Simulations of self-polarization levels in FCC-ee

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Contents:

- Introduction
- Sokolov-Ternov polarization in a 100 km ring
- Polarization wigglers
- Simulations at 45 and 80 GeV ($60^0/60^0$, $90^0/90^0$ optics)
- Summary

FCC Week, Amsterdam, April 12 2018

- *Resonant de-polarization* has been proposed for accurate beam energy calibration ($\ll$ 100 keV) at 45 and 80 GeV beam energy.
  It relies on the relationship $\boldsymbol{\nu_{spin}} = \boldsymbol{a\gamma}$ [a].
- Beam polarization is obtained "for free" through Sokolov-Ternov effect.
  The effect is in practice restricted to a limited range of values of machine size and beam energy because
  - of the build-up rate
  - it is jeopardized by machine imperfections (spin/orbital motion resonances) which affects the reachable level of polarization in particular at high energy.
- 10% beam polarization is estimated to be enough for the purpose of energy calibration.

[a] $\boldsymbol{a}$ = gyromagnetic anomaly

Sokolov-Ternov polarization

Beam get vertically polarized in the ring guiding field

$$P_{\infty}^{\mathrm{ST}} = 92.3\% \qquad \tau_p^{-1} = \frac{5\sqrt{3}}{8}\frac{r_e \gamma^5 \hbar}{m_0 C}\oint \frac{ds}{|\rho|^3}$$

For FCC-e^+e^- with $\rho \simeq 10424$ m,it is

E (GeV)	τ_{pol} (h)	$\tau_{10\%}$ (*) h
45	256	29
80	14	1.6

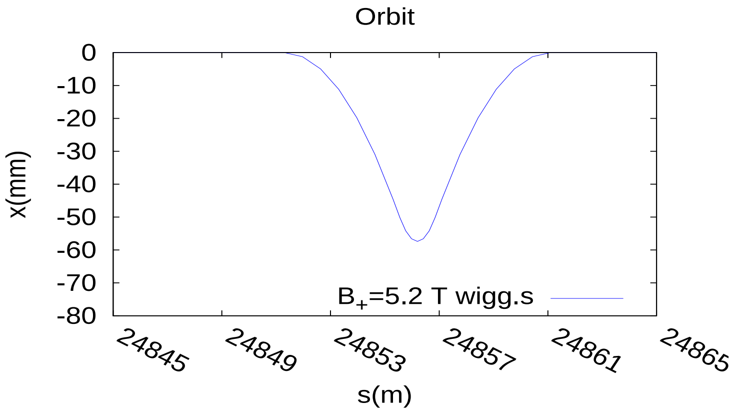
(*) Time needed to reach P=10% for energy calibration

$$\tau_{10\%} = -\tau_p \times \ln(1 - 0.1/P_{\infty})$$

Polarization wigglers

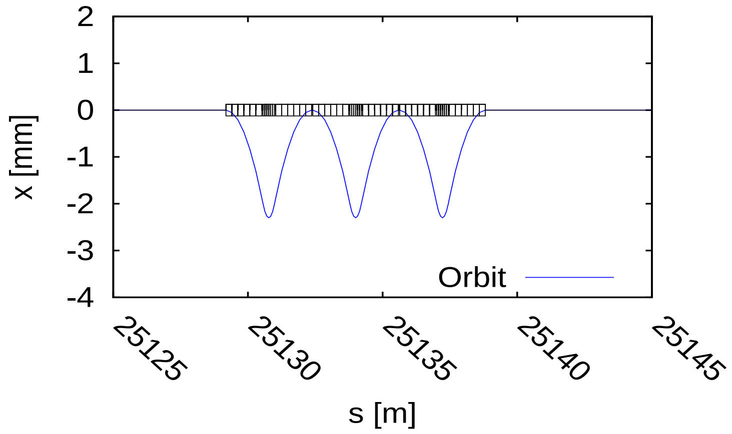
LEP-like

(orbit shown for $\boldsymbol{B^+}$=5.2 T)



3 periods

(for $\boldsymbol{B^+}$=0.7 T)



- $\boldsymbol{\epsilon_x}$ increases from 90 pm to 120 pm with proposed wigglers (field for $\boldsymbol{\tau_{10\%}}$=2.7 h) turned on for the 90/90 deg optics.
- Energy spread as with previous design for the same $\boldsymbol{\tau_p}$.

Polarization in real storage rings

A perfectly planar machine (w/o solenoids) is always *spin transparent*.

Sokolov-Ternov effect in the guiding dipole field	Perturbations (v-bends, vertical orbit in quads etc.)
$\downarrow$	$\downarrow$
Polarisation	Depolarisation
$\searrow$	$\swarrow$

Equilibrium polarisation $(< P_\infty^{\mathrm{ST}})$

Spin diffusion may be particularly large when spin and orbital motions are in resonance

$$\nu_{spin} \pm mQ_x \pm nQ_y \pm pQ_s = \text{integer}$$

Tools

Accurate simulations are necessary for evaluating the polarization level to be expected in presence of misalignments.

- MAD-X used for simulating quadrupole misalignments and orbit correction
- SITROS (by J. Kewish) used for computing the resulting polarization.
  - Tracking code with 2th order orbit description and non-linear spin motion.
  - Used for HERA-e in the version improved by M. Böge and M. Berglund.
  - It contains SITF (fully 6D) for analytical polarization computation with *linearized* spin motion.
    - Useful tool for preliminary checks before embarking in time consuming tracking.
    - Computation of polarization related to the 3 degree of freedom separately: useful for disentangling problems!

Simulations for a toy ring

Preliminary studies with a simplified optics (FODO cells and dispersion-free regions for wigglers) have shown that large polarization could be achieved at 45 GeV (even with very large wiggler fields) and at 80 GeV, in presence of misalignments.

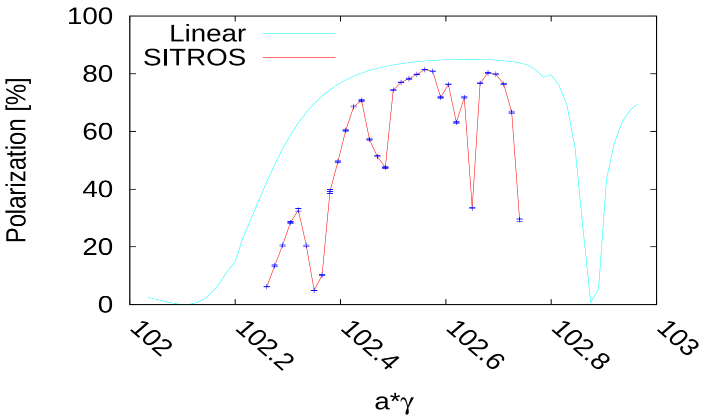
45 GeV beam energy

δy^{Q}_{rms}=200 μm

with BPMs errors

SVD+ harmonic bumps

$|\delta \hat{n}_0|_{rms}$=6.2 mrad



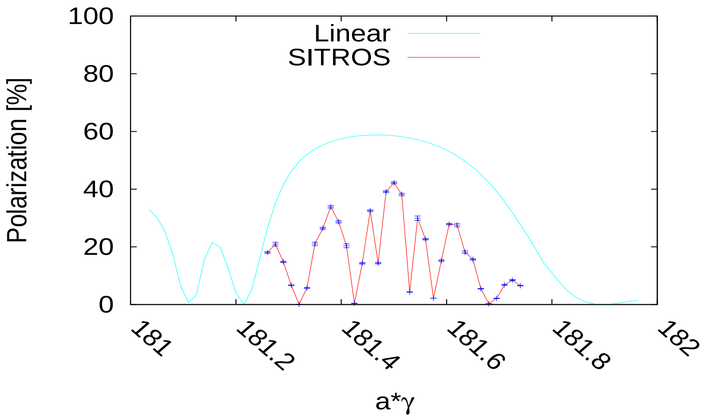
80 GeV beam energy

δy^{Q}_{rms}=200 μm

with BPMs errors

SVD+ harmonic bumps

$|\delta \hat{n}_0|_{rms}$ =14 mrad



Simulations for the "actual" FCC-ee

FCC-$e^{\pm}$ design relies on ultra-flat beams.

(January 2018)

	Z	WW
Beam energy [GeV]	45.6	80
FODO	$60^0/60^0/$	$60^0/\ 60^0$
ϵ_x [nm]	0.27	0.84
ϵ_y [pm]	1	1.7
β_x^* [m]	0.15	0.2
β_y^* [mm]	0.8	1
σ_x^* [μm]	6.4	13
σ_y^* [nm]	28	41

For squeezing β_y^* strong quadrupoles are needed in the IR where β_y is large.

$\rightsquigarrow$ Large impact on chromaticity and response to misalignments in the vertical plane.

Additional related problems

- Beam offsets in the strong IRs sextupoles may produce betatron coupling.
- Small offsets of the IRs quads may lead to an undamped machine.

Optics changed over the years as a result of the feasibility studies.

45 GeV optics – 2017

Optics		ξ_x	ξ_y
90/90	all sexts off	-520	-1751
	IR setxs off	+5	-1177
60/60	all sexts off	-368	-1613
	IR setxs off	+5	-1405

Optics		F_y
90/90	all quads	592
	w/o IPs quads(*)	136
60/60	all quads	830
	w/o IPs quads(**)	132

(*) QC1R(L), QC2R(L)2, QC3, QC3L,QC4, QC4L

(**) QC1R(L)1, QC1R(L)2, QC1R(L)3, QC2R(L)1 and QC2R(L)2

- The IPs quads dominate chromaticity and orbit response to misalignments.

January 2018 Optics (60^0/ 60^0)

Optics		ξ_x	ξ_y
45 GeV	all sexts off	-361	-1540
	IR setxs off	+3.5	-1230
80 GeV	all sexts off	-359	-1331
	IR setxs off	+3	-1017

Optics		F_y
45 GeV	all quads	665
	w/o IPs quads(*)	124
		F_y
80 GeV	all quads	492
	w/o IPs quads(**)	127

- New 60/60 deg optics: IRs less chromatic and contribution to the misalignment response smaller wrt October 2017 60/60 deg optics.

Simulations of orbit distortions

"Tricks" needed for introducing misalignments errors in the simulation (!):

- Move tunes away from integer ("*set up*" tunes)
  - q_x: 0.1 $\rightarrow$ 0.2
  - q_y: 0.2 $\rightarrow$ 0.3
- Switch sextupoles off (linear machine)
- Add errors to "arc" quads in steps of 5-10 μm and correct by each step with large number (some hundreds) correctors
- Add errors to the IR quadrupoles in steps of 5 μm and correct with close by correctors.

A lengthy procedure not feasible in a *real* machine. In practice: use *"relaxed"* optics and one-turn steering through correction dipoles for establishing a closed orbit.

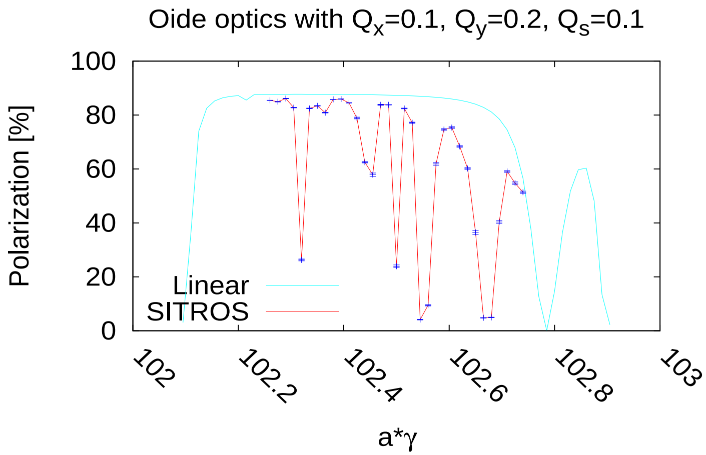
2017 90/90 deg optics

45 GeV case with 4 wigglers (LEP-like).

δy^{Q}_{rms}=200μm, no BPMs errors

y_{rms}=0.049 mm

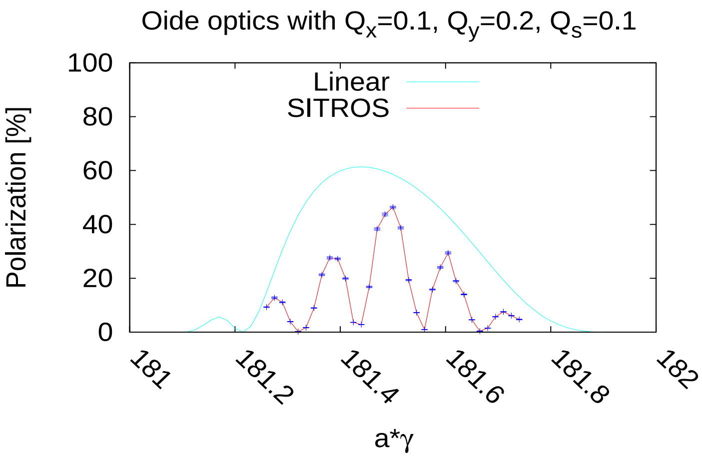
$|\delta\hat{n}|_{0,rms}$=0.4 mrad, no harmonic bumps

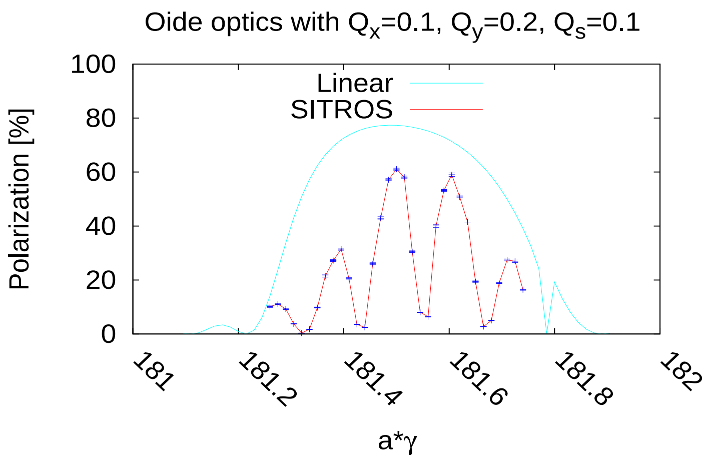


Same error realization at 80 GeV

$|\delta\hat{n}|_{0,rms}$=2 mrad

with harmonic bumps





Introducing BPM errors and quadrupole radial offsets and roll angles, misalignments had to be decreased! Set of errors assumed (but no statistics!)

	IR Quads	IR BPMs	other Quads	other BPMs
δx (μm)	10	10	30	30
δy (μm)	10	10	30	30
$\delta\theta$ (μrad)	10	10	30	30
calibration	-	1%	-	1%

- Although the resulting orbit after correction is in the order of few microns, the vertical emittance may result above specs.

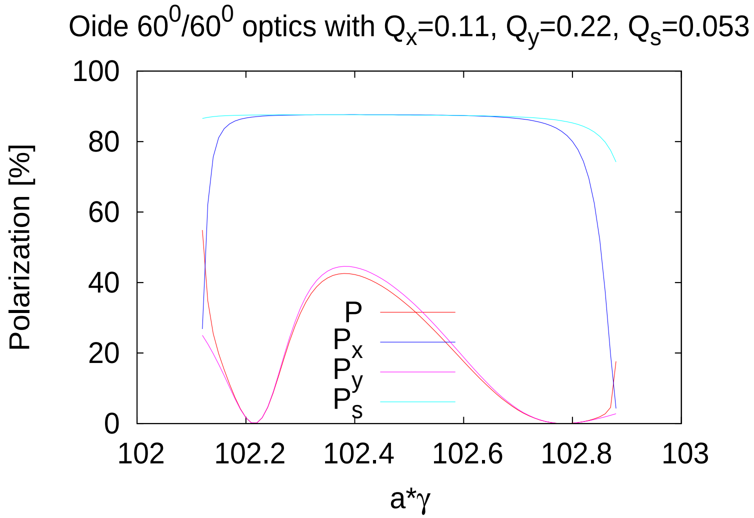
  $\rightsquigarrow$ Skew quadrupoles are used for minimizing the spurious vertical dispersion and the betatron coupling.

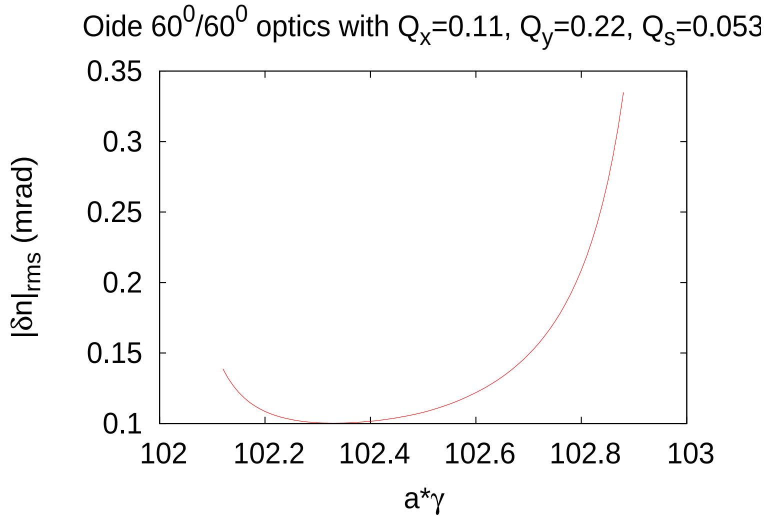
- Many seeds give no stable optics when sextupoles are turned on: the beam position at the sextupoles must be extremely well controlled!

- Some seeds give anti-damped machine when synchrotron radiation is turned on: the beam position at the IR quadrupoles must be extremely well controlled too!

Some optics show a small $\boldsymbol{P_y}$ although $\boldsymbol{\epsilon_y}$ and $\boldsymbol{D_y}$ are small.

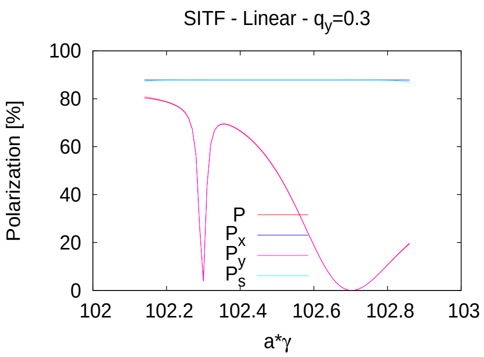
An example (October 2017 60/60 deg optics):

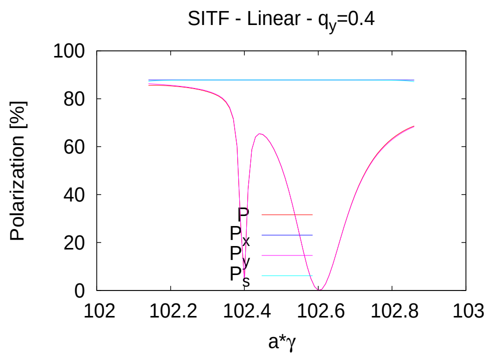


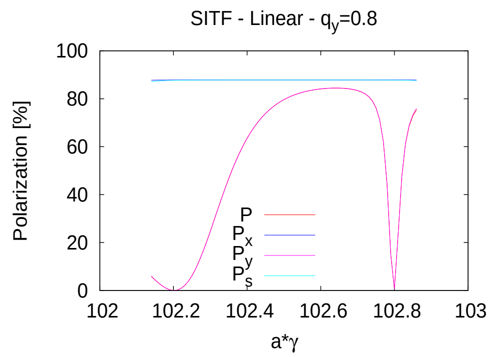


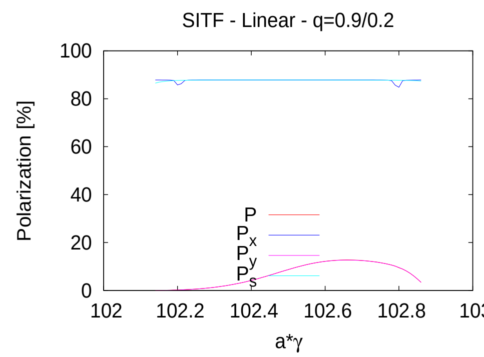
Tune scan

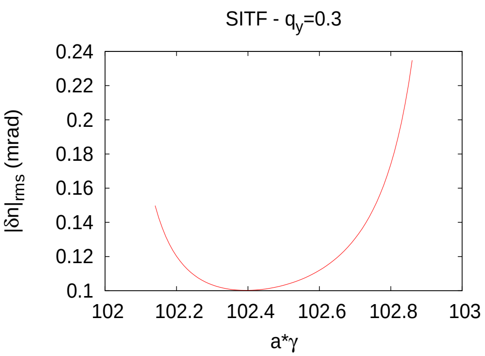
⇓

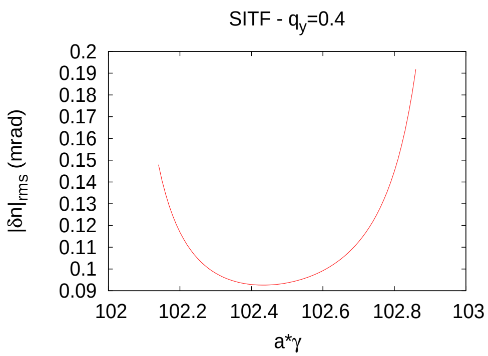


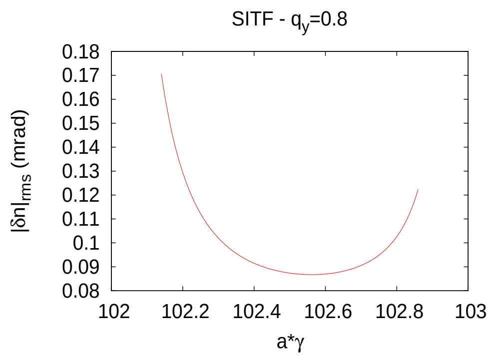


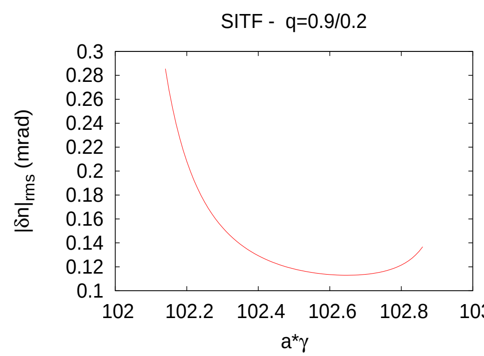












Why is 0.1/0.8 better then 0.1/0.2 ?

Linear approximation for spin diffusion:

$$\frac{\partial \hat{n}}{\partial \delta}(\vec{u};s) = \vec{d}(s) = \frac{1}{2}\Im\Big\{(\hat{m}_0 + i\hat{l}_0)^* \sum_{k=\pm x,\pm y,\pm s} \Delta_k\Big\}$$

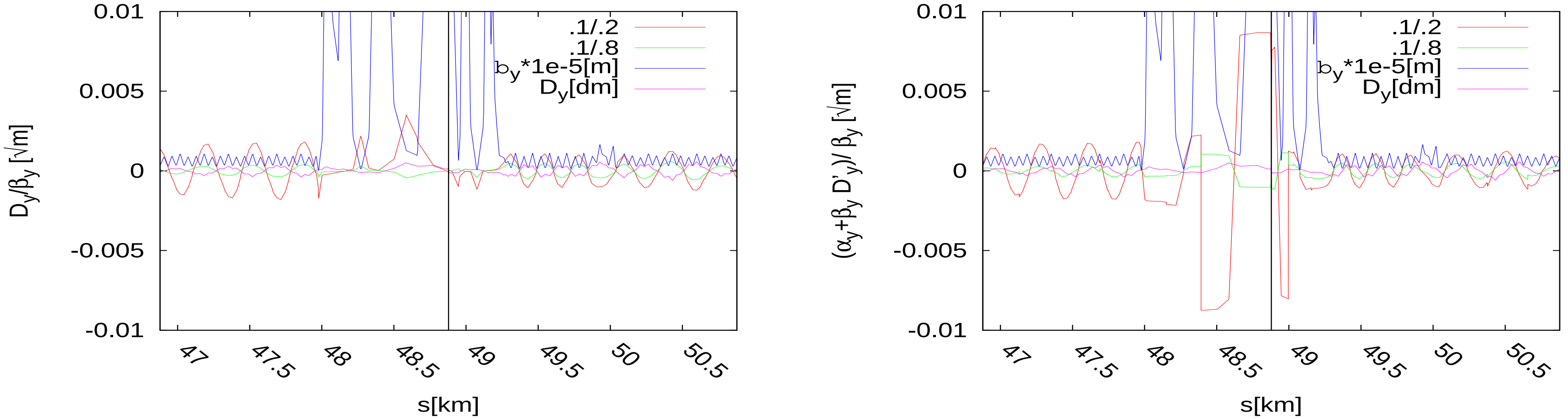
$$\Delta_{\pm x,\pm y} = (1+a\gamma)\frac{e^{\mp i\mu_{x,y}}}{e^{2i\pi(\nu\pm Q_{x,y})}-1}\frac{[-D\pm i(\alpha D+\beta D')]_{x,y}}{\sqrt{\beta_{x,y}}}\ J_{x,y}$$

$$\Delta_{\pm s} = (1+a\gamma)\frac{e^{\pm i\mu_s}}{e^{2i\pi(\nu\pm Q_s)}-1}\ J_s$$

$$J_{\pm x,\pm y} = \int_s^{s+L} ds'(\hat{m}_0 + i\hat{l}_0)\cdot\left\{\begin{array}{c}\hat{y}\sqrt{\beta_x}\\ \hat{x}\sqrt{\beta_y}\end{array}\right\} K e^{\pm i\mu_{x,y}}$$

$$J_s = \int_s^{s+L} ds'(\hat{m}_0 + i\hat{l}_0)\cdot(\hat{y}D_x + \hat{x}D_y)\,K$$

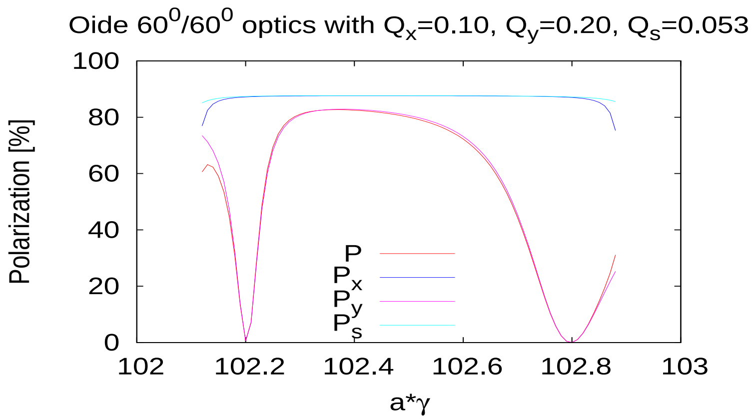
Plotting the factor $\frac{[-D_y \pm i(\alpha_y D_y + \beta D_y')]}{\sqrt{\beta_y}}$ which multiplies $J_{\pm y}$:

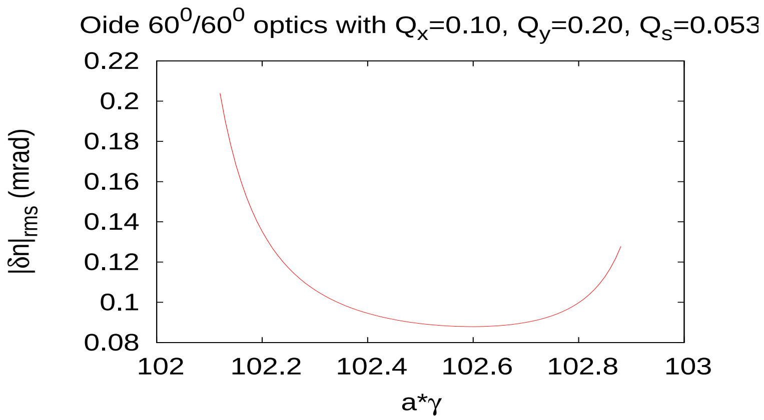


The factor is much smaller when the tunes are moved to .1/.8

$\rightsquigarrow$ Instead of correcting linear coupling and spurious vertical dispersion, we should try minimizing $[-D_y \pm i(\alpha_y D_y + \beta D_y')]/\beta_y$.

Found skew quadrupole settings improving $\boldsymbol{\Delta}_{\pm y}$ at expenses of betatron coupling with .1/.2 tunes:

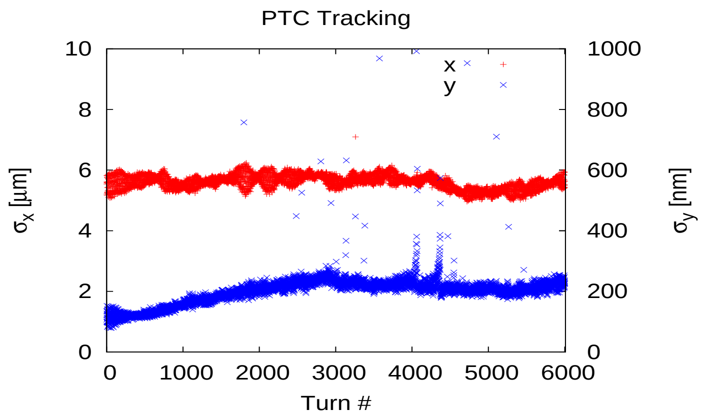




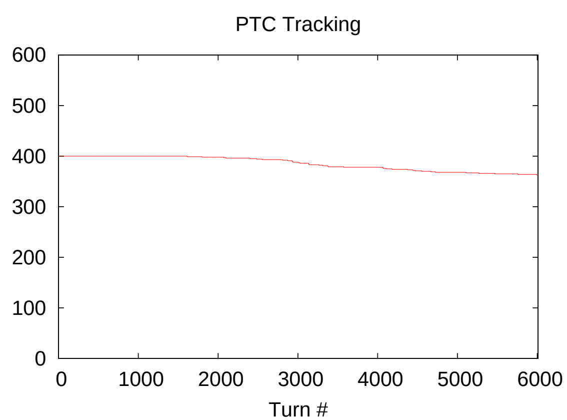
	x_{rms} (μm)	y_{rms} (μm)	D^y_{rms} (mm)	ϵ_x (nm)	ϵ_y (pm)	$\lvert C^- \rvert$
no skews	41	16	12	0.222	12	0.017
with	40	17	7	0.220	5	0.022

But...problems with SITROS tracking: no equilibrium found, many particles "lost"!

SITROS artifact or true? Results of MADX-PTC tracking for the same machine with errors.



	σ_x	σ_y
	(μm)	(nm)
analytical	5.8	78
PTC Tracking	$\simeq$ 5.6	$\simeq$ 230

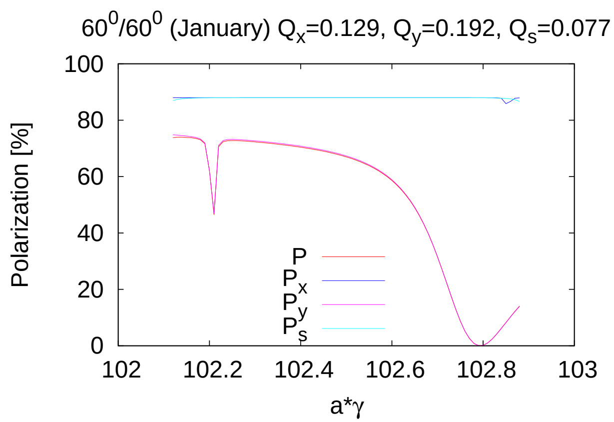


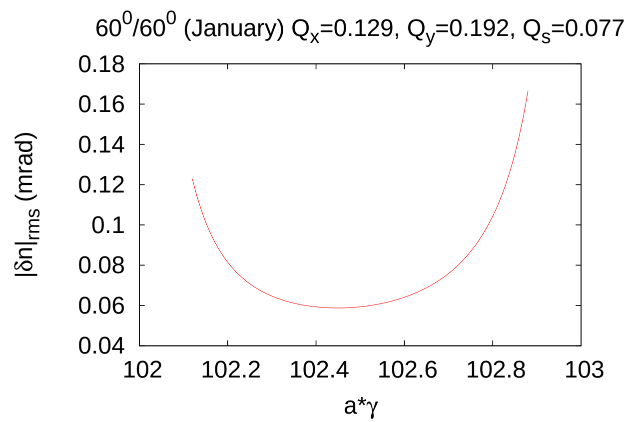
$\rightsquigarrow$ PTC tracking confirms a "disease".

New January 45 GeV optics, 8 wigglers, $\tau_{10\%}$=2.7 h with B^+=0.568 T.

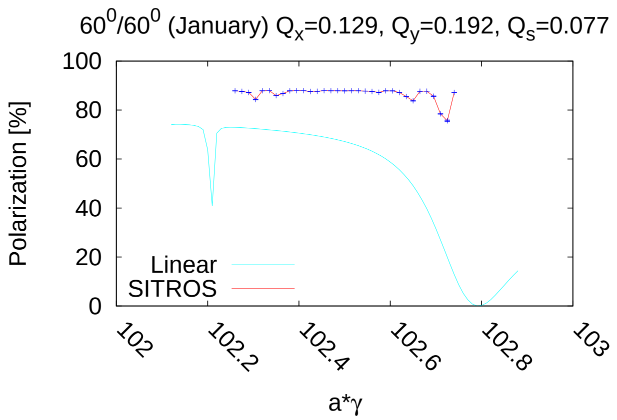
One error realization.

	x_{rms} (μm)	y_{rms} (μm)	D^y_{rms} (mm)	ϵ_x (nm)	ϵ_y (pm)	$\|C^-\|$
no skews	26	13	2	0.215	0.5	0.0014



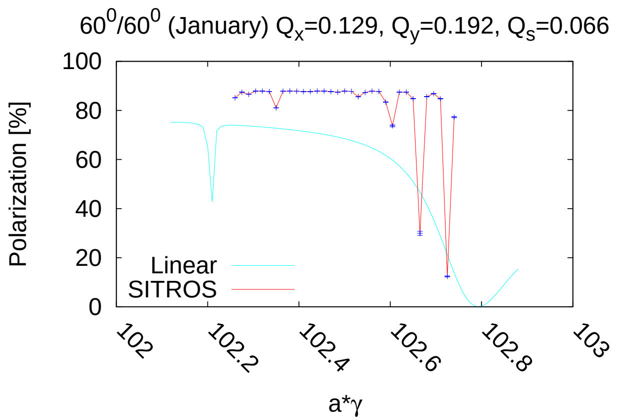


P_y limiting polarization, but P_{lin} large enough.



V=840 MV

	σ_x (μm)	σ_y (nm)	σ_ℓ (mm)
analytical	5.714	23.0	3.356
SITROS Tracking	8.526	23.8	3.439

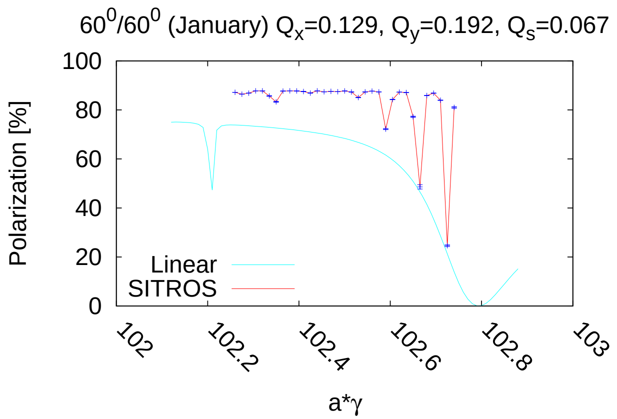


V=620 MV

	σ_x (μm)	σ_y (nm)	σ_ℓ (mm)
analytical	5.716	23.9	3.909
SITROS Tracking	8.629	43.6	3.890

Tracking shows very large polarization!

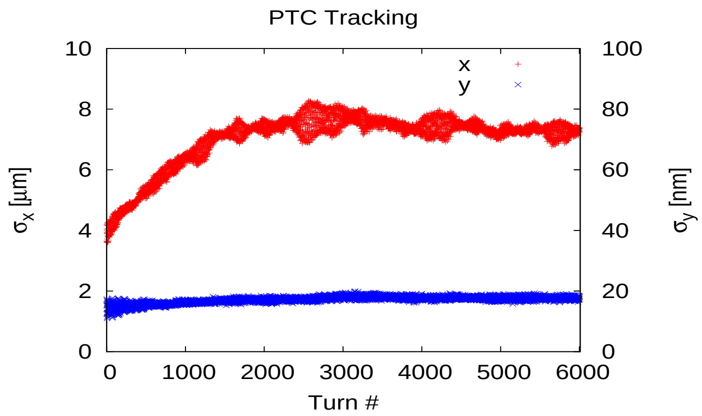
The wiggler field may be increased to $B^{+}= 0.664$ T to obtain $\tau_{10\%}$=1.7 h



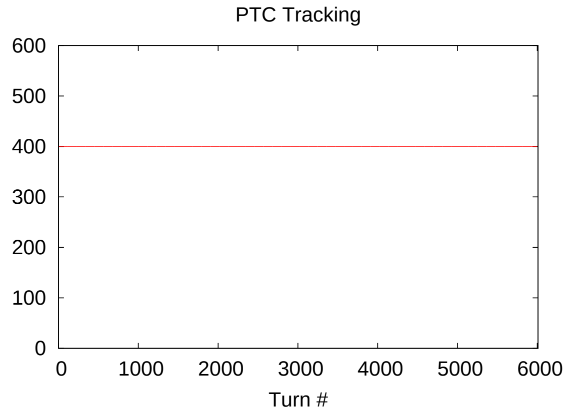
V=700 MV

	σ_x (μm)	σ_y (nm)	σ_ℓ (mm)
analytical	5.803	27.4	4.398
SITROS Tracking	9.602	38	4.461

PTC tracking for the perturbed January 60/60 optics with wigglers on ($\tau_{10\%}$= 2.8 h)



	σ_x (μm)	σ_y (nm)
analytical	7.8	18.9
PTC Tracking	$\simeq$ 7.3	$\simeq$ 18



No losses and good agreement of beam σ !

Polarization – 80 GeV

New January 80 GeV optics, no wigglers.

Tunes in presence of synchrotron radiation for the unperturbed machine:

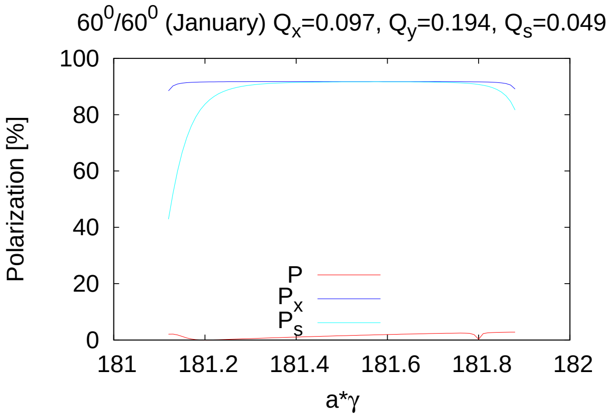
	MADX	SITF	MADX	SITF	MADX	SITF
	S.R. off		S.R. on		S.R. on & SEXTS off	
Q_x	269.100	.100	269.138	.138	269.100	.101
Q_y	269.200	.200	269.128	.129	269.200	.196

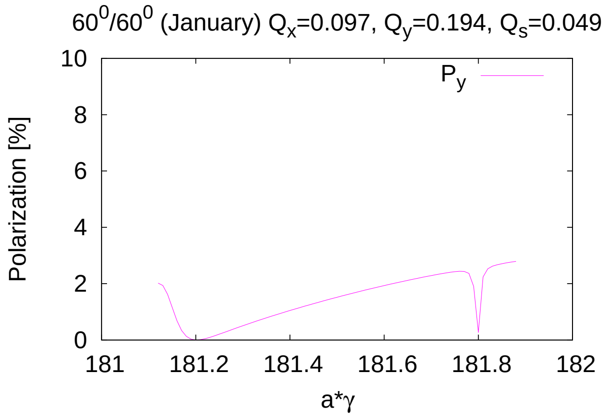
Feed-down effect of sextupoles!

One error realization:

	x_{rms}	y_{rms}	D^y_{rms}	ϵ_x	ϵ_y	$\|C^-\|$
	(μm)	(μm)	(mm)	(nm)	(pm)	
no skews	144	11	2	0.792	0.1	< 0.001

Tunes adjusted taking into account sextupole effect.





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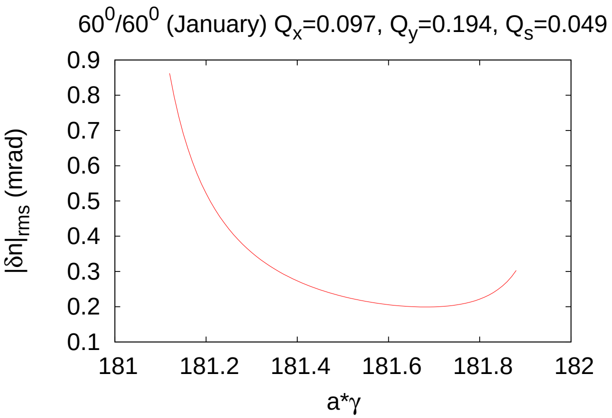
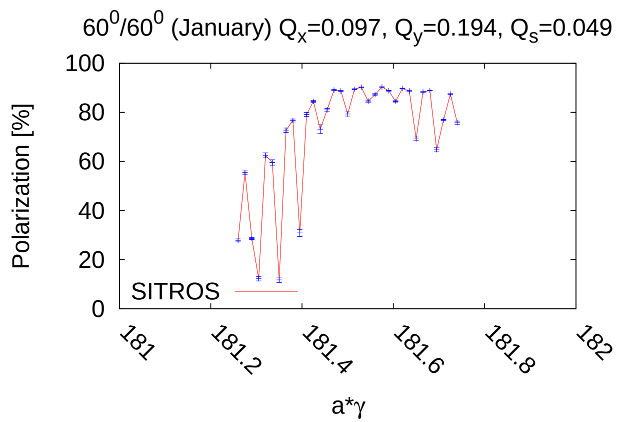
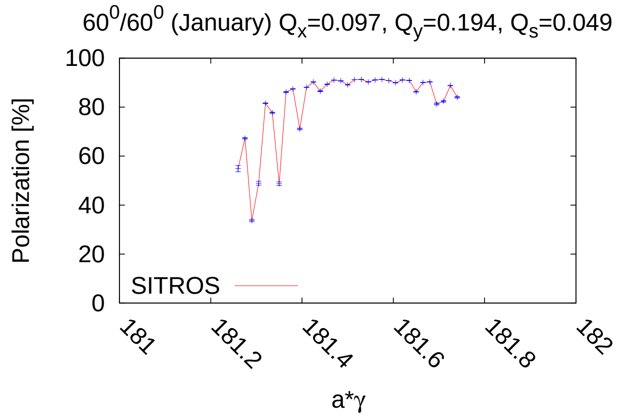




V= 700 MV

	$\boldsymbol{\sigma_x}$ (μm)	$\boldsymbol{\sigma_y}$ (nm)	$\boldsymbol{\sigma_\ell}$ (mm)
analytical	13.22	19.5	3.079
SITROS Tracking	12.66	44.1	3.105

Correctors added after each bending magnet for correcting sawtooth effect: linear polarization shows no improvement, tracking does.

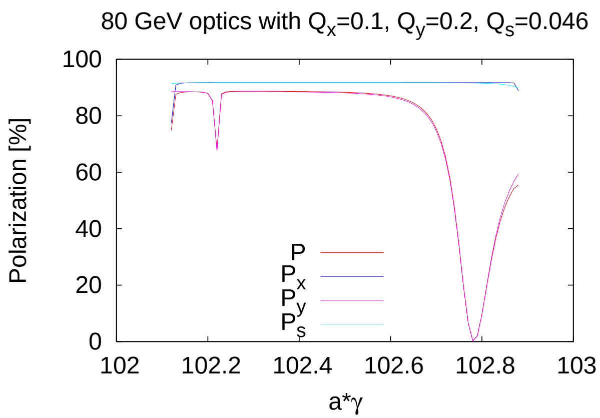


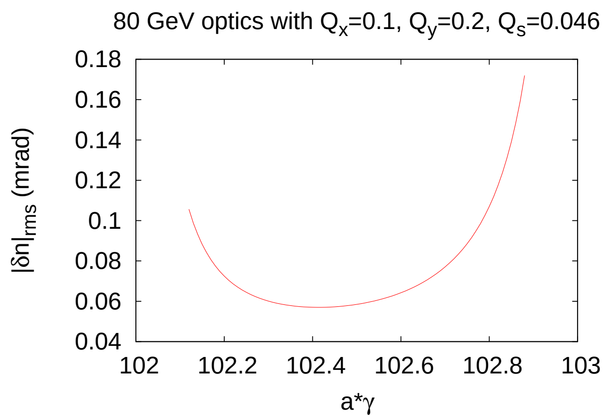
V= 700 MV

	$\boldsymbol{\sigma_x}$	$\boldsymbol{\sigma_y}$	$\boldsymbol{\sigma_\ell}$
	(μm)	(nm)	(mm)
analytical	12.595	19.1	3.083
SITROS Tracking	12.206	26.3	3.108

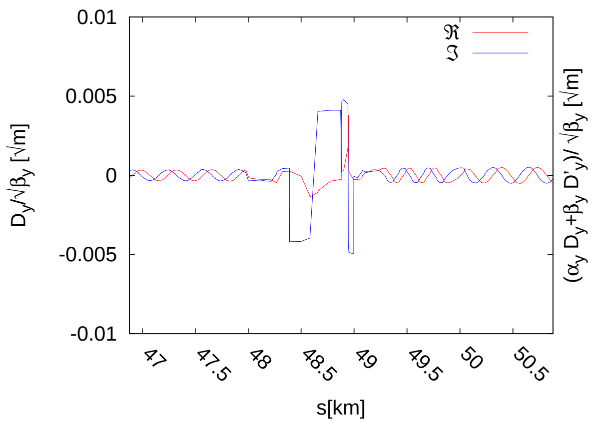
Could it be the result of the larger impact of errors at larger energy?

80 GeV optics, same error realization but at 45 GeV





If *true* it could be cured by correcting $[-D_y \pm i(\alpha_y D_y + \beta D'_y)]/\beta_y$ as shown for the previous 60/60 deg optics.



Consistency check.

$$45\,\mathrm{GeV}: P_\infty = 88\% \quad \text{and} \quad \tau_d = 0.2\times 10^8\,\mathrm{sec} \qquad \text{(from SITF)}$$

Extrapolating at 80 GeV it should be

$$\tau_d^{(80)} \approx \left(\frac{45}{80}\right)^5 \tau_d^{(45)} = 1.3\times 10^6\,\mathrm{sec}$$

From

$$P_\infty \approx P_{\mathrm{ST}} \frac{\tau_d}{\tau_{\mathrm{ST}}+\tau_d}$$

it would be $P_\infty \approx 88\%$ at 80 GeV.

Instead

$$80\,\mathrm{GeV}: P_\infty^{lin} = 2\% \quad \text{and} \quad \tau_d = 0.9\times 10^3\,\mathrm{sec}$$

Summary

- Beam polarization is obtained "for free" through Sokolov-Ternov effect.
  - At 45 GeV *wigglers* are required to get $\tau_{10\%} \approx$ 2-3 h.
- P_∞ depends on how well is the machine aligned/corrected, requirements becoming stricter at high the energy.
  - Extremely well corrected orbit/optics is required for a large chromatic machine with β_y^*=0.8-1 mm as FCC-ee to work and meet required performance.
    - This benefits also polarization, but a special correction may be needed for $[-D_y \pm i(\alpha_y D_y + \beta D'_y)]/\beta_y$.
    - Polarization shouldn't be an issue at 45 GeV.
    - Are there surprises at 80 GeV? Puzzling results to be cross checked.

THANK YOU!

Polarization wigglers

τ_p may be reduced by introducing wigglers:

$$\tau_p^{-1} = F\gamma^5 \Big[\int_{dip} \frac{ds}{|\rho_d|^3} + \int_{wig} \frac{ds}{|\rho_w|^3} \Big] \qquad F \equiv \frac{5\sqrt{3}}{8} \frac{r_e \hbar}{m_0 C}$$

Polarization

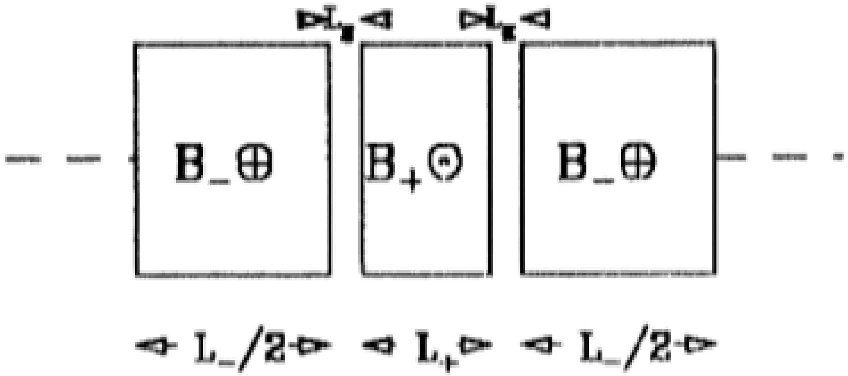
$$P_\infty = \frac{8}{5\sqrt{3}} \frac{\oint ds \frac{\hat{B}\cdot\hat{n}_0}{|\rho|^3}}{\oint ds \frac{1}{|\rho|^3}} \propto \tau_p \Big[\int_{dip} ds \, \frac{\hat{B}_d \cdot \hat{n}_0}{|\rho_d|^3} + \int_{wig} ds \, \frac{\hat{B}_w \cdot \hat{n}_0}{|\rho_w|^3} \Big]$$

$\hat{n}_0 \equiv \hat{y}$ in a perfectly planar ring.

Constraints:

- $x' = 0$ outside the wiggler $\Rightarrow \int_{wig} ds \, B_w = 0$ (vanishing field integral)
- $x = 0$ outside the wiggler $\Rightarrow \int_{wig} ds \, sB_w = 0$ (true for symmetric field)
- P large $\Rightarrow \int_{wig} ds \, B_w^3$ must be large

The LEP polarization wigglers:



For 4 LEP-like wigglers with $B_+/B_-(=L_-/L_+)\simeq$ 6 and B^+=0.7 T it is $\tau_{10\%}$ $\simeq$2.9 h at 45 GeV.

Horizontal emittance

$$\epsilon_x = C_q \gamma^2 \frac{\mathcal{I}_5}{J_x \mathcal{I}_2} \qquad \mathcal{I}_2 \equiv \oint ds \frac{1}{\rho^2}$$

$$\mathcal{I}_5 \equiv \oint ds \frac{\beta_x D_x'^2 + 2\alpha_x D_x D_x' + \gamma_x D_x^2}{|\rho|^3}$$

Even if located where nominally D_x=0, wigglers may increase the horizontal emittance

$$\Delta \mathcal{I}_5 \simeq \frac{1}{15\pi^3} \frac{< \beta_x >_w \ell_w}{\rho_w^5} \lambda_w$$

The effect is small for the $60^0/60^0$ deg FODO.

For the 1 mm β^* optics ($90^0/90^0$ deg FODO) the horizontal emittance at 45 GeV increases from 90 pm to 500 pm.

The emittance increase can be mitigated by choosing a shorter wiggler period, λ_w.

Polarization formulas

The Derbenev-Kondratenko polarization rate

$$\tau_{\mathrm{DK}}^{-1} = \frac{5\sqrt{3}}{8}\frac{r_e\gamma^5\hbar}{m_0 C}\oint < \frac{1}{|\rho|^3}\left[1 - \frac{2}{9}(\hat{n}\cdot\hat{v})^2 + \frac{11}{18}\left(\frac{\partial\hat{n}}{\partial\delta}\right)^2\right] >$$

may be written as

$$\tau_{\mathrm{DK}}^{-1} = \tau_p^{-1} \simeq \tau_{\mathrm{BKS}}^{-1} + \tau_d^{-1}$$

with

$$\tau_{\mathrm{BKS}}^{-1} = \frac{5\sqrt{3}}{8}\frac{r_e\gamma^5\hbar}{m_0 C}\oint ds \frac{1}{|\rho|^3}\left[1 - \frac{2}{9}(\hat{n}\cdot\hat{v})^2\right]$$

and

$$\tau_d^{-1} = \frac{5\sqrt{3}}{8}\frac{r_e\gamma^5\hbar}{m_0 C}\oint ds < \frac{1}{|\rho|^3}\left[\frac{11}{18}\left(\frac{\partial\hat{n}}{\partial\delta}\right)^2\right] >$$

Similarly for $\boldsymbol{P}_\infty$

$$\vec{P}_{\rm DK} = \hat{n}_0 \frac{8}{5\sqrt{3}} \frac{\oint ds < \frac{1}{|\rho|^3} \hat{b}\cdot(\hat{n} - \frac{\partial \hat{n}}{\partial \delta}) >}{\oint ds < \frac{1}{|\rho|^3}\left[1 - \frac{2}{9}(\hat{n}\cdot\hat{v})^2 + \frac{11}{18}(\frac{\partial \hat{n}}{\partial \delta})^2\right] >} \qquad \hat{b} \equiv \vec{v}\times\dot{\vec{v}}/|\vec{v}\times\dot{\vec{v}}|$$

$$P_\infty = P_{\rm DK} \simeq P_{\rm BKS}\frac{\tau_d}{\tau_{\rm BKS}+\tau_d} = P_{\rm BKS}\frac{\tau_p}{\tau_{\rm BKS}}$$

Approximations done

- $\hat{n}\cdot\hat{v}$ is evaluated on the closed orbit
- $\hat{b}\cdot\frac{\partial \hat{n}}{\partial \delta}$ has been neglected. In general it is small.