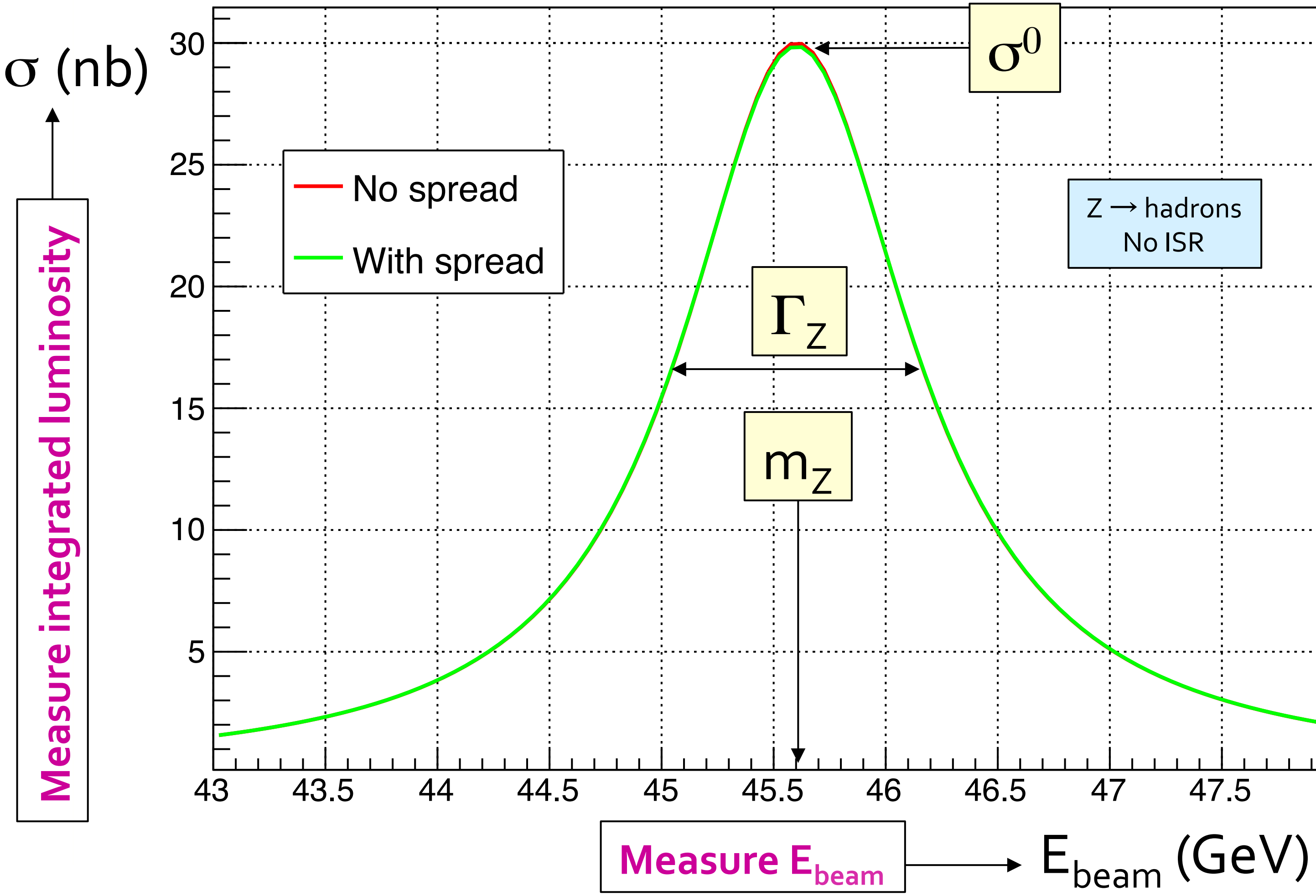
Z pole scan strategy and √s spread measurement

	Z	W	H (ZH)	ttbar
beam energy [GeV]	45.6	80	120	175-182.5
arc cell optics	60/60	60/60	90/90	90/90
emittance hor/vert [nm]/[pm]	0.27/1.0	0.84/1.7	0.63/1.3	1.4/2.8
β* horiz/vertical [m]/[mm]	**0.15/.8**	**0.2/1**	**0.3/1**	**1/1.6**
SR energy loss / turn (GeV)	0.036	0.34	1.72	9.21
total RF voltage [GV]	**0.10**	**0.75**	**2.0**	**8.8-10.3**
energy acceptance [%]	±1.3	±1.3	±1.7	±2.4-2.8
energy spread (SR / **BS) [%]**	0.038 / **0.132**	0.066 / **0.165**	0.099 / **0.165**	0.15 / **0.20**
bunch length (SR / BS) [mm]	3.5 / 12.1	3.0 / 7.5	3.15 / 5.3	2.75 / 3.80
bunch intensity [10^{11}]	1.7	2.3	1.8	3.2-3.35
no. of bunches / beam	**16640**	**1300**	**328**	**40-33**
beam current [mA]	**1390**	**147**	**29**	**6.4-5.4**
SR total power [MW]	**100**	**100**	**100**	**100**
luminosity [10^{34} cm^{-2}s^{-1}]	**230**	**34**	**8.5**	**1.9-1.7**
luminosity lifetime [min]	**70**	**24**	**18**	**25**
allowable asymmetry [%]	±5	±3	±3	±3

Z pole scan strategy: # points ? Energies ?

- **At the Z pole, $\delta E_{beam}/E_{beam} \simeq 0.132\% \Rightarrow \delta E_{beam} \simeq 60$ MeV**



Z pole scan strategy without energy spread

- Three parameters to fit : m_Z, Γ_Z, and σ^0
  - Need (at least) three centre-of-mass energies at and around the Z pole
    - With a calibration of the beam energy to 50 keV (A. Blondel's talk)
    - With a relative precision on the absolute luminosity of 10^{-4} (M. Dam's talk)
      - ➨ And a point-to-point relative precision of 5×10^{-5}
  - Precision targets
    - 100 keV on m_Z ; 100 keV on Γ_Z; dominated by the above.
    - Statistical precision is a few keV with the FCC-ee statistics
  - Possible beam energies around half-integer spin tunes $\nu = E_{beam} / 0.4406486$

ν	99.5	100.5	101.5	102.5	103.5
E_{beam}	43.85	44.29	44.72	45.16	45.61
$\sqrt{s}$	87.69	88.57	89.43	90.31	91.21

ν	103.5	104.5	105.5	106.5	107.5
E_{beam}	45.61	46.04	46.49	46.92	47.36
$\sqrt{s}$	91.21	92.07	92.97	93.83	94.71

Z pole scan strategy without energy spread

- **Result of the 3-parameter fit to a Breit-Wigner**
  - With 100 ab^{-1} at the peak ($\sqrt{s}$ = 91.21 GeV)
  - With 30 ab^{-1} + 30 ab^{-1} at peak ± 1, peak ± 2, peak ± 3, peak ± 4 (in spin tune)

Scan	± 1	± 2	± 3	± 4	(−4,+3) or (−3,+4)
$\sigma(m_Z)$ [keV]	84	85	91	99	95
$\sigma(\Gamma_Z)$ [kev]	313	153	116	100	107

- Standard at LEP
  - ➨ EWPO measurements would have been statistics limited with ±3 or ±4
- All these options meet the (100 kev, 100 keV) target at FCC-ee
  - ➨ Need another criterion to decide

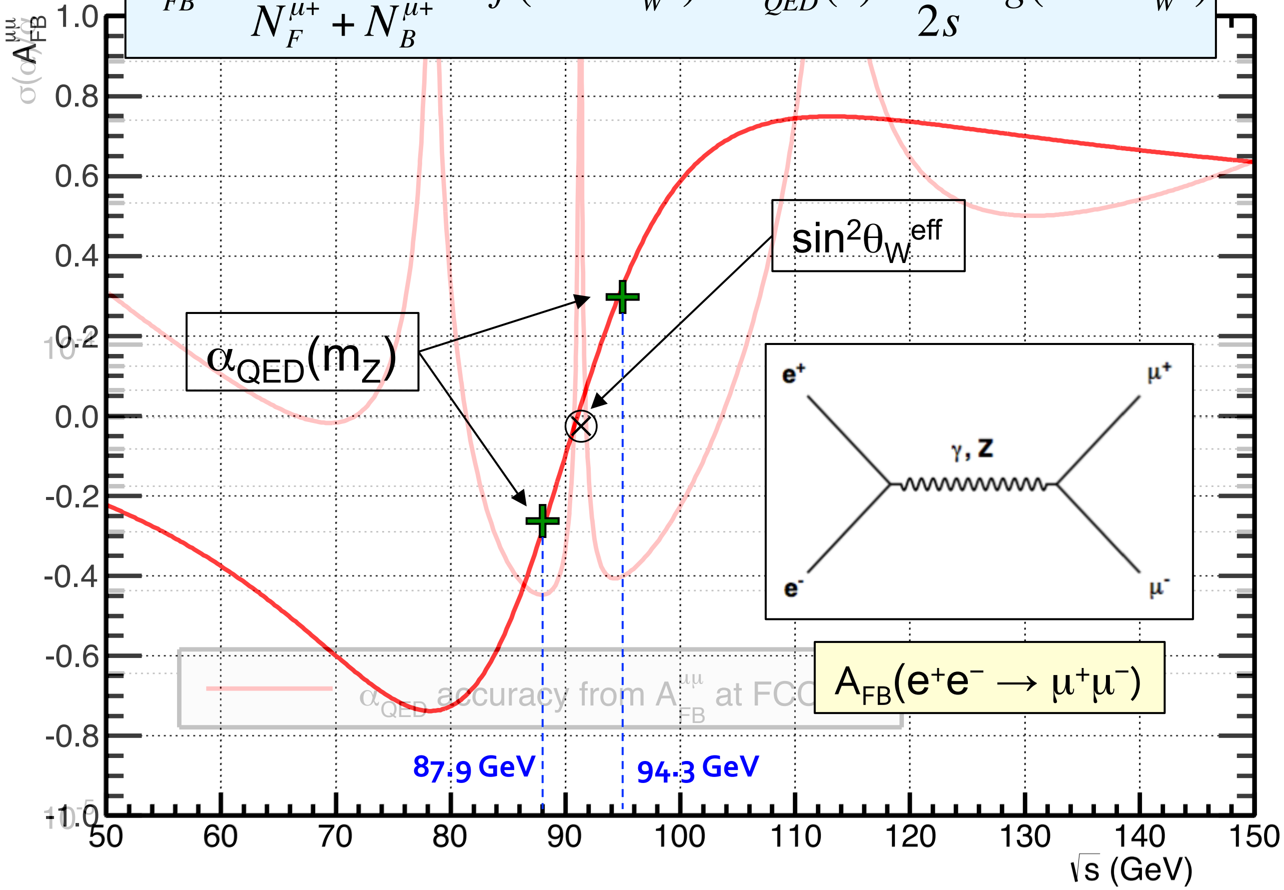
- Note: Results of the fit with Peak, Peak ± 2, Peak ± 4
  - $\sigma(m_Z)$ = 64 keV
  - $\sigma(\Gamma_Z)$ = 97 keV

At the expense of a loss of statistics for rare Z decays

Z pole scan strategy without energy spread

- **Choice driven by $\sin^2\theta_W^{eff}$ and $\alpha_{QED}(m_Z)$ determination from $A_{FB}(\mu\mu)$**

$$A_{FB}^{\mu\mu} = \frac{N_F^{\mu+} - N_B^{\mu+}}{N_F^{\mu+} + N_B^{\mu+}} \approx f(\sin^2\vartheta_W^{eff}) + \alpha_{QED}(s)\frac{s - m_Z^2}{2s} g(\sin^2\vartheta_W^{eff})$$



Z pole scan strategy without energy spread

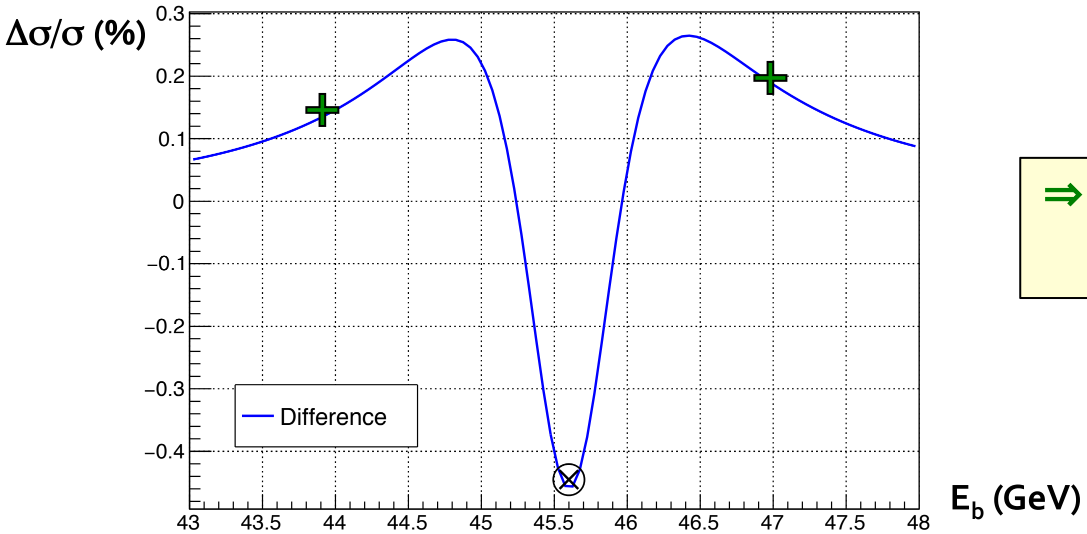
- **Best energies for A_{FB}**
  - 91.2 GeV for the effective weak mixing angle
  - 87.9 GeV and 94.3 GeV for the electromagnetic coupling constant
- **Closest half-integer tunes**
  - ν = 103.5 for the peak
    - $\sqrt{s}$ = 91.2 GeV
  - ν = 99.55 below the peak (Pk – 4)
    - $\sqrt{s}$ = 87.8 GeV
  - ν = 106.55 OR ν = 107.5 above the peak (Pk + 3 or Pk + 4)
    - $\sqrt{s}$ = 93.9 GeV OR $\sqrt{s}$ = 94.7 GeV (equidistant from 94.3 GeV)
      - ➨ Favour Pk + 3 for the sake of statistics (rare decays)
- **Minimal scan strategy within four years and with two detectors**
  - 100 ab^{-1} at the peak ($\sqrt{s}$ = 91.2 GeV)
  - 30 ab^{-1} + 30 ab^{-1} at peak – 4 ($\sqrt{s}$ = 87.8 GeV) and peak + 3 ($\sqrt{s}$ = 93.9 GeV)

Note: more statistics is always welcome

(3 IPs instead of 2; 6 years instead of 4; 5 points instead of 3; etc.)

Z lineshape with energy spread

- The Z lineshape is slightly modified
  - With δE ~ 60 MeV for each beam, $\delta\sigma/\sigma$ ~ −0.45% at the peak and +0.15% off peak



⇒ Apparent increase of the Z width

$\Gamma_Z \rightarrow [\Gamma_Z^2 + 8\ \delta E^2_{peak}]^{1/2}$

- If the energy spread δE_{peak} at the peak is known with an uncertainty $\sigma(\delta E_{peak})$
  - $\sigma(\Gamma_Z)$ ~ 12 MeV × $\sigma(\delta E_{peak})/\delta E_{peak}$
    - ➨ 1% uncertainty of δE_{peak} leads to > 120 keV uncertainty on Γ_Z !
  - Need to find a way to determine δE_{peak} to ~0.2%
    - ➨ Relaxed to ~1% for $\sigma(\delta E_{off\text{-}peak})/\delta E_{off\text{-}peak}$

Z lineshape with energy spread

- Can m_Z, Γ_Z, and σ_o and δE be fit altogether with a five-point scan ?
  - IFF δE is constant in time, identical at all points, independent of the bunch …
    - $\sigma(\delta E)/\delta E \sim 1\%$
    - $\sigma(m_Z) = 85$ keV
    - $\sigma(\Gamma_Z) = 350$ keV
  - And we certainly do not want to assume that the energy spread
    - Does not depend on time
    - Does not depend on energy
    - Does not depend on the bunch
    - Does not depend on anything else

Better idea(s) required

Note: $A_{FB}^{\mu\mu}$ with energy spread

- Convoluting A_{FB} with a Gaussian has almost no effect at the peak
  - The sampling of $\sqrt{s}$ around $\sqrt{s_-}$ = 87.8 and $\sqrt{s_+}$ = 93.9 GeV, however, is not uniform
    - Has to weigh the asymmetry by the production cross section:

$$\Delta A_{\rm FB}^{\mu\mu}(s_\pm) = \frac{\int A_{\rm FB}^{\mu\mu}(s)\sigma_{\mu\mu}(s)\exp -\frac{\left(\sqrt{s}-\sqrt{s_\pm}\right)^2}{2s_\pm\delta^2}d\sqrt{s}}{\int \sigma_{\mu\mu}(s)\exp -\frac{\left(\sqrt{s}-\sqrt{s_\pm}\right)^2}{2s_\pm\delta^2}d\sqrt{s}} - A_{\rm FB}^{\mu\mu}(s_\pm)$$

  - Effect is to increase the effective $\sqrt{s_-}$ and decrease the effective $\sqrt{s_+}$
    - And therefore decrease the absolute value of the asymmetry in both points

$$\frac{\Delta A_{\rm FB}}{A_{\rm FB}}(s_-) = -0.99\times 10^{-3} \quad \text{and} \quad \frac{\Delta A_{\rm FB}}{A_{\rm FB}}(s_+) = -1.03\times 10^{-3}$$

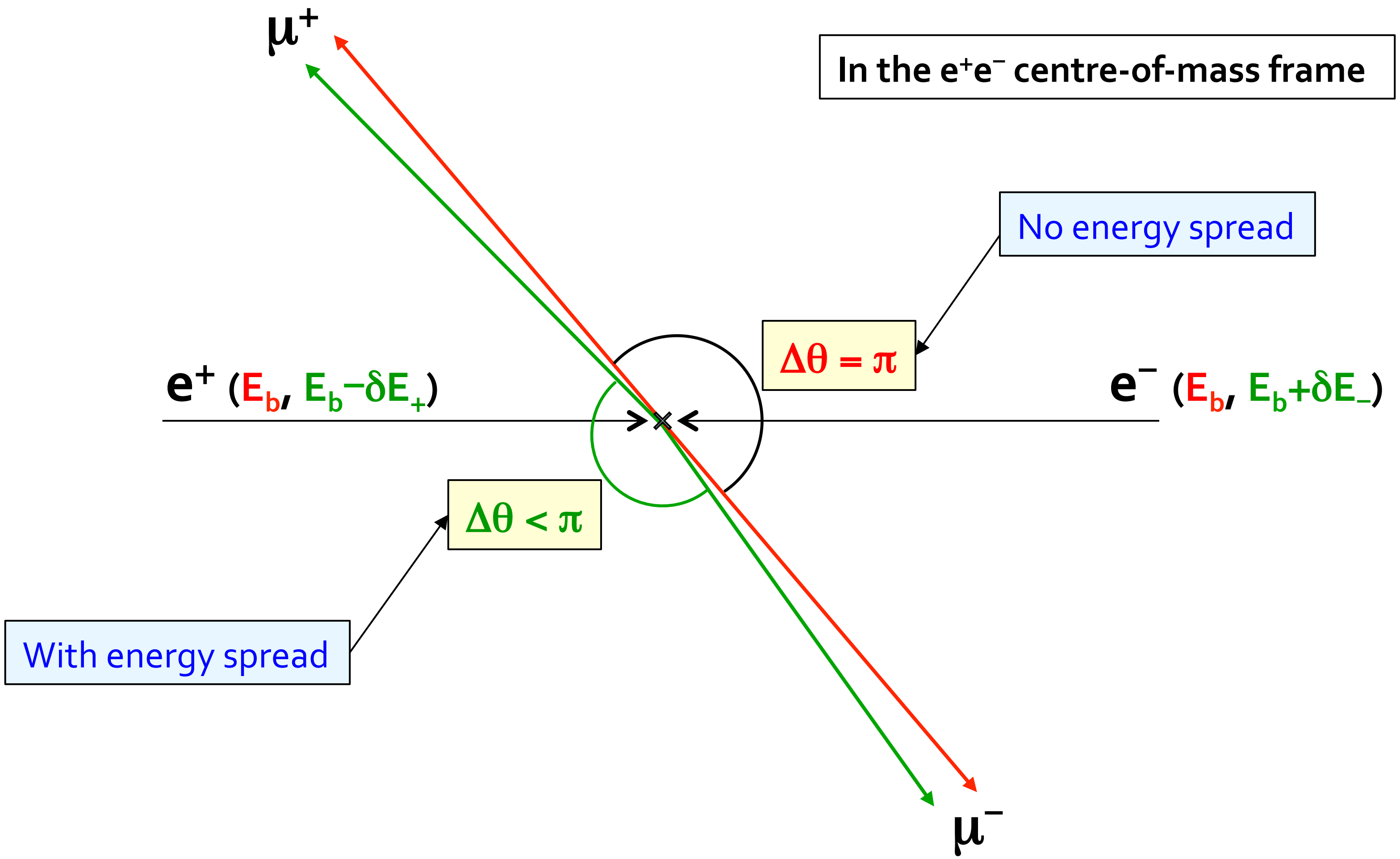
$\Rightarrow \Delta\alpha_{QED}/\alpha_{QED} \sim 10^{-3}$

Stat: 3×10^{-5}

- Need to know the energy spread to better than 1% at $\sqrt{s_\pm}$
  - To limit the effect on α_{QED} to < 10^{-5} (same as the effect from the $\sqrt{s}$ knowledge)
    - Similar constraint as for the Z lineshape

Make use of $e^+e^- \to \mu^+\mu^-$ events

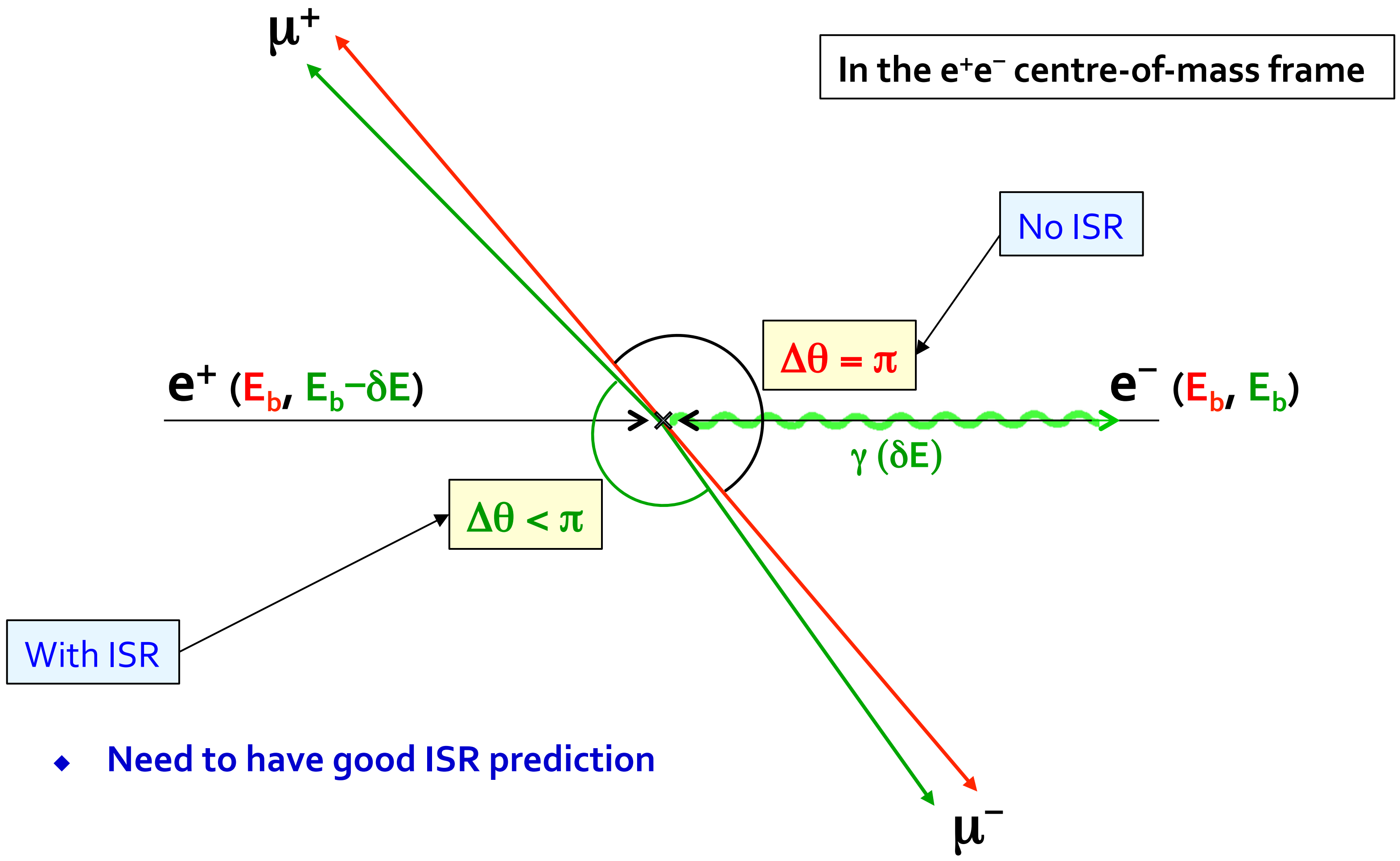
- How are the events modified with energy spread (in the c.o.m.) ?



- Generates a longitudinal boost in the centre-of-mass frame

Make use of $e^+e^- \rightarrow \mu^+\mu^-$ events

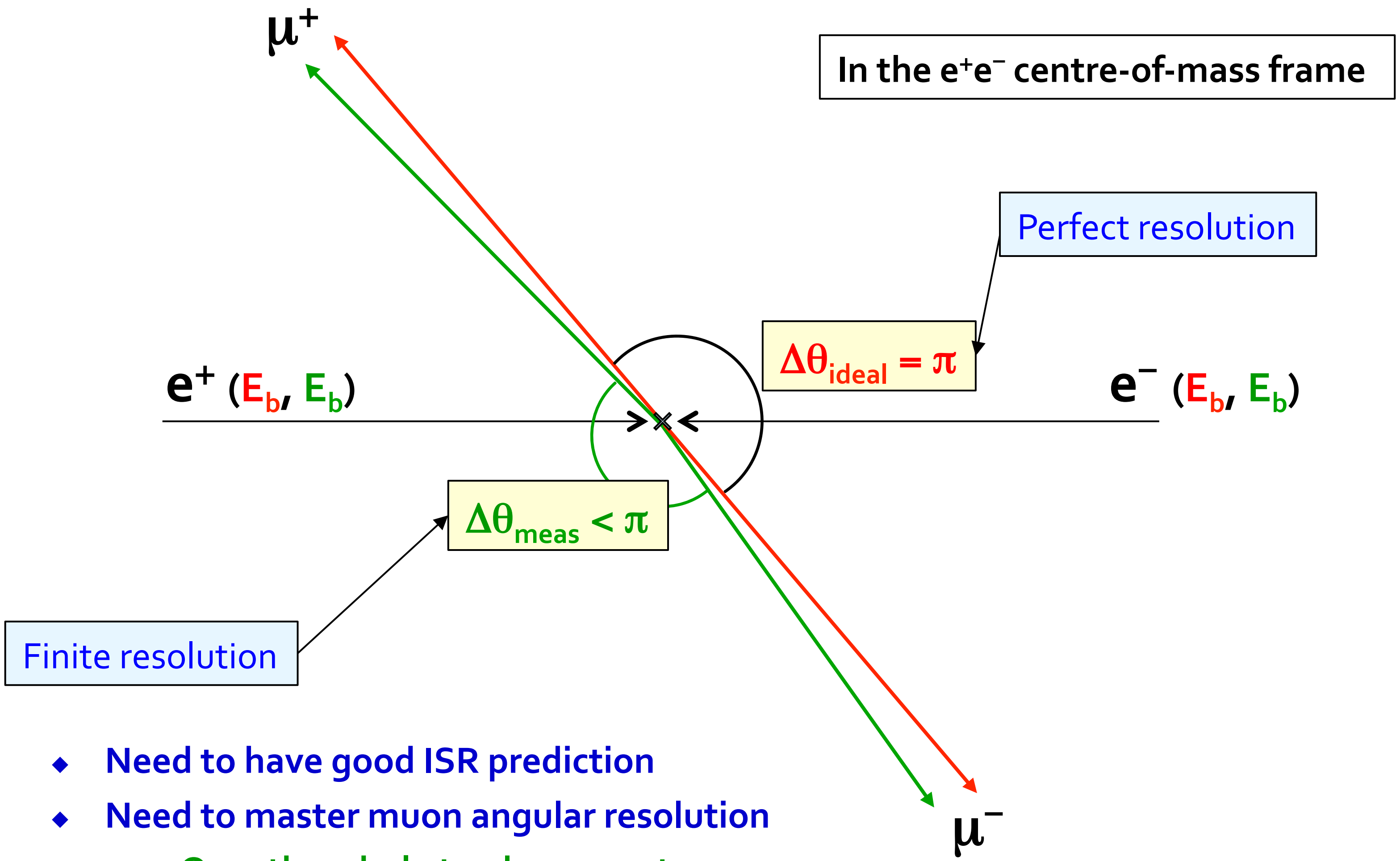
- Competes with initial state radiation (that you cannot get rid of)



- Need to have good ISR prediction

Make use of $e^+e^- \rightarrow \mu^+\mu^-$ events

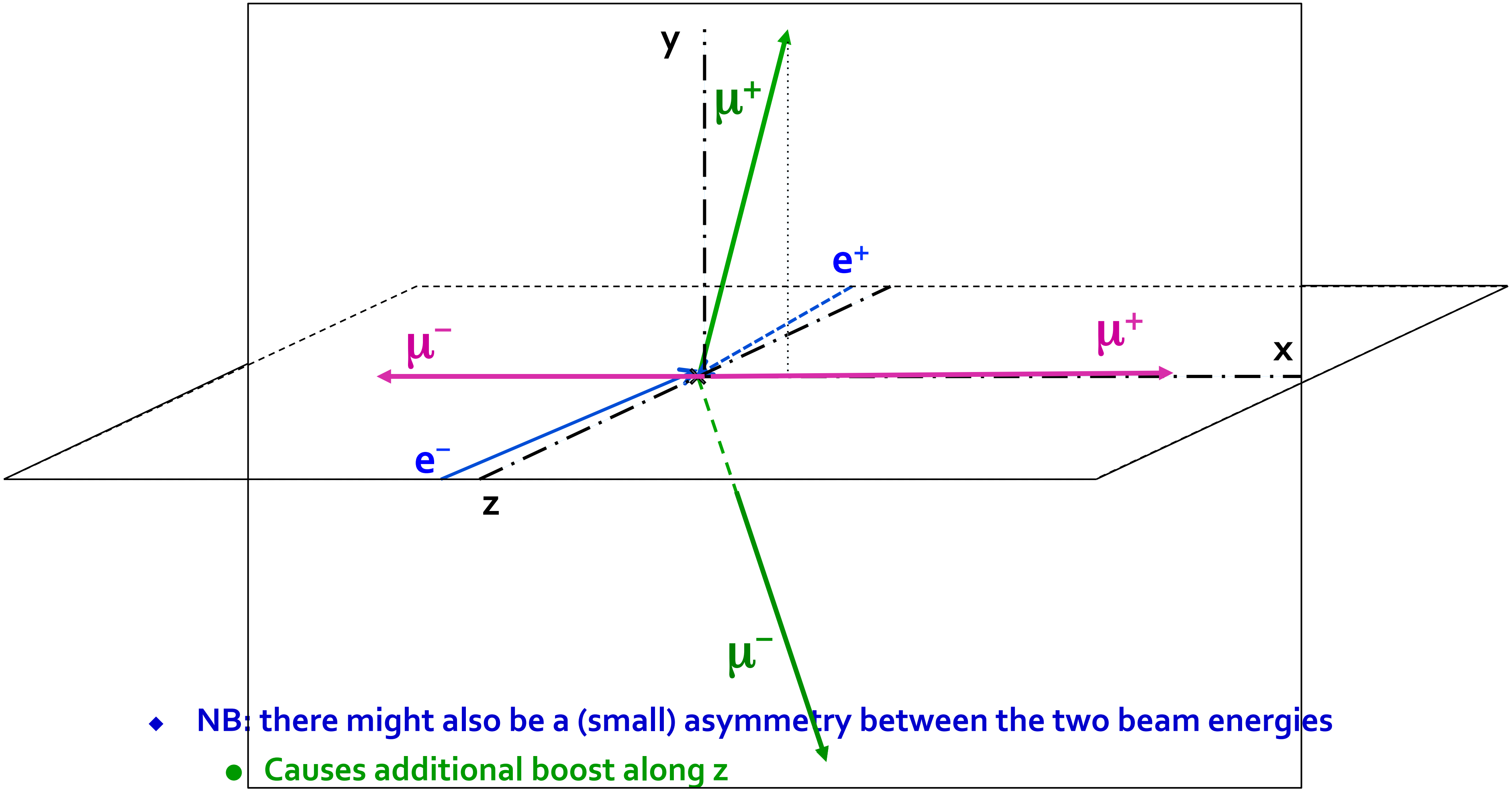
- Also competes with muon angular resolution …



- Need to have good ISR prediction
- Need to master muon angular resolution
  - Over the whole tracker accpetance

The FCC-ee frame is not the e^+e^- c.o.m frame

- An $e^+e^- \rightarrow \mu^+\mu^-$ event produced at $\theta=\pi/2$, with $\phi=0$ or $\phi=\pi/2$



- NB: there might also be a (small) asymmetry between the two beam energies
  - Causes additional boost along z

Total energy-momentum conservation

- Assuming one photon emitted along one of the two beams

$$
\begin{aligned}
E^+ \sin\theta^+ \cos\varphi^+ + E^- \sin\theta^- \cos\varphi^- + |p_z^\gamma| \tan\alpha/2 &= \sqrt{s}\tan\alpha/2, \\
E^+ \sin\theta^+ \sin\varphi^+ + E^- \sin\theta^- \sin\varphi^- \qquad &= 0, \\
E^+ \cos\theta^+ \qquad + E^- \cos\theta^- \qquad + \; p_z^\gamma &= 0, \\
E^+ \qquad + E^- \qquad + \; |p_z^\gamma|/\cos\alpha/2 &= \sqrt{s}/\cos\alpha/2,
\end{aligned}
$$

- Where $E^\pm$ are the measured energies of the $\mu^\pm$
- Where α is the beam crossing angle (nominal : 30 mrad),
- Where the z axis is the bisector of the two beam axes,
- Where the two beam axes form the (x,z) plane,
- Where $\theta^\pm$ are measured with respect to the z axis in the FCC-ee frame,
- Where $\phi^\pm$ are measured with to the x axis in the plane transverse to the z axis,
- Where $\sqrt{s}$ is the centre-of-mass energy of the collision

Longitudinal boost $x_\gamma = p_z(\gamma)/\sqrt{s}$

- **Solve (E,p) conservation equations for x_γ**

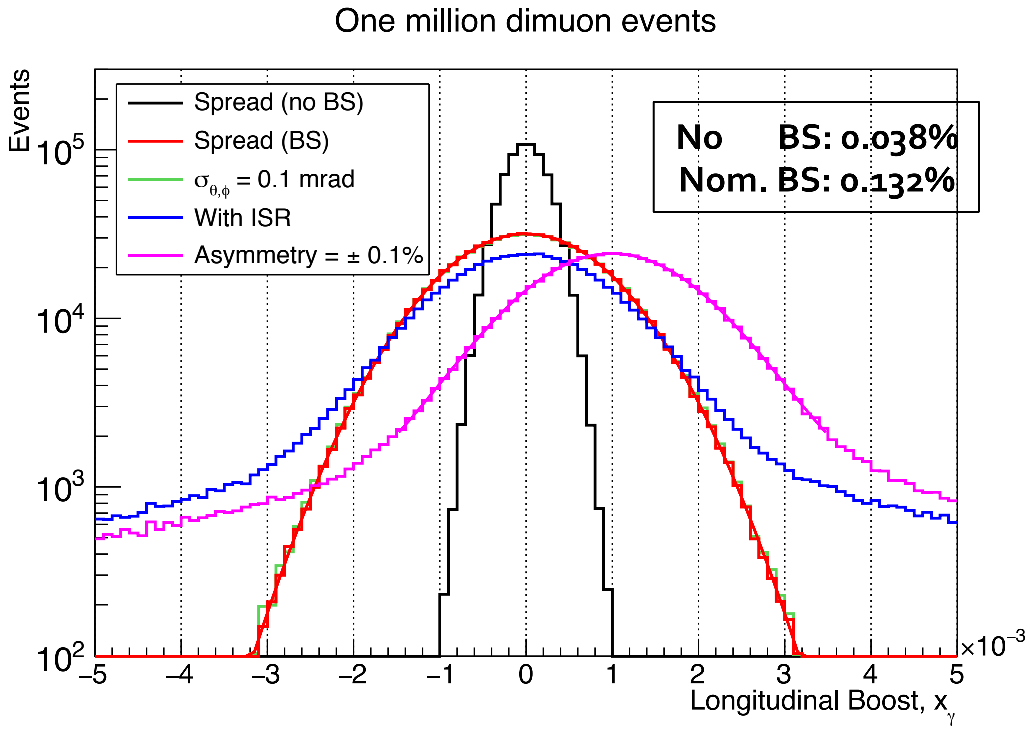
$$x_\gamma = -\frac{x_+\cos\theta^+ + x_-\cos\theta^-}{\cos(\alpha/2) + |x_+\cos\theta^+ + x_-\cos\theta^-|}$$

With $$x_\pm = \frac{\mp\sin\theta^\mp\sin\varphi^\mp}{\sin\theta^+\sin\varphi^+ - \sin\theta^-\sin\varphi^-}$$

- **As a function of the muon angles only (usually better determined than momenta)**
  - **Assume initially that α and the "natural" (x,y,z) axes are known perfectly**
    - **z axis = bissector of the two beam axes**
    - **(x,z) plane = plane that contains the two beam axes**
    - **y = axis going upwards perpendicularly to that plane**
  - **Assume that these axes match the local (detector) axes**

Distribution of x_γ

- With 10^6 dimuon events (every 5 minutes at the Z pole)



- Returns the √s spectrum (with asymmetry) for perfect angular resolution and no ISR
  - An angular resolution of 0.1 mrad seems adequately small (typical of CLD / IDEA)
  - ISR slightly degrades the Gaussian core and add tails, needs to be unfolded
- Returns beam energy relative asymmetry with 10^{-6} precision

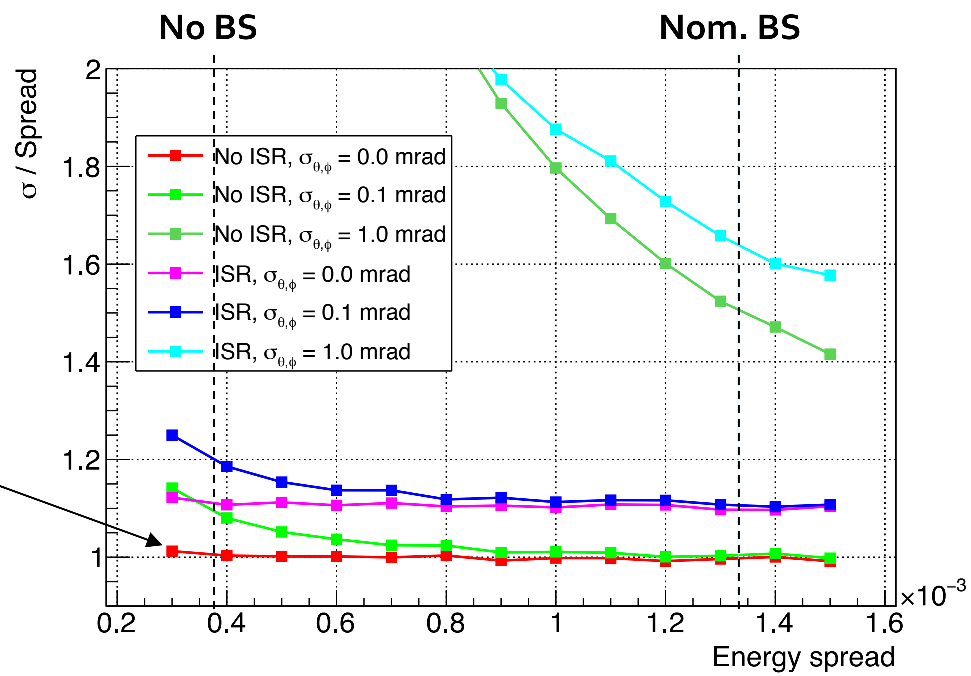
Effect of $\sigma_{\theta,\phi}$ and ISR

- Fit the core of the x_γ distribution with a Gaussian and compare to E_{spread}

  - With 10^6 dimuons
  - With $\sigma_{\theta,\phi}$ = 0 and no ISR

$$\sqrt{2}\sigma = (0.03804 \pm 0.00004)\% \quad \text{No BS}$$
$$\sqrt{2}\sigma = (0.13185 \pm 0.00011)\% \quad \text{Nom. BS}$$

  - ~0.1% relative precision
    - ➨ BINGO !



- Effect of ISR is to increase the x_γ width by 10%
  - Need to be known with a precision better than 1% for a per-mil precision on E_{spread}
- Effect of $\sigma_{\theta,\phi}$ = 0.1 mrad is to increase the x_γ width by 0.5% for nominal BS
  - Need to be known with a precision of ~10% over the whole tracker acceptance
- Effect of $\sigma_{\theta,\phi}$ = 1 mrad is devastating (similar to energy spread, difficult to control)

Control the angular resolution to 0.01 mrad ?

- **Q: How to measure the angular resolution to 10% or better**
  - For any value of θ and ϕ ?

- **A: Take a muon track in dimuon events**
  - Refit it with the odd hits, on the one hand, and with the even hits, on the other
    - And compare the angles
  - Need only 100 tracks in each (θ , ϕ) bin for a 10% precision
    - 10^6 dimuon events = 5 minutes at the Z pole = bins of 3×3 $(\text{mrad})^2$
  - Expected to be stable in time
    - Precision (or bin size) improves with dimuon statistics

Summary (extract from the CDR)

- **A three-point scan of the Z resonance (87.8, 91.2, 93.9 GeV)**
  - **Is adequate for the measurement of m_Z, Γ_Z, $\sin^2\theta_W^{eff}$ and α_{QED} with target precisions**
    - **If the beam energy spread δE is known to a few per mil (for Γ_Z and α_{QED})**
- **Dimuon events offer a permanent monitoring of δE**
  - **With adequate precision, every few minutes**
    - **And with the same as those used to measure the EW observables**

Pseudo Observable	Γ_Z			$\alpha_{QED}(m_Z^2)$		Γ_W	Γ_{top}
Acceptable error	35 keV			10^{-5}		0.5 MeV	9 MeV
$\sqrt{s}$ (GeV)	87.9	91.2	93.8	87.9	93.8	161	350
$\sigma(\delta E)/\delta E$	0.8%	0.2%	0.8%	0.7%		7%	15%
$N_{e^+e^-\to\mu^+\mu^-}$	$5\,10^4$	$8\,10^5$	$5\,10^4$	$6.5\,10^4$		650	150
L (10^{34}cm^{-2}s^{-1})	230					32	1.8
$\sigma_{\mu\mu}$ (pb)	185	1450	460	185	460	4.0	0.8
Dimuon rate (Hz)	425	3325	1050	425	1050	1.3	0.015
Time needed	2 min	4 min	<1 min	3 min	1 min	8 min	2 h 30

Notes added

- **All these numbers are valid for nominal luminosities / energy spread**
  - For smaller energy spread, the precision requirement are quadratically less stringent
- **Example: 0.066% instead of 0.132% at the Z pole (still BS dominated)**
  - Precision required increases by a factor 4 to 0.8% instead of 0.2%
    - Number of events needed decrease by a factor of 16 to 5×10^4 instead of 8×10^5.
  - In the same time, the luminosity is reduced much less, by a factor $2^{2/3} \sim 1.58$
    - If the E spread decrease is achieved by increasing the vertical β^*
      - ➨ Keeping beam currents and vertical emittance unchanged
  - Therefore the time needed decreases to 25 seconds instead of 4 minutes
- **All times displayed in the table are absolute maxima**

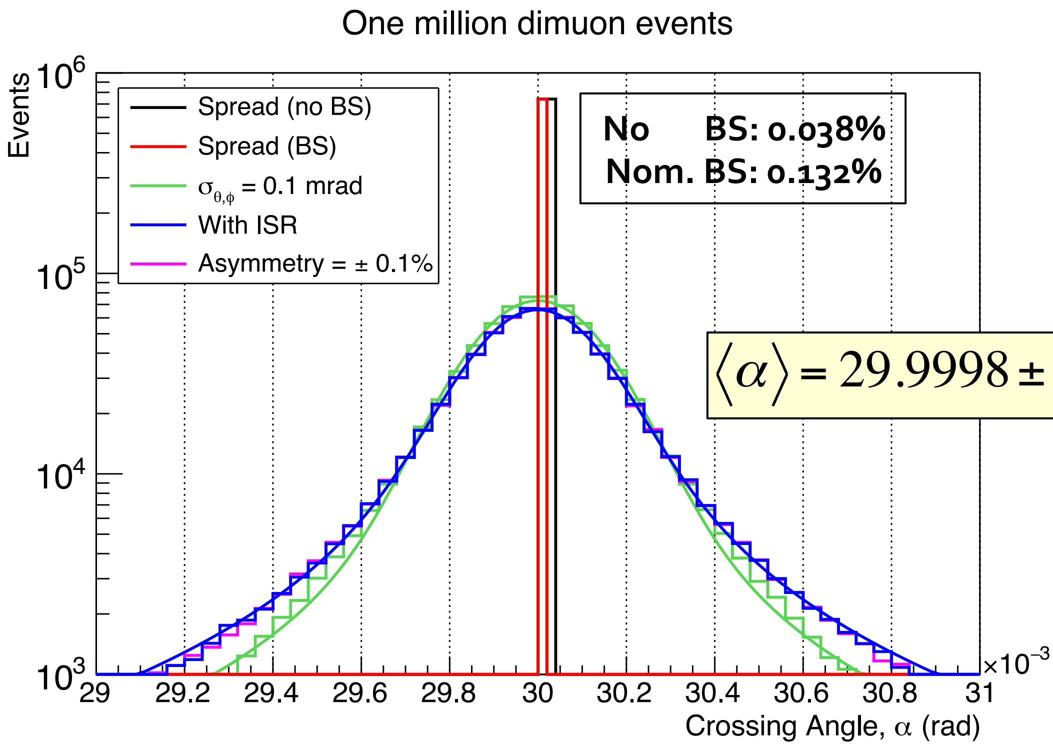
- **All these numbers were derived from the muon angles only**
  - Angular resolution must be better than 0.1 mrad, + measured to better than 0.01 mrad
    - Angular biasses must be kept below 0.01 mrad too (absolute alignment)
  - To be studied: (probably marginal) influence of muon momenta

Beam crossing angle determination

- With 10^6 dimuon events (every 5 minutes at the Z pole)

$$\alpha = 2\arcsin\left[\frac{\sin(\varphi^- - \varphi^+)\sin\theta^+\sin\theta^-}{\sin\varphi^-\sin\theta^- - \sin\varphi^+\sin\theta^+}\right]$$

See also poster
“1000 recipes to use up dimuons”

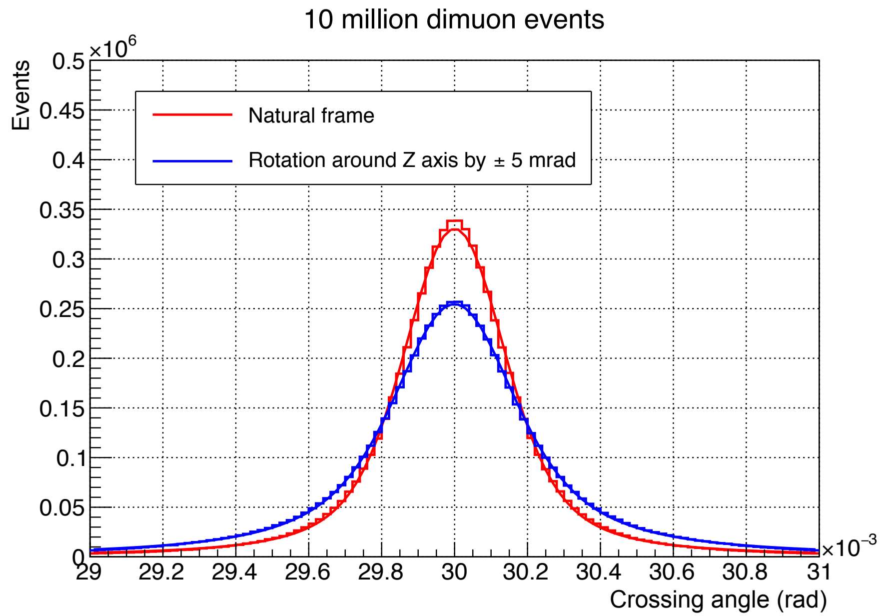


$$\langle\alpha\rangle = 29.9998 \pm 0.0003 \text{ mrad}$$

- Spread sensitive to anything happening in the transverse plane
  - ϕ resolution, p_T of emitted photons, and (x,y,z) axes knowledge

Detector alignment

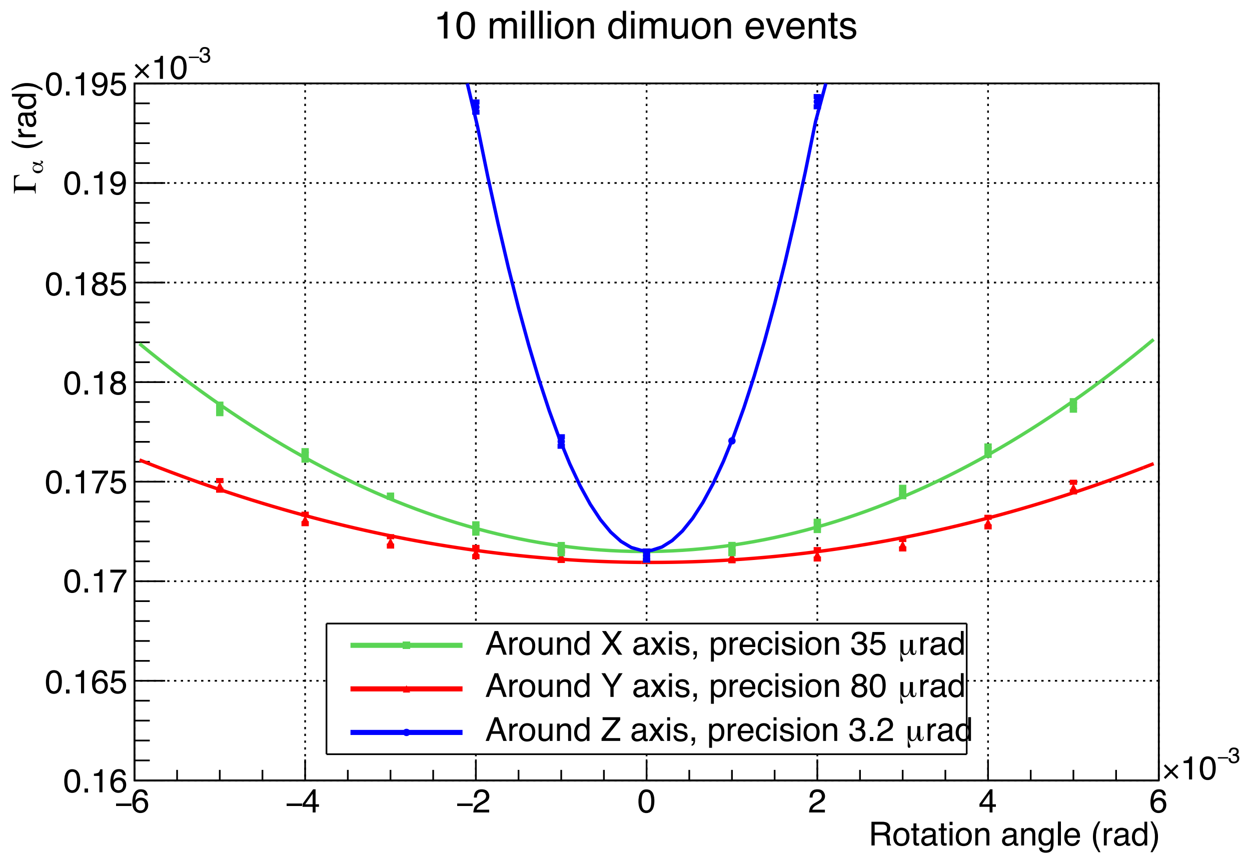
- **Absolute angle determination is (usually) not an easy task**
  - Requires alignment of the local (detector) frame with the natural (FCC-ee) frame
    - Z axis = solenoid axis vs bissector of the two beam axes
    - (X,Z) plane = horizontal plane vs plane containing the two beam axes
- **Spread of α increases with anything happening in the transverse plane**
  - E.g., rotation around the Z axis changes both X and Y directions



    - Similarly, rotation around the X (Y) axis changes Y (X) direction

Detector alignment

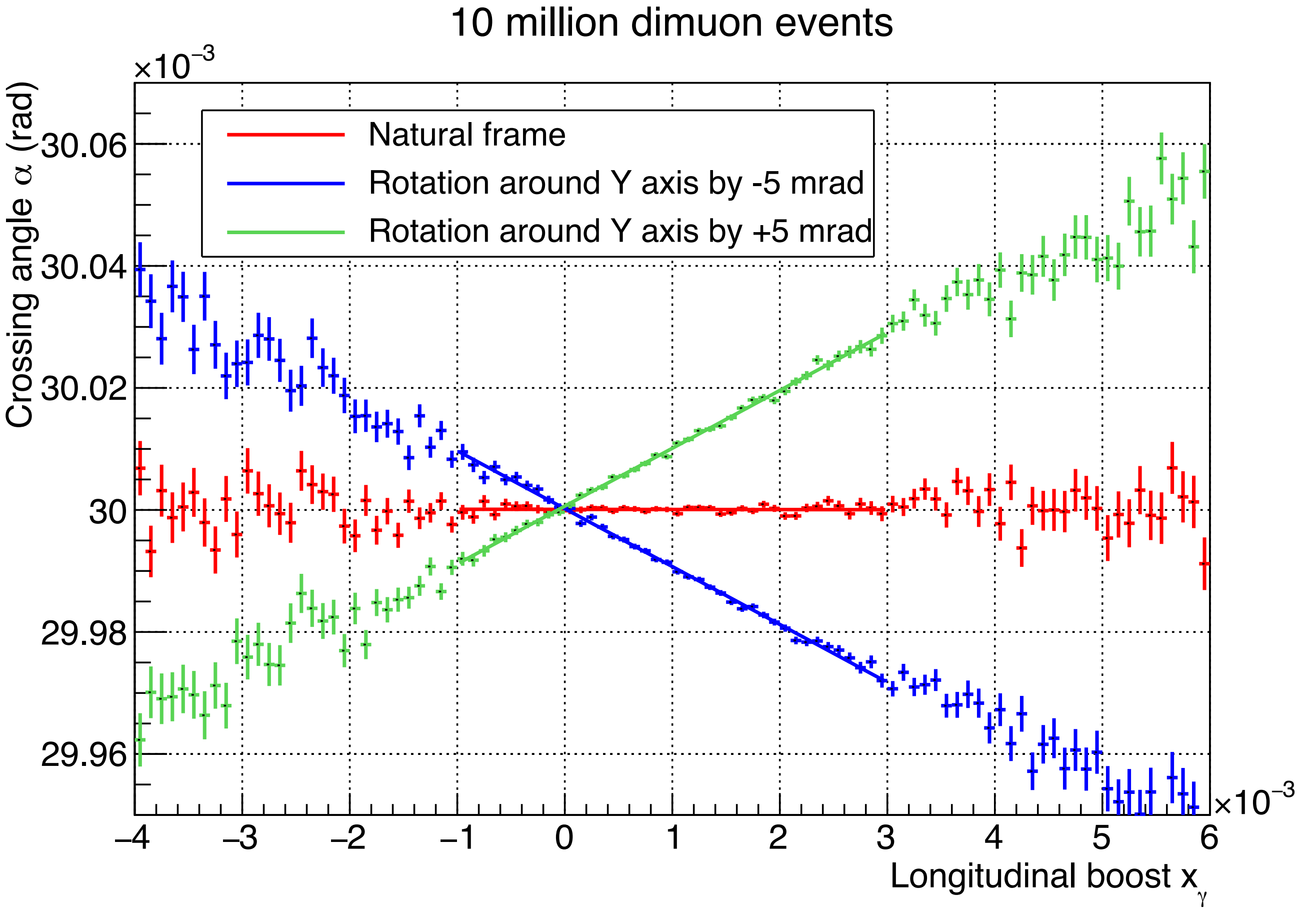
- Minimize the spread of the α distribution to find the three Euler angles



- **Note: α spread dominated by the ϕ resolution (here 0.1 mrad)**
  - **Precisions quadratically improves with the resolution in ϕ (here 0.1 mrad)**

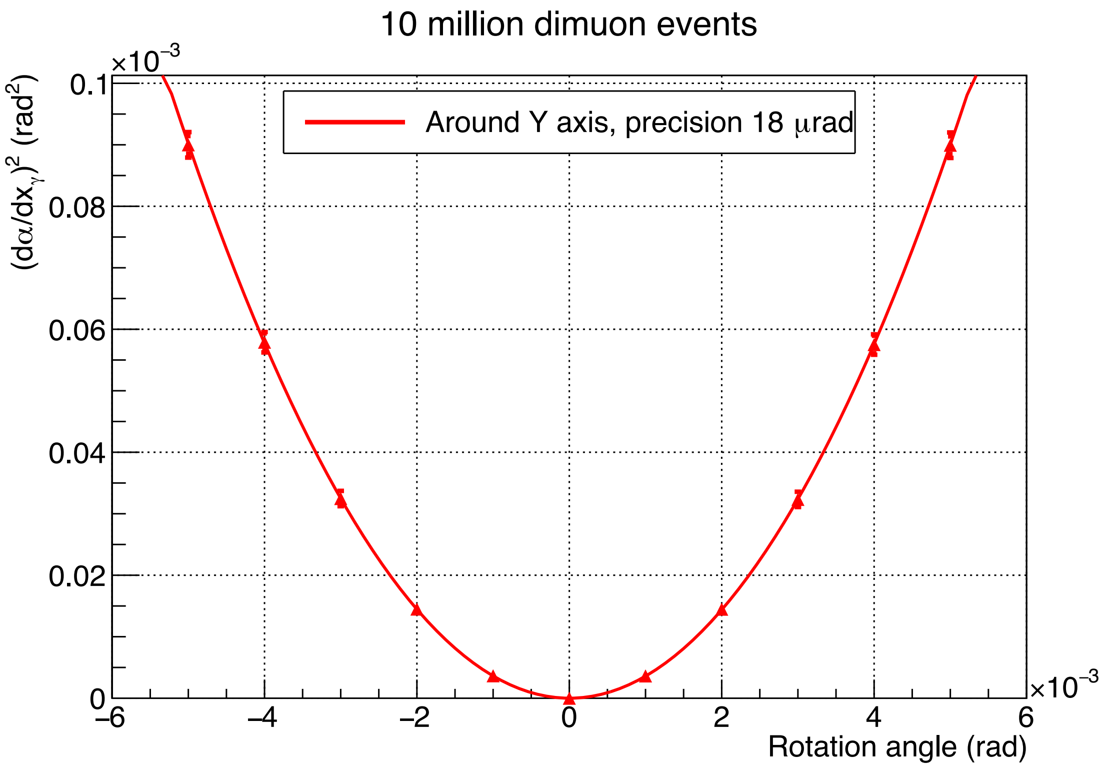
Detector alignment

- **Improve the angle corresponding to a rotation around the Y axis**
  - **X and Z information get mixed by such a rotation**
    - **Resuting in a strong (linear) correlation between x_γ and α:**



Detector alignment

- **Minimize the correlation between x_γ and α:**



- **Improves the precision on that crossing angle by a factor of five.**
  - **Reach a precision of 0.1 μrad on α and of 10^{-7} on x_γ**
  - **Variation of the x_γ spread already insignificant with 100 times less events**

Detector alignment

- A finite crossing angle is the key
  - A zero crossing angle cannot be determined with precision ($\sin\phi^+=\sin\phi^-$, $\sin\theta^+ = \sin\theta^-$)

$$\alpha = 2 \arcsin \left[\frac{\sin(\varphi^- - \varphi^+) \sin\theta^+ \sin\theta^-}{\sin\varphi^- \sin\theta^- - \sin\varphi^+ \sin\theta^+} \right]$$

    - Spread is infinitely large even with a perfect detector alignment
    - Leads to inaccurate absolute alignment
      - ➨ With corresponding biases on the muon angles
  - With a finite crossing angle, only events with $\sin\phi^+ \sim \sin\phi^- \sim 0$ need to be rejected
    - Cut $|\sin\phi^\pm| > 0.2$ was applied in all previous plots
- Possible bonus (to be studied)
  - Built-in energy asymmetry between electrons and positrons