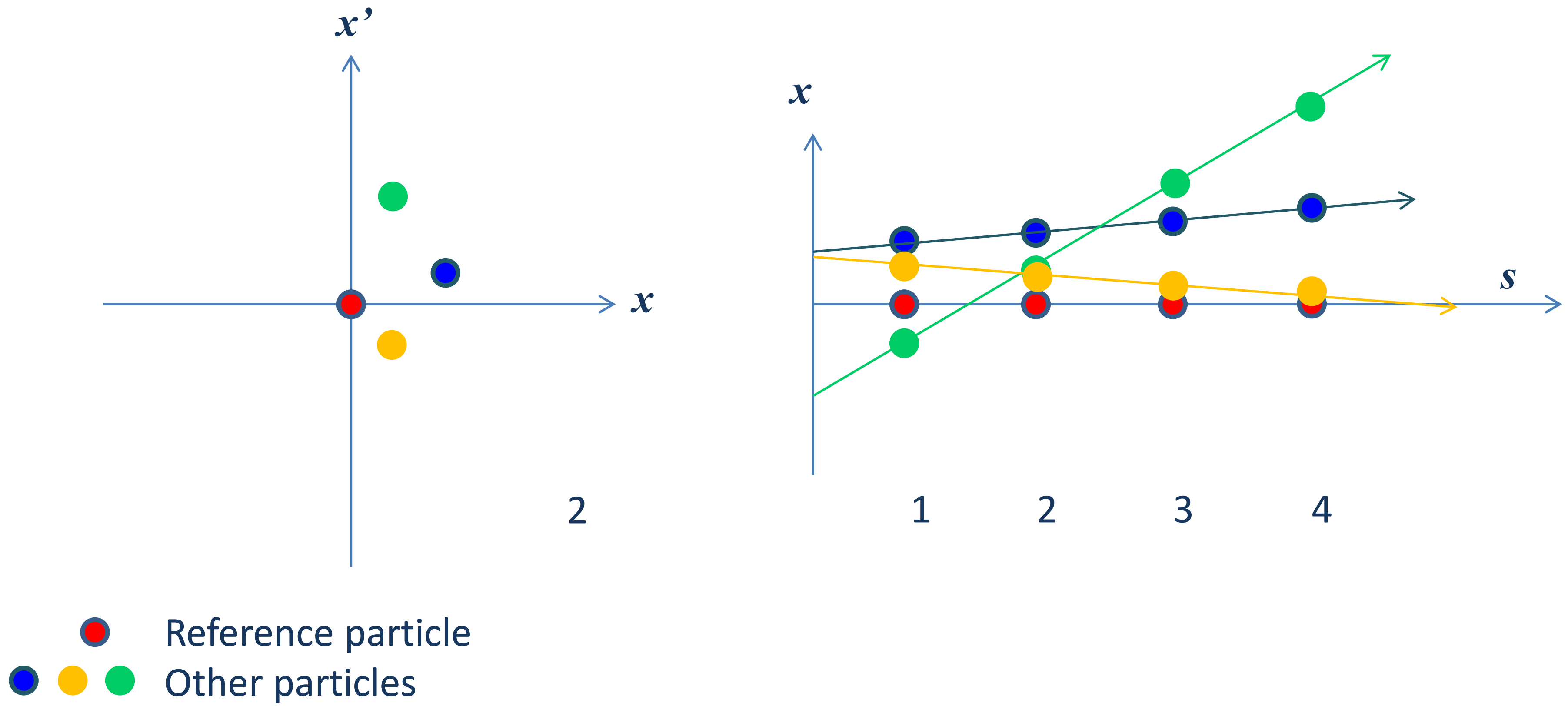
From single particle to emittance



Reference particle

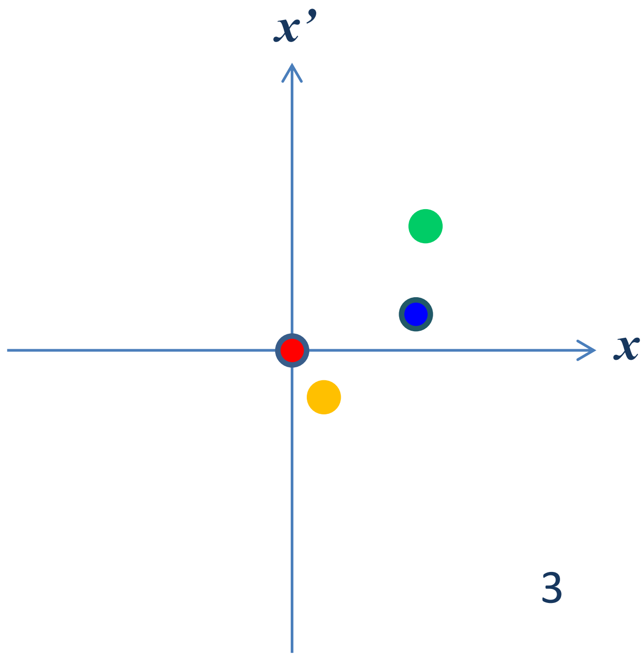
Other particles

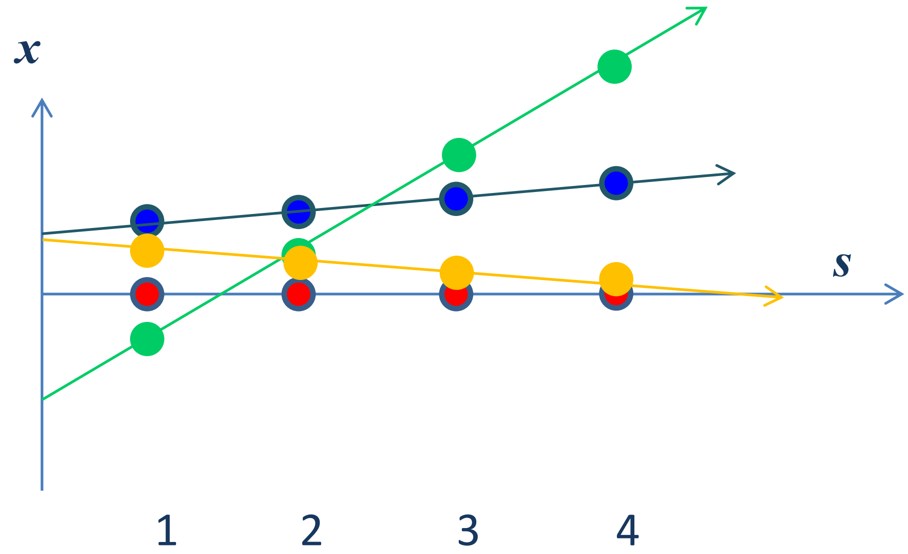
From single particle to emittance



(No magnetic field from 1 to 4)

From single particle to emittance



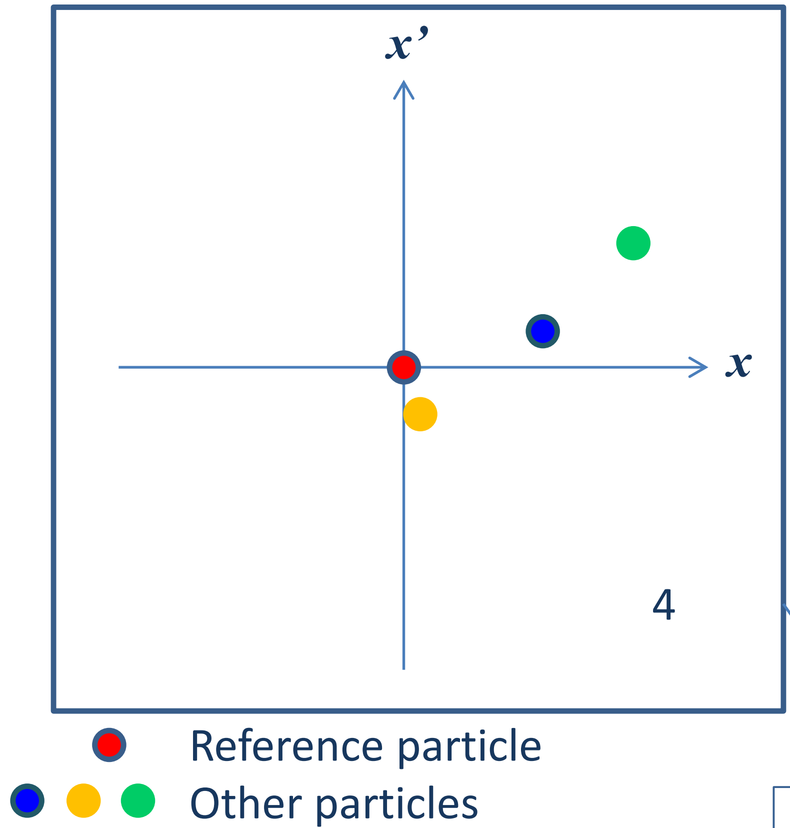


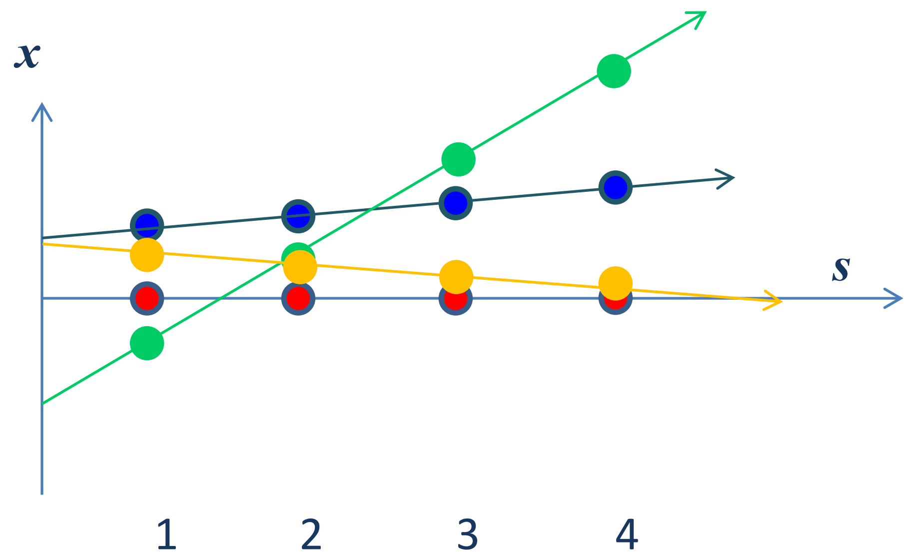
Reference particle

Other particles

(No magnetic field from 1 to 4)

From single particle to emittance

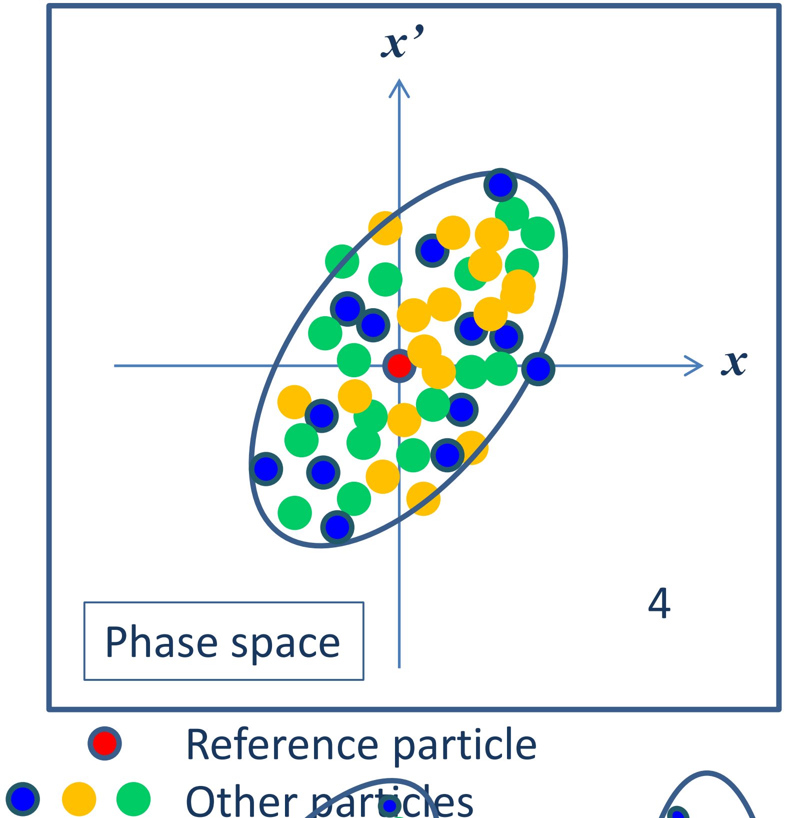




Phase space

(No magnetic field from 1 to 4)

Many particles
each of them with its own position and momentum
in every point of the accelerator

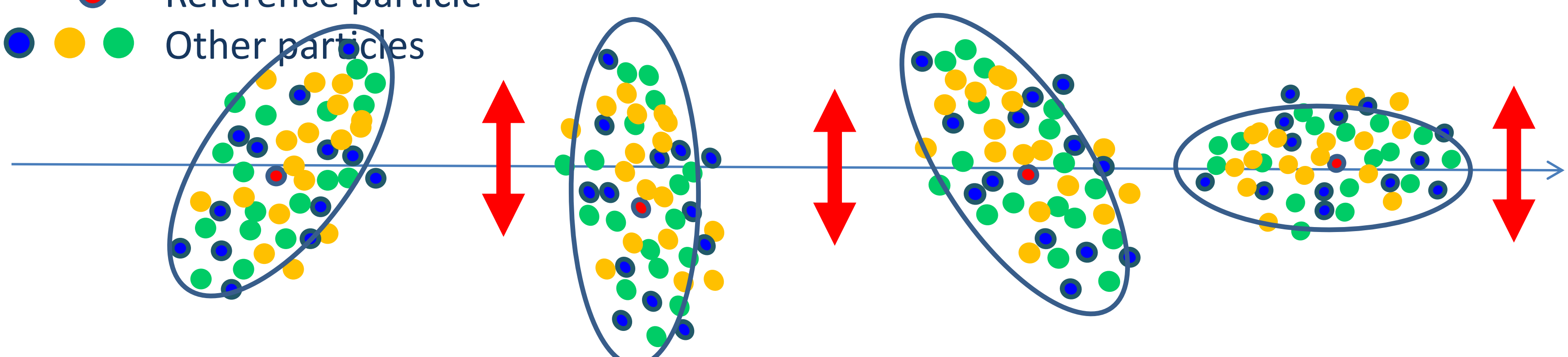


Emittance = Area of phase space

Each beam will have emittances

- Horizontal (x, x')
- Vertical (y,y')
- Longitudinal (Time-Energy)

In the only presence of linear magnetic fields ( ) the emittance will be constant, even if the ellipse orientation and axis ratio aspect will change along s



Twiss parameters – Betatron tune

$x'' + Kx = 0$ If K = constant => motion of harmonic oscillator

$x'' + K(s)x = 0$ If K varies with s: **Hill's equation**

The solution of the Hill equation is given by:

$$x(s) = \sqrt{\varepsilon}\sqrt{\beta(s)}cos(\varphi(s) + \varphi_0)$$

ε and φ_0 integration constants
Inserting $x(s)$ in the equation of motion it can be shown that the phase advance is related to β by

$$\varphi(s) = \int_0^s \frac{ds}{\beta(s)}$$

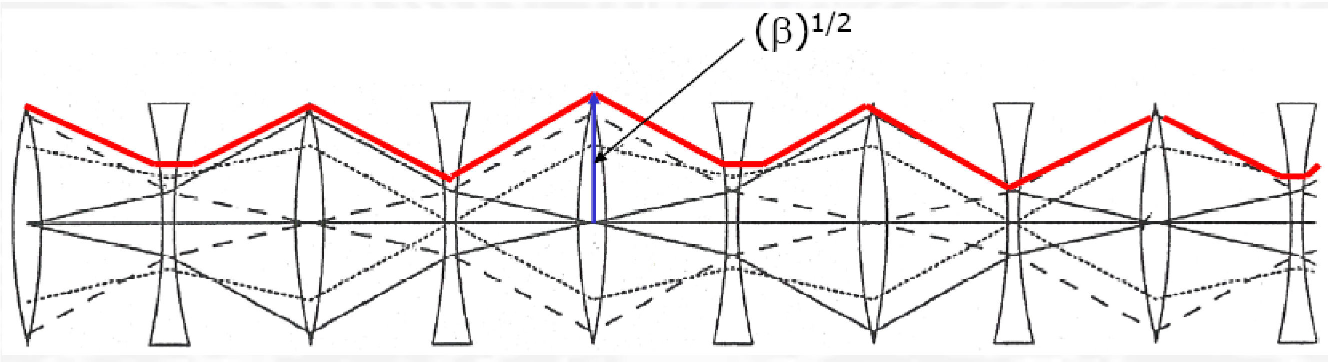
In storage rings (length of circumference = L) beta is periodic

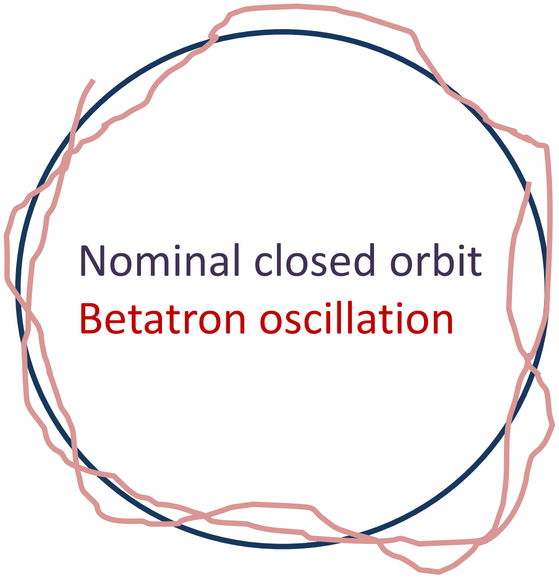
$$\beta(s + L) = \beta(s)$$

One complete turn: phase advance in one turn: ***Betatron Tune***

$$\boldsymbol{Q_{x,y} = \frac{1}{2\pi}\oint \frac{ds}{\beta_{x,y}(s)}}$$

Betatron oscillation





Particles oscillate around the closed orbit, a number of times which is given by the betatron tune.
The square of the β function by the emittance represents the envelope of the betatron oscillations

Twiss parameters

Amplitude of an oscillation

$$\hat{x}(s) = \sqrt{\varepsilon}\,\sqrt{\beta(s)}$$

$\beta(s)$ represents the envelope of all particle trajectories at a given position s in a storage ring

$$x(s) = \sqrt{\varepsilon}\sqrt{\beta(s)}cos(\varphi(s) + \varphi_0)$$

$$x'(s) = -\frac{\sqrt{\varepsilon}}{\sqrt{\beta(s)}}\{\alpha(s)\cos(\varphi(s) + \varphi_0) + sin(\varphi(s) + \varphi_0)\}$$

$$\alpha(s) = -\frac{1}{2}\beta'(s)$$

$$\gamma(s) = \frac{1 + \alpha^2(s)}{\beta(s)}$$

α, β and γ are the Twiss parameters

Emittance

Inserting in x' eq.

$$\varepsilon = \gamma(s)x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s){x'}^2(s)$$

ε is a constant of motion, not depending on s.

Parametric representation of an ellipse in x,x' phase space defined by alfa, beta, gamma: **Courant-Snyder invariant** emittance ε

For a single particle, different positions in the storage ring and different turns:

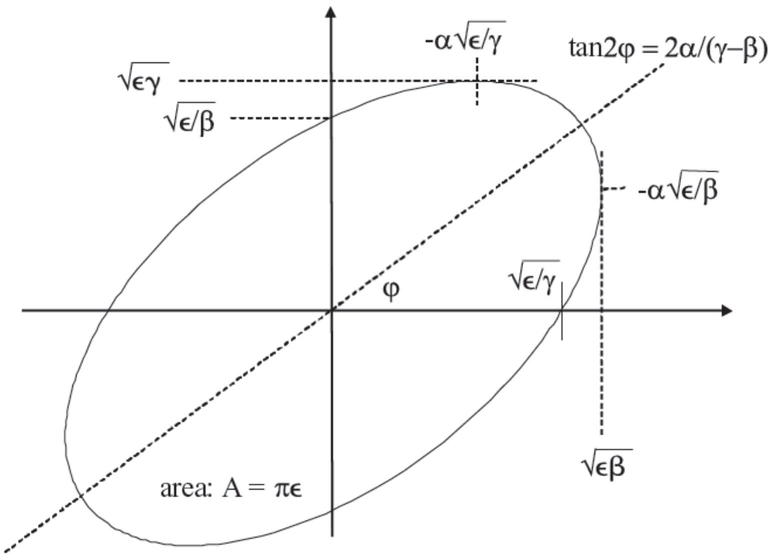
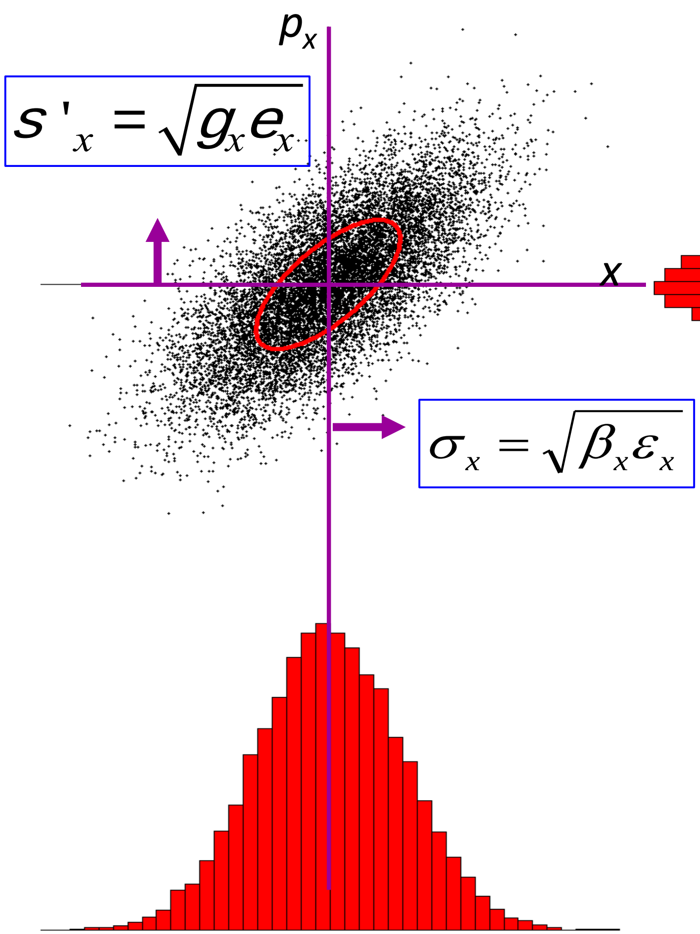


Fig. 5.2. Phase space ellipse

Emittance and beam dimensions

- The emittance is the area of the phase space occupied by the particles
  Knowing the emittance and the Twiss parameters in a point of the accelerator, the beam dimensions are obtained : $\sigma_{x,y}$ e $\sigma'_{x,y}$



$$\varepsilon_x = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle x x' \rangle^2}$$

Ellipse area= $\pi \varepsilon_x$

$$\langle x^2 \rangle = \beta_x \varepsilon_x$$

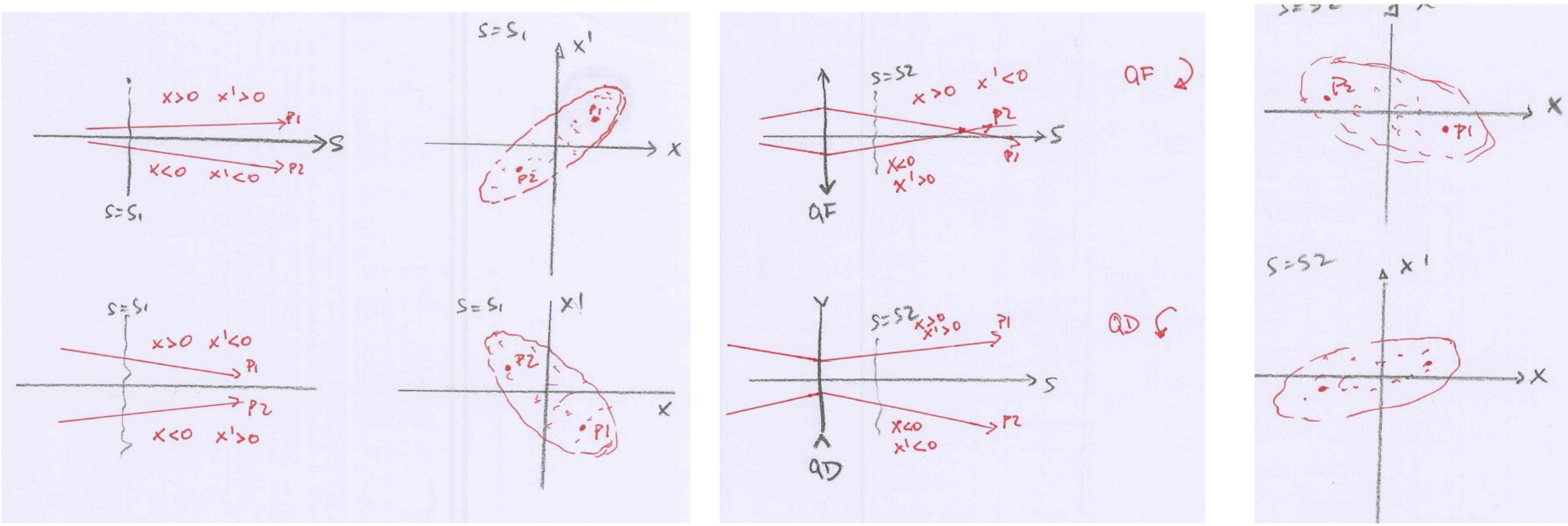
$$\langle x'^2 \rangle = \gamma_x \varepsilon_x$$

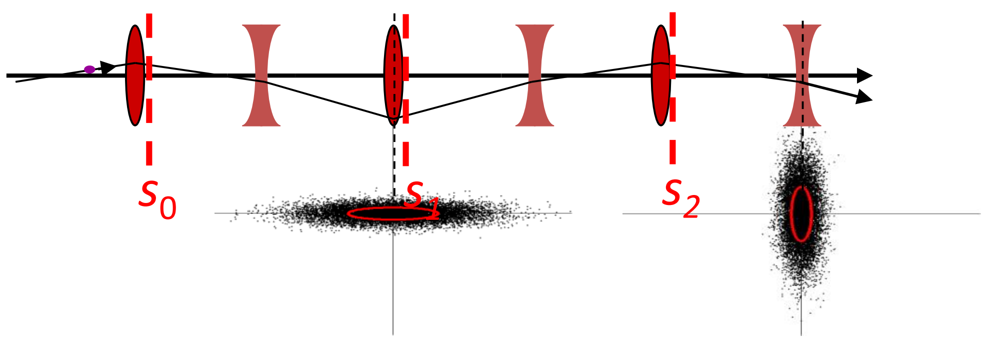
$$\langle x x' \rangle = -\alpha_x \varepsilon_x$$

$$\beta_x \gamma_x - \alpha_x^2 = 1$$

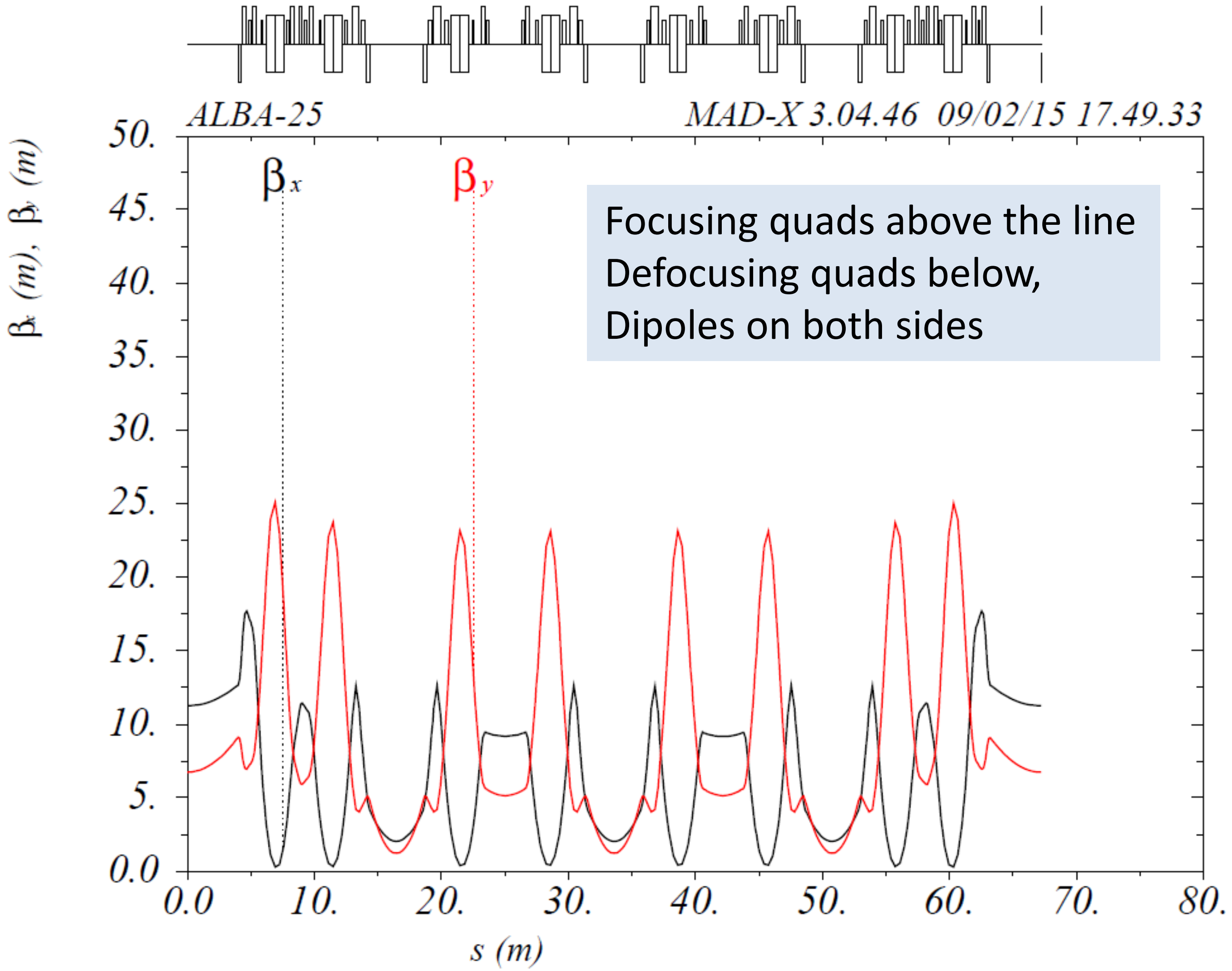
Effect of a quad on the phase space

The phase space orientation indicates if the beam trajectories are focused or defocused (at rms values):





Example of betatron functions in a storage ring - ALBA



How the emittance changes (or does not change)

Under the influence of only conservative forces the phase space area is constant.
Magnetic fields of dipoles and quadrupoles are conservative:
In a beam the emittance is constant (**Liouville's theorem**)

When there is acceleration the emittance decreases: **Adiabatic damping**:
Transverse emittance while accelerating decreases proportional to increase in momentum

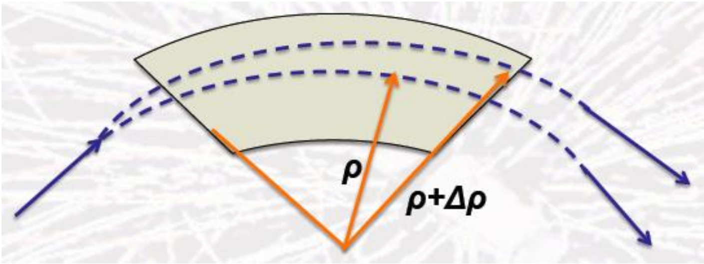
$$x' = \frac{dx}{dx} = \frac{dp_x}{dp}$$

Increasing the longitudinal momentum with increasing energy decreases x' and therefore the transverse emittance
Normalized emittance is defined as the invariant part:

$$\varepsilon_n = \varepsilon\beta\gamma$$

Where now β and γ are not the Twiss parameters, but: $\beta = \frac{v}{c}$ $and\ \gamma = \sqrt{\frac{1}{1-\beta^2}}$

Off-Momentum Particles



Magnets are chromatic elements:

Dipoles: $\rho = \frac{p}{eB} \rightarrow \frac{\Delta\rho}{\rho_0} = \frac{\Delta p}{p_0} = \frac{\Delta\theta}{\theta_0}$

Quads: $K = \frac{G}{B\rho} = \frac{Ge}{p}$

$$\Delta\theta = -\theta_0 \frac{\Delta p}{p_0}$$

$$\Delta K = -K_0 \frac{\Delta p}{p_0}$$

$\Delta p/p \ll 1$

Off-momentum particles are not oscillating around design orbit, but around a different closed orbit (chromatic closed orbit). The displacement between the design and chromatic orbits is regulated by the dispersion function *D*(*s*), which is periodic in a synchrotron.

For particles with energy deviation the Hill's equation has an extra term and is **not homogeneous**:

$$x'' + K(s)x = \frac{1}{\rho}\frac{\Delta p}{p_0}$$

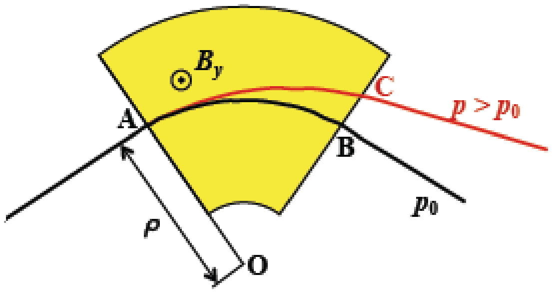
The solution is the sum of the solution of the homogenous equation + a term of dispersion:

$$x = x_{Hom} + D(s)\frac{\Delta p}{p_0}$$

Path length dependence on energy with dipoles – Storage rings

Particles with higher energies do longer paths since they are less bent.

The momentum compaction in a storage ring measures how much the total trajectory length depends on the energy deviation from the reference particle, and it is related with the Dispersion function



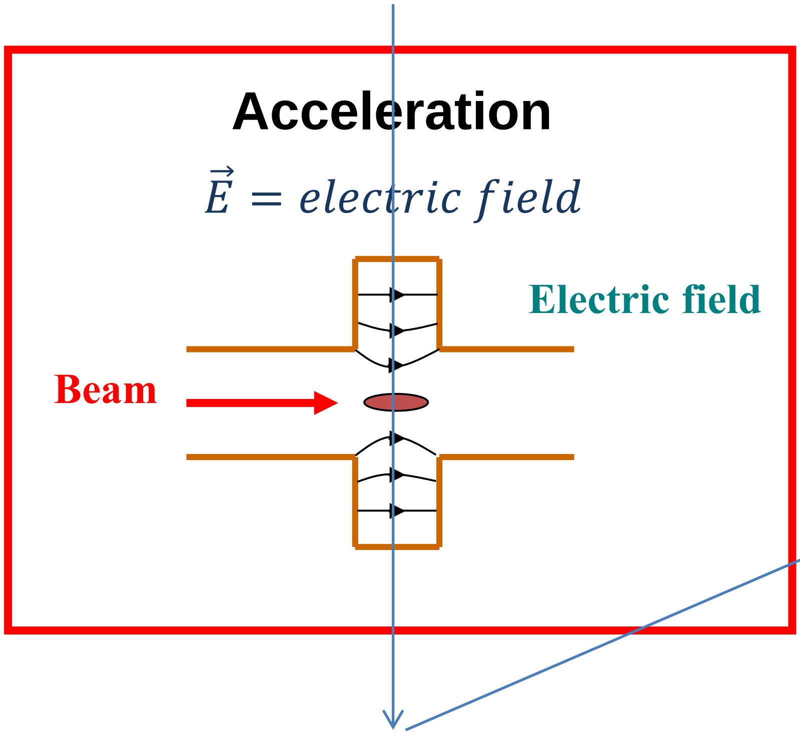
Momentum compaction in a storage ring:

$$\alpha_C = \frac{1}{L_o} \int_0^{L_0} \frac{D}{\rho} ds$$

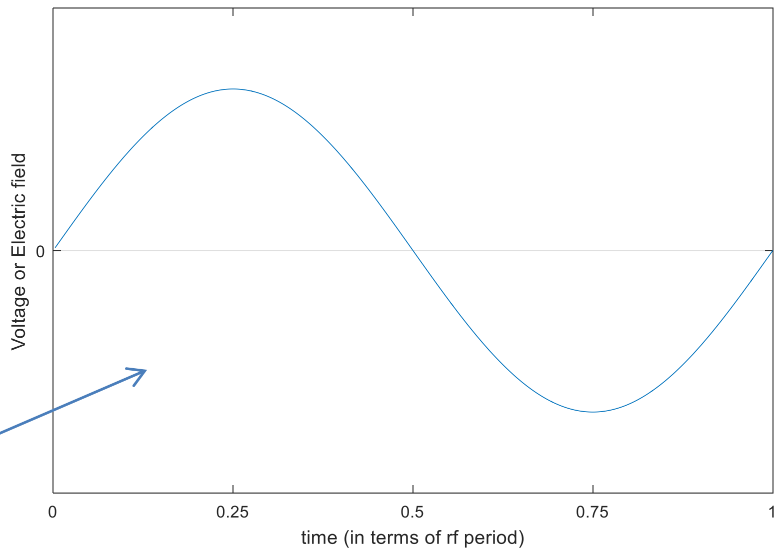
measures the relative change in circumference per unit relative momentum offset. If α_C is small the different trajectories are 'packed'. If it is zero all trajectory have the same length (ring is isochronous)

Electric field and particles

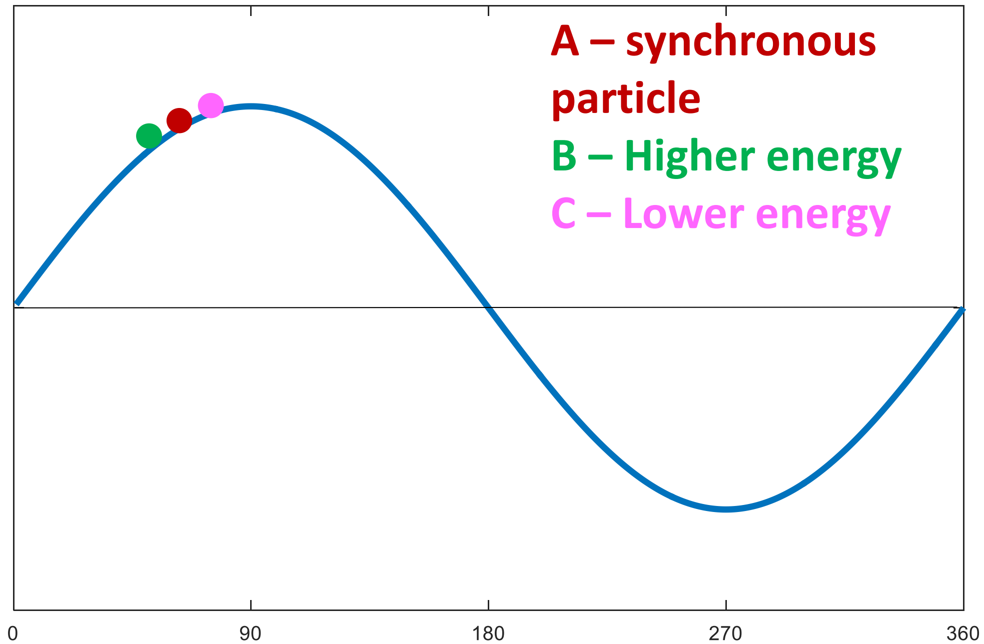
Lorentz force $\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right) \Rightarrow \vec{F} = q\vec{E}$



at the cavity center



Synchrotron Oscillation



The cavity field is seen by the particles one per turn, and the turn is a multiple of the rf period, so that particles reach the cavity always with the same phase.

The B particle arrives at $t_B<t_s$ and gains energy $\Delta W_B<\Delta W_s$

The C particle arrives at $t_C>t_s$ and gains energy $\Delta W_C>\Delta W_s$

The synchronous particle arrives at the synchronous phase

B and C oscillate in phase (time) about the synchronous particle =>

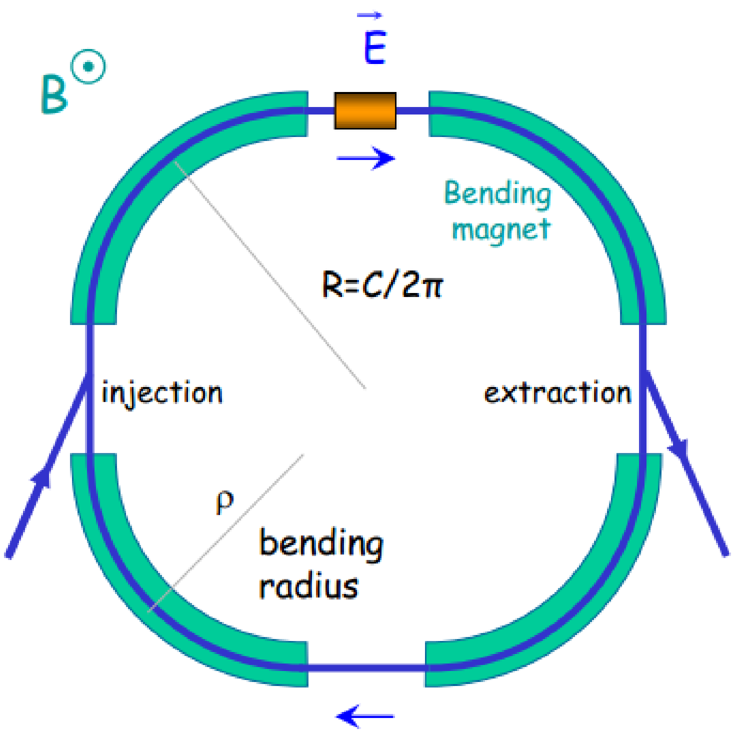
synchrotron oscillation

Synchrotron motion

- Energy and phase are related through the rf acceleration. The nominal particle is the one which is in phase with the rf and has the nominal energy. The variation of phase and energy with respect to the nominal ones represents the synchrotron motion

Synchrotrons – storage rings

The synchrotron is a synchronous accelerator since there is a synchronous RF phase for which the energy gain fits the increase of the magnetic field at each turn. That implies the following operating conditions:



$e\hat{V}\sin\phi \longrightarrow$ Energy gain per turn

$\phi = \phi_s = cte \longrightarrow$ Synchronous particle

$\omega_{RF} = h\omega_r \longrightarrow$ RF synchronism (h - harmonic number)

$\rho = cte \quad R = cte \rightarrow$ Constant orbit

$B\rho = P/e \Rightarrow B \longrightarrow$ Variable magnetic field

Colliders

Colliders have made the history of particle physics

Beam beam collision *Fixed target collision*

$$W \cong 2\sqrt{E_1 E_2}$$

$$W \cong \sqrt{2E\, m_t}$$

W = 1 GeV in the center of mass

$E_1 = E_2 = 0.5$ GeV *E = 1000 GeV (e-)*

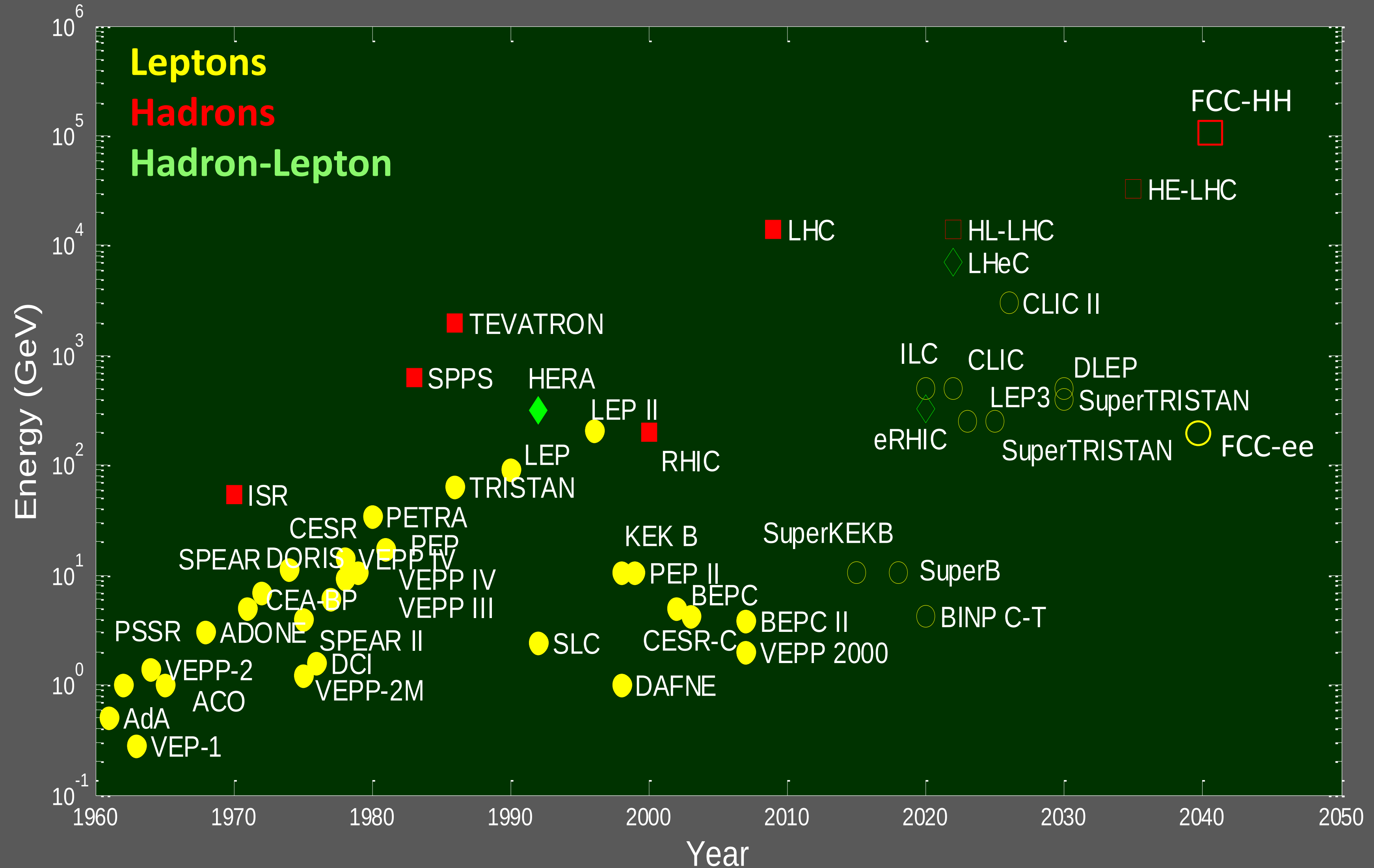
W = 100 GeV in the center of mass

$E_1 = E_2 = 50$ GeV *E = 10^7 GeV (e-)*

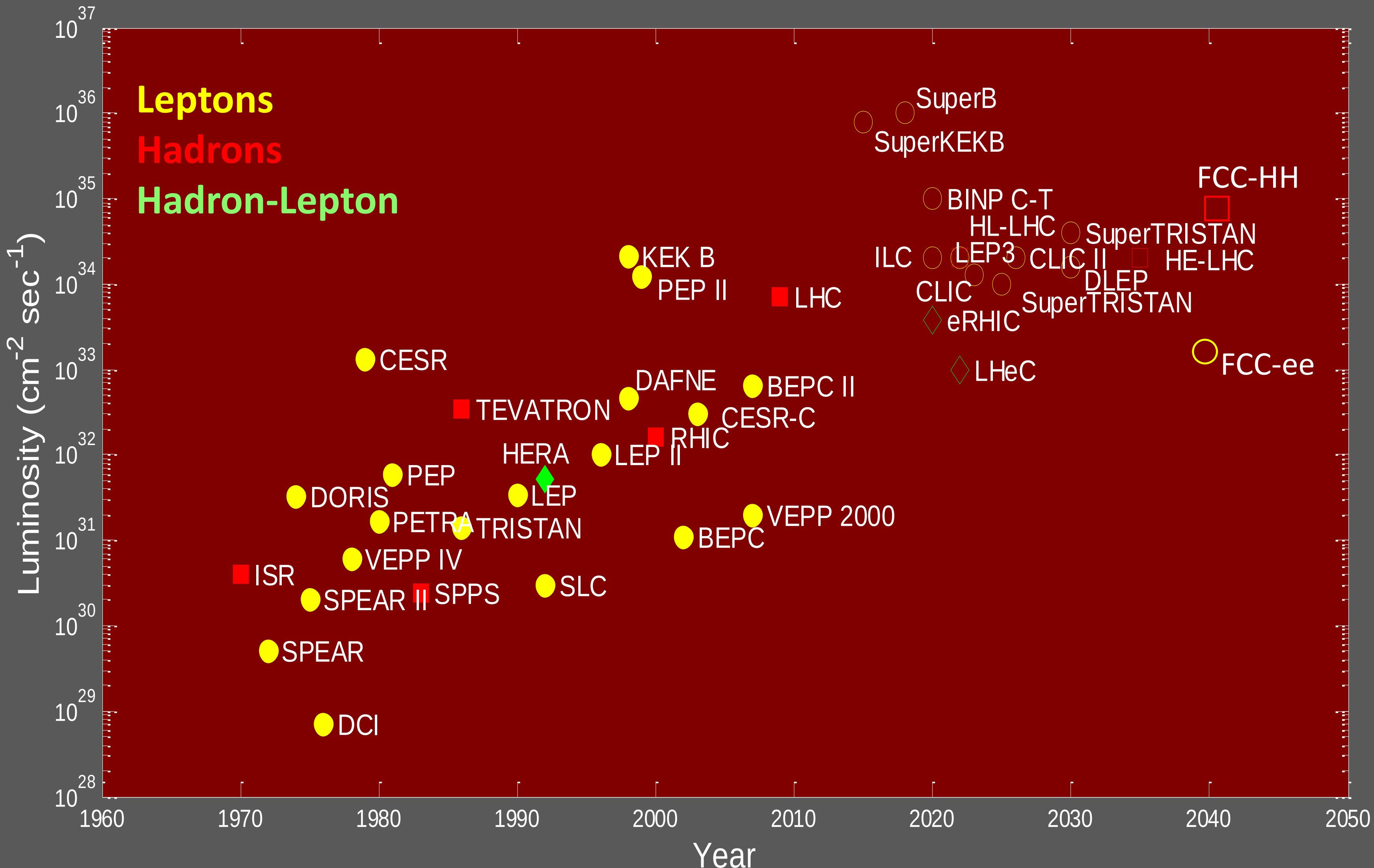
Colliders

Hadron colliders ->	Discovery experiments
Lepton colliders - >	Precision experiments
Hadron-Lepton Colliders ->	Leptons used to probe hadron structures

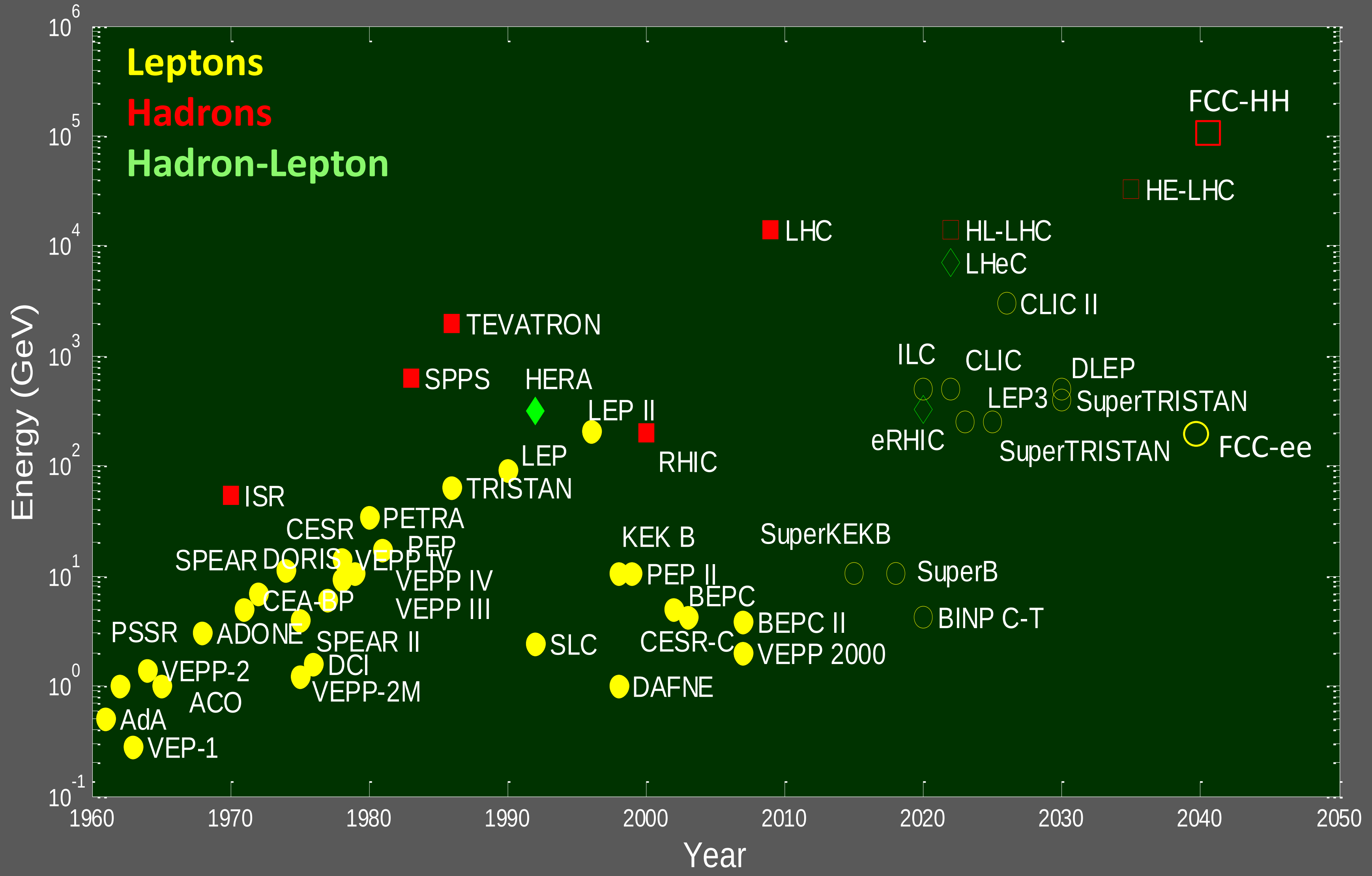
Collider energies



Collider luminosities



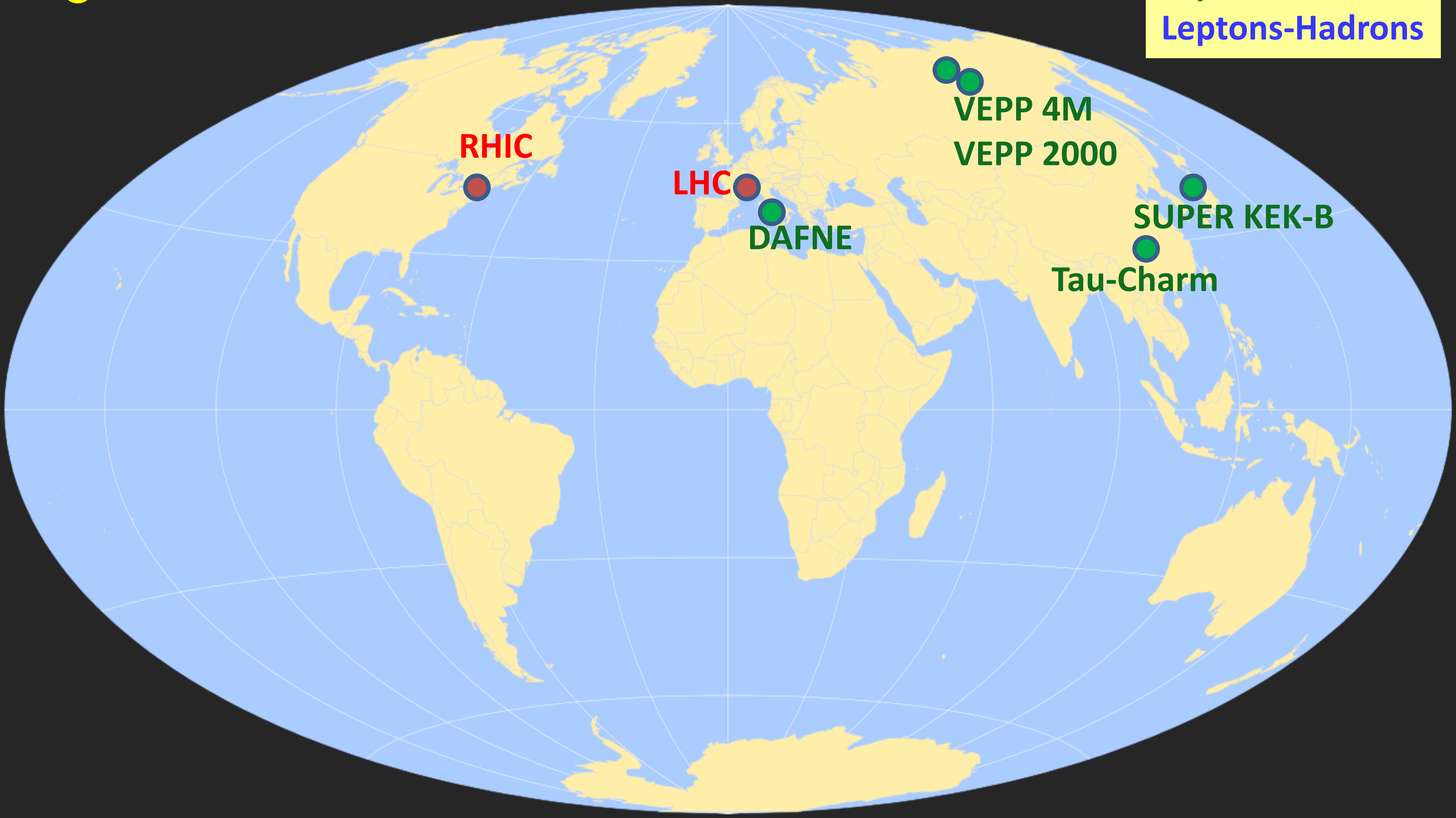
Collider energies



Particle Colliders - 2018

- In operation
- In commissioning

Hadrons
Leptons
Leptons-Hadrons



Luminosity
How many particles collide in each crossing point?

Many particles per bunch

Low density

High density

Luminosity in a circular collider

intuitively: higher L if there are more particles and more tightly packed

Peak luminosity

$$L = f_{rev}\frac{kN_1N_2}{4\pi\sigma_x\sigma_y}$$

$N_{1,2}$ = # of particle per bunch
$\sigma_{x,y}$ = horizontal and vertical beam size at IP
k = # of colliding bunches
f_{rev} = revolution frequency

Low β* at IPs

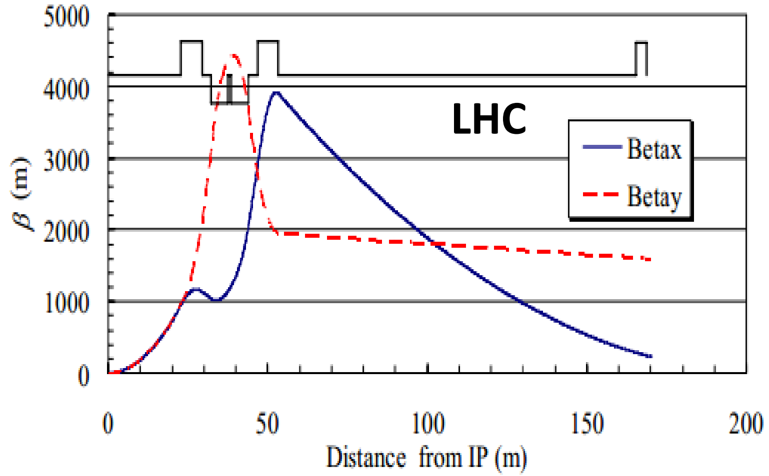


Fig. 2: β-functions near the IP with nominal LHC lay-out.

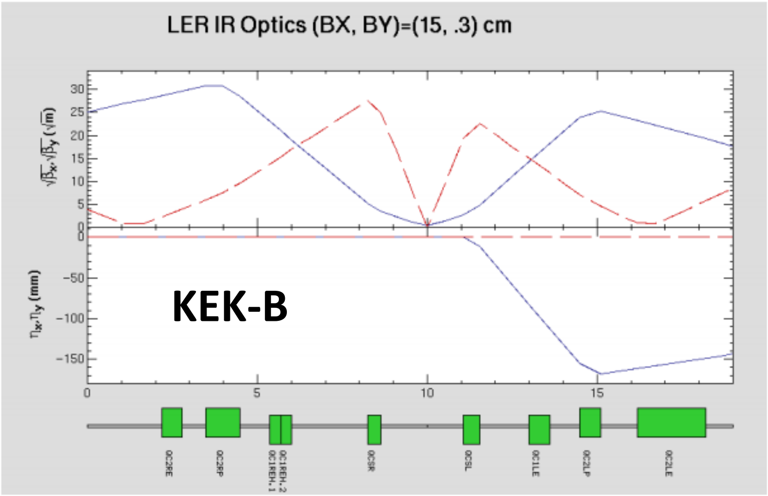


Figure 6: LER optics parameters around IP.

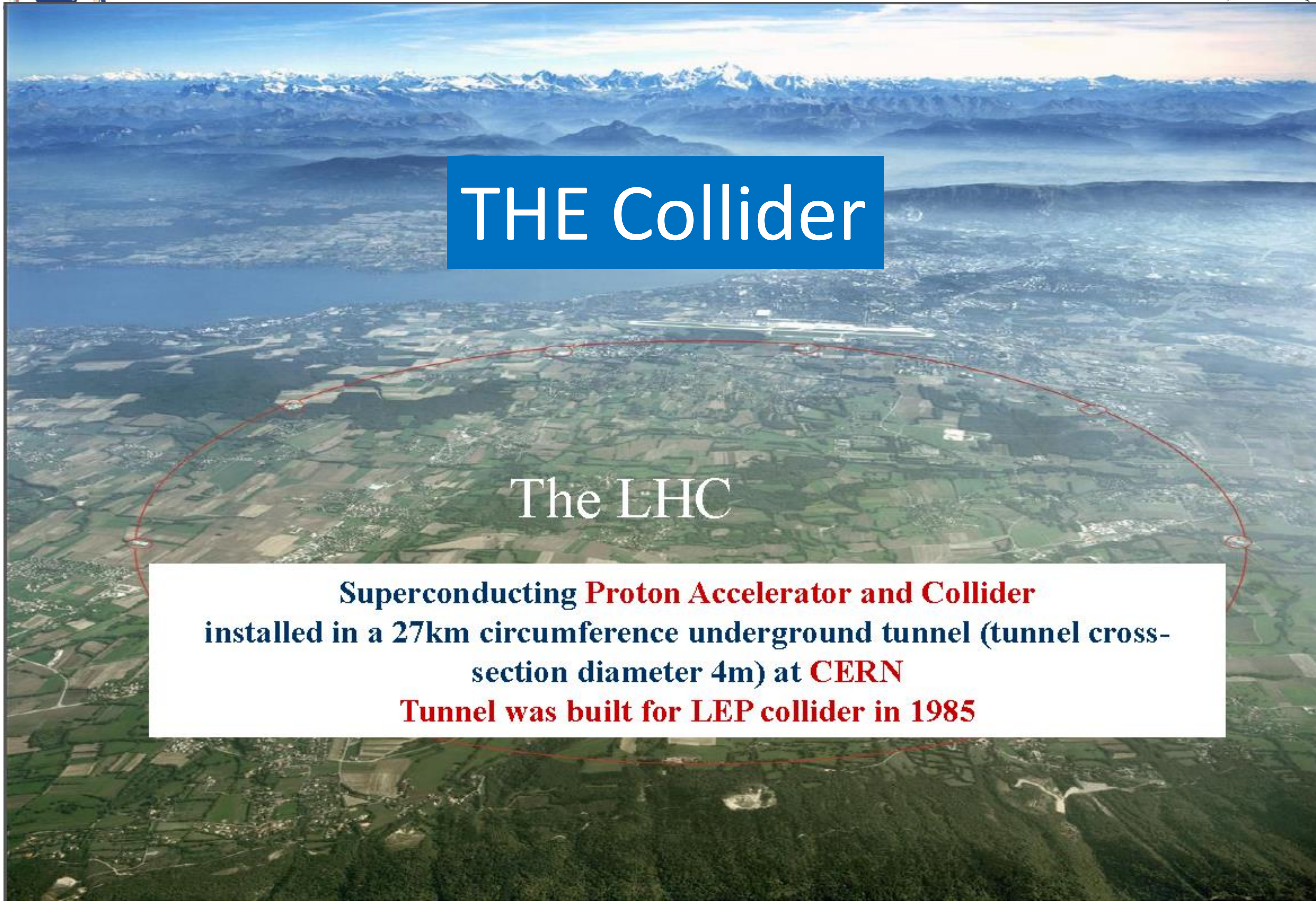
Proton beams:
Horizontal emittance = vertical emittance
Horizontal β^* = Vertical β^*

Electron-positron beams
Horizontal emittance >> Vertical emittance
Usually Horizontal β^* > Vertical β^*

$$\sigma^*{}_{x,y} = \sqrt{\varepsilon_{x,y}\,\beta^*{}_{x,y}}$$

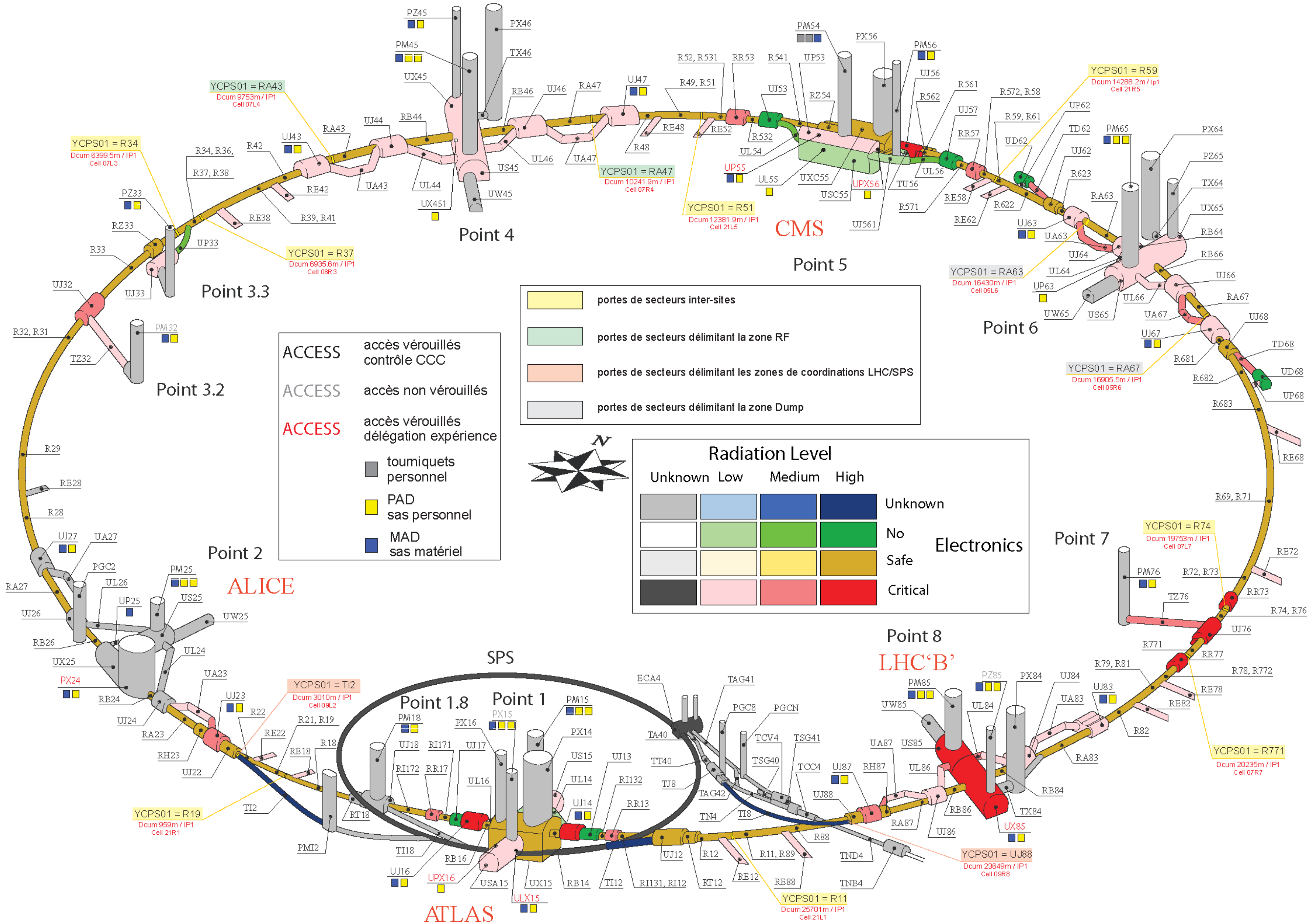
$$L = \frac{k f N_1 N_2}{4\pi\sqrt{\beta^*{}_x\,\beta^*{}_y\,\varepsilon_x\varepsilon_y}}$$

THE Collider



Superconducting Proton Accelerator and Collider
installed in a 27km circumference underground tunnel (tunnel cross-section diameter 4m) at CERN
Tunnel was built for LEP collider in 1985

LHC Layout



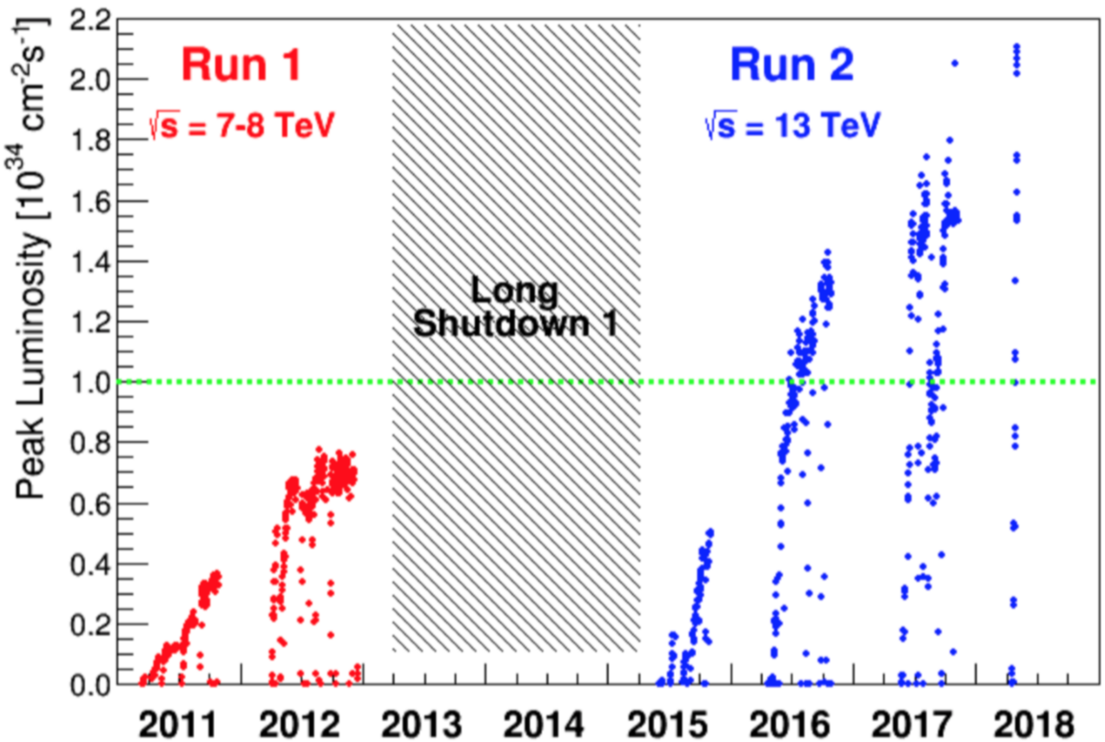
Point 1
Point 1.8
Point 2
Point 3.2
Point 3.3
Point 4
Point 5
Point 6
Point 7
Point 8
SPS
ATLAS
ALICE
CMS
LHC'B'
ACCESS accès vérouillés contrôle CCC
ACCESS accès non vérouillés
ACCESS accès vérouillés délégation expérience
tourniquets personnel
PAD sas personnel
MAD sas matériel
portes de secteurs inter-sites
portes de secteurs délimitant la zone RF
portes de secteurs délimitant les zones de coordinations LHC/SPS
portes de secteurs délimitant la zone Dump
N
Radiation Level
Unknown Low Medium High
Unknown
No
Safe
Critical
Electronics
YCPS01 = R34
YCPS01 = RA43
YCPS01 = R37
YCPS01 = RA47
YCPS01 = R51
YCPS01 = R59
YCPS01 = RA63
YCPS01 = RA67
YCPS01 = R74
YCPS01 = R771
YCPS01 = UJ88
YCPS01 = R11
YCPS01 = R19
YCPS01 = TI2

LHC arc view

27 km cooled down to 2K
The coolest place in the world

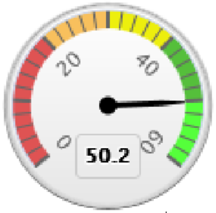
Latest news on LHC luminosity (from Fabiola Gianotti comm)

- First beams on 30 March → intensity ramp-up completed 5 May (10 days ahead of schedule)
- BCMS scheme, ~1.2 10^{11} p/bunch, 2550 bunches (max number, limited by transfer lines)
- Achieved peak luminosity 2.1×10^{34} cm^{-2} s^{-1}
- Pile-up in ATLAS, CMS: average ~ 50 events/x-ing, max ~60 events/x-ing
- ~ 50% of time in stable beams
- Total integrated luminosity to ATLAS and CMS: ~ 5.2 fb^{-1} (goal for the year is 60 fb^{-1})



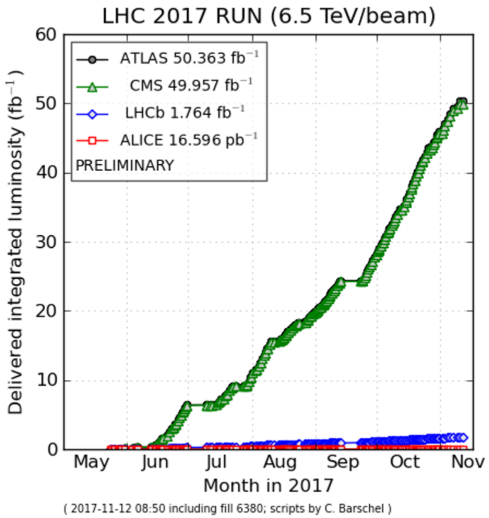
LHC 2017 : Integrated Performance

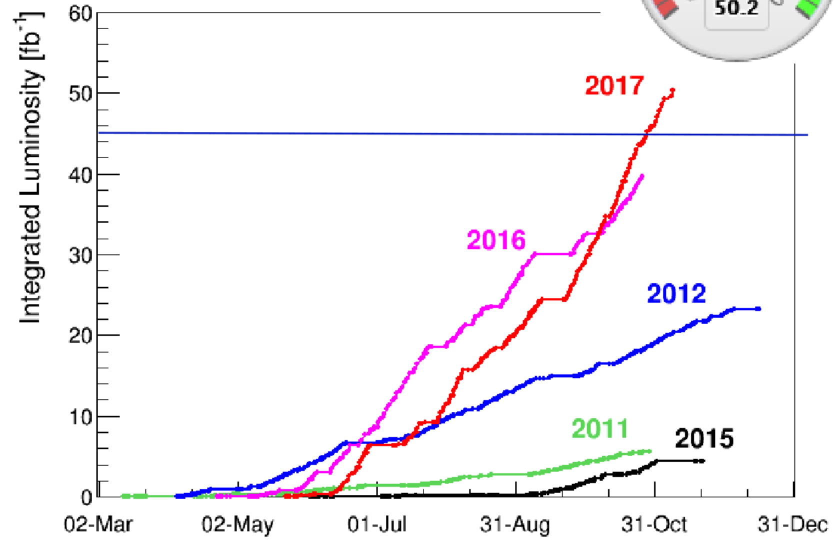
50 fb^{-1}



2017 goal:

45 fb^{-1}





From Bordry's comm

Peak luminosity
2.2 10^{34} cm^{-2} s^{-1}

With luminosity
levelling at
1.5 10^{34} cm^{-2} s^{-1}

Stable beams: 50%

Luminosity is measured in cm^{-2} sec^{-1}
Integrated luminosity is measured in $barn^{-1}$

1 barn = 10^{-24} cm^2; 1 pbarn = 10^{-36} cm^2; 1 barn = 10^{-39} cm^2

If <L> = 10^{34} cm^{-2} sec^{-1} => Integrated L/day = 864 $pbarn^{-1}$ = 0.864 $fbarn^{-1}$

50 fb-1 => 60 days at 10^{34} cm^{-2} sec^{-1} average

Linear colliders not abandoned waiting for decisions in Japan and for the next European and Japanese strategy for HEP

ILC

- Well extablished SC rf technology (TESLA, FLASH, EXFEL…)
- 2004: decision on technology for next linear collider up to 1 TeV
- 1.3 GHz, 31.5 MV/m
- Maximum energy 1 TeV cm - Phase I at 0.5 TeV
- GDE (Global Design Effort) - International collaboration
- Site independent

CLIC

- Dual beam acceleration technology
- R&D at CERN ~ 25 y
- Normal conducting cavities
- 12 GHz, 100 MV/m
- Maximum energy 3 TeV cm – Phase I at 0.5 TeV
- International collaboration around CTF3

After LHC: FCC?

new circular collider in Geneve area

FCC: 97.5 km

The future

time ←

parameter	FCC-hh		HE-LHC	HL-LHC	LHC
collision energy cms [TeV]	100		27	14	14
dipole field [T]	16		16	8.33	8.33
circumference [km]	97.75		26.7	26.7	26.7
beam current [A]	0.5		1.1	1.1	0.58
bunch intensity $[10^{11}]$	1	1	2.2	2.2	1.15
bunch spacing [ns]	25	25	25	25	25
synchr. rad. power / ring [kW]	2400		101	7.3	3.6
SR power / length [W/m/ap.]	28.4		4.6	0.33	0.17
long. emit. damping time [h]	0.54		1.8	12.9	12.9
beta* [m]	1.1	0.3	0.25	0.15 (min.)	0.55 (0.25)
normalized emittance [μm]	2.2		2.5	2.5	3.75
peak luminosity $[10^{34}\ cm^{-2}s^{-1}]$	5	30	28	5 (lev.)	1
events/bunch crossing	170	1000	800	132	27
stored energy/beam [GJ]	8.4		1.3	0.7	0.36

2043 – on 2040 – on
Preliminary design stage

2026-2037
funded program

2012-2023
in operation

HL-LHC
High Luminosity LHC

To extend its discovery potential, the LHC will need a major upgrade around 2020 to increase its luminosity (rate of collisions) by a factor of 10 beyond the original design value (from 300 to 3000 fb-1). As a highly complex and optimized machine, such an upgrade of the LHC must be carefully studied and requires about 10 years to implement.

Key innovative technologies, such as cutting-edge 13 Tesla superconducting magnets, very compact and ultra-precise superconducting cavities for beam rotation, and 300-metre-long high-power superconducting links with zero energy dissipation.

HL-LHC: L= 5×10^{34} $cm^{-2}s^{-1}$ → 3000 fb^{-1} by 2037 to ATLAS, CMS

Ultimate (from HW margins): 7×10^{34} $cm^{-2}s^{-1}$ → 4000 fb^{-1}

→ Major intervention on 1.2 km of accelerator: Nb_3Sn inner triplet quadrupoles and 11 T dipoles (tests of full-size prototypes of both in 2018); collimators; crab cavities; cryogenics; civil engineering, etc.

CtC materials: 950 MCHF → 15% spent (35% committed)

Summary

- Introduction to existing accelerators
- Electromagnetic fields
- Single particle dynamics – Beam emittances and dynamics
- Luminosity in a circular collider
- + (not covered in this introduction) collective effects: interaction between particles and their environment and among particles themselves

Simple exercises for Wednesday

A proton beam is injected in a synchrotton with the energy of 700keV, accelerated and extracted at the energy of 100 MeV. The dipolar magnetic field in the synchrotron during the acceleration

A) Increases quadratically with the magnetic rigidity
B) Is kept constant
C) Increases linearly with the magnetic rigidity

The revolution frecuency of a 600 MeV in a synchrotron with a 100 m circumference is

A) Higher than 100 kHz
B) Equal to 100 kHz
C) Lower than 100 kHz

During the acceleration at the LHC from 400 GeV to 6.5 TeV the transverse emittance

A) Is kept constant
B) Increases
C) Decreases

Simple exercises for Wednesday

A synchrotron for electrons at 2 GeV with a circumference of 408 m has a rf system with the frecuency of 500 MHz. The maximum number of bunches which can be stored is

A - > 500

B - = 500

C - < 500

At LHC the luminosity per bunch is $L_{bunch} = 10^{32}$ $cm^{-2}seg^{-1}$. To increase it is more effective:

A – decreasing by 10% the β^* value in both transverse plane

B – increase by 10% the n. of particles per bunch

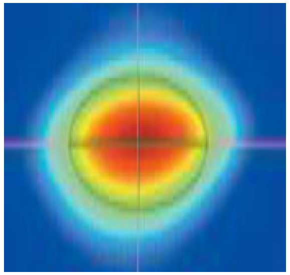
C – they are two equivalent effects

In a target used for diagnostics along a Linac the beam spot appears as in the figure. If you have a single image, and don't know the emittances, can you deduce:

A – $\beta_x = \beta_y$

B – $\beta_x > \beta_y$

C – the relationship between β_x and β_y cannot be deduced



Energy emitted in a ring

$$P_{SR} = \frac{2cr_e}{3\left(m_0c^2\right)^3}\frac{E^4}{\rho^2} = \frac{2r_ecm_0c^2}{3\rho^2}\gamma^4$$

Larmor formula:
Instantaneous power emitted by a particle (by integrating the Poynting vector)

$$U_0 = \int_{finite\ \rho} P_{SR}\,dt \quad = \frac{2}{3}r_em_oc^2\beta^3\gamma^4\oint\frac{ds}{\rho^2} = C_\gamma\frac{E^4\left(GeV^4\right)}{\rho(m)} \propto \gamma^4 I_2$$

$$C_\lambda = \frac{4\pi}{3}\frac{r_e}{\left(mc^2\right)^3} = 8.846\cdot10^{-5}\frac{m}{GeV^3} \qquad for\ e^-, e^+$$

Energy emitted per turn by every particle. Note the strong dependence on γ

Emitted power per turn by N_{tot} electrons (positrons) and protons (antiprotons)

$$P_e(kW) = \frac{e\gamma^4}{3\varepsilon_0\rho}I_b = 88.46\frac{E(GeV)^4 I(A)}{\rho(m)}$$

$$P_p(kW) = \frac{e\gamma^4}{3\varepsilon_0\rho}I_b = 6.03\frac{E(TeV)^4 I(A)}{\rho(m)}$$

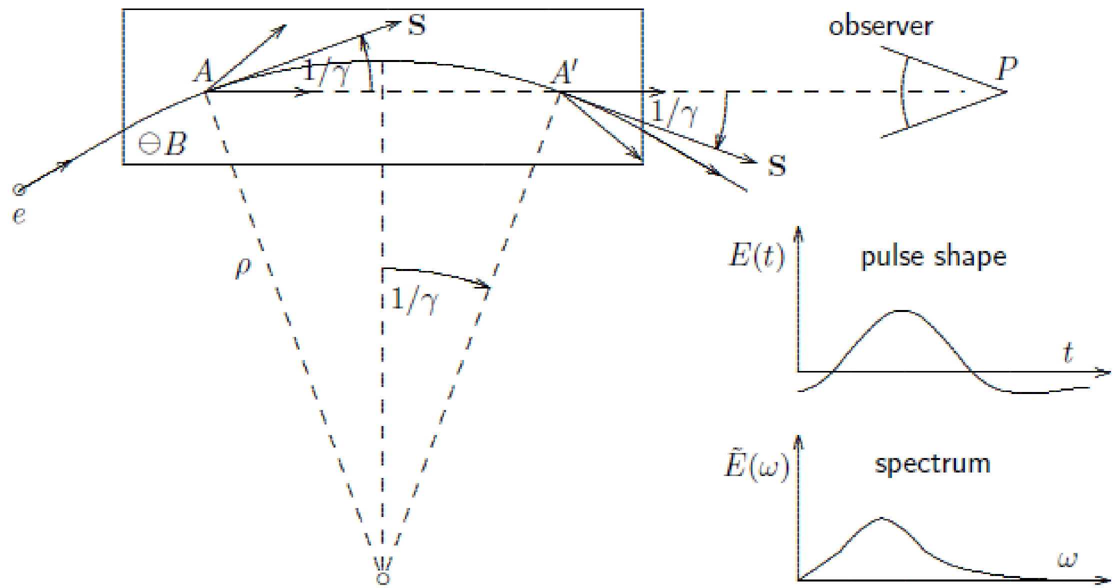
$$I_2 = \oint\frac{ds}{\rho^2}$$

$$r_e = \frac{e^2}{4\pi\varepsilon_o m_o c^2}$$

$$N_{tot} = \frac{I\cdot T_o}{e}$$

Typical emission frequency

The photon emitted at A is seen when passing by A', after t_γ
The electron which emits at A' arrives at A', after t_e



Difference in time between the electron and the photon travel

For γ>>1

$$\Delta t = t_e - t_\gamma = \frac{2\rho}{\beta\gamma c} - \frac{2\rho\sin(1/\gamma)}{c} \sim \frac{2\rho}{\beta\gamma c}\left(1 - \beta\gamma\left(\frac{1}{\gamma} - \frac{1}{3!\gamma^3}\right)\right) \sim \frac{2\rho}{\gamma c}\left(\frac{1}{2\gamma^2} + \frac{1}{6\gamma^2}\right) = \frac{4}{3}\frac{\rho}{c\gamma^3}$$

The frequency corresponding to half the pulse length is the critical frequency

$$\omega_c \sim \frac{1}{\frac{1}{2}\Delta t} \sim \frac{3c\gamma^3}{2\rho}$$

Critical energy

The energy at which the SR is higher is the critical energy, which is obtained from the critical frequency

$$\varepsilon_c = \hbar\omega_c = C_c \frac{E^3}{\rho} \qquad C_c = \frac{3\hbar c}{2(mc^2)^3}$$

For electrons we can write

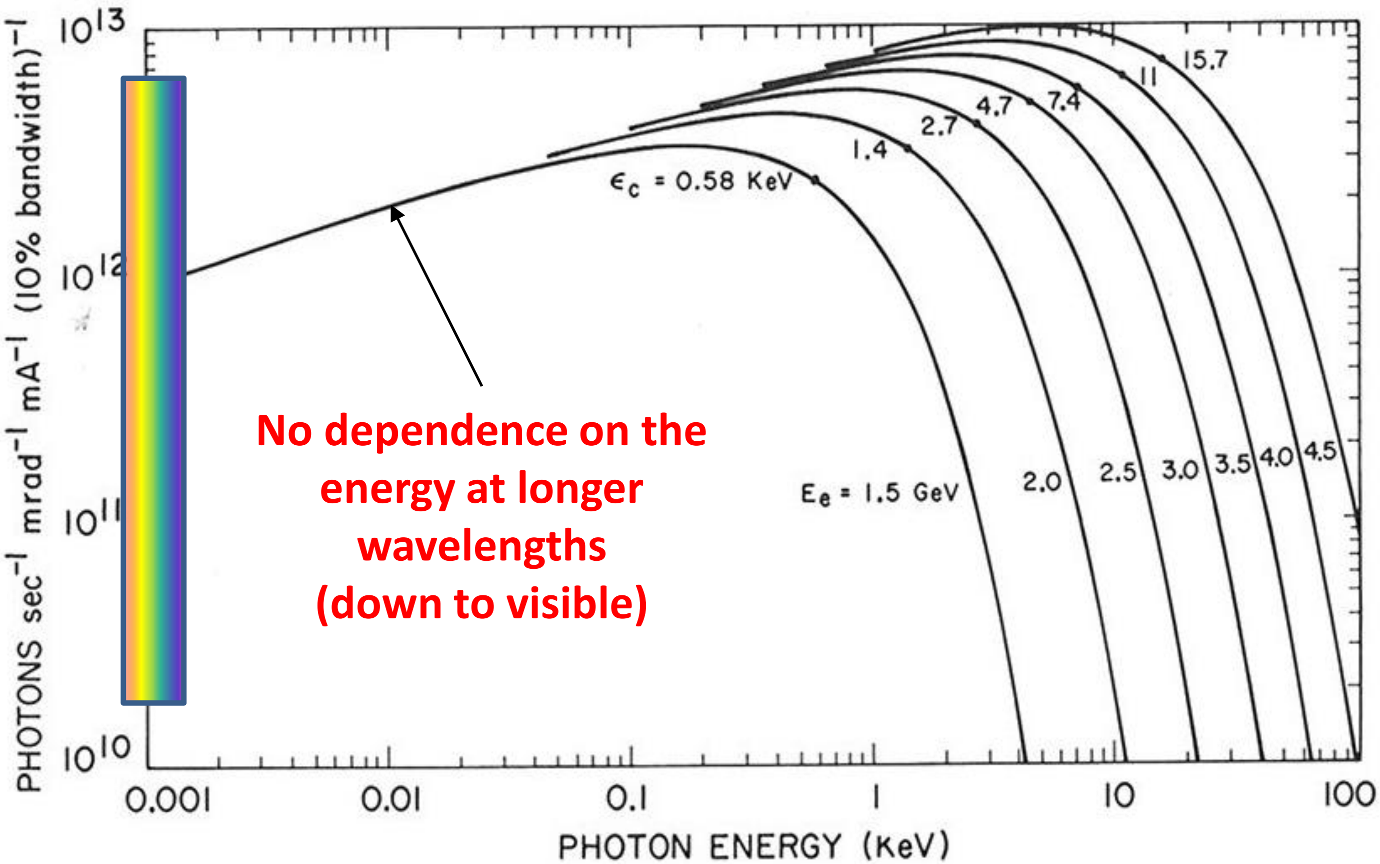
$$\varepsilon_c(keV) = 2.2183 \frac{E^3(GeV^3)}{\rho(m)} = 0.66503 E^2(GeV^2) B(T)$$

The higher the bending field the higher the SR photon critical energy

The SR spectrum in a circular accelerator is made up of harmonics of the particle revolution frequency up and beyond the critical frequency, not much separated and with beamline spread, so that the spectrum appears continuous.

Synchrotron radiation emission as a function of beam energy

Dependence of the frequency distribution of the energy radiated via synchrotron emission on the electron beam energy (same ρ)



Effect of synchrotron radiation emission on the emitting particle beam dynamics

Quantum nature of radiation

Radiation damping

Emittance going versus zero

Quantum excitation

Equilibrium distribution

Emittance

The emittance is determined by a balance between two competing processes: quantum excitation of betatron oscillations from photon emission and longitudinal re-acceleration within the RF cavities

The emittance depends on the dispersion and on the betatron functions in the dipoles, and on the energy

$$\varepsilon_x = \frac{55}{32\sqrt{3}} \frac{\hbar}{mc} \frac{\gamma^2}{J_x} \frac{\langle H/\rho^3 \rangle}{\langle 1/\rho^2 \rangle}$$

$$H(s) = \gamma D_x^{\,2} + 2\alpha D_x D_x' + \beta D_x'^{\,2}$$

J_x is the Robinson partition number evaluated for the horizontal plane
The emission of photons is done in bendings, where there is dispersion. The electron amplitude oscillation afterwards is given by the dispersion, the original amplitude oscillation and energy loss
The smaller the dispersion the smaller the final equilibrium emittance: increasing the n. of dipoles in a ring the dispersion decreases and so does the emittance