Beam-cavity interaction challenges for FCC-ee cavities

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Overview of challenges in FCC-ee Z machine

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam current, $I_{b, DC}$</td>
<td>1.39 A</td>
</tr>
<tr>
<td>Number of bunches, $M$</td>
<td>16640</td>
</tr>
<tr>
<td>Minimum abort gap, $\tau_{\text{gap}}$</td>
<td>2 µs</td>
</tr>
<tr>
<td>RF frequency, $f_{RF}$</td>
<td>400.79 MHz</td>
</tr>
<tr>
<td>$R/Q$ of fundamental mode</td>
<td>42.3 Ω</td>
</tr>
<tr>
<td>Cavity voltage, $V_{\text{cav}}$</td>
<td>1.91 MV</td>
</tr>
<tr>
<td>Number of cavities, $N_{\text{cav}}$</td>
<td>52</td>
</tr>
<tr>
<td>Harmonic number, $h$</td>
<td>130680</td>
</tr>
<tr>
<td>Radiation damping time, $\tau_{\text{SR}}$</td>
<td>414 ms</td>
</tr>
</tbody>
</table>

→ Power losses due to high order modes (HOM)

→ Transient beam loading

→ Longitudinal coupled-bunch instability
HOM power loss calculations

Simulated cavity impedance

\[ P = I_{b,DC}^2 \sum_{k=-\infty}^{\infty} \text{Re}[Z_{||}(kf_{\text{rev}})]|I_k|^2 \]

Normalized Fourier harmonics of beam current

- \( I_{b,DC} \) – average beam current
- \( f_{\text{rev}} \) – revolution frequency
- \( k \) – revolution harmonic number

Estimations of the power loss are required to determine parameters for HOM absorbers (max 1 kW per coupler).
Beam spectrum for different filling schemes

Bunch trains

Train spacing, $t_{tt}$

Bunch spacing, $t_{bb} = 2.5 \div 17.5$ ns

Abort gap, $t_{gap} = 2$ μs

Number of trains, $n_{tr}$

Number of bunches per train, $M_b$
Impedance of LHC-like single-cell cavity

Impedance calculation using ABCI

Axisymmetric structure +
Gaussian bunch

Wake potential

→ Only one mode below cut-off frequency with parameters:
$f_r \approx 694$ MHz, $R/Q \approx 12$ Ω (CST EMS simulations), quality factor $Q = ?$

Cut-off frequency

1/e – decrease of the beam power spectrum, $1/2\pi \sigma$

*Beamstrahlung effect
Power loss above cut-off frequency

Constant parameters: total current ≤ 1.4 A, abort gap 2 μs, bunch population $1.7 \times 10^{11}$

Variable parameters: number of bunches in the train, number of trains, train spacing

→ Power loss is moderate for the present cavity design for bunches in collisions ($\approx 3$ kW)
→ There is a weak dependence on train spacing and bunch spacing
Power loss for HOM below cut-off frequency

Longitudinal coupled-bunch instability growth rate due to HOM

\[
\frac{1}{\tau} = \frac{e|\eta|I_{b,DC}}{2EQ_s} f_R R
\]

If \( \tau > \tau_{SR} \rightarrow \text{stability} \)

\( \tau \) – growth time
\( \tau_{SR} \) – radiation damping time
\( \eta \) – slip factor
\( E \) – beam energy
\( Q_s \) – synchrotron tune

Power losses of about 1 kW are for small \( Q \) + “resonant” cases with high \( Q \)
→ Damping of the mode for longitudinal stability should be moderate
→ Resonant cases should be identified
Power losses for different filling schemes

Resonant case when the beam spectral line overlaps with HOM if:

\[ 1 - \left| \frac{f_r t_{tt}}{f_r t_{tt}} \right| < \frac{1}{Q} \]

\[ f_r = 694 \pm 5 \, \text{MHz}, \, R/Q = 12 \, \Omega, \, Q = 640 \]

→ Some filling schemes should be avoided in machine operation (restrictions for train and bunch spacings)

*I.Karpov et al., CERN-ACC-NOTE-2018-0005 (2018)*
More “general” case

Operation settings define recommendations for the cavity geometry

$n/t_{bb}$

Resonant frequency scan, $f_r$

$t_{bb} = 10.0 \text{ ns}$, $t_{tt}f_{RF} = 48$

$\text{Normalized power spectrum}$

$\text{Frequency (MHz)}$

$\text{Power loss < 1 kW for } t_{tt}f_{rf} > 100$

$\text{Dangerous}$

$\text{Safe}$

$\text{Present design}$
Transient beam loading

Lumped circuit model for superconducting RF cavity*

\[ I_g(t) = \frac{V(t)}{2R/Q} \left( \frac{1}{Q_0} + \frac{1}{Q_{\text{ext}}} - \frac{2i\Delta\omega}{\omega_{RF}} \right) + \frac{dV(t)}{dt} \frac{1}{\omega_{RF}R/Q} + \frac{I_{b,RF}(t)}{2} \]

Optimum detuning

\[ \Delta\omega_{\text{opt}} = \omega_0 - \omega_{RF} = -\omega_{RF} \frac{\langle I_{b,RF} \rangle R/Q \sin(\phi_s)}{2V_{\text{cav}}} \]

\( (11.4 \text{ kHz}) \)

Optimum loaded quality factor

\[ Q_{L,\text{opt}} = \left( \frac{1}{Q_0} + \frac{1}{Q_{\text{ext}}} \right)^{-1} \approx \frac{V_{\text{cav}}}{\langle I_{b,RF} \rangle R/Q \cos(\phi_s)} \]

\( (44000) \)

*For example in J. Tückmantel, CERN-ATS-Note-2011-002, 2011
Numerical calculations of transients

Assuming optimum detuning and

\[ I_{b,RF}(t) = 2I_{b,DC}[1 + a_b(t)]e^{i[\phi(t) - \phi_s]} \]

\[ V(t) = V_{cav}[1 + a_V(t)]e^{i\phi(t)} \]

\( a_b \) – beam current modulation

We get:

\( a_V(t) \) – amplitude modulation → spread of \( \sigma \) and \( Q_s \)

\( \phi(t) \) – phase modulation → collision point shift

Modulation is dominated by abort gap

→ For \( t_{gap} > 2 \mu s \), peak-to-peak \( a_V > 6\% \), and peak-to-peak \( \phi > 60 \) ps

→ Collision point shift can be eliminated, if gap transients are matched (PEP-II, LHC)
Longitudinal coupled-bunch instability driven by the fundamental impedance

For short Gaussian bunches the growth rate of the mode $m$ is*

\[
\frac{1}{\tau_m} \approx \frac{e \eta \omega_{RF}}{4\pi E Q_s} I_{b,DC} N_{cav} (\text{Re}\{Z_\parallel [\omega_{RF} + (m + Q_s)\omega_{rev}]\} - \text{Re}\{Z_\parallel [\omega_{RF} - (m + Q_s)\omega_{rev}]\})
\]

**Fundamental cavity impedance**

\[
Z_\parallel(\omega) = \frac{R/Q Q_L}{1 + i Q_L \left[ \frac{\omega_{RF}}{\omega_0} - \frac{\omega_0}{\omega_{RF}} \right]}
\]

→ For optimum detuning (about $4 \times f_{rev}$) the most unstable mode is $m = -4$

*For example in A. Chao, Physics of Collective Beam Instabilities in High Energy Accelerators, 1993
Mitigation by direct RF feedback*

The flat response is achieved for
\[ 1/G = R/Q \omega_{RF} \tau_d \]

Loop delay \( \tau_d \approx 700 \text{ ns} \) (similar to LHC)

→ growth rates of all unstable modes are smaller than synchrotron radiation damping rate
→ To increase stability margins one-turn delay feedback (similar to SPS, LHC) or more sophisticated double peaked comb filter (PEP II) can be used

*D. Boussard, Control of Cavities with High Beam Loading, IEEE NS-32 (1985)
Conclusions

• HOM power loss contributions:
  • From impedance above cutoff frequency is about 3 kW,
  • From overlap of HOM below cutoff frequency with beam spectral line is below 1 kW for train spacing larger than 100 RF buckets, if 10 ns and 17.5 ns bunch spacing are excluded from operation.

• HOM frequency ranges for new cavity designs which are “safe” for given bunch spacings were identified.

• Transient beam loading is dominated by abort gap. For $t_{\text{gap}} > 2$ µs:
  • peak-to-peak cavity amplitude modulation $> 6\%$,
  • peak-to-peak cavity phase modulation $> 60$ ps, but collision point shift still can be eliminated by matching abort gap transients.

• Longitudinal coupled-bunch instability due to fundamental cavity impedance can be mitigated using direct RF feedback with loop delay of 700 ns.
Thank you for your attention!
Shift of the resonant frequency

There are many cases when the spectrum line hits the resonant line.
Not all of them are dangerous.

\[ f_r = 694 \text{ MHz}, \quad R/Q = 12 \Omega, \quad Q = 640, \quad t_{bb} = 2.5 \text{ ns} \]

"Resonant" condition\*:
\[ 1 - \frac{|f_r t_{tt}|}{f_r t_{tt}} < \frac{1}{Q} \]

Comparison with Pedersen model

$t_{tt} f_{HF} = 100$, $t_{bb} = 2.5$ ns, $M_b = 12$, $n_{tr} = 1298$, $t_{gap} = 2.4$ $\mu$s

- **Simulations**
- **Pedersen model**

### Amplitude Modulation $a_v$ (%)
- $t_{tt} f_{HF} = 100$, $t_{bb} = 2.5$ ns, $M_b = 12$, $n_{tr} = 1298$, $t_{gap} = 2.4$ $\mu$s

### Phase Modulation $\phi$ (deg.)
- $t_{tt} f_{HF} = 100$, $t_{bb} = 2.5$ ns, $M_b = 12$, $n_{tr} = 1298$, $t_{gap} = 2.4$ $\mu$s
Dependence on the train spacing for bunch spacing of 17.5 ns
Beam current for different filling schemes