Quantum Walks of Two Particles: Interaction and Interference
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What are Quantum Walks?

- Example of a classical random walk

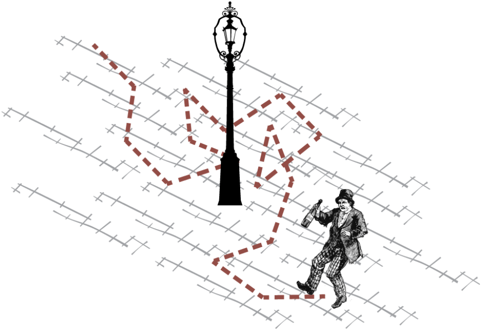

Figure: (https://rijusarkar.files.wordpress.com/2013/07/random-walk.png)

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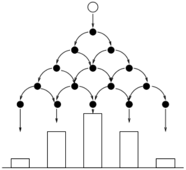

Figure: (Julia Kempe. Quantum random walks: an introductory overview. Contemporary Physics, 44(4):307-327, 2003.)

- **Superposition of state** If a particle can be in the state $|\psi\rangle$ and also in the state $|\phi\rangle$ then it can be in the state $|\psi\rangle + |\phi\rangle$
- **Single-particle interference** $(\langle\psi| + \langle\varphi|)\circ(|\psi\rangle + |\varphi\rangle) = \langle\psi|\circ|\psi\rangle + \langle\varphi|\circ|\varphi\rangle + 2Re\,(\langle\varphi|\circ|\psi\rangle)$

What motivate us to this study?

1. One particle QWs
   - perform quantum algorithm
   - implement quantum cryptography protocols
2. Two particles QWs $\Longrightarrow$ Understand the many-body problem.

OUTLINE

1. Introduction
   - Description of Continuous Time Quantum Walks (QW)
   - Importance of the topic
2. Quantum Walks With One Particle
   - Analytical description of the dynamics
   - Numerical description of the dynamics
3. Quantum Walks With Two Particles
   - Analytical description
   - Numerical description
4. Conclusion

Hamiltonian for one particle

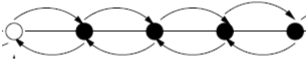

Figure: Quantum walk with one particle on a line (Julia Kempe. Quantum random walks: an introductory overview. Contemporary Physics, 44(4):307-327, 2003.)

- Schrödinger equation :

$$i\hbar \frac{d}{dt} |\psi_t\rangle = \hat{H} |\psi_t\rangle .$$

- Hamiltonian describing the tunnelling of a particle to a neighboring site:

$$\hat{H}_{1p} = -J \sum_{\ell=1}^{L-1} (|\ell\rangle \langle \ell+1| + |\ell+1\rangle \langle \ell|).$$

Diagonalization of the Hamiltonian

- Diagonalization for hard wall boundary conditions

$$\hat{H}_{1p} = \sum_{n=1}^{L} \underbrace{-2\cos\left(\frac{n\pi}{L+1}\right)}_{E_n} |k_n\rangle\langle k_n| .$$

where $|k_n\rangle = \sum_{\ell=1}^{L} \sqrt{\frac{2}{L+1}} \sin\left(\frac{n\pi\ell}{L+1}\right) |\ell\rangle$

- We plot the probability density distribution $P(\ell, t)$ for the particle to be on a site ℓ at a time t.

Probability density distribution

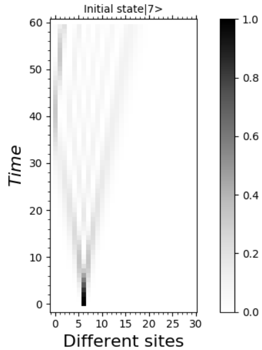

Figure: Probability density distribution $P(\ell, t)$ for the quantum walk of one particle on a lattice with L=30 sites.

Hamiltonian for two particles

- $|\ell, m\rangle$= state of the system with two particles
  1. For distinguishable particles:$|\ell, m\rangle = |l\rangle \otimes |m\rangle$
  2. For indistinguishable particles

$$\begin{cases} |\ell,\ell\rangle \quad \text{for} \quad 1 \leq \ell \leq L \\ \text{and} \quad \frac{|\ell,m\rangle + |m,\ell\rangle}{\sqrt{2}} \quad \text{for} \quad 1 \leq \ell < m \leq L \end{cases}$$

- Hamiltonian

$$\hat{H}_{2p} = \hat{H}_{1p} \otimes \hat{I} + \hat{I} \otimes \hat{H}_{1p} + \hat{V}, \quad \text{where} \quad \hat{V} = U \sum_{\ell=1}^{L} |\ell,\ell\rangle \langle \ell,\ell| .$$

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Effect of interference

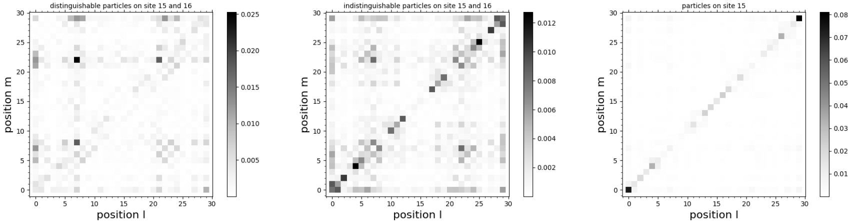

Figure: Probability density $P(\ell, m, t)$ of two particles undertaking a walk on 30 sites, at $t = 40\hbar/J$ and for $U = 2J$. Different panels refer to different initial state and different nature of particles as specified in the heading. Numbers into bracket are particles'initial positions.

Proposals for future works

- Include more walkers
- Experiment
- Investigate the effect of entanglement
- Include the decoherence

END

THANK YOU FOR YOUR KIND ATTENTION !!!