AUGMENTED LAGRANGIAN METHOD IN A NON STATIONARY FLOW

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INTRODUCTION

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- about numerical resolution of a non stationnary flow problem

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- The method is Augmented Lagrangian Method(ALM)
- Aim : to show the efficiency of ALM.

OUTLINE

- PRINCIPLE OF ALM
- RESOLUTION ALGORITHM AND FINITE ELEMENT METHOD
- RESULTS
- CONCLUSION

PRINCIPLE OF ALM

- Principle :

$$\text{Physical problem} \longrightarrow \text{saddle points search problem}$$

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- Physical problem : two-dimensional flow of a newtonian fluid between two non-concentric cylinders

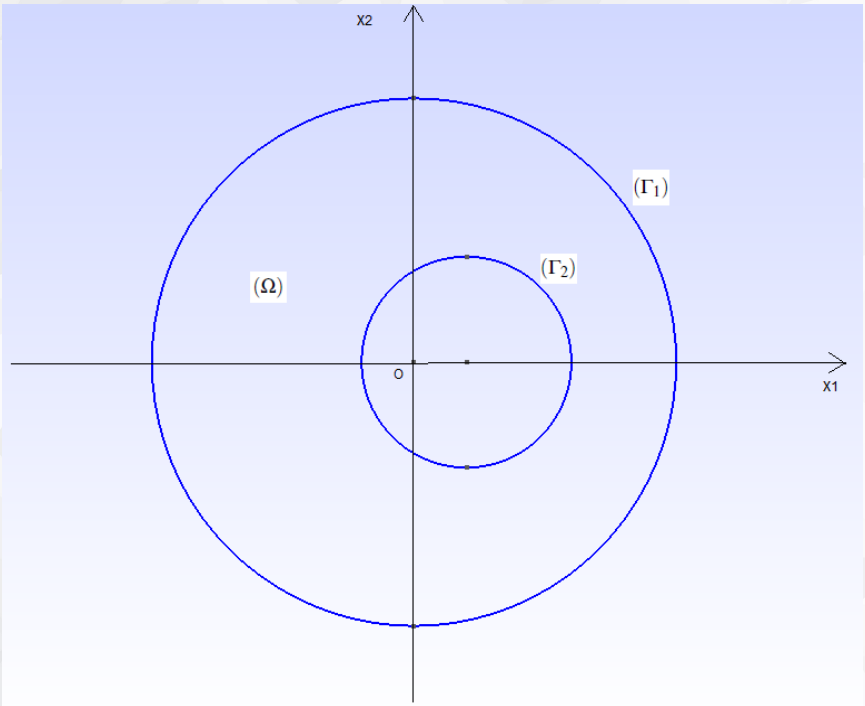

FIGURE 1 : Non-concentric cylinders

- Navier-Stokes equations

$$\frac{\partial \vec{u}}{\partial t} - \frac{1}{Re}\Delta \vec{u} + (\vec{u}.\vec{\nabla})\vec{u} + \vec{\nabla} p = \vec{f} \tag{1}$$

$$\vec{\nabla}.\vec{u} = 0 \tag{2}$$

$$\vec{u}(\vec{x}, 0) = \vec{u_0}(\vec{x}) \tag{3}$$

$$\vec{u_\Gamma} = \vec{U} \tag{4}$$

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- General form of an optimization problem :

$$V : \text{ linear space }, U \subset V, J : V \to \mathbb{R}$$

$$u \in U \qquad \text{et} \qquad J(u) = \inf_{v \in U} J(v) \qquad (\mathscr{P}).$$

AUGMENTED LAGRANGIAN METHOD

- based on the search for saddle points of the augmented Lagrangian $\mathscr{L}_r$:

$$\mathscr{L}_r(v,\lambda) = J(v) + \sum_{i=1}^{m} \lambda_i \varphi_i(u) + \frac{r}{2}\|\varphi(v)\|^2, \qquad r > 0. \tag{5}$$

where φ_i are the constraints, and a saddle point (u,p) is defined as point satisfying the following condition

$$\begin{aligned} \inf_{v\in\mathbb{R}^n} \sup_{\lambda\in\mathbb{R}^m_+} \mathscr{L}(v,\lambda) &= \sup_{\lambda\in\mathbb{R}^m_+} \mathscr{L}(u,\lambda) = \mathscr{L}(u,p) \\ &= \inf_{v\in\mathbb{R}^n} \mathscr{L}(v,p) = \sup_{\lambda\in\mathbb{R}^m_+} \inf_{v\in\mathbb{R}^n} \mathscr{L}(v,\lambda) \end{aligned} \tag{6}$$

RESOLUTION ALGORITHM AND FINITE ELEMENT METHOD

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RESOLUTION ALGORITHM

$$u^{n+1,0},\ p^1 \text{ arbitrarily chosen,} \quad (7)$$

$u^{n+1,s-1}$ et p^s being known, we search for $u^{n+1,s}$ solution of

$$u^{n+1,s} - k\frac{1}{Re}\Delta u^{n+1,s} - kr\nabla(\nabla.u^{n+1,s}) + k(u^{n+1,s-1}.\nabla)u^{n+1,s}$$

$$+\frac{k}{2}(\nabla.u^{n+1,s-1})u^{n+1,s} + k\nabla p^s = F \quad (8)$$

$$u_{\Gamma_1}^{n+1,s} = w((n+1)\Delta t) \quad (9)$$

$$u_{\Gamma_2}^{n+1,s} = 0 \quad (10)$$

$$\text{then } p^{s+1} \text{ by : } p^{s+1} = p^s - \zeta(\nabla.u^{n+1,s}) \quad (11)$$

- Convergence condition : k little enough.

FINITE ELEMENT METHOD

PRINCIPLE :

$$PDE \quad -\text{finite element method} \longrightarrow \quad [\mathrm{M}][\mathrm{U}]=[\mathrm{V}]$$

where $[U]$ is the unknown vector, $[M]$ a matrix and $[V]$ a vector.

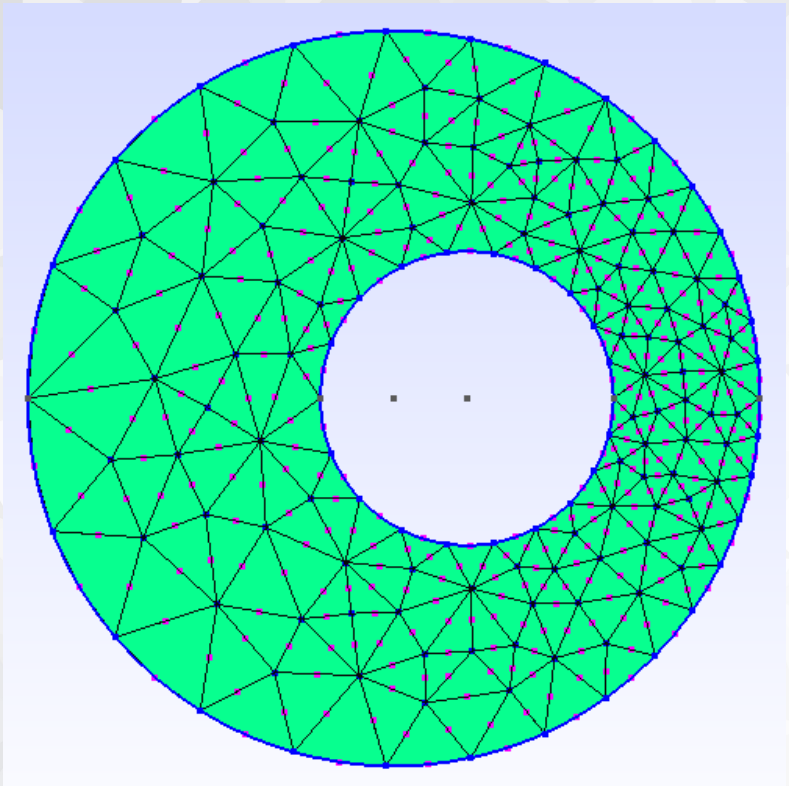

FIGURE 2 : The meshed domain

RESULTS

- optimal parameters
- Velocity and pressure field

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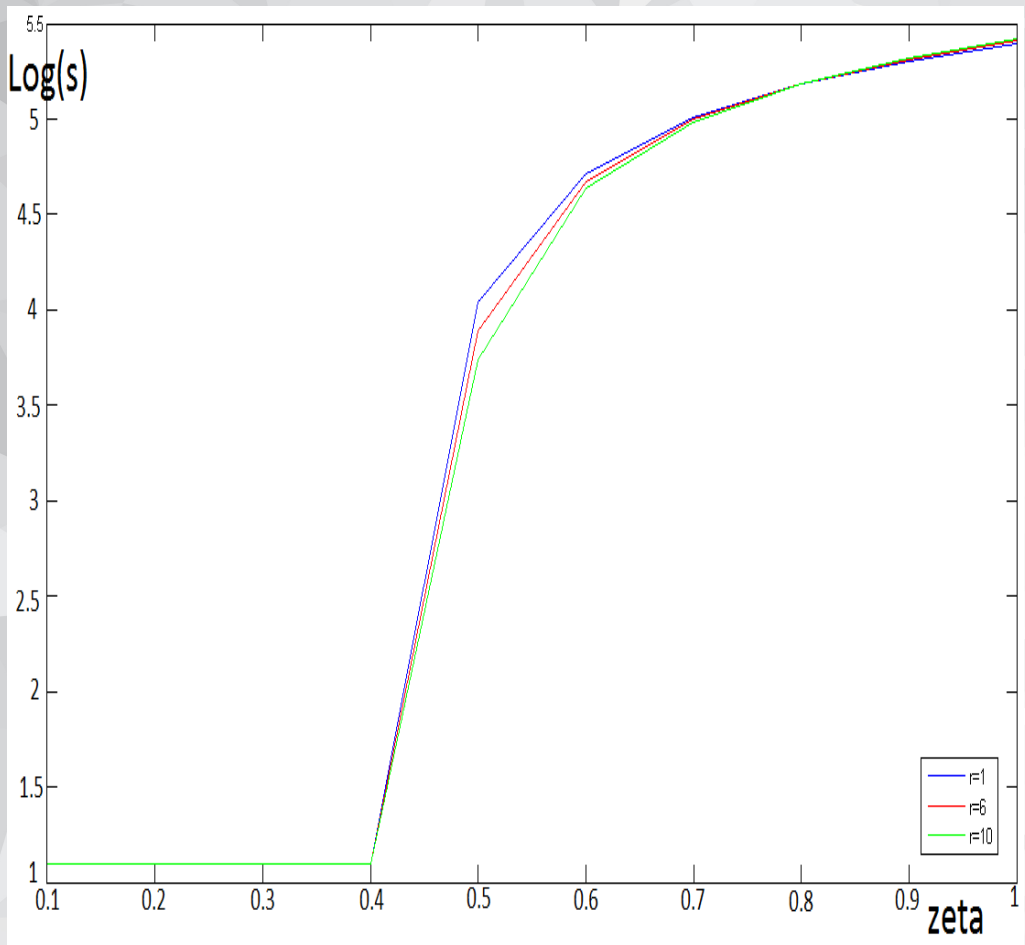

FIGURE 3 : log(s)=f(ζ)

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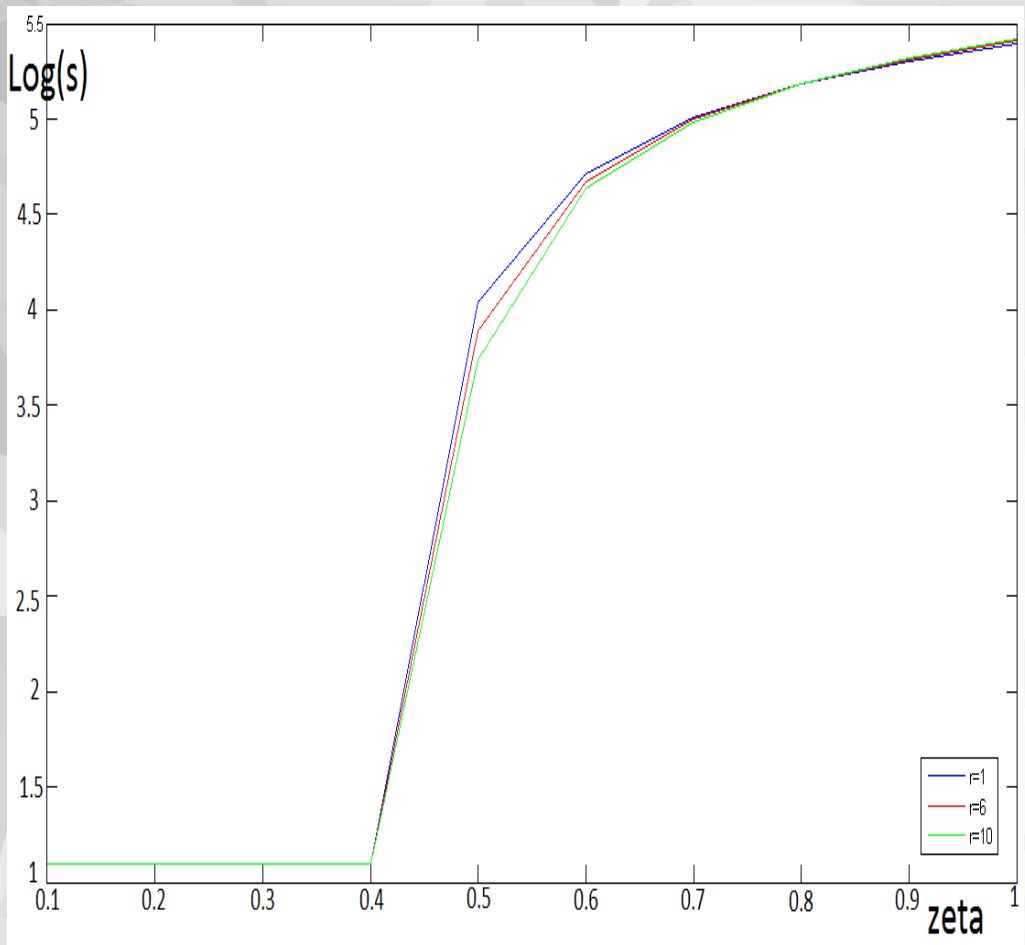

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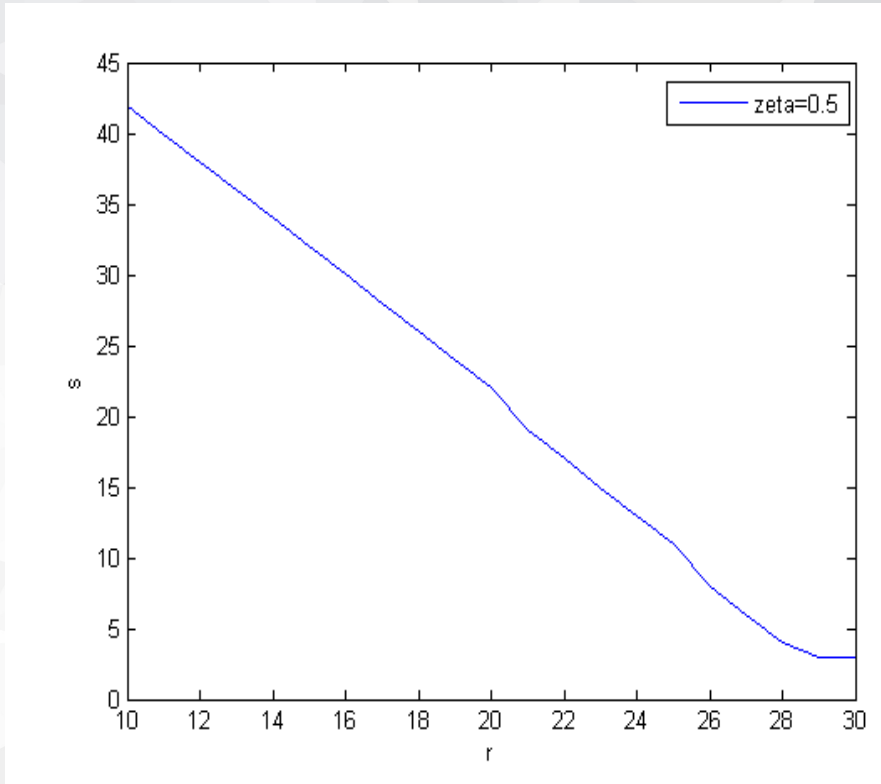

FIGURE 4 : s=g(r)

VELOCITY AND PRESSURE

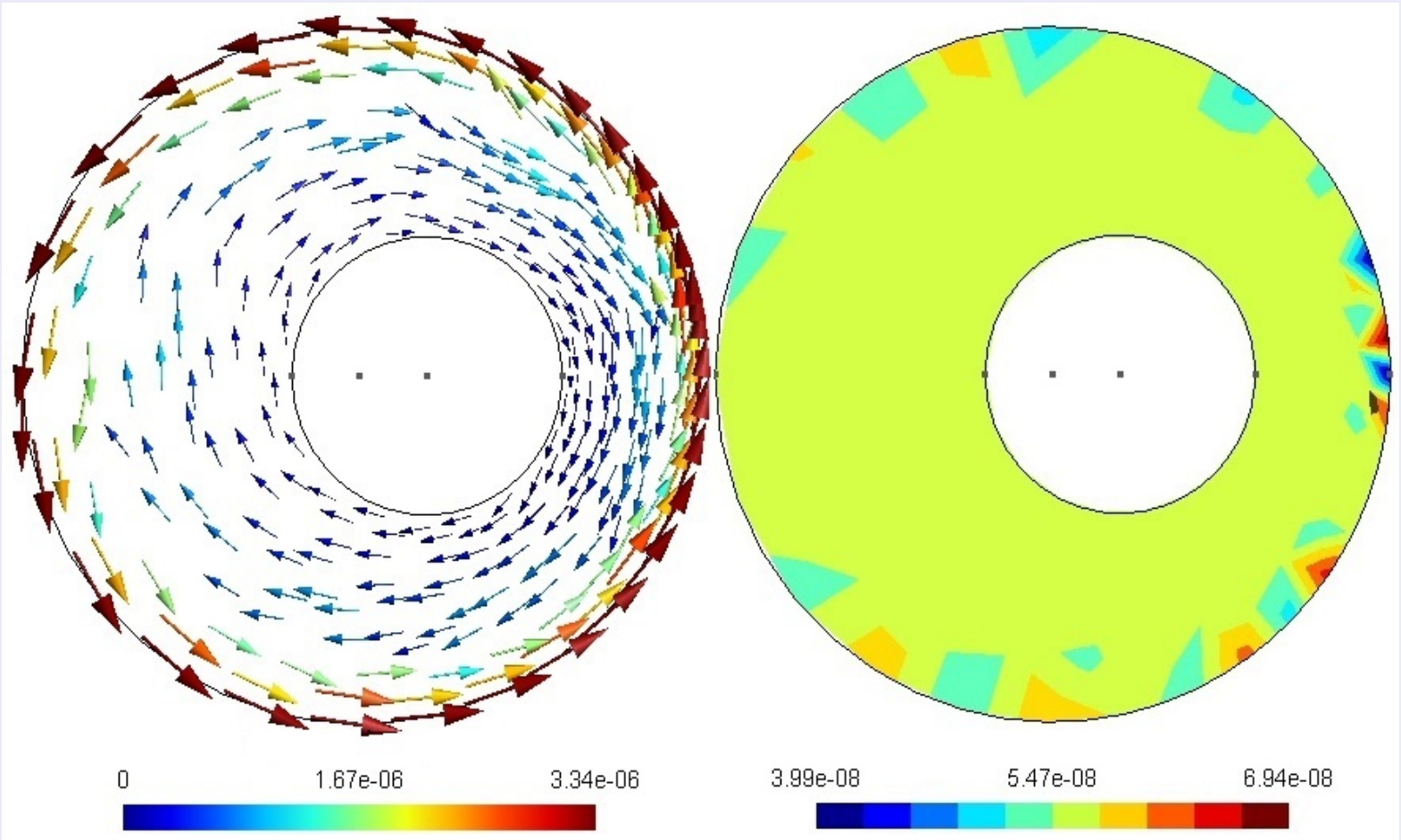

FIGURE 5 : Vitesse and pressure field for Re=0.1

VELOCITY AND PRESSURE

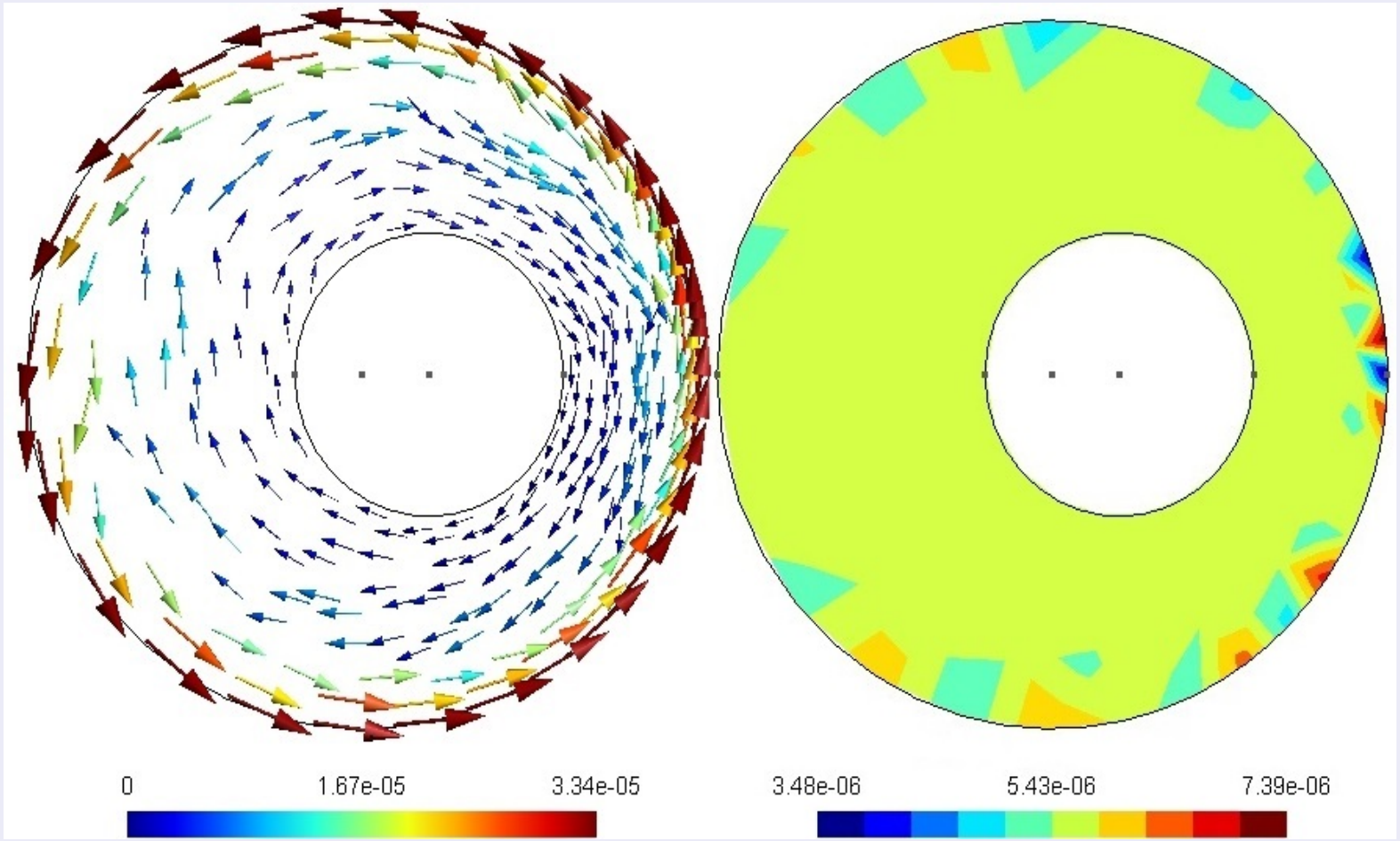

FIGURE 6 : Vitesse and pressure field for Re=1

VELOCITY AND PRESSURE

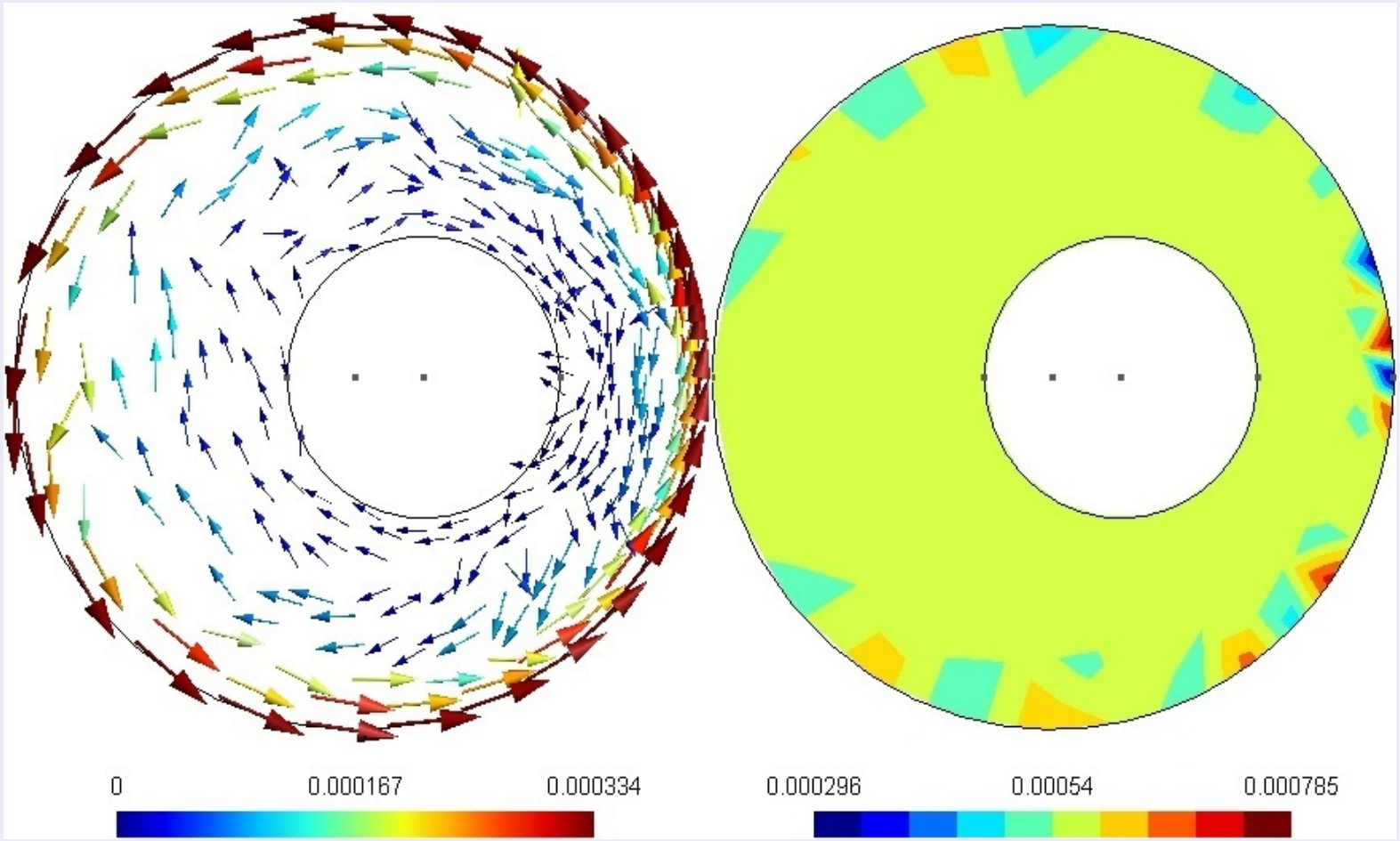

FIGURE 7 : Vitesse and pressure field for Re=10

VELOCITY AND PRESSURE

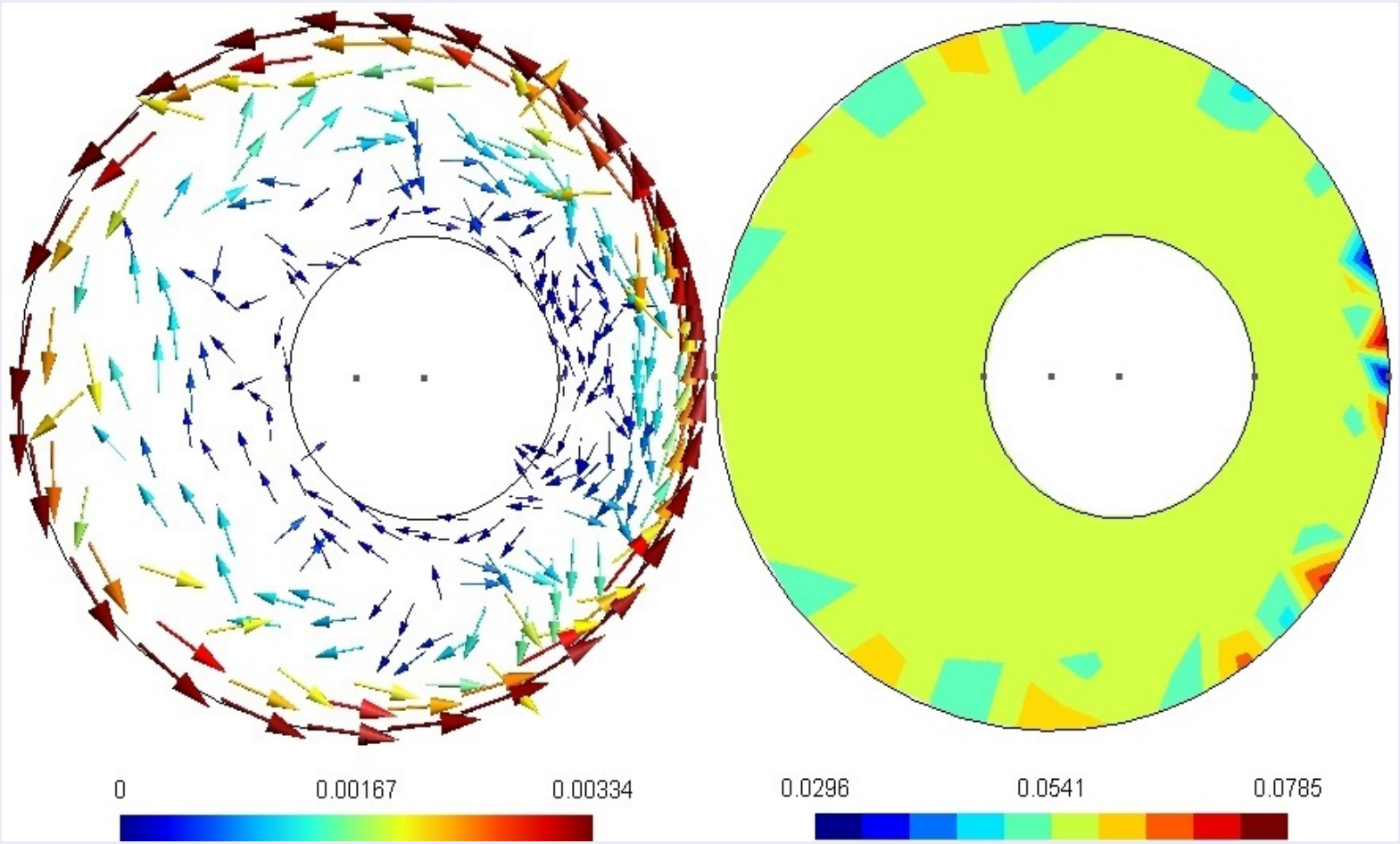

FIGURE 8 : Vitesse and pressure field for Re=100

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  - k determines the algorithm convergence ($k = 10^{-4}$)
  - r influences the speed convergence : it increases with r
  - ζ influences the speed convergence : it decreases with ζ.
- The minimum iteration number at which the convergence is reached is $\mathbf{s=3}$ $\Rightarrow$ ALM is efficient.

THANK YOU FOR YOUR ATTENTION