



# Bright Solitons in the Electrooxidation of CO on Platinum Thin Film Electrode

A Work done by  
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African School of Fundamental Physics and Applications (ASP) Windhoek Namibia  
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 Introduction  
 The Chemistry of electrooxidation  
 The Strasser–Eiswirth– model  
 Derivation of Korteweg de-Vries (KdV) equation  
 Complex Ginzburg–Landau  
 Stationary solitary

- 1 Introduction
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- The combustion of fossil fuels releases emissions into the air which causes pollution.
- W. R. Grove in 1839 [1], using the knowledge of electrolysis discovered the fuel cell.
- Fuel cells are considered as a prime candidate for the "green" energy production : clean, quiet.
- Fuel cells are electrochemical devices, that use chemical energy from a fuel (hydrogen, methanol, etc.) and an oxidant (air or oxygen) and PLATINUM catalyst to produce electrical energy.
- The 'reforming' fuels such as natural gas or methanol, introduces CO into the hydrogen gas, which poisons the platinum catalyst.
- The electrochemical oxidation of CO on Pt is an electrolytic reaction through which the CO is removed from the Pt surface.
- SOLITARY waves were first observed in this process in 1992 by Rotermund et . al [2] using PEEM spectroscopy.
- In 2005, Bauer et.al experimentally observed dissipative SOLITONS in the electrooxidation of CO on Pt using FTIR spectroscopy in the ATR configuration.

## Purpose and method

- Analytical proof of bright solitons in the electrooxidation of CO on Platinum electrode observed experimentally.
- The perturbation analysis ; reductive perturbation and multiple scale expansion.

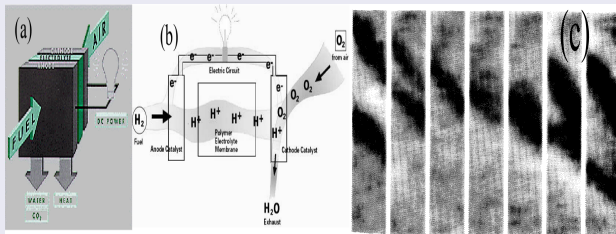
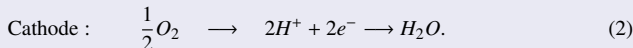
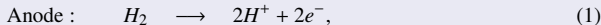


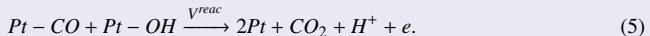
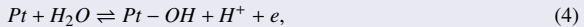
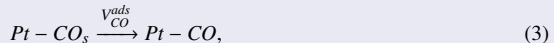
FIGURE: (a.) Fuel cell. (b) Schematic diagram .(c) Experimental observation of soliton collision of CO coverage [2].



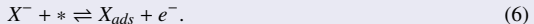
# The Chemistry of electrooxidation

## The Chemistry of electrooxidation

- Mean field Langmuir–Hindshelwood (L-H) mechanism is used to describe the reaction.
- Simulations are usually based on three elementary reaction steps; adsorption of CO, oxidative adsorption of water and reaction of adsorbed CO and OH molecules



- In Souradip et al. [4] it was shown that, the additional competitive adsorption of anions such as  $Cl^-$ , blocks free surface sites for OH and CO, and may induce oscillations;



- As long as the OH coverage remains very small, the reaction rate can be expressed without taking explicitly the coverage of OH into account, Zhang et al. [5].

# The Strasser–Eiswirth–Ertl (KEE) model

- The temporal evolution of the chemical subsystem, consisting of  $\Theta_{CO}$ , the CO coverage of the electrode, and  $\Theta_X$ , the anion coverage is then given by [4]

$$\begin{aligned} \partial_t \Theta_{CO} &= v_{CO}^{ads} - v^{reac} + D_0 \partial_{xx} \Theta_{CO}, \\ \partial_t \Theta_X &= v_X^{ads} - v_X^{des} + D_0 \partial_{xx} \Theta_X, \\ C \partial_t \phi_{DL} &= -S_{tot} F (2v^{reac} + (v_X^{ads} - v_X^{des})) - \sigma \partial_z \phi_{DL}|_{z=WE}, \end{aligned} \quad (7)$$

where the corresponding adsorption, desorption and reaction rates are given by the following expressions :

$$\begin{aligned} v_{CO}^{ads} &= k_{CO}^{ads} c_s (0.99 - \Theta_{CO} - \Theta_X), \\ c_s &= \frac{c_b D_0}{D_0 + S_{tot} k_{CO}^{ads} \delta (1 - \Theta_{CO} - \Theta_X)}, \\ v^{reac} &= k^{reac} (1 - \Theta_{CO} - \Theta_X) \Theta_{CO} \exp\left(\alpha \frac{F}{RT} \phi_{DL}\right), \\ v_X^{des} &= k_X^{des} \Theta_X \exp\left[(\alpha - 1) \frac{F}{RT} \phi_{DL}\right], \\ v_X^{ads} &= k_X^{ads} c_X \mathfrak{K} \left(0.99 - \Theta_{CO} - \frac{\Theta_X}{\Theta_X^{max}}\right) \exp\left(\alpha \frac{F}{RT} \phi_{DL}\right). \end{aligned} \quad (8)$$

# The model

- Let  $\Theta_{CO} = U$ ,  $\Theta_X = V$ ,  $\phi_{DL} = W$  and the equations reduce to

$$U_t = \frac{\bar{\alpha}(0.99 - U - V)}{D_0 + \beta(1 - U - V)} - k_X^{ads}(1 - U - V)U \exp(\alpha fW) + D_0 U_{xx}, \quad (9)$$

$$V_t = \gamma(0.99 - U - V/V^{max}) \exp(\alpha fW) - k_X^{des} V \exp(gW) + D_0 V_{xx}, \quad (10)$$

$$W_t = -\gamma_1(1 - U - V)U \exp(\alpha fW) - \gamma_2(0.99 - U - V/V^{max}) \exp(\alpha fW) + \gamma_3 V \exp(gW) - \sigma \partial_z W|_{z=WE}, \quad (11)$$

- where  $\bar{\alpha} = k_{CO}^{ads} c_b D$ ,  $\beta = S_{tot} k_{CO}^{ads} \delta$ ,  $\gamma = k_X^{ads} c_X \mathfrak{R}$ ,  $f = F/RT$ ,  $g = (\alpha - 1)F/RT$ ,  
 $\gamma_1 = \frac{2S_{tot} F k^{reac}}{C}$ ,  $\gamma_2 = \gamma \frac{S_{tot} F}{C}$ ,  $\gamma_3 = \frac{k_X^{ads} S_{tot} F}{C}$ ,  $\sigma_1 = \frac{\sigma}{C}$ .

# The model

## Hypothesis

- Since the coverages are very small, we can take the Binomial expansion of the quotient.
- We assume that the coverages are very small and that the electrode potential is close to the strictly potentiostatic case ( $W \ll 1$ ) and hence  $\exp(W) \approx 1$ .
- We neglect the surface diffusion of adsorbed CO, since it is small compared to the lateral diffusion of the bulk CO [4].
- We consider the homogeneous case of the migration coupling, where it vary linearly with the electrode potential.
- Using these assumptions and putting the equation in the Linard, we have

$$V_{tt} - \Omega_0 V_{xx} - (k_0 + k_1 V + k_2 V^2) V_t - (\bar{\Omega}_8 + \bar{\Omega}_9) V_t^2 - (\Lambda_0 V + \ell_0 V^2) V_{xx} - (\chi_2 + \chi_3) V_{xx}^2 - \eta_0 - \eta_1 V - \eta_2 V^2 - \eta_3 V^3 - (\Lambda_2 - \Lambda_3 V) V_t V_{xx} - \varpi_0 V_t V_{xx}^2 + \varpi_1 V_t^2 V_{xx} + \Gamma V_t^3 + \varpi V_{xx}^3 + D_0 V_{xxt} = 0,$$

$$W_t + (\tau_3 + \tau_4 V) V_t - (\pi_0 + \pi_1 V) V_{xx} - \pi_2 V_t^2 + \rho_0 V_t V_{xx} - \tau_4 V^2 - \tau_1 V - \tau_0 - \sigma_1 (W - U_0) - D_2^2 V_{xx}^2 = 0,$$

$$U = 0.99 - \beta_2 V - \alpha_2 V_t + D_0 V_{xx}. \quad (12)$$



# Derivation of Korteweg de-Vries (KdV) equation

## Reductive perturbation method

- We seek for weak amplitude wave, by applying the reductive perturbation technique.
- Constraint : nonlinearity balances dispersion.
- At the order  $O(\epsilon^{3/2})$ , we have ;

$$-k_0 V_T + k_1 u_0 VV_S + D_0 u_0 V_{SSS} = \Omega_0 V_{SS} + u_0 (\varpi_0 + \Lambda_2) V_S V_{SS}, \quad (13)$$

- where  $\Omega_0 = \epsilon^{1/2} \Omega_0$ ,  $\Lambda_2 = \Lambda_2/\epsilon$ ,  $\varpi_0 = \varpi_0/\epsilon$ .
- After applying the scaling on this equation, we have the perturbed kdV given by

$$V'_\tau - 6V'V'_\epsilon + V'_{\epsilon\epsilon\epsilon} = \gamma V'_{\epsilon\epsilon} - \beta V'_\epsilon V'_{\epsilon\epsilon}, \quad (14)$$

where  $\tau = \frac{u_0 k^{3/2} D_0}{k_0} T$ ,  $\epsilon = -\sqrt{k_1} S$ ,  $V = -6D_0 V'$ .

- This equation is known as the Modified kdV-Burger (MkdVB) equation.

# Derivation of KdV

- The unperturbed equation is given by ;

$$V'_\tau - 6V'_\epsilon V'_\epsilon + V'_{\epsilon\epsilon\epsilon} = 0. \quad (15)$$

- This equation has a one soliton solution given by Kivshar et al. [5] which has the form

$$V' = -2k^2 \operatorname{sech}^2 z, \quad (16)$$

where  $z = k(\epsilon - \zeta)$  and  $\zeta$  is the phase.

- Then, using the perturbation theory based on the inverse scattering transform that predicts the temporal evolution of the amplitude  $k$ , and the phase  $\zeta$  we have

$$\frac{dk}{d\tau} = -\frac{12\gamma k}{15}, \quad (17)$$

$$\frac{d\zeta}{d\tau} = 4k^2 - \frac{112\beta k}{105}. \quad (18)$$

- Solving these, we see that  $k(\tau) = A_0 \exp(-\alpha\tau)$  which implies that the dissipation brings about the exponential decay of the amplitude.
- Also, the phase  $\zeta(\tau) = 4k^2\tau + \sigma k\tau$ .

# Derivation of KdV

- Substituting these expression in the unperturbed solution, it then become

$$V'(\varepsilon, \tau) = -2a_0^2 \exp(-2\alpha\tau) \operatorname{sech}^2 \left[ a_0 \exp(-\alpha\tau) (\varepsilon - 4a_0^2 \exp(-2\alpha\tau)\tau + a_0\sigma \exp(-\alpha\tau)\tau) \right]. \quad (19)$$

In terms of the original variable, it is given by

$$\Theta_X = Ae^{-2\lambda t} \operatorname{sech}^2(X), \quad (20)$$

- where  $X = a_0 e^{-\lambda t} (-qx + (p + qu_0)t - 4e^{-\lambda t} \lambda)$  and  $A = 12a_0 D_2$ .

- From the coupling equation, the CO coverage is given by

$$\begin{aligned} \Theta_{CO} = & 0.99 - \beta_2 A e^{-2\lambda t} \operatorname{sech}^2(X) - 2\lambda \bar{\alpha}_2 A e^{-\lambda t} \operatorname{sech}(X) - 2a_0 \lambda c A e^{-3\lambda t} \operatorname{sech}^2(X) \tanh(X) \\ & + 8\lambda A e^{-4\lambda t} \operatorname{sech}^2(X) \tanh(X) + 4D_2 q^2 a_0^2 A e^{-4\lambda t} \operatorname{sech}^2(X) \tanh^2(X) \\ & - 4D_2 a_0^2 q^2 A e^{-4\lambda t} \operatorname{sech}^4(X). \end{aligned} \quad (21)$$

# Derivation of kdv

## Model graphs

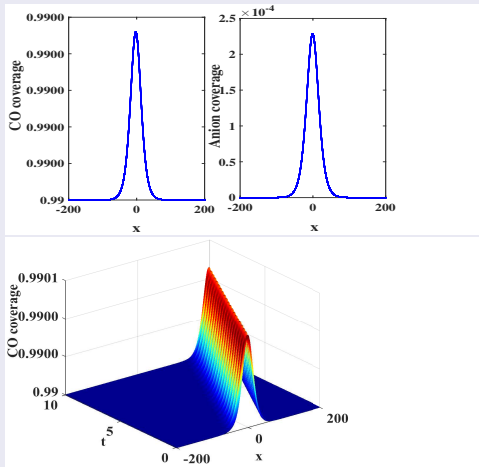


FIGURE: CO and Anion coverage for the KdV-Burger equation :  $c = 100, \lambda = 0.005, q = 5, \alpha = 0.005$

# Complex Ginzburg–Landau (CGL) equation

## Multiple scale expansion method

- This is a perturbation technique in which, the wavelength of periodic oscillations of the carrier wave is comparable to the envelope width, the latter looks like it is breathing [6].
- Constraint : nonlinearity and dispersion are not balanced.
- In order to determine the order of the different terms, we introduce the variable  $V = \epsilon\phi$  and  $W = \epsilon\varphi$ .
- We suppose that  $D_0$  is perturbed to the order  $\epsilon^2$ . This is due to the roughness of the anode which increases the process of reaction and the equations become

$$\begin{aligned}
 \phi_{tt} & - \Omega_0\phi_{xx} - \epsilon^2 D_0\phi_{xxt} - (k_0 + \epsilon k_1\phi + \epsilon^2\phi)\phi_t - \epsilon(\bar{\Omega}_8 + \epsilon\bar{\Omega}_9\phi)\phi_t^2 - (\epsilon\Lambda_0\phi + \epsilon^2\ell_0\phi^2)\phi_{xx} \\
 & - \epsilon(\chi_2 + \epsilon\chi_3\phi)\phi_{xx}^2 - \eta_0/\epsilon - \eta_1\phi - \epsilon\eta_2\phi^2 - \epsilon^2\eta_3\phi^3 - \epsilon(\Lambda_2 - \epsilon\Lambda_3\phi)\phi_t\phi_{xx} \\
 & - \varpi_0\epsilon^2\phi_t\phi_{xx}^2 + \epsilon^2\varpi_1\phi_t^2\phi_{xx} + \epsilon^3\Gamma\phi_t^3 + \epsilon^2\varpi\phi_{xx}^3 = 0,
 \end{aligned} \tag{22}$$

$$\begin{aligned}
 \varphi_t & + (\tau_3 + \epsilon\tau_4\phi)\phi_t - (\pi_0 + \epsilon\pi_1\phi)\phi_{xx} - \epsilon\pi_2\phi_t^2 + \epsilon\rho_0\phi_t\phi_{xx} - \epsilon\tau_2\phi^2 - \tau_1\phi - \tau/\epsilon \\
 & + \sigma_1(W - U_0) - \epsilon D_2^2 v_{xx}^2 = 0.
 \end{aligned} \tag{23}$$

# Equation of Motion of the Amplitude

- We now look for modulated solution of the form

$$\phi = A(X_1, T_1, X_2, T_2)e^{i\theta} + c.c + \epsilon[C(X_1, T_1, X_2, T_2) + D(X_1, T_1, X_2, T_2)e^{2i\theta}] + c.c + O(\epsilon^2), \quad (24)$$

$$\varphi = F(X_1, T_1, X_2, T_2)e^{i\theta} + c.c + \epsilon[G(X_1, T_1, X_2, T_2) + H(X_1, T_1, X_2, T_2)e^{2i\theta}] + c.c + O(\epsilon^2). \quad (25)$$

# Multiple scale expansion

- At the order  $\epsilon^0$ , the annihilation of terms in  $e^{i\theta}$ , gives the dispersion relation of linear waves of the system.

$$\omega^2 = \Omega_0 k^2 + \eta_1 \quad (26)$$

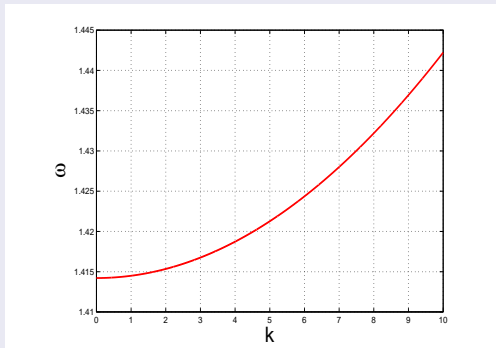


FIGURE: The dispersion relation of the reaction pulse :  $\Omega_0 = 0.008, \eta_1 = 2$ .

# Multiple scale expansion

- At the order  $\epsilon^1$ , the cancellation of terms in  $e^{i\theta}$  gives the solvability condition

$$\frac{\partial A}{\partial T_1} + V_g \frac{\partial A}{\partial X_1} = 0. \quad (27)$$

- At the order  $\epsilon^2$ , the cancellation of terms in  $e^{i\theta}$  and using the transformation  $\xi_i = X_i - V_g T_i$  and  $\tau_i = T_i$  yields

$$i \frac{\partial A}{\partial \tau_2} + P \frac{\partial^2 A}{\partial \xi_1^2} + Q |A|^2 A = i \frac{R}{2} A. \quad (28)$$

- This equation shows that, the evolution of modulated waves in this flow cell model is described by the Complex Ginzburg-Landau equation.

- where the nonlinearity and dispersion coefficients Q is complex while R and P are respectively real and are given by  $Q = \bar{R} - i\bar{F}$  and  $R = \frac{\bar{G}}{\omega}$ , where the expressions of the coefficients are

$$\bar{G} = k_0 - D\omega k, \quad P = \frac{\Omega_0 - V_g}{2\omega}, \quad \bar{F} = \frac{\varpi_0 \omega^4 - \omega k - 2k_2}{2}, \quad (29)$$

$$\bar{R} = \frac{3\ell_0 \omega^2 - 3\eta_3 - \Omega_9 \omega^{-3} \chi_3 - \varpi_1 - 3\omega_2^6 + \frac{6\eta_2^2}{\eta_1}}{2\omega}. \quad (30)$$



- We look for a solution of the form  $A(\tau_2, \xi_1) = B(\xi_1)e^{-i\omega\tau_2}$  using the method in Soto-Crespo et al [3], we have our solution in the original reference frame to be

$$\Theta_X = N \operatorname{sech}(Z_1) \cos(Z_2) - \frac{\eta_2}{\eta_1} N^2 \operatorname{sech}^2(Z_1) \cos^2(Z_2), \quad (31)$$

where  $Z_1 = \epsilon \sqrt{\beta}(x - x_0)$ ,  $Z_2 = \Phi + kx - \delta\omega t$ ,  $N = 2\epsilon \sqrt{\frac{\beta}{\alpha}}$  and  $M = (k + \Phi_x)$  with  $b = V_g + \epsilon V_p$  and  $\delta = 1 + \epsilon^2$ .

- Similarly,

$$\begin{aligned} \phi_{DL} &= \frac{-(\omega^2\tau_3 + \tau)}{\omega} N \operatorname{sech}(Z_1) \cos(Z_2) \\ &+ 2\epsilon^2 \left[ \frac{4\tau_3 k \eta_2 \beta \sqrt{\beta}}{\eta_1 \alpha \omega} \operatorname{sech}^2(Z_1) \tanh(Z_1) \cos(2\Phi - 2\omega\tau_2) \cos(2\theta) \right. \\ &+ (\omega^2\tau_4 - \pi_2\omega^2 - \frac{\eta_2}{\eta_1}(4\tau_3\omega^2 + \tau)) \cos(2\theta) + \Phi' \omega k^2 \rho_0 \sin(2\theta) \\ &\left. + \frac{\beta}{\alpha} \Phi' \operatorname{sech}^2(Z_1) \sin(2\theta) (1 + \sqrt{\beta} \tanh(Z_1) \sin(2\Phi - 2\omega\tau_2)) \right]. \quad (32) \end{aligned}$$

From the coupling equation, we have

$$\begin{aligned}
 \Theta_{CO} = & 0.99 - N \operatorname{sech}(Z_1) \cos(Z_2) [\beta_2 + \bar{\alpha}_2 \epsilon b \tanh(Z_1)] + \frac{\eta_2 \beta_2}{\eta_1} N^2 \operatorname{sech}^2(Z_1) \cos^2(Z_2) \\
 & + \bar{\alpha}_2 N (\Phi_t - \delta \omega) \operatorname{sech}(Z_1) [\sin(Z_2) - \frac{2\eta_2}{\eta_1} N \sin(2Z_2)] + D_2 [\beta \epsilon^2 N \operatorname{sech}^3(Z_1) \cos(Z_2) \\
 & - \epsilon^2 N \operatorname{sech}(Z_1) \tanh(Z_1) \cos^2(Z_2) + (1 - \epsilon \sqrt{\beta}) M \operatorname{sech}(Z_1) \tanh(Z_1) \sin(Z_2) \\
 & - N \Phi_{xx} \operatorname{sech}(Z_1) (\sin(Z_2) - \frac{\eta_1}{\eta_2} \sin(2Z_2))] + M^2 \operatorname{sech}(Z_1) (\frac{2\eta_2}{\eta_1} N \cos(2Z_2) - \cos(Z_2)) \\
 & + \frac{2\beta \epsilon^2 \eta_2}{\eta_1} N^2 \operatorname{sech}^2(Z_1) \cos^2(Z_2) (2 \tanh(Z_1) - \operatorname{sech}(Z_1)) \\
 & + \frac{2\beta \epsilon^2 \eta_2}{\eta_1} N^2 M \tanh(Z_1) \operatorname{sech}^2(Z_1) \sin(2Z_2) \\
 & - \frac{\sqrt{\beta} \eta_2}{\eta_1} N^2 M \tanh(Z_1) \operatorname{sech}(Z_1) \sin(2Z_2)]. \tag{33}
 \end{aligned}$$

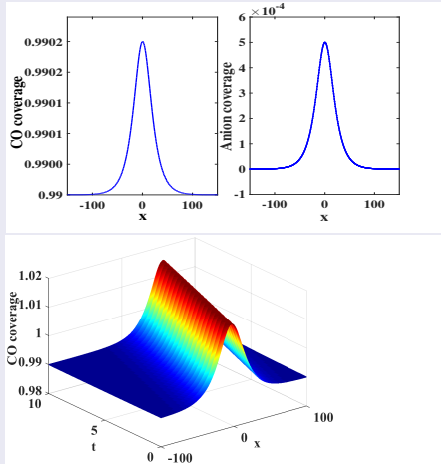


FIGURE: CO and Anion coverage for the CGL equation :  $\epsilon = 0.1, k = 0.003, x_0 = 0.03, \delta = 0.4, \Omega = 0.005$

## Conclusion

- We have analytically proven that, the reaction pulses in the electrooxidation process are solitons, a conjecture made by Krischer et al.
- We have considered the two different configurations of our system.
- We were able to show that this solitons are dissipative using the perturbation method.

## Opening

- We intend to study the backfiring effect observed experimental, by doing collision studies.

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Introduction

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Stationary  
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# THANK YOU

