## Z pole scan strategy and $\sqrt{s}$ spread measurement

<table>
<thead>
<tr>
<th></th>
<th>Z</th>
<th>W</th>
<th>H (ZH)</th>
<th>ttbar</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>beam energy [GeV]</strong></td>
<td>45.6</td>
<td>80</td>
<td>120</td>
<td>175-182.5</td>
</tr>
<tr>
<td><strong>arc cell optics</strong></td>
<td>60/60</td>
<td>60/60</td>
<td>90/90</td>
<td>90/90</td>
</tr>
<tr>
<td><strong>emittance hor/vert [nm]/[pm]</strong></td>
<td>0.27/1.0</td>
<td>0.84/1.7</td>
<td>0.63/1.3</td>
<td>1.4/2.8</td>
</tr>
<tr>
<td><strong>$\beta^*$ horiz/vertical [m]/[mm]</strong></td>
<td><strong>0.15/0.8</strong></td>
<td><strong>0.2/1</strong></td>
<td><strong>0.3/1</strong></td>
<td><strong>1/1.6</strong></td>
</tr>
<tr>
<td><strong>SR energy loss / turn (GeV)</strong></td>
<td>0.036</td>
<td>0.34</td>
<td>1.72</td>
<td>9.21</td>
</tr>
<tr>
<td><strong>total RF voltage [GV]</strong></td>
<td>0.10</td>
<td>0.75</td>
<td>2.0</td>
<td>8.8-10.3</td>
</tr>
<tr>
<td><strong>energy acceptance [%]</strong></td>
<td>±1.3</td>
<td>±1.3</td>
<td>±1.7</td>
<td>±2.4-2.8</td>
</tr>
<tr>
<td><strong>energy spread (SR / BS) [%]</strong></td>
<td>0.038 / <strong>0.132</strong></td>
<td>0.066 / <strong>0.165</strong></td>
<td>0.099 / <strong>0.165</strong></td>
<td>0.15 / <strong>0.20</strong></td>
</tr>
<tr>
<td><strong>bunch length (SR / BS) [mm]</strong></td>
<td>3.5 / 12.1</td>
<td>3.0 / 7.5</td>
<td>3.15 / 5.3</td>
<td>2.75 / 3.80</td>
</tr>
<tr>
<td><strong>bunch intensity $[10^{11}]$</strong></td>
<td>1.7</td>
<td>2.3</td>
<td>1.8</td>
<td>3.2-3.35</td>
</tr>
<tr>
<td><strong>no. of bunches / beam</strong></td>
<td>16640</td>
<td>1300</td>
<td>328</td>
<td>40-33</td>
</tr>
<tr>
<td><strong>beam current [mA]</strong></td>
<td>1390</td>
<td>147</td>
<td>29</td>
<td>6.4-5.4</td>
</tr>
<tr>
<td><strong>SR total power [MW]</strong></td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td><strong>luminosity $[10^{34} \text{ cm}^{-2} \text{s}^{-1}]$</strong></td>
<td>230</td>
<td>34</td>
<td>8.5</td>
<td>1.9-1.7</td>
</tr>
<tr>
<td><strong>luminosity lifetime [min]</strong></td>
<td>70</td>
<td>24</td>
<td>18</td>
<td>25</td>
</tr>
<tr>
<td><strong>allowable asymmetry [%]</strong></td>
<td>±5</td>
<td>±3</td>
<td>±3</td>
<td>±3</td>
</tr>
</tbody>
</table>
Z pole scan strategy: # points? Energies?

- At the Z pole, \( \frac{\delta E_{\text{beam}}}{E_{\text{beam}}} \approx 0.132\% \Rightarrow \delta E_{\text{beam}} \approx 60 \text{ MeV} \)
Z pole scan strategy without energy spread

- Three parameters to fit: $m_Z$, $\Gamma_Z$, and $\sigma^o$
  - Need (at least) three centre-of-mass energies at and around the Z pole
    - With a calibration of the beam energy to 50 keV (A. Blondel’s talk)
    - With a relative precision on the absolute luminosity of $10^{-4}$ (M. Dam’s talk)
      - And a point-to-point relative precision of $5 \times 10^{-5}$
  - Precision targets
    - 100 keV on $m_Z$; 100 keV on $\Gamma_Z$; dominated by the above.
    - Statistical precision is a few keV with the FCC-ee statistics
  - Possible beam energies around half-integer spin tunes $\nu = E_{\text{beam}} / 0.4406486$

<table>
<thead>
<tr>
<th>$\nu$</th>
<th>99.5</th>
<th>100.5</th>
<th>101.5</th>
<th>102.5</th>
<th>103.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{\text{beam}}$</td>
<td>43.85</td>
<td>44.29</td>
<td>44.72</td>
<td>45.16</td>
<td>45.61</td>
</tr>
<tr>
<td>$\sqrt{s}$</td>
<td>87.69</td>
<td>88.57</td>
<td>89.43</td>
<td>90.31</td>
<td>91.21</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\nu$</th>
<th>103.5</th>
<th>104.5</th>
<th>105.5</th>
<th>106.5</th>
<th>107.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{\text{beam}}$</td>
<td>45.61</td>
<td>46.04</td>
<td>46.49</td>
<td>46.92</td>
<td>47.36</td>
</tr>
<tr>
<td>$\sqrt{s}$</td>
<td>91.21</td>
<td>92.07</td>
<td>92.97</td>
<td>93.83</td>
<td>94.71</td>
</tr>
</tbody>
</table>
Z pole scan strategy without energy spread

- **Result of the 3-parameter fit to a Breit-Wigner**
  - With 100 ab⁻¹ at the peak (\(\sqrt{s} = 91.21\) GeV)
  - With 30 ab⁻¹ + 30 ab⁻¹ at peak ± 1, peak ± 2, peak ± 3, peak ± 4 (in spin tune)

<table>
<thead>
<tr>
<th>Scan</th>
<th>± 1</th>
<th>± 2</th>
<th>± 3</th>
<th>± 4</th>
<th>(−4,+3) or (−3,+4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma(m_Z)) [keV]</td>
<td>84</td>
<td>85</td>
<td>91</td>
<td>99</td>
<td>95</td>
</tr>
<tr>
<td>(\sigma(\Gamma_Z)) [keV]</td>
<td>313</td>
<td>153</td>
<td>116</td>
<td>100</td>
<td>107</td>
</tr>
</tbody>
</table>

- Standard at LEP
  - EWPO measurements would have been statistics limited with ±3 or ±4
- All these options meet the (100 keV, 100 keV) target at FCC-ee
  - Need another criterion to decide

- **Note: Results of the fit with Peak, Peak ± 2, Peak ± 4**
  - \(\sigma(m_Z) = 64\) keV
  - \(\sigma(\Gamma_Z) = 97\) keV

At the expense of a loss of statistics for rare Z decays
Choice driven by $\sin^2 \theta_W^{\text{eff}}$ and $\alpha_{\text{QED}} (m_Z)$ determination from $A_{FB}(\mu \mu)$
Z pole scan strategy without energy spread

- **Best energies for** $A_{FB}$
  - 91.2 GeV for the effective weak mixing angle
  - 87.9 GeV and 94.3 GeV for the electromagnetic coupling constant

- **Closest half-integer tunes**
  - $\nu = 103.5$ for the peak
    - $\sqrt{s} = 91.2$ GeV
  - $\nu = 99.55$ below the peak ($Pk - 4$)
    - $\sqrt{s} = 87.8$ GeV
  - $\nu = 106.55$ OR $\nu = 107.5$ above the peak ($Pk + 3$ or $Pk + 4$)
    - $\sqrt{s} = 93.9$ GeV OR $\sqrt{s} = 94.7$ GeV (equidistant from 94.3 GeV)
    - Favour $Pk + 3$ for the sake of statistics (rare decays)

- **Minimal scan strategy within four years and with two detectors**
  - 100 ab$^{-1}$ at the peak ($\sqrt{s} = 91.2$ GeV)
  - 30 ab$^{-1}$ + 30 ab$^{-1}$ at peak $- 4$ ($\sqrt{s} = 87.8$ GeV) and peak $+ 3$ ($\sqrt{s} = 93.9$ GeV)

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Note: more statistics is always welcome
(3 IPs instead of 2; 6 years instead of 4; 5 points instead of 3; etc.)
The Z lineshape is slightly modified

- With $\delta E \sim 60$ MeV for each beam, $\delta \sigma/\sigma \sim -0.45\%$ at the peak and $+0.15\%$ off peak

\[ \Gamma_Z \rightarrow [\Gamma_Z^2 + 8 \delta E_{peak}^2]^{1/2} \]

- If the energy spread $\delta E_{peak}$ at the peak is known with an uncertainty $\sigma(\delta E_{peak})$
  - $\sigma(\Gamma_Z) \sim 12$ MeV $\times \sigma(\delta E_{peak})/\delta E_{peak}$
    - 1% uncertainty of $\delta E_{peak}$ leads to $> 120$ keV uncertainty on $\Gamma_Z$!
  - Need to find a way to determine $\delta E_{peak}$ to $\sim 0.2\%$
    - Relaxed to $\sim 1\%$ for $\sigma(\delta E_{off-peak})/\delta E_{off-peak}$
Z lineshape with energy spread

- Can $m_Z$, $\Gamma_Z$, and $\sigma_0$ and $\delta E$ be fit altogether with a five-point scan?
  - IFF $\delta E$ is constant in time, identical at all points, independent of the bunch ...
    - $\sigma(\delta E)/\delta E \sim 1\%$
    - $\sigma(m_Z) = 85$ keV
    - $\sigma(\Gamma_Z) = 350$ keV

- And we certainly do not want to assume that the energy spread
  - Does not depend on time
  - Does not depend on energy
  - Does not depend on the bunch
  - Does not depend on anything else

Better idea(s) required
Note: $A_{FB}^{\mu\mu}$ with energy spread

- Convoluting $A_{FB}$ with a Gaussian has almost no effect at the peak
  - The sampling of $\sqrt{s}$ around $\sqrt{s_-} = 87.8$ and $\sqrt{s_+} = 93.9$ GeV, however, is not uniform
    - Has to weigh the asymmetry by the production cross section:

\[
\Delta A_{FB}^{\mu\mu}(s_\pm) = \frac{\int A_{FB}^{\mu\mu}(s)\sigma_{\mu\mu}(s)\exp\left(-\frac{(\sqrt{s}-\sqrt{s_\pm})^2}{2s_\pm\delta^2}\right)d\sqrt{s}}{\int \sigma_{\mu\mu}(s)\exp\left(-\frac{(\sqrt{s}-\sqrt{s_\pm})^2}{2s_\pm\delta^2}\right)d\sqrt{s}} - A_{FB}^{\mu\mu}(s_\pm)
\]

- Effect is to increase the effective $\sqrt{s_-}$ and decrease the effective $\sqrt{s_+}$
  - And therefore decrease the absolute value of the asymmetry in both points

\[
\frac{\Delta A_{FB}}{A_{FB}}(s_-) = -0.99 \times 10^{-3} \quad \text{and} \quad \frac{\Delta A_{FB}}{A_{FB}}(s_+) = -1.03 \times 10^{-3}
\]

$\Rightarrow \Delta \alpha_{QED}/\alpha_{QED} \sim 10^{-3}$

Stat: $3 \times 10^{-5}$

- Need to know the energy spread to better than 1% at $\sqrt{s_+}$
  - To limit the effect on $\alpha_{QED}$ to $< 10^{-5}$ (same as the effect from the $\sqrt{s}$ knowledge)
    - Similar constraint as for the Z lineshape
How are the events modified with energy spread (in the c.o.m.)?

- Generates a longitudinal boost in the centre-of-mass frame
Make use of $e^+e^- \rightarrow \mu^+\mu^-$ events

- Competes with initial state radiation (that you cannot get rid of)

$e^+ (E_b, E_b-\delta E) \rightarrow \mu^+$

$e^- (E_b, E_b) \rightarrow \mu^-$

$\Delta \theta = \pi$

In the $e^+e^-$ centre-of-mass frame

$\Delta \theta < \pi$

No ISR

With ISR

$\gamma (\delta E)$

Need to have good ISR prediction
Make use of $e^+e^- \rightarrow \mu^+\mu^-$ events

- Also competes with muon angular resolution …

- Need to have good ISR prediction
- Need to master muon angular resolution
  - Over the whole tracker acceptance

$e^+ (E_b, E_b)$

In the $e^+e^-$ centre-of-mass frame

$\Delta \theta_{\text{ideal}} = \pi$

$\Delta \theta_{\text{meas}} < \pi$

$e^- (E_b, E_b)$

Perfect resolution

Finite resolution
The FCC-ee frame is not the e⁺e⁻ c.o.m frame

- An $e^+e^- \rightarrow \mu^+\mu^-$ event produced at $\theta=\pi/2$, with $\phi=0$ or $\phi=\pi/2$

- NB: there might also be a (small) asymmetry between the two beam energies
- Causes additional boost along $z$
Total energy-momentum conservation

- Assuming one photon emitted along one of the two beams

\[
\begin{align*}
E^+ \sin \theta^+ \cos \varphi^+ + E^- \sin \theta^- \cos \varphi^- + |p^\gamma_z| \tan \alpha/2 &= \sqrt{s} \tan \alpha/2, \\
E^+ \sin \theta^+ \sin \varphi^+ + E^- \sin \theta^- \sin \varphi^- &= 0, \\
E^+ \cos \theta^+ + E^- \cos \theta^- + p^\gamma_z &= 0, \\
E^+ + E^- + |p^\gamma_z|/ \cos \alpha/2 &= \sqrt{s}/ \cos \alpha/2,
\end{align*}
\]

- Where \( E^\pm \) are the measured energies of the \( \mu^\pm \)
- Where \( \alpha \) is the beam crossing angle (nominal: 30 mrad),
- Where the \( z \) axis is the bisector of the two beam axes,
- Where the two beam axes form the \((x,z)\) plane,
- Where \( \theta^\pm \) are measured with respect to the \( z \) axis in the FCC-ee frame,
- Where \( \phi^\pm \) are measured with respect to the \( x \) axis in the plane transverse to the \( z \) axis,
- Where \( \sqrt{s} \) is the centre-of-mass energy of the collision
Longitudinal boost $x_γ = p_z(γ)/√s$

- Solve $(E,p)$ conservation equations for $x_γ$

$$x_γ = -\frac{x_+ \cos θ^+ + x_- \cos θ^-}{\cos(α/2) + |x_+ \cos θ^+ + x_- \cos θ^-|}$$

**With**

$$x_± = \frac{∓ \sin θ^± \sin φ^±}{\sin θ^+ \sin φ^+ - \sin θ^- \sin φ^-}$$

- As a function of the muon angles only (usually better determined than momenta)

  - Assume initially that $α$ and the "natural" $(x,y,z)$ axes are known perfectly
    - $z$ axis = bissector of the two beam axes
    - $(x,z)$ plane = plane that contains the two beam axes
    - $y$ = axis going upwards perpendicularly to that plane
  - Assume that these axes match the local (detector) axes
With $10^6$ dimuon events (every 5 minutes at the Z pole)

- Returns the $\sqrt{s}$ spectrum (with asymmetry) for perfect angular resolution and no ISR
  - An angular resolution of 0.1 mrad seems adequately small (typical of CLD / IDEA)
  - ISR slightly degrades the Gaussian core and add tails, needs to be unfolded
- Returns beam energy relative asymmetry with $10^{-6}$ precision
**Effect of $\sigma_{\theta,\phi}$ and ISR**

- Fit the core of the $x_\gamma$ distribution with a Gaussian and compare to $E_{\text{spread}}$

- With $10^6$ dimuons
- With $\sigma_{\theta,\phi} = 0$ and no ISR

\[
\sqrt{2}\sigma = (0.03804 \pm 0.00004)\% \quad \text{No BS}
\]
\[
\sqrt{2}\sigma = (0.13185 \pm 0.00011)\% \quad \text{Nom. BS}
\]

- $\sim 0.1\%$ relative precision
  - **BINGO!**

- Effect of ISR is to increase the $x_\gamma$ width by 10%
  - Need to be known with a precision better than 1% for a per-mil precision on $E_{\text{spread}}$
- Effect of $\sigma_{\theta,\phi} = 0.1$ mrad is to increase the $x_\gamma$ width by 0.5% for nominal BS
  - Need to be known with a precision of $\sim 10\%$ over the whole tracker acceptance
- Effect of $\sigma_{\theta,\phi} = 1$ mrad is devastating (similar to energy spread, difficult to control)
Control the angular resolution to 0.01 mrad?

Q: How to measure the angular resolution to 10% or better
- For any value of $\theta$ and $\phi$?

A: Take a muon track in dimuon events
- Refit it with the odd hits, on the one hand, and with the even hits, on the other
  - And compare the angles
- Need only 100 tracks in each $(\theta, \phi)$ bin for a 10% precision
  - $10^6$ dimuon events = 5 minutes at the Z pole = bins of $3 \times 3$ (mrad)$^2$
- Expected to be stable in time
  - Precision (or bin size) improves with dimuon statistics
Summary (extract from the CDR)

- A three-point scan of the Z resonance (87.8, 91.2, 93.9 GeV)
  - Is adequate for the measurement of $m_Z$, $\Gamma_Z$, $\sin^2\theta_W^{\text{eff}}$ and $\alpha_{\text{QED}}$ with target precisions
  - If the beam energy spread $\delta E$ is known to a few per mil (for $\Gamma_Z$ and $\alpha_{\text{QED}}$)

- Dimuon events offer a permanent monitoring of $\delta E$
  - With adequate precision, every few minutes
  - And with the same as those used to measure the EW observables

<table>
<thead>
<tr>
<th>Pseudo Observable</th>
<th>$\Gamma_Z$</th>
<th>$\alpha_{\text{QED}}(m_Z^2)$</th>
<th>$\Gamma_W$</th>
<th>$\Gamma_{\text{top}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acceptable error</td>
<td>35 keV</td>
<td>$10^{-5}$</td>
<td>0.5 MeV</td>
<td>9 MeV</td>
</tr>
<tr>
<td>$\sqrt{s}$ (GeV)</td>
<td>87.9</td>
<td>91.2</td>
<td>93.8</td>
<td>87.9</td>
</tr>
<tr>
<td>$\sigma(\delta E)/\delta E$</td>
<td>0.8%</td>
<td>0.2%</td>
<td>0.8%</td>
<td>0.7%</td>
</tr>
<tr>
<td>$N_{e^+e^-\rightarrow\mu^+\mu^-}$</td>
<td>$5 \times 10^4$</td>
<td>$8 \times 10^5$</td>
<td>$5 \times 10^4$</td>
<td>$6.5 \times 10^4$</td>
</tr>
<tr>
<td>$L$ ($10^{34} \text{cm}^{-2}\text{s}^{-1}$)</td>
<td>230</td>
<td></td>
<td>32</td>
<td>1.8</td>
</tr>
<tr>
<td>$\sigma_{\mu\mu}$ (pb)</td>
<td>185</td>
<td>1450</td>
<td>460</td>
<td>185</td>
</tr>
<tr>
<td>Dimuon rate (Hz)</td>
<td>425</td>
<td>3325</td>
<td>1050</td>
<td>425</td>
</tr>
<tr>
<td>Time needed</td>
<td>2 min</td>
<td>4 min</td>
<td>&lt; 1 min</td>
<td>3 min</td>
</tr>
</tbody>
</table>

Patrick Janot
FCC Week in Amsterdam
10 Apr 2018
All these numbers are valid for nominal luminosities / energy spread
- For smaller energy spread, the precision requirement are quadratically less stringent

Example: 0.066% instead of 0.132% at the Z pole (still BS dominated)
- Precision required increases by a factor 4 to 0.8% instead of 0.2%
  - Number of events needed decrease by a factor of 16 to $5 \times 10^4$ instead of $8 \times 10^5$.
- In the same time, the luminosity is reduced much less, by a factor $2^{2/3} \approx 1.58$
  - If the E spread decrease is achieved by increasing the vertical $\beta^*$
    - Keeping beam currents and vertical emittance unchanged
- Therefore the time needed decreases to 25 seconds instead of 4 minutes

All times displayed in the table are absolute maxima

All these numbers were derived from the muon angles only
- Angular resolution must be better than 0.1 mrad, + measured to better than 0.01 mrad
  - Angular biases must be kept below 0.01 mrad too (absolute alignment)
- To be studied: (probably marginal) influence of muon momenta
Beam crossing angle determination

- With $10^6$ dimuon events (every 5 minutes at the Z pole)

\[
\alpha = 2 \arcsin \left( \frac{\sin (\varphi^- - \varphi^+) \sin \theta^+ \sin \theta^-}{\sin \varphi^- \sin \theta^- - \sin \varphi^+ \sin \theta^+} \right)
\]

One million dimuon events

- Spread sensitive to anything happening in the transverse plane
  - \( \varphi \) resolution, \( p_T \) of emitted photons, and (\( x, y, z \)) axes knowledge

See also poster “1000 recipes to use up dimuons”

\[ \langle \alpha \rangle = 29.9998 \pm 0.0003 \text{ mrad} \]
Detector alignment

- Absolute angle determination is (usually) not an easy task
  - Requires alignment of the local (detector) frame with the natural (FCC-ee) frame
    - Z axis = solenoid axis vs bisector of the two beam axes
    - (X,Z) plane = horizontal plane vs plane containing the two beam axes

- Spread of $\alpha$ increases with anything happening in the transverse plane
  - E.g., rotation around the Z axis changes both X and Y directions

  ![Graph showing events vs crossing angle](image)

  - Similarly, rotation around the X (Y) axis changes Y (X) direction
Minimize the spread of the $\alpha$ distribution to find the three Euler angles.

- Note: $\alpha$ spread dominated by the $\phi$ resolution (here 0.1 mrad)
- Precisions quadratically improves with the resolution in $\phi$ (here 0.1 mrad)

10 million dimuon events
Detector alignment

- Improve the angle corresponding to a rotation around the Y axis
  - X and Z information get mixed by such a rotation
  - Resulting in a strong (linear) correlation between $x_\gamma$ and $\alpha$:

10 million dimuon events
Detector alignment

- Minimize the correlation between $x_\gamma$ and $\alpha$:
  - Improves the precision on that crossing angle by a factor of five.
  - Reach a precision of 0.1 $\mu$rad on $\alpha$ and of $10^{-7}$ on $x_\gamma$
  - Variation of the $x_\gamma$ spread already insignificant with 100 times less events
Detector alignment

- A finite crossing angle is the key
  - A zero crossing angle cannot be determined with precision \((\sin\phi^+ = \sin\phi^-, \sin\theta^+ = \sin\theta^-)\)

\[
\alpha = 2 \arcsin \left[ \frac{\sin (\varphi^- - \varphi^+) \sin \theta^+ \sin \theta^-}{\sin \varphi^- \sin \theta^- - \sin \varphi^+ \sin \theta^+} \right]
\]

- Spread is infinitely large even with a perfect detector alignment
- Leads to inaccurate absolute alignment
  - With corresponding biases on the muon angles

- With a finite crossing angle, only events with \(\sin \phi^+ \sim \sin \phi^- \sim 0\) need to be rejected
  - Cut \(|\sin \phi^*| > 0.2\) was applied in all previous plots

- Possible bonus (to be studied)
  - Built-in energy asymmetry between electrons and positrons