FCC-ee beam polarization and Energy Calibration

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Some references (not a complete set!):

B. Montague, Phys.Rept. 113 (1984) 1-96;
Polarization at LEP, CERN Yellow Report *88-02;*
Beam Polarization in e+e- , AB, CERN-PPE-93-125 Adv.Ser.Direct.High Energy Phys. 14 (1995) 277-324;
L. Arnaudon et al., Accurate Determination of the LEP Beam Energy by resonant depolarization,
Z. Phys. C 66, 45-62 (1995).
Spin Dynamics in LEP http://dx.doi.org/10.1063/1.1384062
Precision EW Measts on the Z Phys.Rept.427:257-454,2006 arXiv:0509008v3
D.P. Barber and G. Ripken ``Handbook of Accelerator Physics and Engineering" World Scientific (2006), (2013)
D.P. Barber and G. Ripken, Radiative Polarization, Computer Algorithms and Spin Matching in Electron Storage Rings arXiv:physics/9907034

for FCC-ee:

First look at the physics case of TLEP arXiv:1308.6176, **JHEP 1401 (2014) 164**
DOI: 10.1007/JHEP01(2014)164
M. Koratzinos FCC-ee: Energy calibration IPAC'15 arXiv:1506.00933
E. Gianfelice-Wendt: Investigation of beam self-polarization in the FCC-ee arXiv:1705.03003

October EPOL workshop: https://indico.cern.ch/event/669194/

Requirements from physics

1. Center-of-mass energy determination with precision of $\pm$ 100 keV around the Z peak
2. Center-of-mass energy determination with precision of $\pm$ 300 keV at W pair threshold
3. For the Z peak-cross-section and width, require energy spread uncertainty $\Delta\sigma_E/\sigma_E$ =0.2%

NB: at 2.3 10^{36}/cm^2/s/IP : **full LEP statistics** 10^6 $\mu\mu$ 2.10^7 qq in **6 minutes** in each expt

-- use resonant depolarization as main measuring method
-- use pilot bunches to calibrate during physics data taking: 100 calibrations per day each 10^{-6} rel.
-- long lifetime at Z requires the use of wigglers at beginning of fills
➔ take data at points where self polarization is expected

$$\nu_s = \frac{g-2}{2}\frac{E_b}{m_e} = \frac{E_b}{0.4406486(1)} \approx N + (0.5\pm0.1)$$

$$E_{CM} = \left(N + (0.5\pm0.1)\right) \text{ x } 0.8812972 \text{ GeV}$$

Given the Z and W widths of 2 GeV, this is easy to accommodate with little loss of statistics.
It might be more difficult for the Higgs 125.09+-0.2 corresponds to ν_s = 141.94+-022

Simulations of polarization level with SITROS

Some results of coupling/dispersion correction

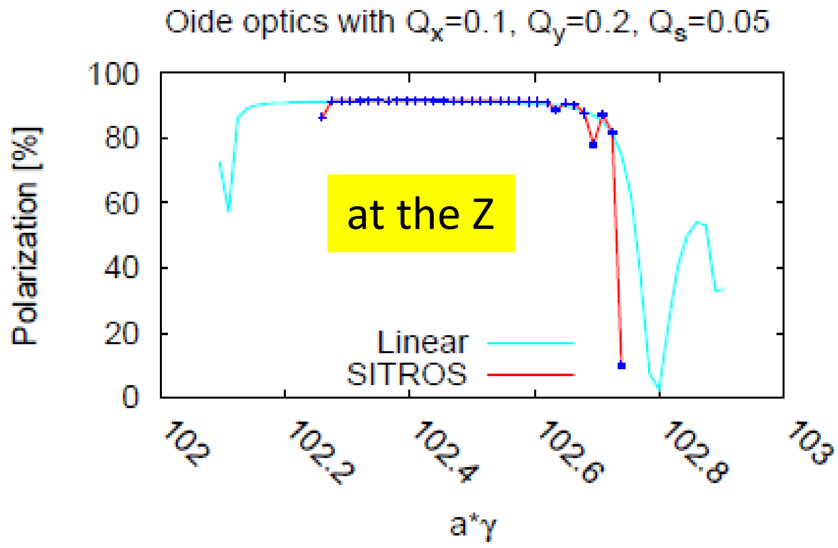
E. Gianfelice

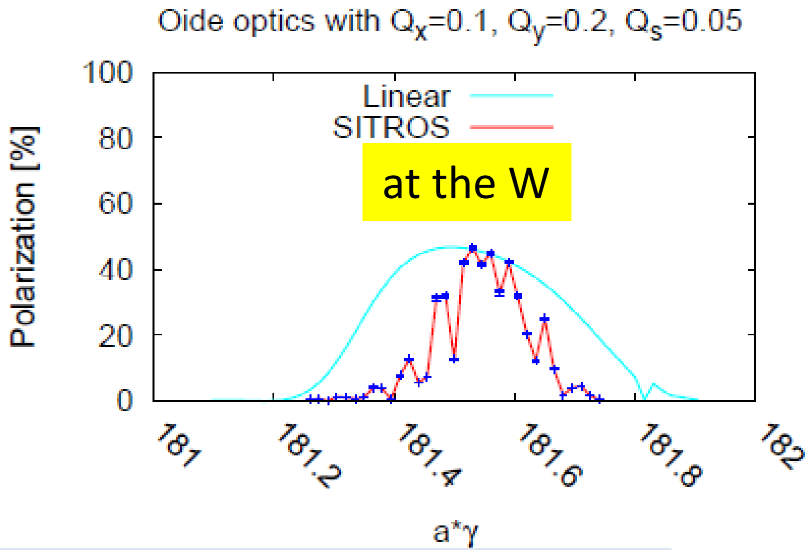
- δy^Q_{rms}=200 μm (including doublets)
- 250 μrad quadrupole roll angle (including doublets)
- 1086 BPMs w/o errors
- orbit corrected with 1086 CVs down to y_{rms}=0.05 mm
- coupling/dispersion correction with 289 skew quadrupoles

1. orbit and emittance corrections needed for the FCC-ee luminosity are sufficient to ensure useful levels of polarization.

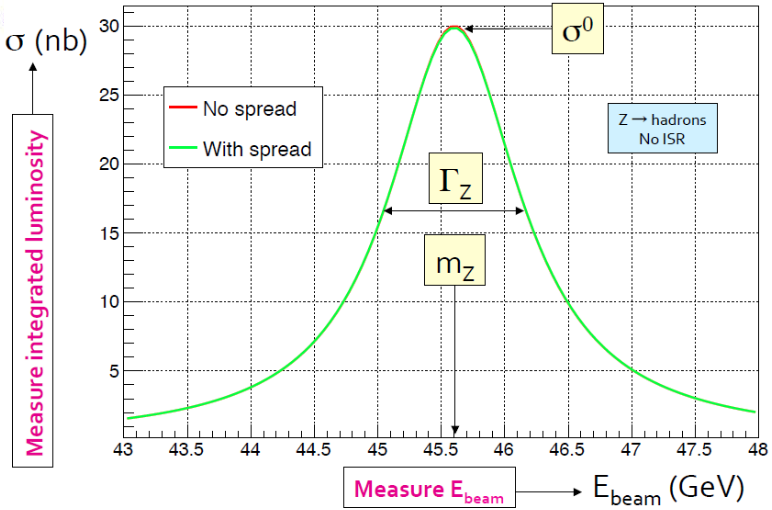
2. HOWEVER: same simulation does not produce luminosity and polarization,

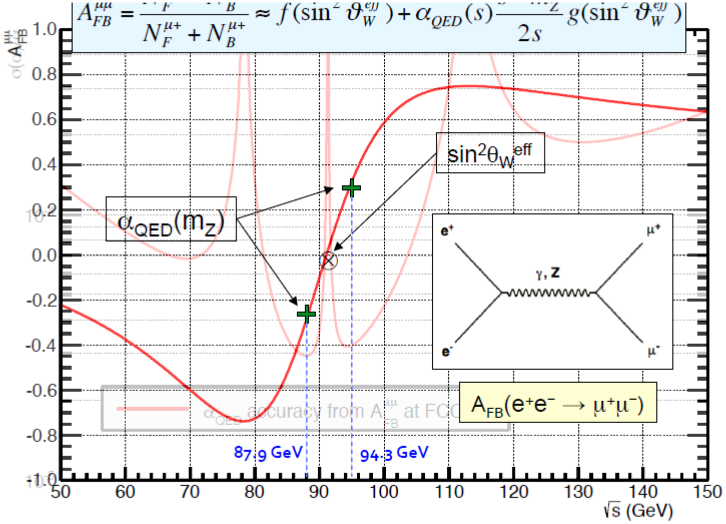
➔ effect of simultaneous optimization could not be simulated





Excellent level of polarization at the Z (even with wigglers) and sufficient at the W.





scan proposed for FCC-ee

E(peak)= 91.2 GeV spin tune = 103.5

E(-4) = 87.9 GeV spin tune = 99.5 `-4'

E(+4) = 93.8 GeV spin tune = 107.5 `+4'

E(+5) = 94.7 GeV spin tune = 108.5 `+5'

2/3 at peak 1/3 off peak.

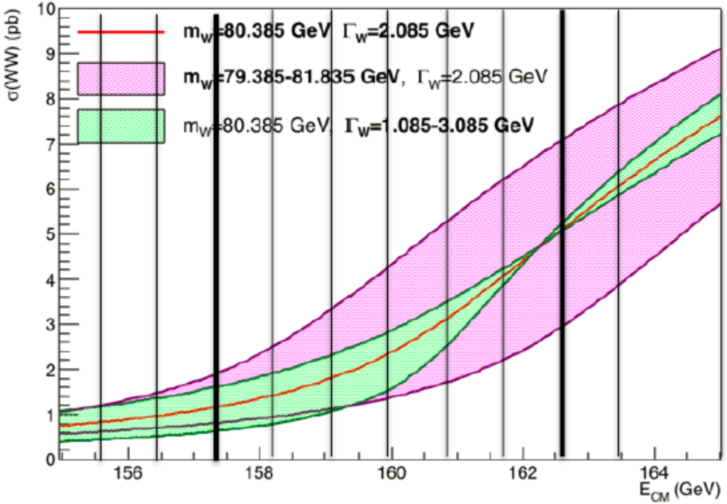
P. Janot

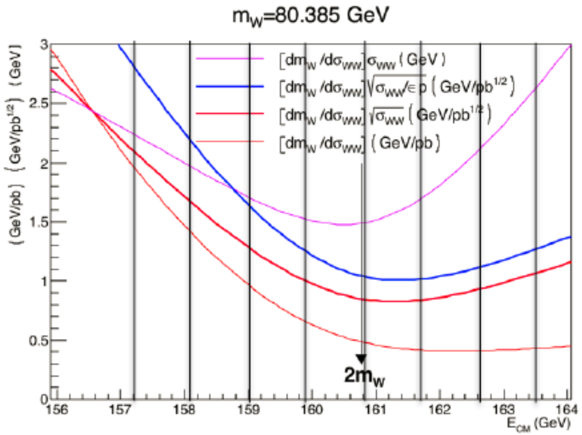
These are the beam energies for the W threshold measurement

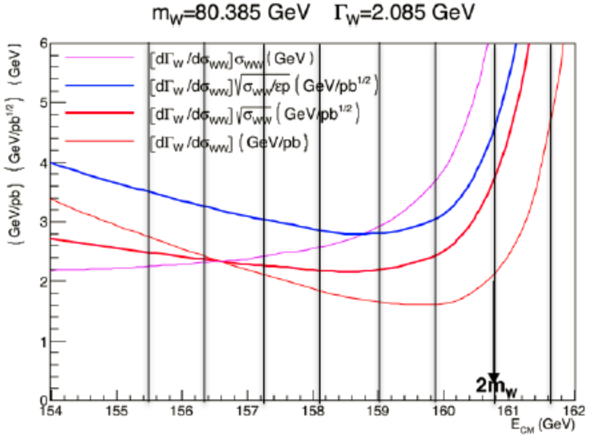
with half-integer spin tunes

P. Azzurri

limiting data taking points to
$E_{CM} = (2n+1)*0.4406486$ GeV







min $\Delta m_W + \Delta\Gamma_W$

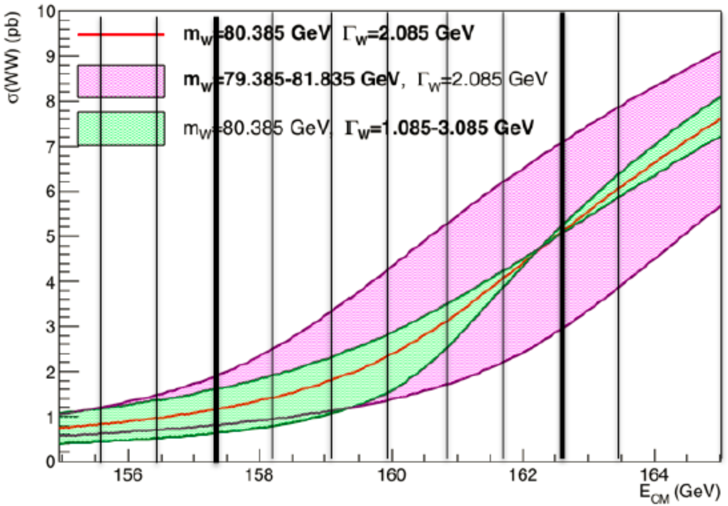
with E_1=157.3 GeV E_2=162.6 GeV f=0.4
Δm_W=0.65 $\Delta\Gamma_W$=1.6 Δm_W=0.60 (MeV)

~10% loss of stat precision

These are the beam energies for the W threshold measurement with half-integer spin tunes

P. Azzurri

limiting data taking points to
$E_{CM} = (2n+1)*0.4406486$ GeV



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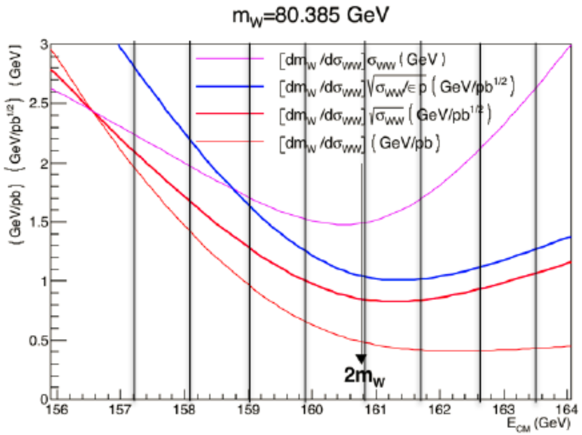
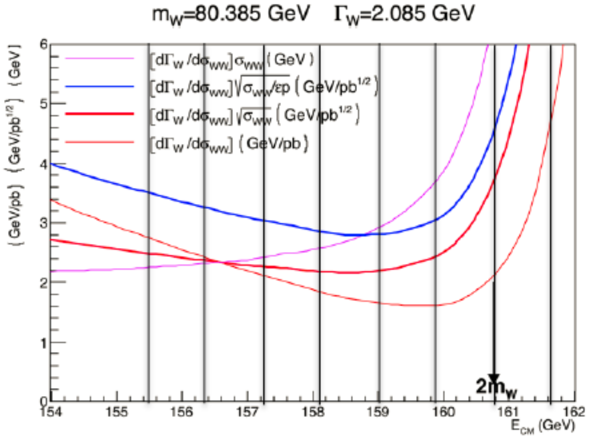




~10% loss of stat precision

A limitation: Touschek effect

Oide-san pointed out that the ‘pilot bunches’ would lose particles due to Touschek effct

Indeed they have such small emittance that the bunch population reduces fast if it is larger that 4 10^{10} at the Z.

➔ **limit pilot bunch intensity to that value**

this is less of an issue at the W
Tobias Tydecks has calculated the effect and written it up in the CDR!

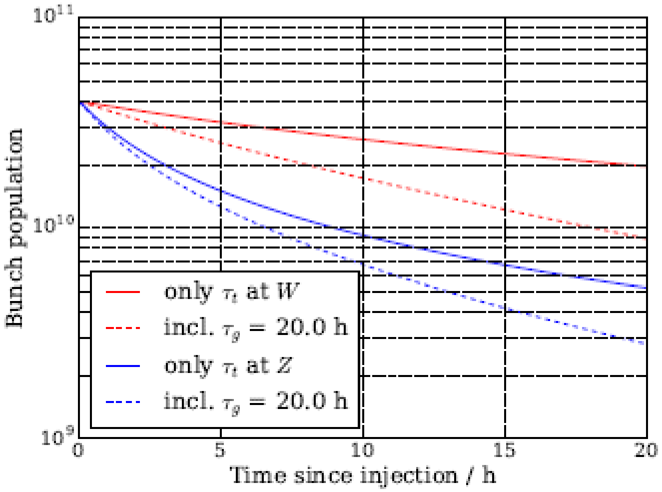


Fig. 1.6: Simulated intensity drop in non-colliding bunches due to Touschek lifetime and combination of Touschek and assumed gas scattering lifetime of $\tau_g = 20$ h for Z and W energies.

Hardware requirements: wigglers

Given the long polarization time at Z, wigglers will be necessary.
An agreement was reached on a set of **8 wiggler units per beam**

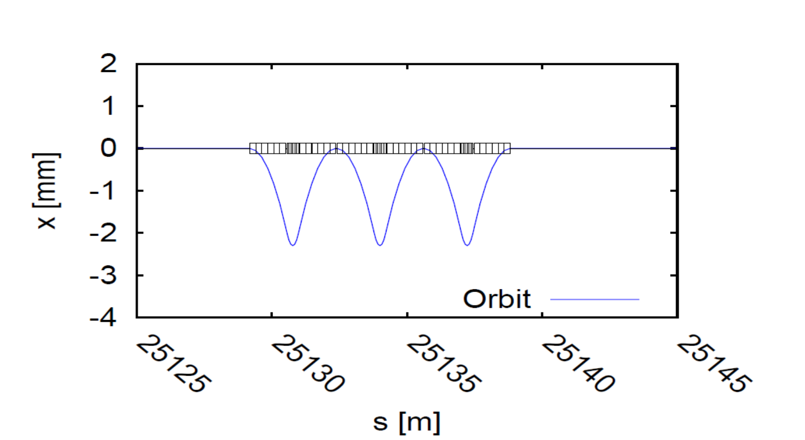
Polarization wigglers

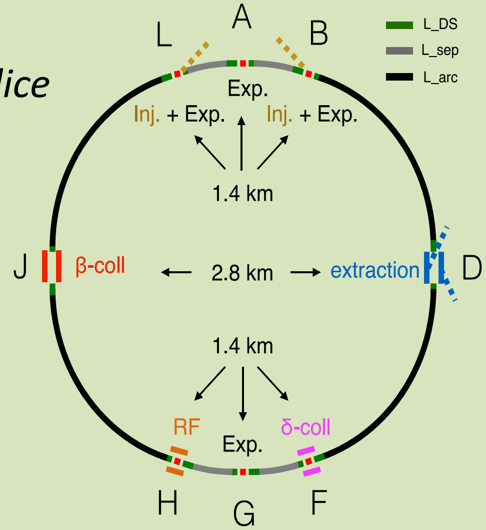
8 units per beam, as specified by *Eliana Gianfelice*

$B+=0.7$ T $L+ = 43$cm $L-/L+ = B+/B- = 6$

at $E_b= 45.6$ GeV and $B+= 0.67$ T

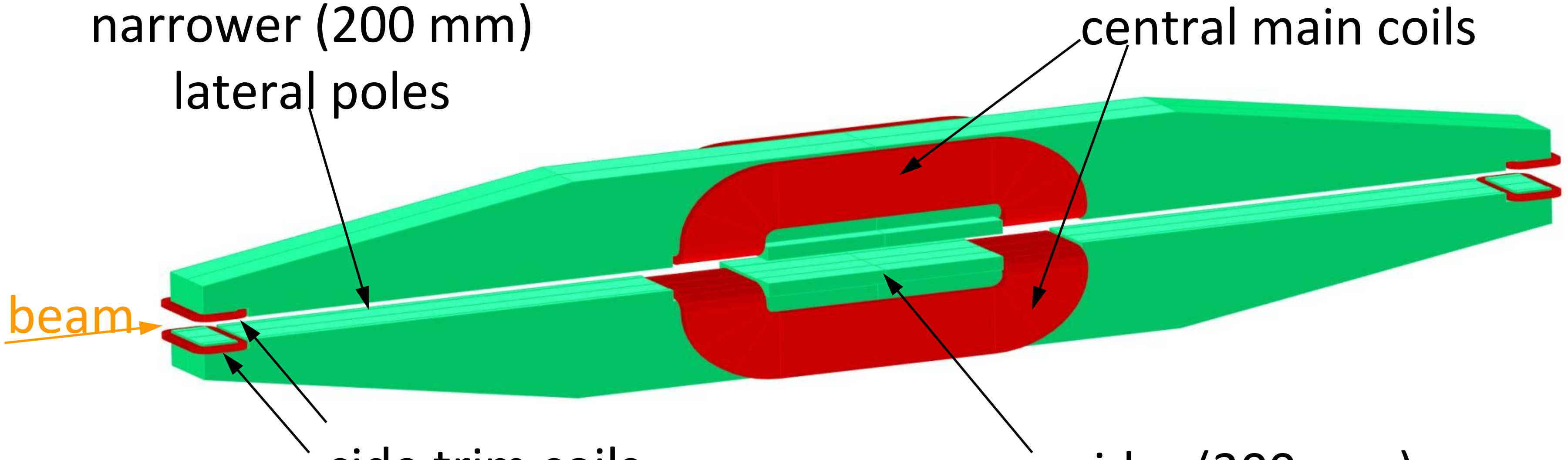
=> $P=10\%$ in 1.8H $\sigma_{Eb} = 60$ MeV $E_{crit}=902$ keV





placed e.g. in dispersion-free straight section H and/or F

First single pole magnetic concept, keeps some of the ideas of the LEP design, in particular the “floating” poles



mass ≈ 4 tons

A. Milanese

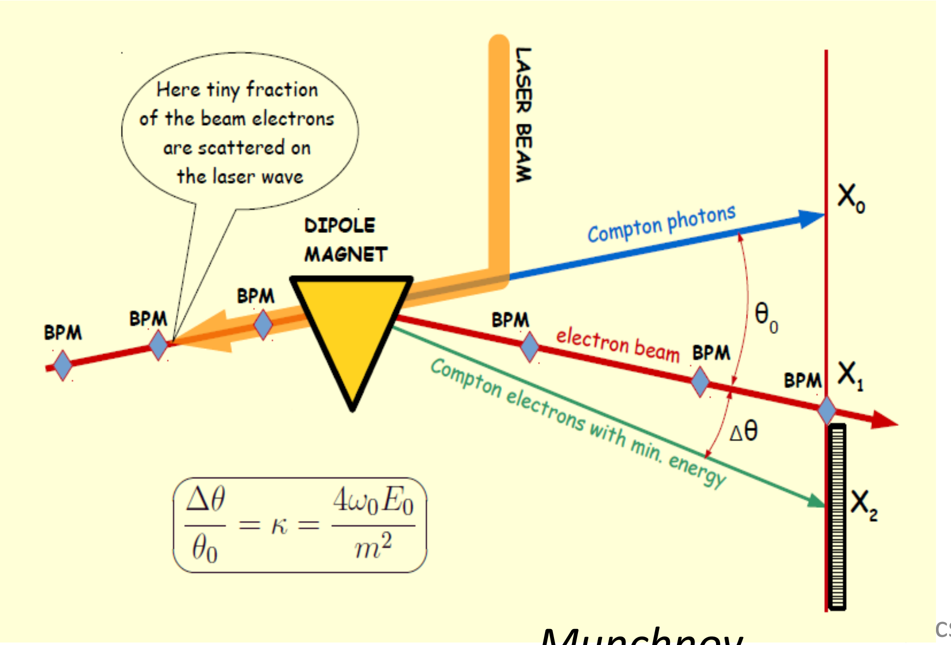
Hardware requirements: polarimeters

2 Polarimeters, one for each beam

Backscattered Compton γ +e → γ + e **532 nm (2.33 eV) laser; detection of photon and electron.**

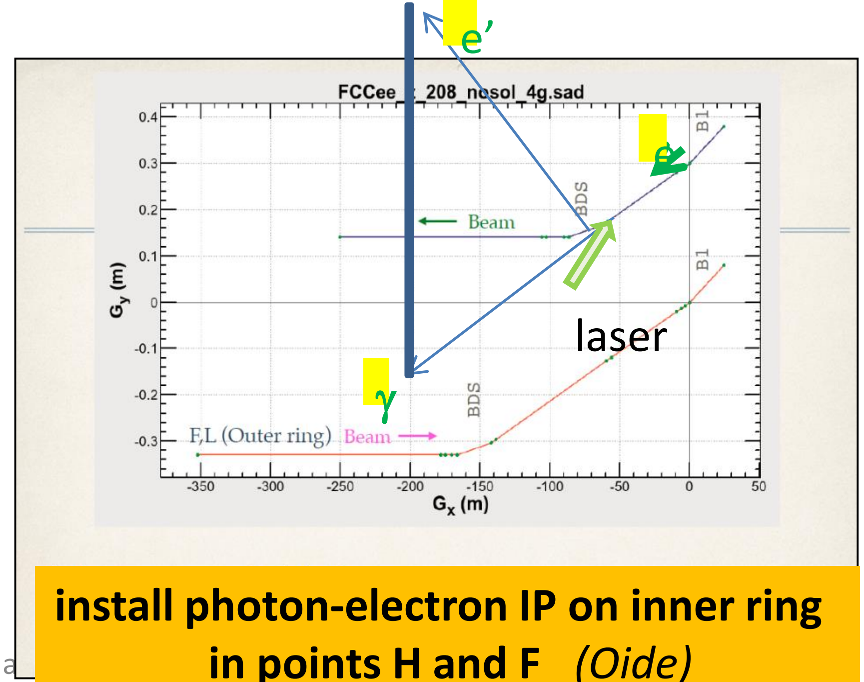
Change upon flip of laser circular polarization → beam Polarization ±0.01 per second

End point of recoil electron → beam energy monitoring ± 4 MeV per second



$$\frac{\Delta\theta}{\theta_0} = \kappa = \frac{4\omega_0 E_0}{m^2}$$

Munchnoy



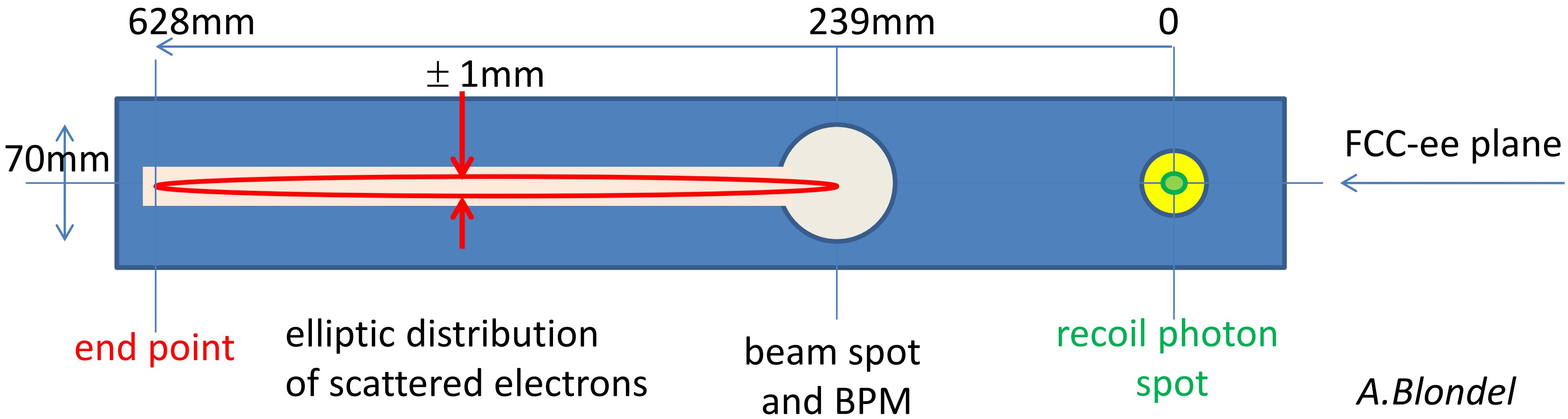
install photon-electron IP on inner ring in points H and F ***(Oide)***

polarimeter-spectrometer situated 100m from end of dipole.

Using the dispersion suppressor dipole with a lever-arm of **100m** from the end of the dipole, one finds
-- minimum compton scattering energy at 45.6 GeV is 17.354 GeV
-- distance from photon recoil to Emin electron is 0.628m

	laser (eV)	beam (GeV)	mc2(MeV)	B field	R	LM	theta	L	true beam
	2.33	45.6	0.511	0.013451	11300	24.119	0.002134	100	45.60005
nominal kappa = 4. E_laser.Ebeam_nom/mc2	1.627567296								
true kappa = 4. E_laser.Ebeam_true/mc2	1.627568924								
nominal Emin	17.35445561								
true Emin	17.35446221								
position of photons	0								
nominal position of beam (m)	0.239182573								
true position of beam (m)	0.239182334	2.39182E-07							
nominal position of min (m)	0.628468308								
true position of min (m)	0.628468069	2.39182E-07							

mouvement of beam and end point are the same:
0.24microns for $\delta Eb/Eb=10^{-6}$ (δEb=45keV)

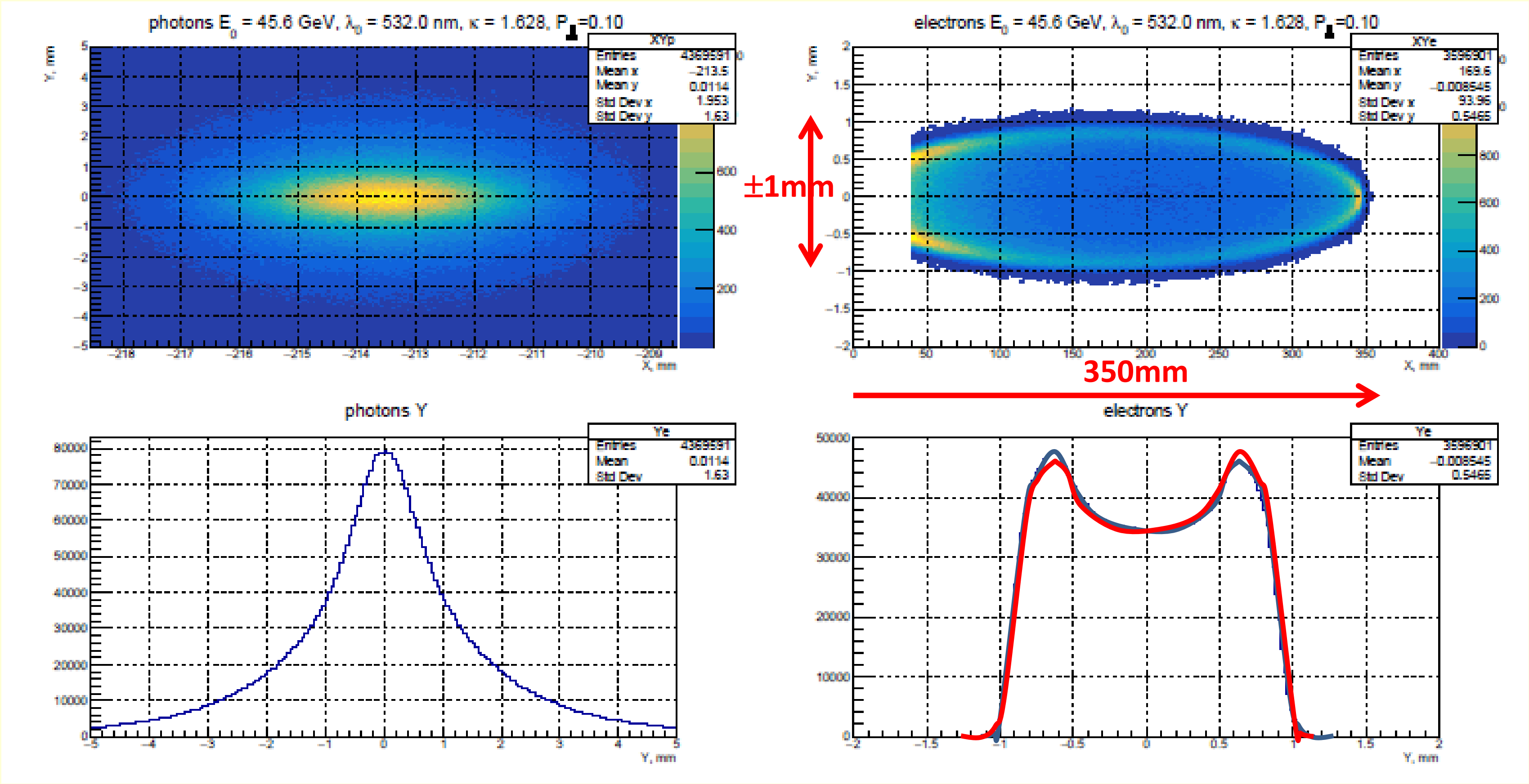


A.Blondel

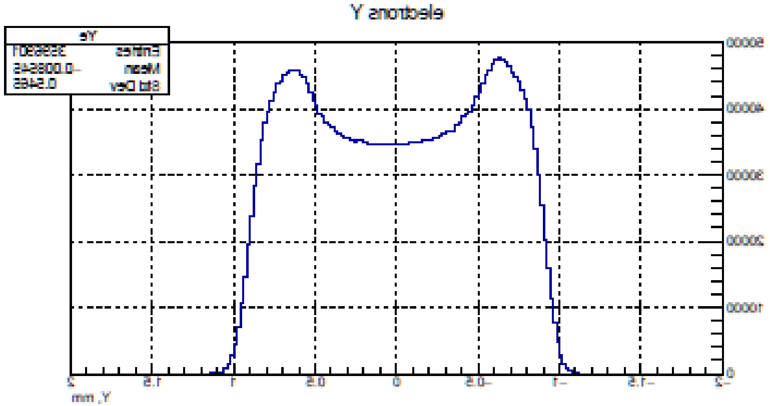
Compton Polarimeter: Rates

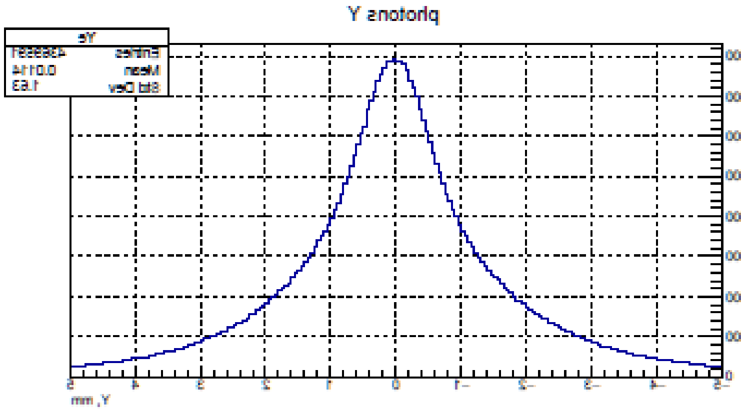
- Laser wavelength $\lambda = 532$ nm.
- Waist size $\sigma_0 = 0.250$ mm. Rayleigh length $z_R = 148$ cm.
- Far field divergence $\theta = 0.169$ mrad
- Interaction angle $\alpha = 1.000$ mrad
- Compton cross section correction 0.5
- Pulse energy: $E_L = 1$ [mJ]; τ_L=5 [ns] (sigma)
- Pulse power: $P_L = 80$ [kW]
- Ratio of angles $R_a = 5.905249$
- Ratio of lengths $R_l = 0.984208$
- $P_L/P_c = 1.1 \cdot 10^{-6}$
- "efficiency" $= 0.13$
- Scattering probability $W \simeq 7 \cdot 10^{-8}$

- With 10^{10} electrons and 3 kHz rep. rate: $\dot{N}_\gamma \simeq 2 \cdot 10^6$


photons E0 = 45.6 GeV, λ0 = 532.0 nm, κ = 1.628, P⊥=0.10
XYp
Entries 4369591
Mean x −213.5
Mean y 0.0114
Std Dev x 1.953
Std Dev y 1.63
Y, mm
X, mm
electrons E0 = 45.6 GeV, λ0 = 532.0 nm, κ = 1.628, P⊥=0.10
XYe
Entries 3596901
Mean x 169.6
Mean y −0.008545
Std Dev x 93.96
Std Dev y 0.5465
±1mm
350mm
photons Y
Ye
Entries 4369591
Mean 0.0114
Std Dev 1.63
electrons Y
Ye
Entries 3596901
Mean −0.008545
Std Dev 0.5465
Y, mm

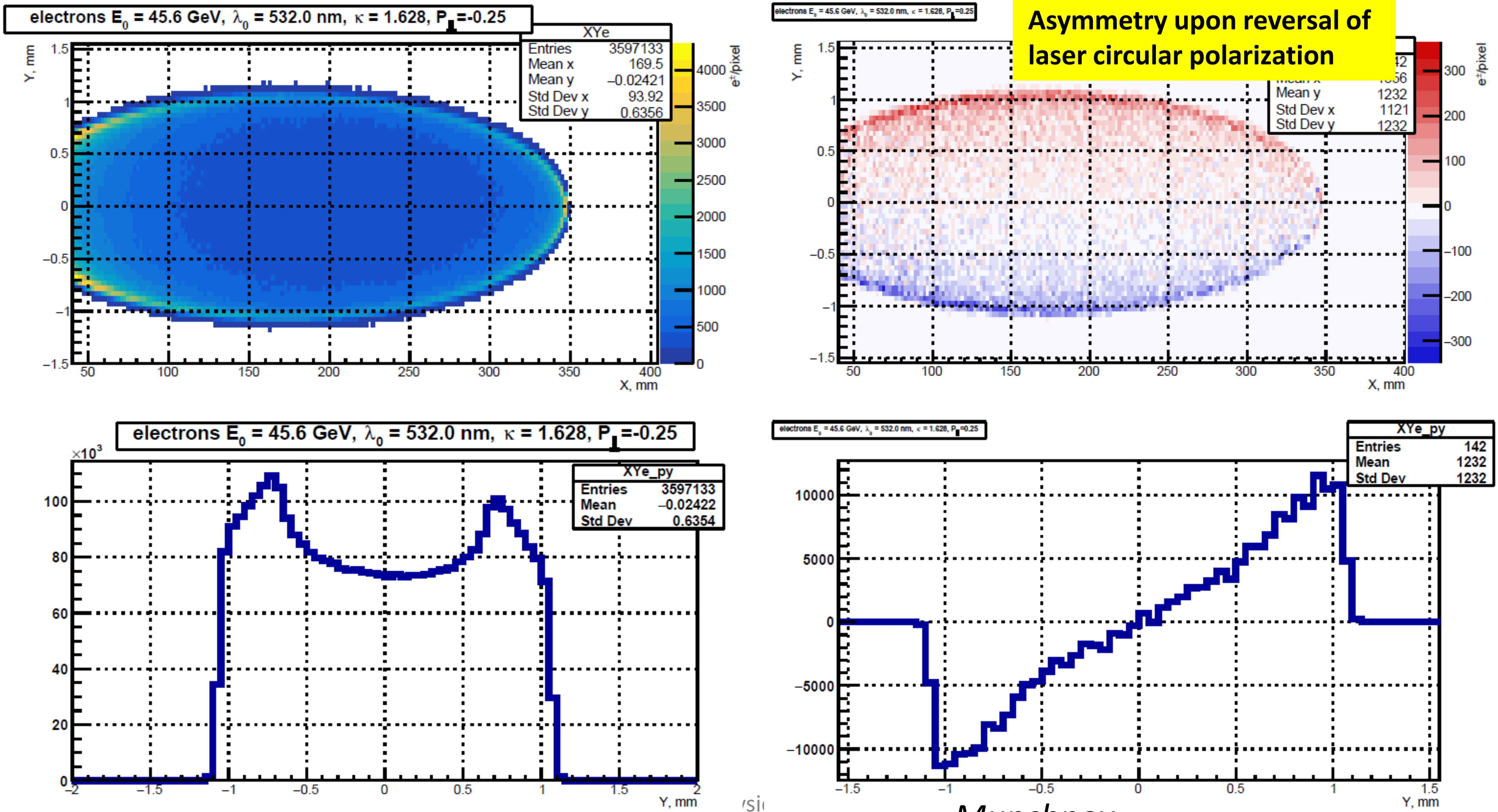
Munchnoi





it is expected that beam polarization can be measured to P $\pm$ 1% (absolute)
in a few seconds. (if the level is 5%, this is 5σ). To be verified with improved fitter (Nickolai)

Asymmetry upon reversal of laser circular polarization



Munchnoy

Compton Polarimeter: Laser

http://laser-export.com/prod/527.html

Last updating: 3.4.201

...Laser-compact Group specializes in research, development and manufacturing of **ultra-violet (UV), green and infrared (IR)** diode-pumped solid-state (DPSS) lasers....

ISO 9001:2008 certified

TECH-527

Application fields: materials micromachining, laser marking, photoacoustics, LIBS (laser-induced breakdown spectroscopy), DLIP (Direct Laser Interference Patterning), LIBD (laser-induced breakdown detection), OPO pumping, remote sensing, high technologies R&D, ablation.

Features:

- Active Q-switched mode of operation with nanosecond pulse duration
- High pulse energy and peak power
- Perfect beam quality
- High pulse-to-pulse stability
- Ultra-compact design
- Conductive cooling of laser head
- External / internal triggering, PC control via RS-232
- Fiber-coupling option is available on request

TECH-series datasheet

Depolarization

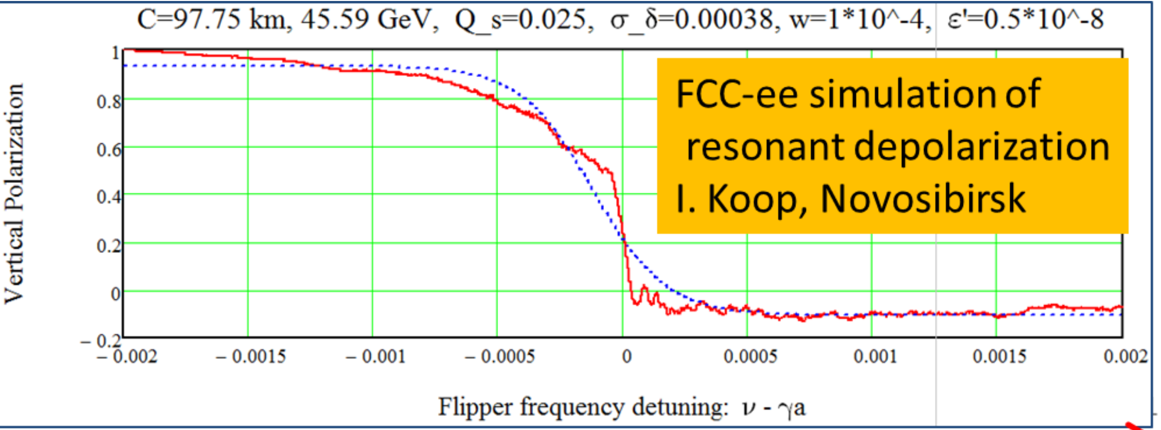
This is not-so trivial in FCC-ee!
16700 bunches circulate
time-between-bunches = 19ns,
depolarize one-and-only-one
of them.
Kicker must have fast (<9ns) rise.

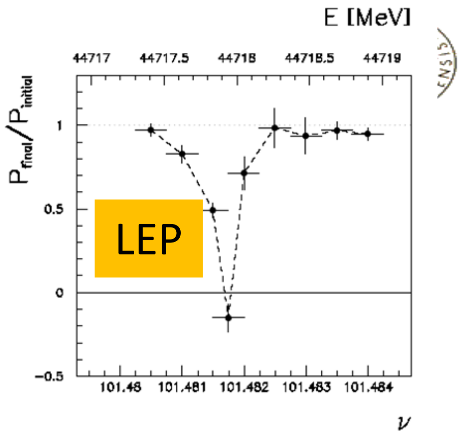
The LHC TF system works essentially on a bunch by bunch basis for 25ns.
They would provide a transverse kick of up to ~20 mrad at the Z peak with ~10 MHz bandwidth. This is 10x more than what we may need-
→ **a priori OK !**

LHC transverse feedback system

- Four kickers per beam, per plane, located in RF zone (UX451) at point 4
  - **Electrostatic kicker**, length 1.5 m.
  - Providing a **kick of ~2 μrad @ 450 GeV** (all 4 units combined).
  - Useful bandwidth ~1 kHz – 20 MHz.

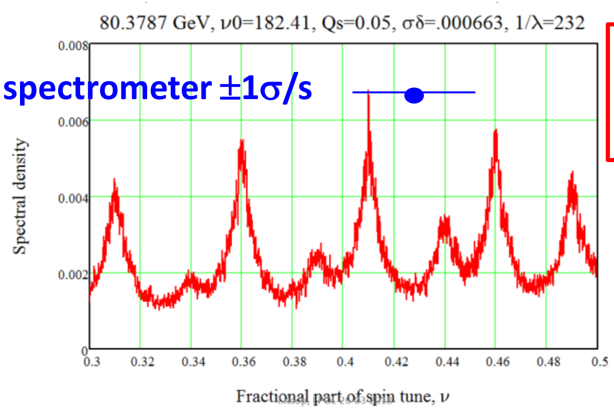
ADT kickers and power amplifiers at point 4





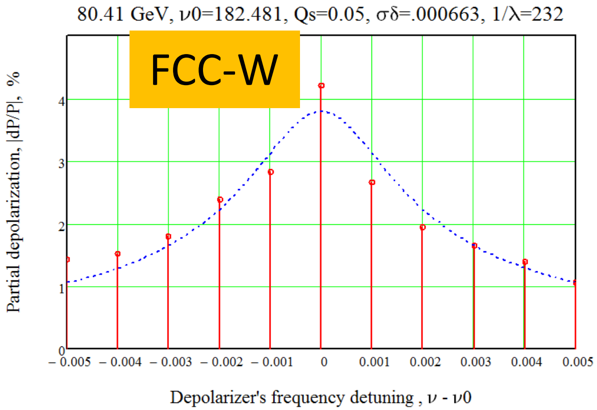
260 seconds sweep of depolarizer frequency

long sweep works well at the Z. Several depolarizations needed: eliminate Qs side band and 0.5 ambiguity
Less well at the W: the Qs side bands are much more excited because of energy spread, need iterations with smaller and smaller sweeps – work in progress. see *I. Koop* presentation.



← Fourier analysis shows the side band situation at W.

First attempt at 'LEP' multiple sweep technique →



From resonant depolarization to Center-of-mass energy
-- 1. from spin tune to beam energy--

The spin tune may not be en exact measurement of the average of the beam energy along the magnetic trajectory of particles. Additional spin rotations may bias the issue. *Anton Bogomyagkov* and *Eliana Gianfelice* have made many estimates.

synchrotron oscillations	$\Delta E/E$	$-2\ 10^{-14}$
Energy dependent momentum compaction	$\Delta E/E$	10^{-7}
Solenoid compensation		$2\ 10^{-11}$
Horizontal betatron oscillations	$\Delta E/E$	$2.5\ 10^{-7}$
Horizontal correctors*)	$\Delta E/E$	$2.5\ 10^{-7}$
Vertical betatron oscillations **)	$\Delta E/E$	$2.5\ 10^{-7}$
Uncertainty in chromaticity correction $O(10^{-6})$	$\Delta E/E$	$5\ \ 10^{-8}$
invariant mass shift due to beam potential		$4\ 10^{-10}$

*) $2.5\ 10^{-6}$ if horizontal orbit change by >0.8mm between calibration is unnoticed or if quadrupole stability worse than 5 microns over that time. consider that 0.2 mm orbit will be noticed

**) $2.5\ 10^{-6}$ for vertical excursion of 1mm. Consider orbit can be corrected better than 0.3 mm.

Vertical orbit distortions

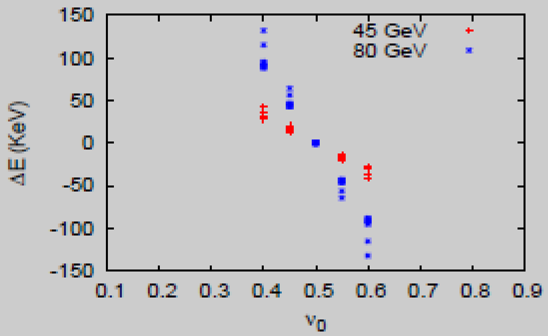
The relationships $\nu_{spin} = a\gamma$ holds for a purely planar ring.

The effect of closed orbit distortion has been evaluated for LEP by using a simplified model by R. Assmann who found that for half-integer ν_s^0 it is $\Delta\nu_s$=0 in first and second order in the extra-spin rotations. For $\nu_s^0 \neq 0.5$ it is

$$< \Delta\nu_s >= \frac{\cot \pi\nu_s^0}{8\pi}(a\gamma)^2\left[< \Sigma_q (K\ell)_q^2 y_q^2 > + < \Sigma_k \theta_k^2 >\right]$$

y_q =*effective* beam position at the quadrupole

Evaluating this expression over 10 seeds



Eliana

10^{-6} at the Z
and $2 \cdot 10^{-6}$ at the W

Energy gains (RF) and energy losses (Arcs and Beamsstrahlung)

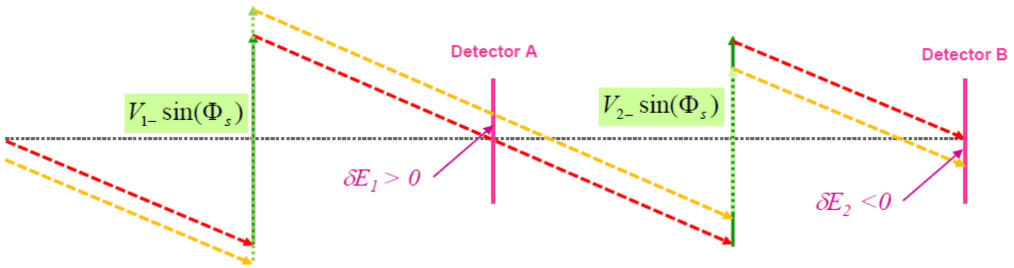
At LEP the disposition of the RF units on each side of the experiments had the effect that any asymmetry in the RF would change the energy of the beams at the IP, but not the average energy in the arcs.

At FCC-ee, because the sequence is RF – energy loss – IP – energy loss- RF such errors have little effect on the relationship between average energy in the arcs and that at the IP.
They can induce a difference between e+ and e- (can be measured in expt!)

RF errors

- If the RF voltage or phase changes in one RF group, the local energy gain will change, the difference must be compensated by the second group → strong correlation of changes / errors between the 2 RF groups.
- To first order the energy change has opposite signs at the 2 experiments !

$\delta E_1 + \delta E_2 \approx 0$



- By averaging the Z mass of the 2 experiments one can cancel out some of the RF errors (is that 'legal'?).
  - This correlation could also be observed by other means (event asymmetries etc).

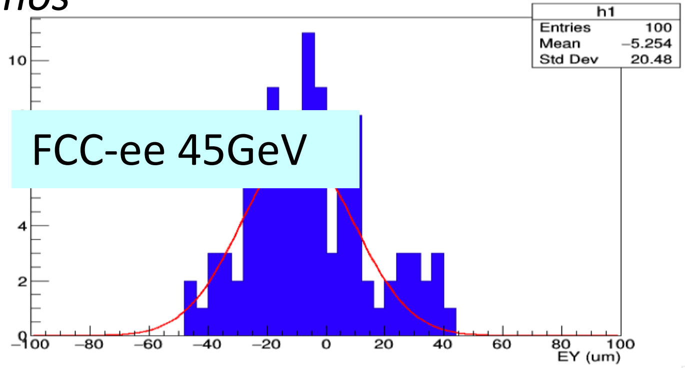
Jorg Wenninger.

Oide-san has shown that one can indeed put all the RF in one straight section for Z and W running

Opposite sign dispersion at IP

$$\Delta E_{CM} = -\frac{1}{2}\cdot\frac{\delta y}{\sigma_y^2}\cdot\frac{\sigma_{E_b}^2}{E_b}\cdot\Delta D_y^*$$

M. Koratzinos



Dima El Khechen

For FCC-ee at the Z we have:

- Dispersion of e+ and e- beams at the IP is 20um (uncorrelated average) –the difference in dispersion matters in this calculation –m'ply by SQRT(2), so $\Delta D_y^* = 28\mu m$.
- Sigma_y is 30nm
- Sigma_E is 0.132%*45000MeV=60MeV
- Delta_ECM is therefore **4MeV** for a 10% offset
- Note that we cannot perform Vernier scans like at LEP, we can only displace the two beams by ~10%sigma_y
- Assume each Vernier scan accurate to 1% sigma_y,
- we need 100 vernier scans to get an E_{CM} accuracy of 40keV – suggestion: vernier scan every hour
- It is likely that Van der Meer scans will be performed regularly at least once per hour or more. (→100 per week)

note that this is an issue both for horizontal and vertical dispersion

Beamstrahlung

Beamstrahlung is emission of photons by (e.g. e^{+-}) in the field of the other (e^{-+})
In a linear collider →low energy tail of the collision energy distribution and a systematic bias.

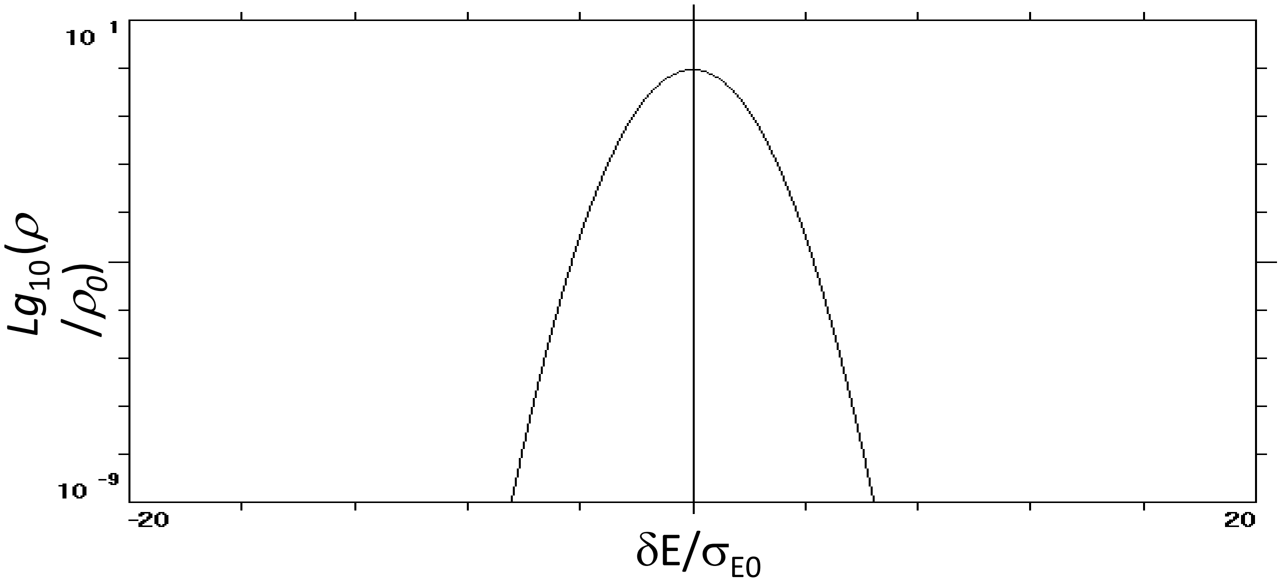
BUT In a circular collider it initiates a synchrotron oscillation! The particle energy distribution remains symmetric, **but the energy spread is very much enlarged.**

Quantitatively the energy loss at the the IP in presence of beamstrahlung is **0.62 MeV**
As Dmitry Shatilov points out this energy loss is compensated by the RF and
the difference between colliding bunches and non-colliding bunches will remain small
the uncertainty is assumed to be less than a few percent of this (~ 20keV)

D. Shatilov

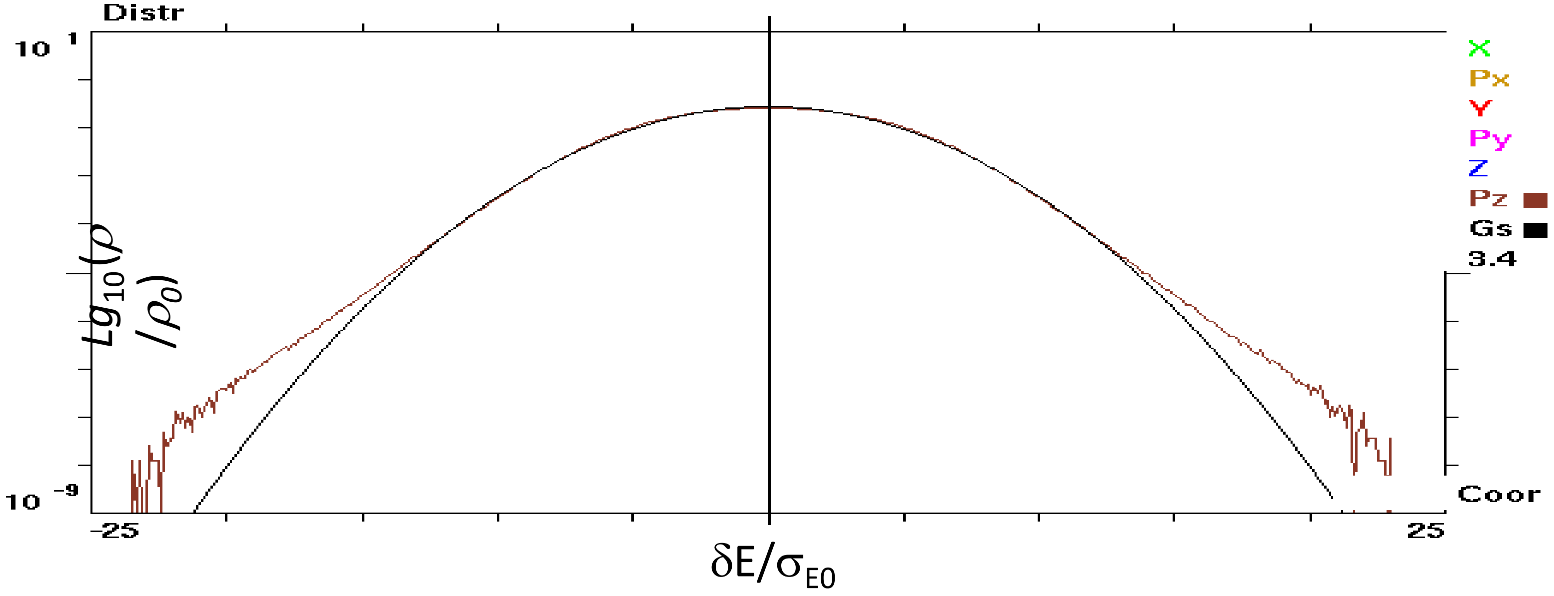
Without Beamstrahlung

Gauss with $\sigma_E = \sigma_{E0}$



with Beamstrahlung E = 45.6 GeV

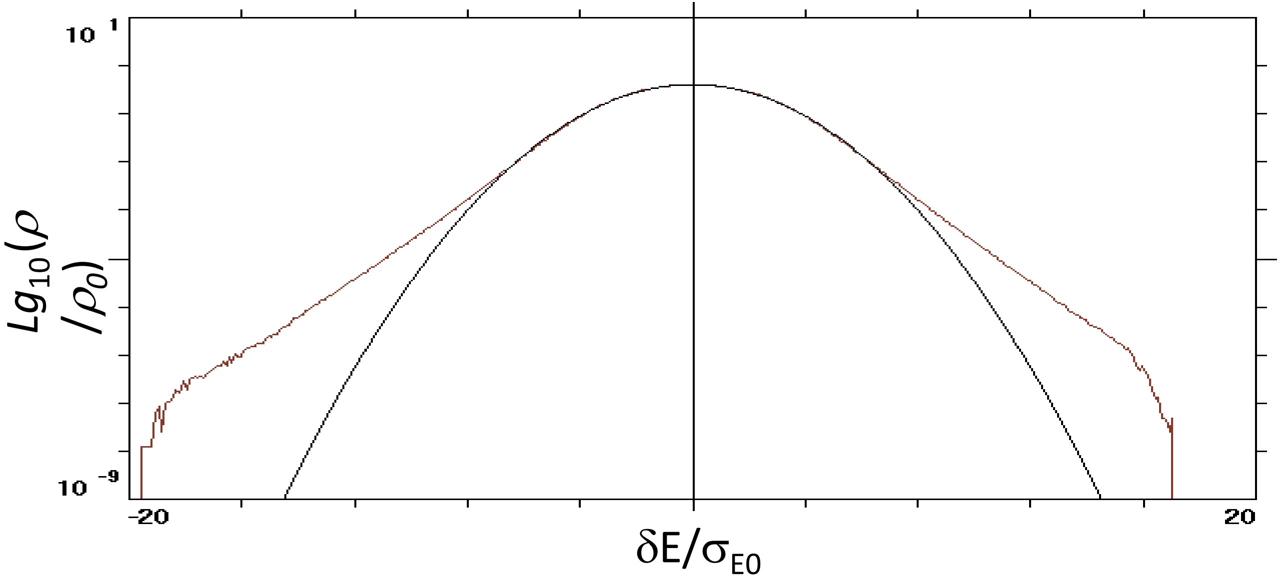
$\sigma_{E0}/E = 0.00038$, $\sigma_E/E = 0.00132$, Black line: Gauss with $\sigma_E = 3.4\ \sigma_{E0}$



Energy acceptance: 1.3% = 34.2 σ_{E0}

E = 80 GeV

σ_{E0} = 0.00066, σ_E = 0.00153, Black line: Gauss with σ_E = 2.3 σ_{E0}



Energy acceptance: 1.3% = 19.7 σ_{E0}

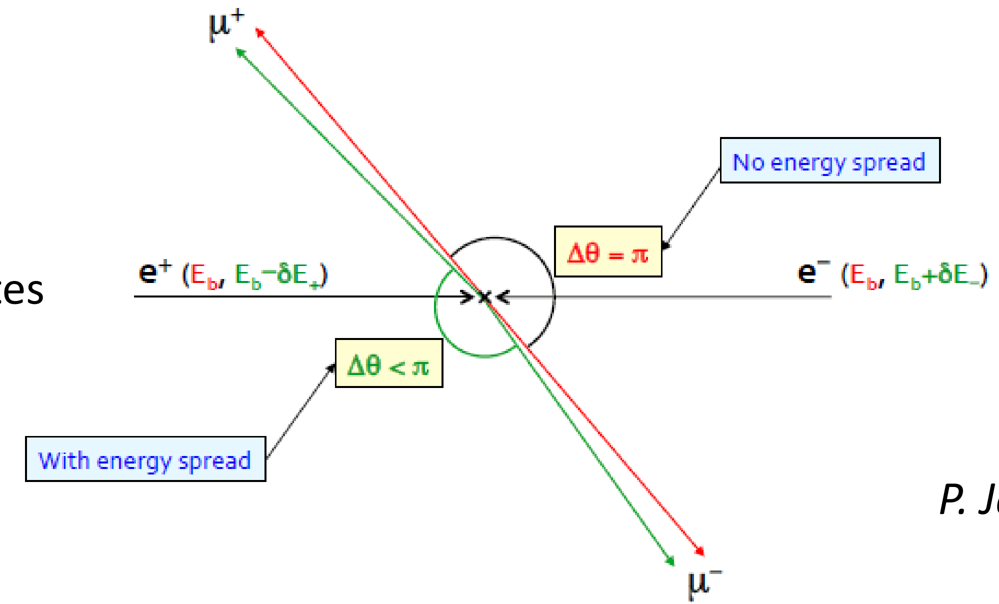
Determination of Energy spread

At the Z peak we collect 10^6 $\mu\mu$ events every 5 minutes
their kinematics is affected by
-- energy spread
-- e+ vs e- energy difference.

Patrick has shown that indeed both can be determined with extremely sufficient precision with a few minutes up to a few hours. OK OK

Make use of $e^+e^- \rightarrow \mu^+\mu^-$ events

- How are the events modified with energy spread ?



P. Janot

Point-to-point errors

	$A_{FB}^{\mu\mu}$ @ FCC-ee	$A_{FB}^{\mu\mu}$ @ FCC-ee 90% correlation
visible Z decays	$5\ 10^{12}$	
muon pairs	$2.5\ 10^{11}$	
$\Delta A_{FB}^{\mu\mu}$ (stat)	$1.2\ 10^{-6}$	
ΔE_{cm} (MeV)	0.1	**0.01 ? 0.023**
$\Delta A_{FB}^{\mu\mu}$ (E_{CM})	$9.2\ 10^{-6}$	**$9.2\ 10^{-7}$? $2.4\ 10^{-6}$**
$\Delta A_{FB}^{\mu\mu}$	$1.0\ 10^{-5}$	**$2.3\ 10^{-6}$? $3.2\ 10^{-6}$**
$\Delta \sin^2\theta^{lept}_W$	$5.9\ 10^{-6}$	**$1.3\ 10^{-6}$? $1.9\ 10^{-6}$**

est. by M.K.

What matters for $A_{FB}^{\mu\mu}$ is the relative error between the Z peak point and the two off-peak points which determine the Z mass. Understanding the point-to-point errors in the energy calibration will be crucial. Presumably quite smaller.

This question has been touched on by M. Koratzinos, needs revisiting.

Conclusions

We have had a very sucessful workshop in October 2017, and the group has been working hard and unveiled a number of aspects of the question of energy calibration.

Several good news
-- polarization levels at Z and W
-- running scenario
-- polarimeter-spectrometer set-up
-- direct measurements of energy spread and energy asymmetries
-- smallness of effects of beamstrahlung and RF effects
-- CDR section of 45 pages and typing!

We are well on track to achive center-of-mass Energy calibration systematics at the level of 100 keV at the Z, 300 keV at the W.

There remains a number of issues
-- -- Opposite sign vertical dispersion : size of effect, correction strategy
-- anti correlation of ECM between expts due to RF
-- correlation matrix of sum and difference between experiments
-- Depolarization for W
-- general issue of software codes: polarization and orbit corrections are not integrated.

THANK YOU!

Measure vertical dispersion at the IP

According to Katsunobu Oide:

- Use BPMs at the high beta points on both sides of the IP
- If relative BPM resolution is 1 um, then resolution on dispersion is 1um/(dp/p)
- (dp/p) (achieved through change of RF frequency) cannot be more than 1% to avoid non-linearities
  ➔ leading to a resolution on D_y of 100um on both sides of the IP
- The dispersion at the IP is the sum of the dispersions on both sides of the IP, which have opposite signs as they are about 180 degrees apart.
- Thus the dispersion at the IP is the subtraction of two big numbers, so relative cross calibration of the two BPMs is also important
- knowing the optics it may be possible to perform a fit to the dispersion function…
- More work is needed here. The required resolution (around 5um) is not yet there.

Beam Polarization can provide main ingredient to Physics Measurements

1. **Transverse beam polarization provides beam energy calibration by resonant depolarization**

→ low level of polarization is required (~10% is sufficient)

→ at Z & W pair threshold comes naturally

→ at Z use of asymmetric wigglers at beginning of fills since polarization time is otherwise very long.

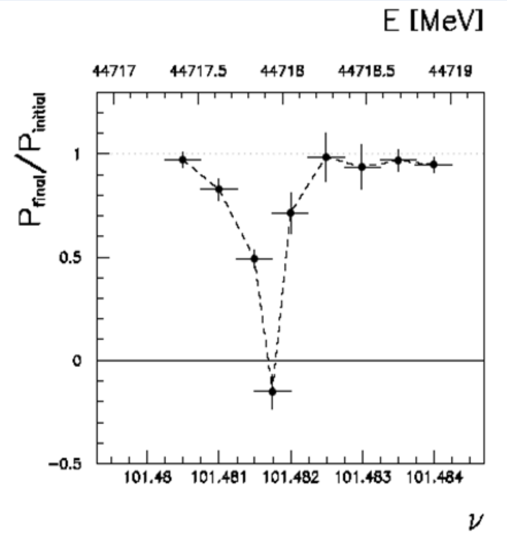
→ could be used also at ee $\rightarrow$ H(126) (depending on exact m_H !)

→ use 'single' non-colliding bunches and calibrate continuously during physics fills to avoid issues encountered at LEP

→ this is possible with e+ and e- Compton polarimeter (commercial laser)

→ should calibrate at energies corresponding to half-integer spin tune

→ must be complemented by analysis of «average E_beam» to E_CM relationship

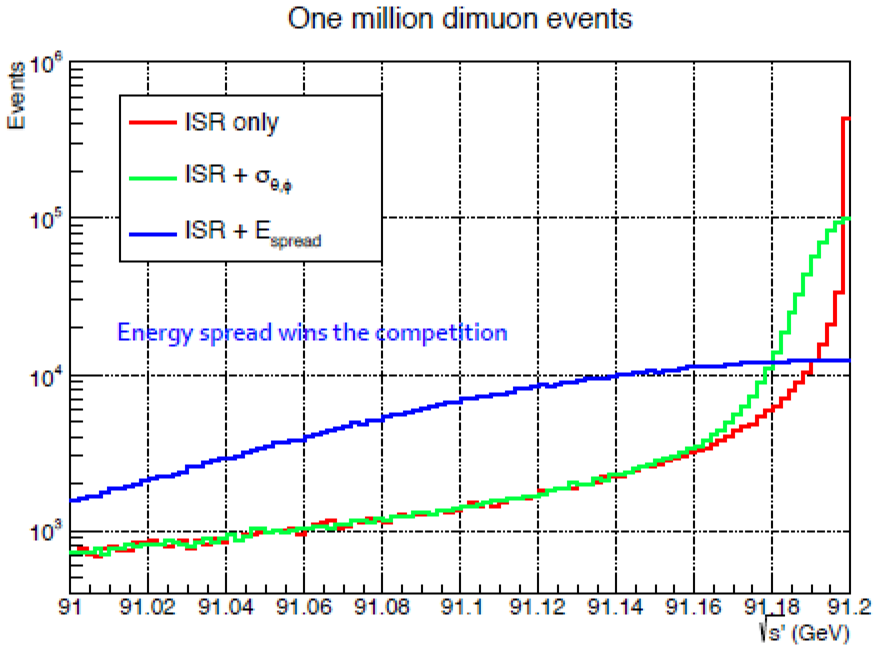


Aim: Z mass & width to ~100 keV (stat: 10 keV) W mass & width to ~500 keV (stat : 300 keV)

For beam energies higher than ~90 GeV can use ee $\rightarrow$ Z γ or ee $\rightarrow$ WW events to calibrate E_{CM} at $\pm$2-4 MeV level: matches requirements for m_H and m_{top} measts

The competition

- Distributions of $\sqrt{s'}$ with 10^6 $e^+e^- \to \mu^+\mu^-$ events at $\sqrt{s}$ = 91.2 GeV
  - With ISR and 0.132% of beam energy spread



Patrick Janot | FCC-ee Polarization Workshop 21 Oct 2017 | 13

Beam Polarization can provide two main ingredients to Physics Measurements

2. Longitudinal beam polarization provides chiral e+e- system

-- High level of polarization is required (>40%)

-- Must compare with natural e+e- polarization due to chiral couplings of electrons (15%) or with final state polarization analysis for CC weak decays (100% polarized) (tau and top)

-- Physics case for Z peak is very well studied and motivated:

A_{LR} , A_{FB}^{Pol}(f) etc... (CERN Y.R. 88-06)

figure of merit is L.P² --> must not lose more than a factor ~10 in lumi.

self calibrating polarization measurement *→ spares

-- uses : enhance Higgs cross section (by 30%)

top quark couplings? final state analysis does as well (Janot **arXiv:1503.01325)**

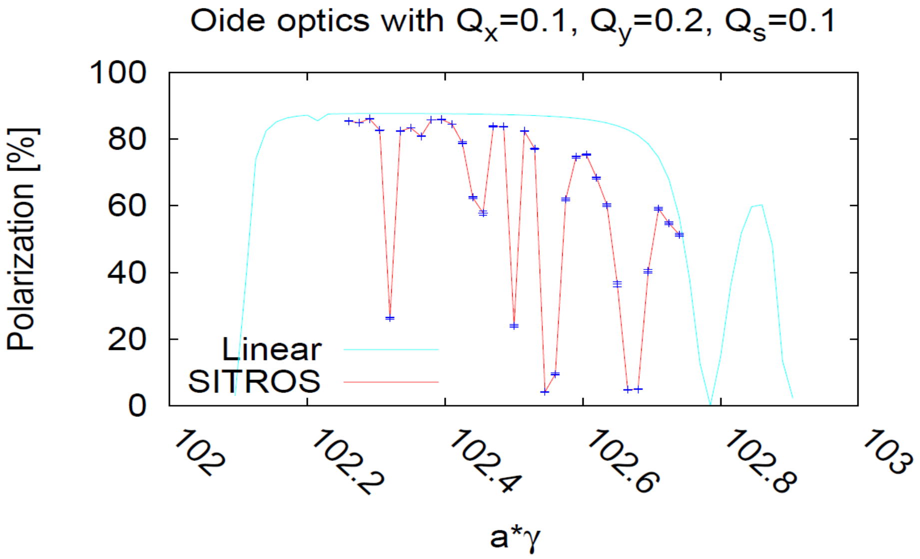
enhance signal, subtract/monitor backgrounds, for ee→WW , ee →H

-- requires High polarization level and often both e- and e+ polarization

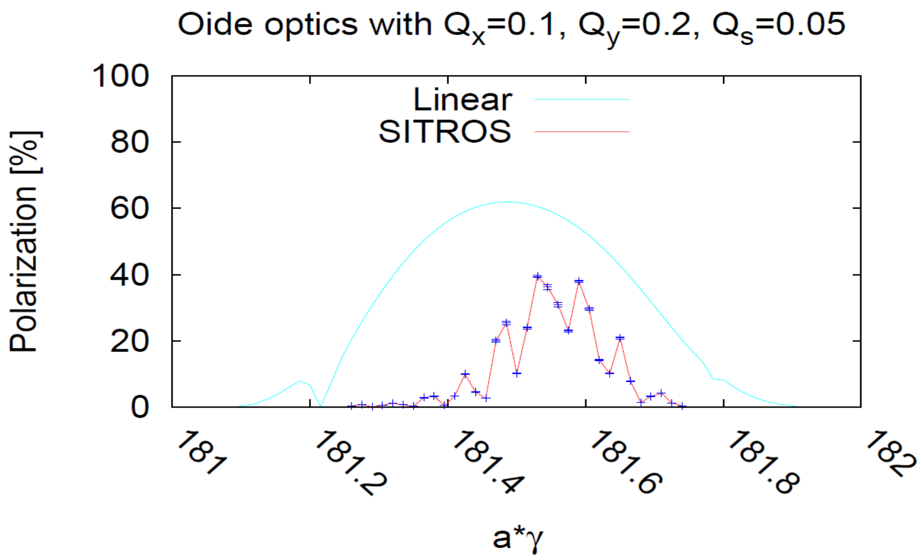
➔ **not interesting If loss of luminosity is too high**

-- Obtaining high level of polarization in high luminosity collisions is delicate in top-up mode

45 GeV



80 GeV



At the Z obtain excellent polarization level but too slow for polarization in physics need wigglers for Energy calibration

At the W expectation similar to LEP at Z → enough for energy calibration

Simulations by Eliana Gianfelice

EUROPEAN ORGANIZATION FOR PARTICLE PHYSICS

CERN–EP/98–40
CERN–SL/98–12
March 11, 1998

Calibration of centre-of-mass energies at LEP1 for precise measurements of Z properties

The LEP Energy Working Group

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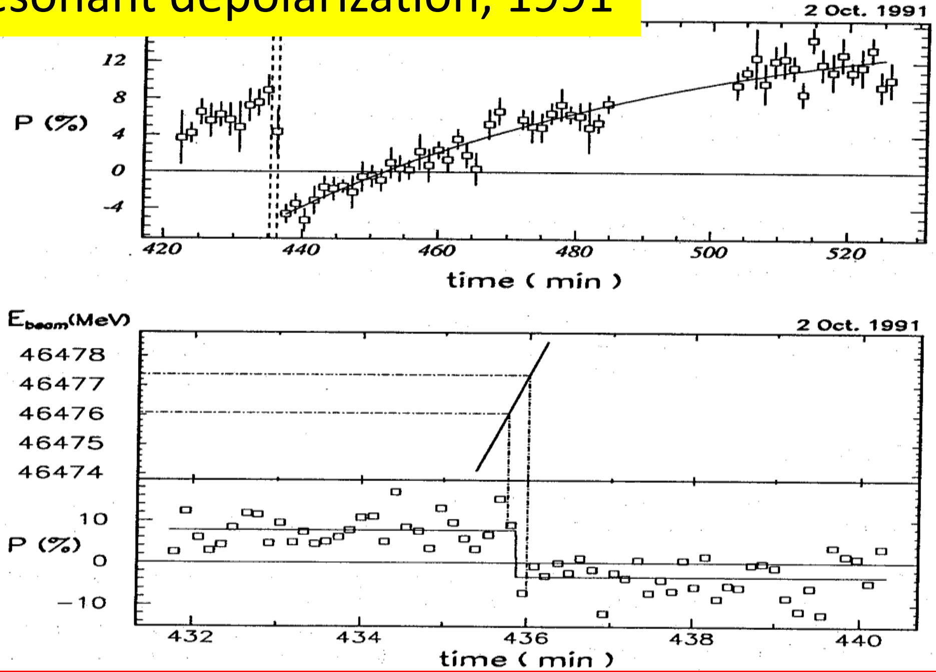
Abstract

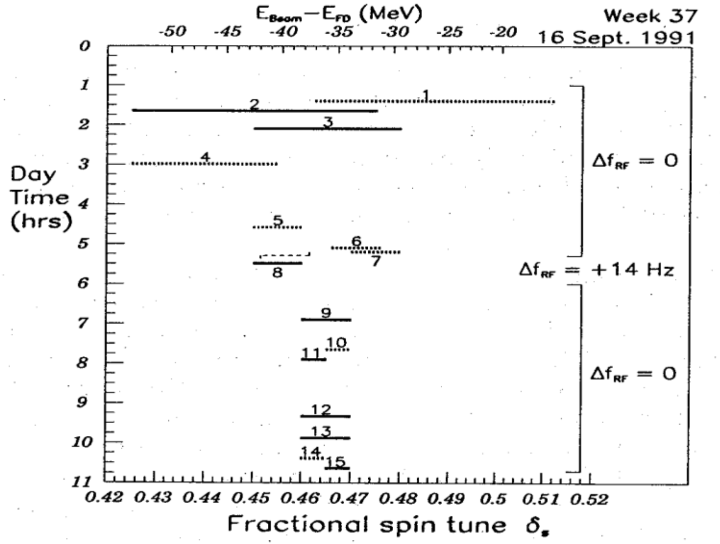
The determination of the centre-of-mass energies from the LEP1 data for 1993, 1994 and 1995 is presented. Accurate knowledge of these energies is crucial in the measurement of the Z resonance parameters. The improved understanding of the LEP energy behaviour accumulated during the 1995 energy scan is detailed, while the 1993 and 1994 measurements are revised. For 1993 these supersede the previously published values. Additional instrumentation has allowed the detection of an unexpectedly large energy rise during physics fills. This new effect is accommodated in the modelling of the beam-energy in 1995 and propagated to the 1993 and 1994 energies. New results are reported on the magnet temperature behaviour which constitutes one of the major corrections to the average LEP energy.
The 1995 energy scan took place in conditions very different from the previous years. In particular the interaction-point specific corrections to the centre-of-mass energy in 1995 are more complicated than previously: these arise from the modified radiofrequency-system configuration and from opposite-sign vertical dispersion induced by the bunch-train mode of LEP operation.
Finally an improved evaluation of the LEP centre-of-mass energy spread is presented. This significantly improves the precision on the Z width.

LEP Resonant depolarization, 1991



$E_{beam} = 46{,}466.6 \pm 0.6$ MeV, e.g. precise to $\pm$ 1.5 10^{-5}.

Figure 20: Polarization signal on 2 October 1991, showing the localization of the depolarizing frequency within the sweep.
Top: display of data points, with the frequency sweep indicated with vertical dashed lines. The full line represents the result of a fit with starting polarization $(-4.9 \pm 1.)\%$, polarization rise-time (60 ± 13) minutes, asymptotic polarization $(18.4 \pm 4.1)\%$.
Bottom: expanded view of the sweep period, with the individual data sets displayed (there are 10 sets per point); The frequency sweep lasted 7 data sets. The corresponding beam energy is shown in the upper box. Spin flip occurred between the two vertical dash-dotted lines.



variation of RF frequency to eliminate half integer ambiguity

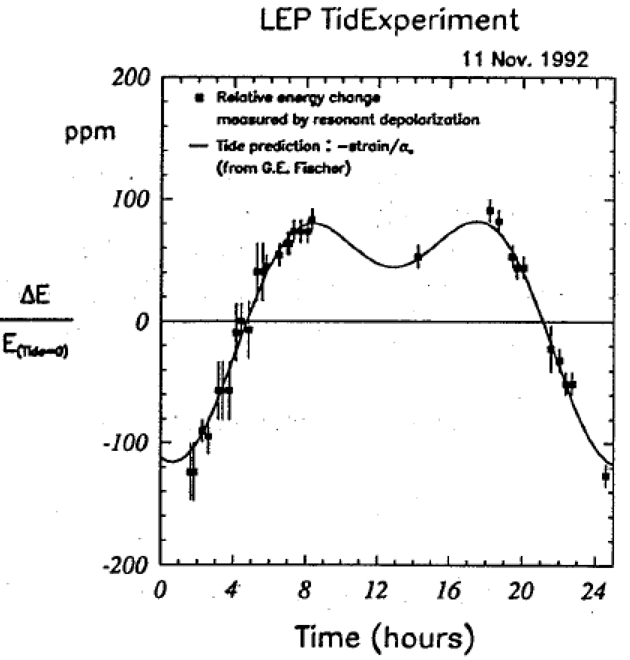
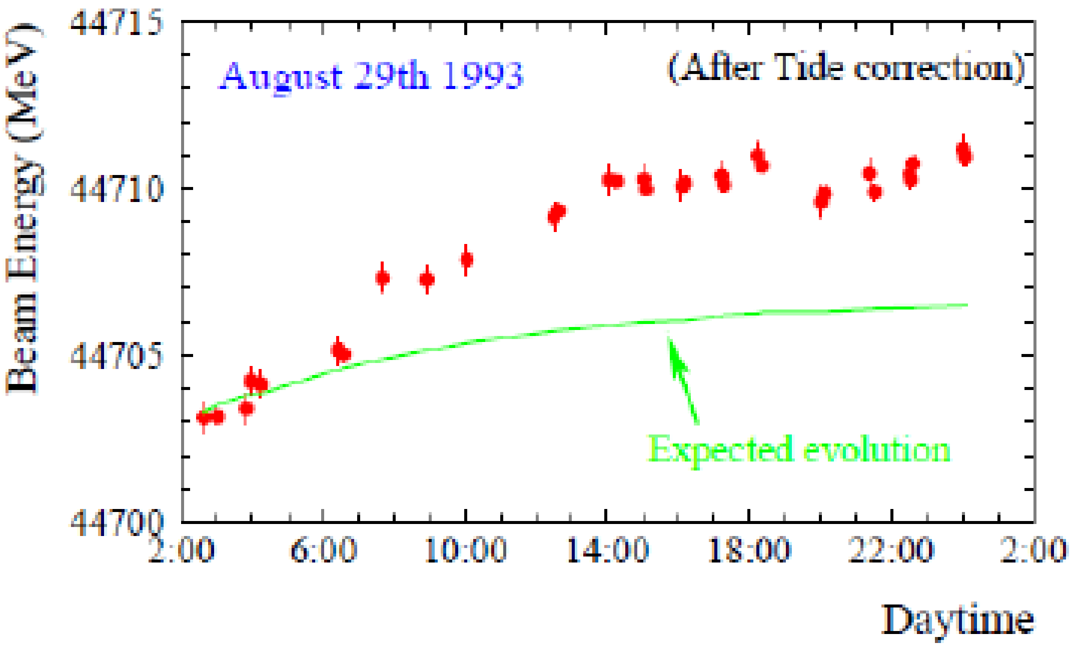


Figure 23: Beam energy variations measured over 24 hours compared to the expectation from the tidal LEP deformation.



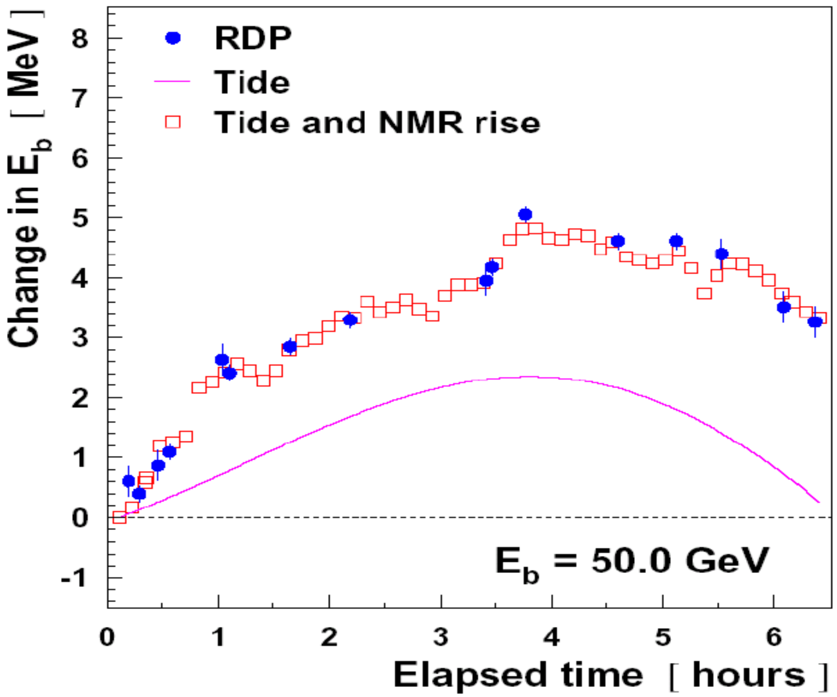
Many effects spoil the calibration if it is performed outside physics time

-- tides and other ground motion

-- RF cavity phases

-- histeresis effects and environmental effects (trains…etc)

Modelling of energy rise by (selected) NMR sampling of B-field is excellent !



(Experiment from 1999)

by 1999 we had an excellent model of the energy variations…
but we were not measuring the Z mass and width anymore
– we were hunting for the Higgs boson!

EXPERIMENTS ON BEAM-BEAM DEPOLARIZATION AT LEP

R. Assmann*, A. Blondel*, B. Dehning, A. Drees°, P. Grosse-Wiesmann, H. Grote, M. Placidi, R. Schmidt, F. Tecker†, J. Wenninger

PAC 1995

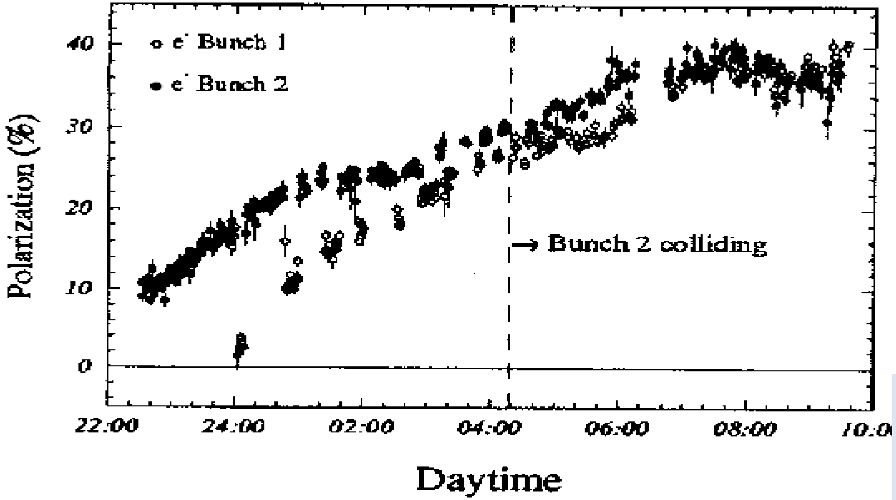


Figure. 3. Polarization level during third experiment

- With the beam colliding at one point, a polarization level of 40 % was achieved. The polarization level was about the same for one colliding and one non colliding bunch.
- It was observed that the polarization level depends critically on the synchrotron tune : when Q_s was changed by 0.005, the polarization strongly decreased.

experiment performed at an energy of 44.71 GeV the polarization level was 40 % with a linear beam-beam tune shift of about 0.04/IP. This indicates, that the beam-beam depolarization does not scale with the linear beam-beam tune shift at one crossing point. Other parameters as spin tune and synchrotron tune are also of importance

LEP:

This was only tried 3 times!

Best result: P = 40% , ξ^*_y= 0.04 , one IP

FCC-ee

Assuming 2 IP and ξ^*_y= 0.01 ➔

reduce luminosity, 10^{10} Z @ P~30%

Longitudinal polarization at FCC-Z?

Main interest: measure EW couplings at the Z peak most of which provide measurements of $\sin^2\theta^{lept}_W = e^2/g^2\ (m_z)$

(-- not to be confused with -- $\sin^2\theta_W = 1 - m_w^2/m_z^2$

Useful references from the past:

«polarization at LEP» CERN Yellow Report 88-02

Precision Electroweak Measurements on the Z Resonance

Phys.Rept.427:257-454,2006 http://arxiv.org/abs/hep-ex/0509008v3

GigaZ @ ILC by K. Moenig

Longitudinal polarization: reduction of polarization due to continuous injection

The colliding bunches will lose intensity continuously due to collisions.
In FCC-ee with 4 IPs, L= 28 10^{34}/cm2/s beam lifetime is 213 minutes
In FCC-ee with 2 IPs, L= 1.4 10^{36}/cm2/s beam life time is 55minutes

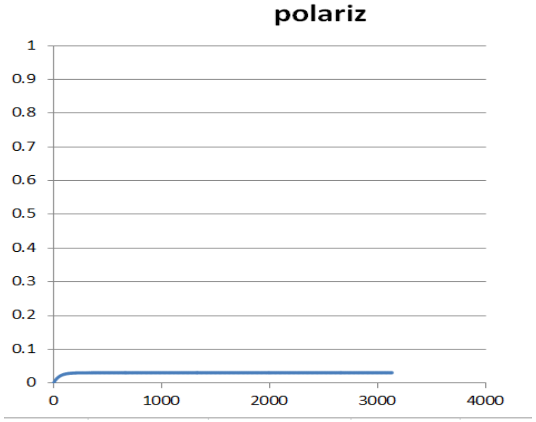
Luminosity scales inversely to beam life time.
The injected e+ and e- are not polarized → asymptotic polarization is reduced.
Assume here that machine has been well corrected and beams (no collisions, no injection) can be polarized to nearly maximum.
(***Eliana Gianfelice in Rome talk***)

- 45 GeV
  - limit ΔE=50 MeV (extrapolating from LEP)
  - 4 wigglers with B^{+}= 0.7 T
  - 10% polarization in 2.9 h for energy calibration

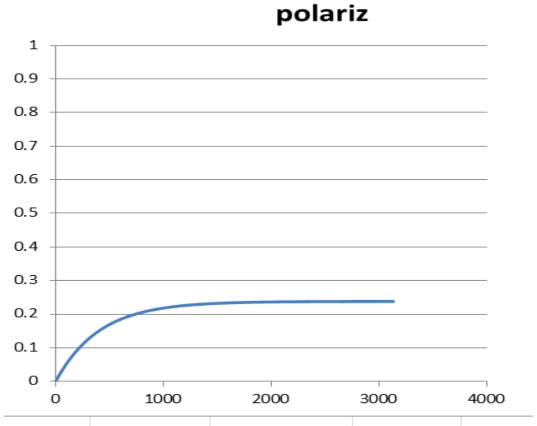
(polarization time is 26h)

We have simulated the simultaneous effect of
-- natural polarization
-- beam consumption by e+e- interactions
-- replenishment with unpolarized beams
assuming ***optimistically*** a maximal 90% asymptotic polarization

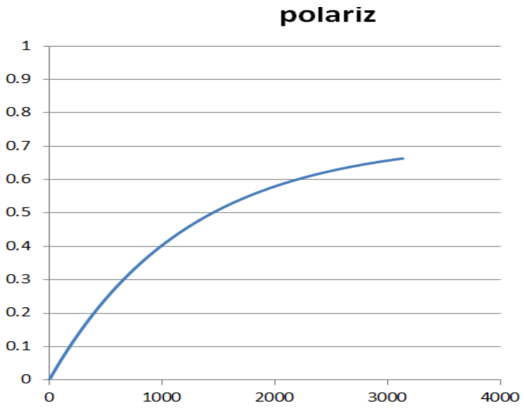
Running at full luminosity
P_max=0.03! P_eff=0.03



Running at 10% Lumi
P_max=0.24, P_eff=0.21



Running at 1% Lumi
P_max=0.66, P_eff=0.5



ΔA_{LR} scales as $1/\sqrt{(P^2L)}$

Lumi loss factor	L.10^34	**Figure of merit: sum(P²L)**	Peff	Pmax
1	220	0.195	0.03	0.03
2	110	0.367	0.059	0.06
4	55	0.627	0.1078	0.11
6	37	0.805	0.149	0.16
8	27	0.924	0.184	0.2
10	22	1.003	0.214	0.24
12	18	1.053	0.24	0.27
15	15	1.09	0.27	0.32
18	12	**1.101**	0.3	0.35
22	10	1.088	0.33	0.4
26	8	1.059	0.354	0.43
30	7	1.023	0.37	0.46
40	5	0.92	0.41	0.52

Optimum around a reduction of luminosity by a factor 18.

This is still a luminosity of ~10^{35} per IP… and the effective polarization is 30%.
This is equivalent to a 100% polarization expt with luminosity reduced by 180.

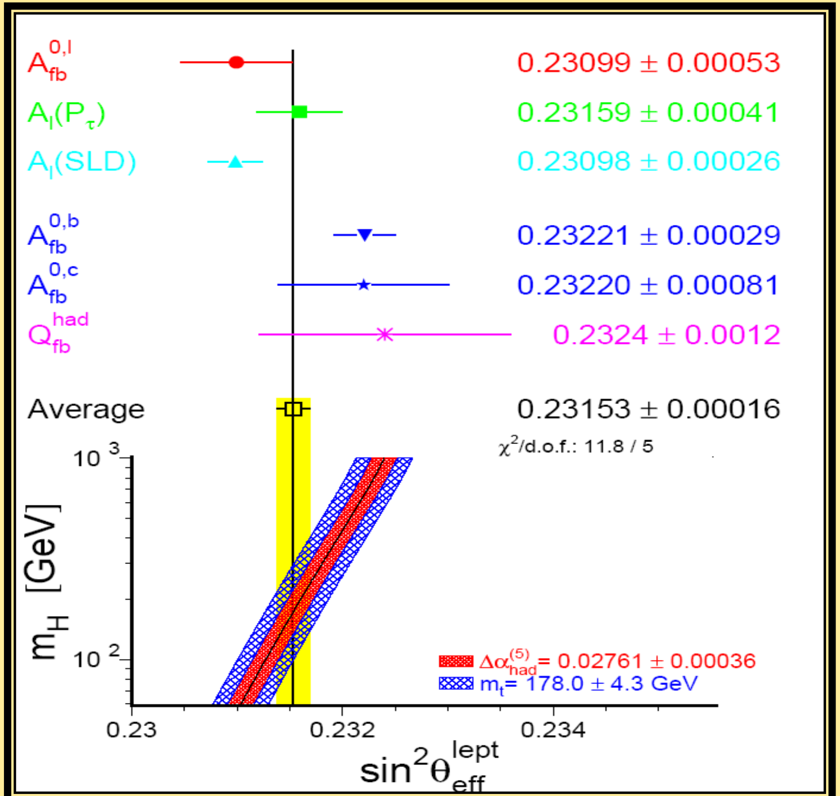
observable	Physics	Present precision		FCC-ee stat Syst Precision	FCC-ee key	Challenge
M_Z MeV/c2	Input	91187.5 ±2.1	Z Line shape scan	0.005 MeV <±0.1 MeV	E_cal	QED corrections
Γ_Z MeV/c2	$\Delta\rho$ (T) (no $\Delta\alpha$!)	2495.2 ±2.3	Z Line shape scan	0.008 MeV <±0.1 MeV	E_cal	QED corrections
$R_l \equiv \frac{\Gamma_h}{\Gamma_l}$	α_s , δ_b	20.767 (25)	Z Peak	0.0001 (2-20)	Statistics	QED corrections
N_ν	Unitarity of PMNS, sterile ν's	2.984 ±0.008	Z Peak Z+γ(161 GeV)	0.00008 (40) 0.001	->lumi meast Statistics	QED corrections to Bhabha scat.
R_b	δ_b	0.21629 (66)	Z Peak	0.000003 (20-60)	Statistics, small IP	Hem. correlations
A_{LR}	$\Delta\rho$, ε_3 ,$\Delta\alpha$ (T, S)	$\sin^2\theta_w^{eff}$ 0.23098(26)	Z peak, Long. polarized	$\sin^2\theta_w^{eff}$ ±0.000006	4 bunch scheme	Design experiment
A_{FB}^{lept}	$\Delta\rho$, ε_3 ,$\Delta\alpha$ (T, S)	$\sin^2\theta_w^{eff}$ 0.23099(53)		$\sin^2\theta_w^{eff}$ ±0.000006	E_cal & Statistics	
M_W MeV/c2	$\Delta\rho$, ε_3 , ε_2, $\Delta\alpha$ (T, S, U)	80385 ± 15	Threshold (161 GeV)	0.3 MeV <0.5 MeV	E_cal & Statistics	QED corections
m_{top} MeV/c2	Input	173200 ± 900	Threshold scan	~10 MeV	E_cal & Statistics	Theory limit at 50 MeV?

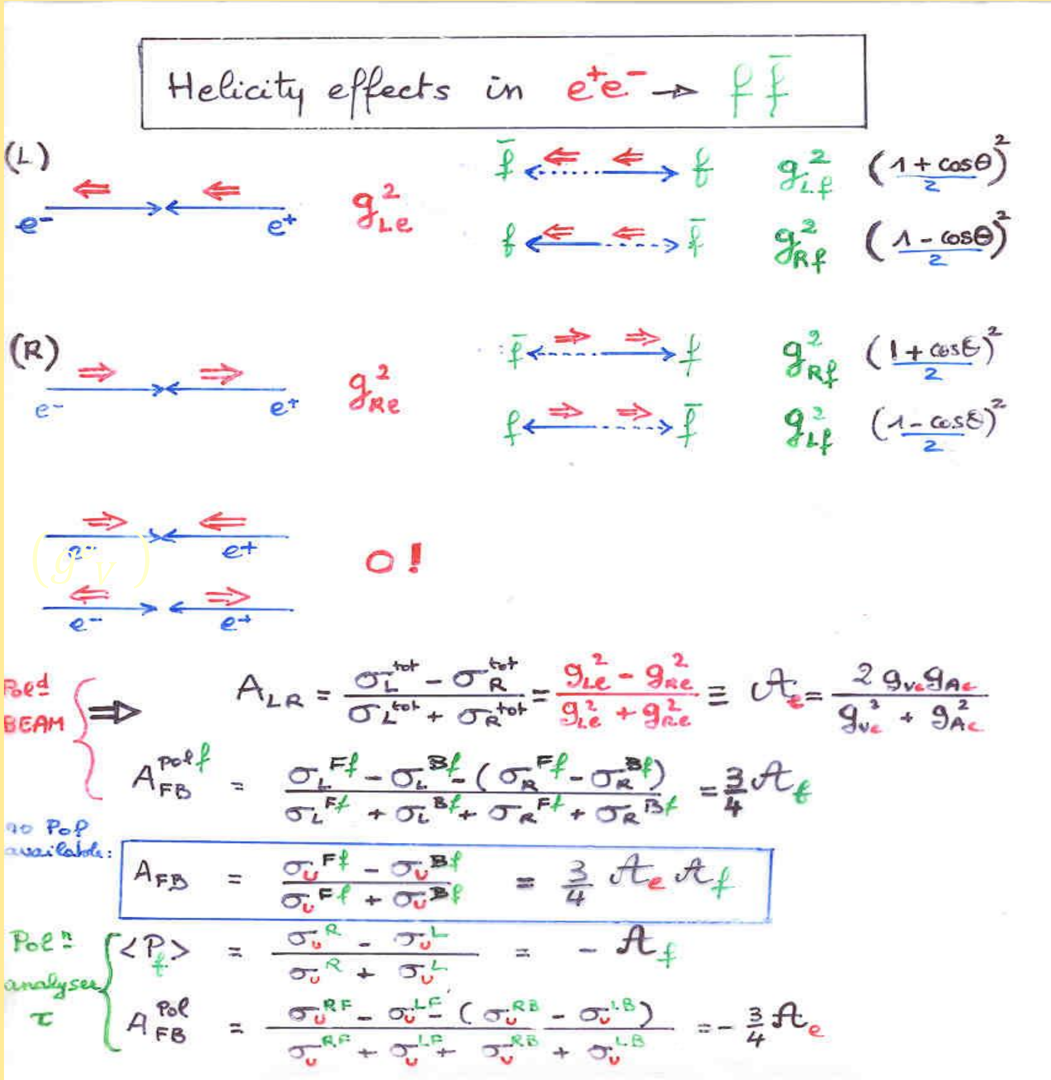
Measuring $\sin^2\theta_W^{eff}$ (m_Z)

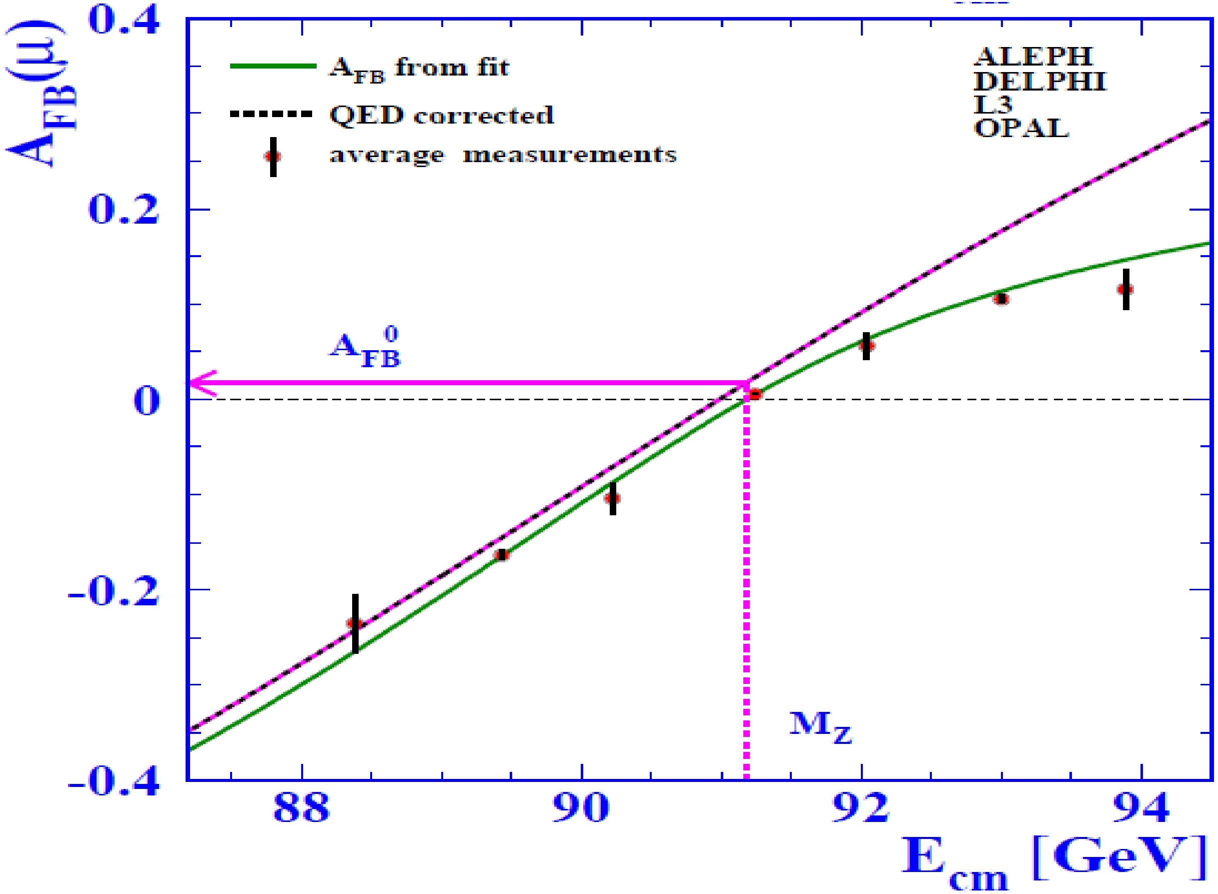
$$\sin^2\theta_W^{eff} \equiv \frac{1}{4}\,(1 - g_V/g_A)$$

$$g_V = g_L + g_R$$

arXiv:0509008







$A_{FB}(\mu)$
0.4
0.2
0
-0.2
-0.4
A_{FB} from fit
QED corrected
average measurements
ALEPH
DELPHI
L3
OPAL
A_{FB}^0
M_Z
88
90
92
94
E_{cm} [GeV]

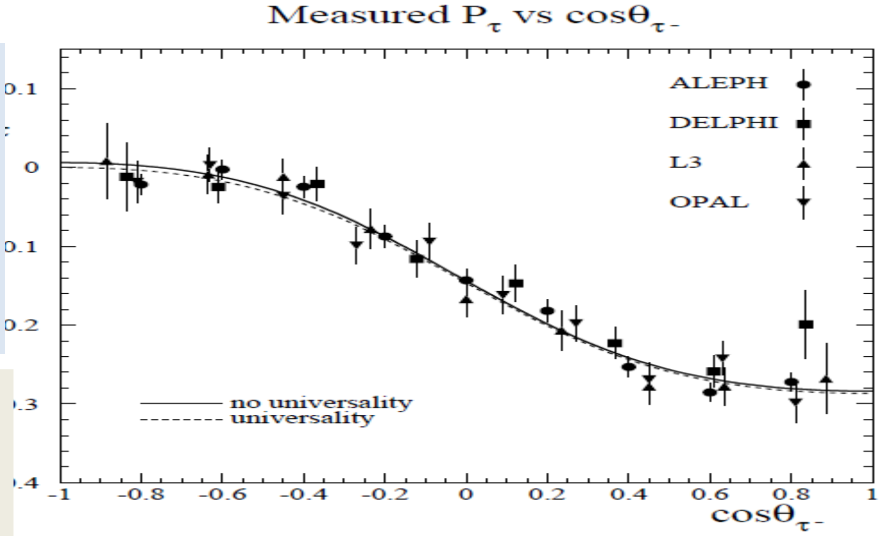
	$A_{FB}^{\mu\mu}$ @ FCC-ee		A_{LR} @ ILC	A_{LR} @ FCC-ee
visible Z decays	10^{12}	visible Z decays	10^{9}	5.10^{10}
muon pairs	10^{11}	beam polarization	90%	30%
$\Delta A_{FB}^{\mu\mu}$ (stat)	$3\ 10^{-6}$	ΔA_{LR} (stat)	$4.2\ 10^{-5}$	$4.5\ 10^{-5}$
$\Delta\ E_{cm}$ (MeV)	0.1		2.2	?
$\Delta A_{FB}^{\mu\mu}$ (E_{CM})	$9.2\ 10^{-6}$	ΔA_{LR} (E_{CM})	$4.1\ 10^{-5}$	
$\Delta A_{FB}^{\mu\mu}$	$1.0\ 10^{-5}$	ΔA_{LR}	$5.9\ 10^{-5}$	
$\Delta \sin^2\theta^{lept}_W$	$5.9\ 10^{-6}$		$7.5\ 10^{-6}$	$6\ 10^{-6}$ +?

All exceeds the theoretical precision from $\Delta\alpha(m_Z)$ (310^{-5}) or the comparison with m_W (500keV)

But this precision on $\Delta \sin^2\theta^{lept}_W$ can only be exploited at FCC-ee!

The forward backward tau polarization asymmetry is very clean.
Dependence on E_{CM} same as A_{LR} negl.
At FCC-ee
ALEPH data 160 pb^{-1} (80 s @ FCC-ee !)

Already syst. level of 6 10^{-5} on $\sin^2\theta^{eff}_W$
much improvement possible
by using dedicated selection
e.g. tau→ π ν to avoid had. model



4.7: The values of $\mathcal{P}_\tau$ as a function of $\cos\theta_{\tau^-}$ as measured by each of the LEP ex- ...nts. Only the statistical errors are shown. The values are not corrected for radiation, ...ence or pure photon exchange. The solid curve overlays Equation 4.2 for the LEP values of $\mathcal{A}_\tau$ and $\mathcal{A}_e$. The dashed curve overlays Equation 4.2 under the assumption of lepton universality for the LEP value of $\mathcal{A}_\ell$.

	ALEPH		DELPHI		L3		OPAL	
	$\delta\mathcal{A}_\tau$	$\delta\mathcal{A}_e$	$\delta\mathcal{A}_\tau$	$\delta\mathcal{A}_e$	$\delta\mathcal{A}_\tau$	$\delta\mathcal{A}_e$	$\delta\mathcal{A}_\tau$	$\delta\mathcal{A}_e$
ZFITTER	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
τ branching fractions	0.0003	0.0000	0.0016	0.0000	0.0007	0.0012	0.0011	0.0003
two-photon bg	0.0000	0.0000	0.0005	0.0000	0.0007	0.0000	0.0000	0.0000
had. decay model	0.0012	0.0008	0.0010	0.0000	0.0010	0.0001	0.0025	0.0005

Table 4.2: The magnitude of the major common systematic errors on $\mathcal{A}_\tau$ and $\mathcal{A}_e$ by category for each of the LEP experiments.

Concluding remarks

1. There are very strong arguments for precision energy calibration with transverse polarization at the Z peak and W threshold.

2. Given the likely loss in luminosity, and the intrinsic uncertainties in the extraction of the weak couplings, the case for longitudinal polarization is limited

➔ **We have concluded that first priority is to achieve transverse polarization** in a way that allows continuous beam calibration by resoenant depolarization

- this is all possible with a very high precision, both at the Z and the W. calibration at higher energies can be made from the data themselves at sufficient level.

- the question of the residual systematic error requires further studies of the relationship between beam energy and center-of-mass energy with the aim of achieving a precision of O(100 keV) on E_CM

EWRCs

relations to the well measured

G_F m_Z α_{QED}

at first order:

$\Delta\rho = \alpha/\pi \; (m_{top}/m_Z)^2$

$- \alpha/4\pi \; \log \, (m_h/m_Z)^2$

$\varepsilon_3 = \cos^2\theta_w \; \alpha/9\pi \; \log \, (m_h/m_Z)^2$

$\delta_{vb} = 20/13 \; \alpha/\pi \; (m_{top}/m_Z)^2$

complete formulae at 2d order including strong corrections are available in fitting codes

e.g. ZFITTER , GFITTER

$\Delta\rho$: $\equiv \varepsilon_1$

$$\Gamma_\ell = (1+\Delta\rho) \frac{G_F \, M_Z^3}{24\pi\sqrt{2}} \left(1 + \left(\frac{g_{Ve}}{g_{Ae}}\right)^2\right)\left(1 + \frac{3}{4}\frac{\alpha}{\pi}\right)$$

ε_3

$$\sin^2\theta_W^{eff} \cos^2\theta_W^{eff} = \frac{\pi\alpha(M_Z^2)}{\sqrt{2}\, G_F\, M_Z^2} \frac{1}{1+\Delta\rho} \frac{1}{1 - \frac{\varepsilon_3}{\cos^2\theta_W}}$$

δ_{vb}

$$\Gamma_b = (1+\delta_{vb})\, \Gamma_d \left(1 - \underset{\propto\, m_b^2/M_Z^2}{\text{mass corrections}}\right)$$

ε_2

$$M_W^2 = \frac{\pi\alpha(M_Z^2)}{\sqrt{2}\, G_F \sin^2\theta_W^{eff}} \cdot \frac{1}{(1-\varepsilon_3+\varepsilon_2)}$$

$\sin^2\theta_W^{eff}$ is defined from

$$\sin^2\theta_W^{eff} = \frac{1}{4}\left(1 - \frac{g_{V\ell}}{g_{A\ell}}\right) = \sin^2\theta_W^{eff}{}_{lept}$$

obtained from asymmetries at the Z.

also Δr

$$m_W^2 = \frac{\pi\alpha}{\sqrt{2}\, G_F} \cdot \frac{1}{\left(1 - \frac{M_W^2}{M_Z^2}\right)} \frac{1}{(1-\Delta r)}$$

$$\Delta r = \Delta\alpha - \frac{\cos^2\theta_W}{\sin^2\theta_W}\Delta\rho + 2\frac{\cos^2\theta_W}{\sin^2\theta_W}\varepsilon_3 + \frac{c^2-s^2}{s^2}\varepsilon_2$$

Extracting physics from $\sin^2\theta^{lept}_W$

1. Direct comparison with m_Z

$$\sin^2\theta_W^{eff}\cos^2\theta_W^{eff} = \frac{\pi\alpha(M_Z^2)}{\sqrt{2}\,G_F\,m_Z^2}\;\frac{1}{1+\Delta\rho}\;\frac{1}{1-\frac{\varepsilon_3}{\cos^2\theta_W}}$$

Uncertainties in m_{top} , $\Delta\alpha(m_z)$, m_H , etc….

$\Delta\sin^2\theta^{lept}_W$ ~ $\Delta\alpha(m_z)$ /3 = 10^{-5} if we can reduce $\Delta\alpha(m_z)$ (see P. Janot)

2. Comparison with m_W/m_Z

Compare above formula with similar one:

$$\sin^2\theta_W\cos^2\theta_W = \frac{\pi\alpha(M_Z^2)}{\sqrt{2}\,G_F\,m_Z^2}\cdot\frac{1}{1-\left(-\frac{\cos^2\theta_W}{\sin^2\theta_W}\Delta\rho + 2\frac{\cos^2\theta_W}{\sin^2\theta_W}\varepsilon_3 + \frac{c^2-s^2}{s^2}\varepsilon_2\right)}$$

Where it can be seen that **$\Delta\alpha(m_z)$** cancels in the relation.

The limiting error is the error on m_W.

For **Δm_W= 0.5 MeV this corresponds to $\Delta\sin^2\theta^{lept}_W = 10^{-5}$**

Will consider today the contribution of the Center-of-mass energy systematic errors

Today: step I, compare

ILC measurement of A_{LR} with 10^9 Z and P_{e-} =80%, P_{e+} =30%

FCC-ee measurement of $A_{FB}^{\mu\mu}$ and $A_{FB}^{Pol}(\tau)$ with 2.10^{12} Z

Comparing A_{LR} (P) and A_{FB} ($\mu\mu$)

Both measure the weak mixing angle as **defined** by the relation $A_\ell = \frac{(g^e_L)^2-(g^e_R)^2}{(g^e_L)^2+(g^e_R)^2}$

with $(g^e_L) = \frac{1}{2} - \sin^2\theta^{lept}_W$ and $(g^e_R) = -\sin^2\theta^{lept}_W$ $A_\ell \approx 8(1/4 - \sin^2\theta^{lept}_W)$

$A_{LR} = A_e$

$A_{FB}^{\mu\mu} = \frac{3}{4} A_e A_\mu = \frac{3}{4} A_\ell^2$

-- $\mathbf{A_{FB}^{\mu\mu}}$ is measured using muon pairs   (5% of visible Z decays) and unpolarized beams

-- $\mathbf{A_{LR}}$ is measured using all statistics of visible Z decays with beams of
alternating longitudinal polarization

both with very small experimental systematics

-- **parametric sensitivity** $\frac{dA_{FB}^{\mu\mu}}{d\sin^2\theta^{lept}_W} = 1.73$ **vs** $\frac{dA_{LR}}{d\sin^2\theta^{lept}_W} = 7.9$

Measurement of A_{LR}

electron bunches	1$\Leftarrow$	2	3	4$\Leftarrow$
positron bunches	1	2$\Rightarrow$	3	4$\Rightarrow$
cross sections	σ_1	σ_2	σ_3	σ_4
event numbers	N_1	N_2	N_3	N_4

$$\sigma_1 = \sigma_u (1 - P^-_e A_{LR})$$

$$\sigma_2 = \sigma_u (1 + P^+_e A_{LR})$$

$$\sigma_3 = \sigma_u$$

$$\sigma_4 = \sigma_u [1 - P^+_e P^-_e + (P^+_e - P^-_e) A_{LR}]$$

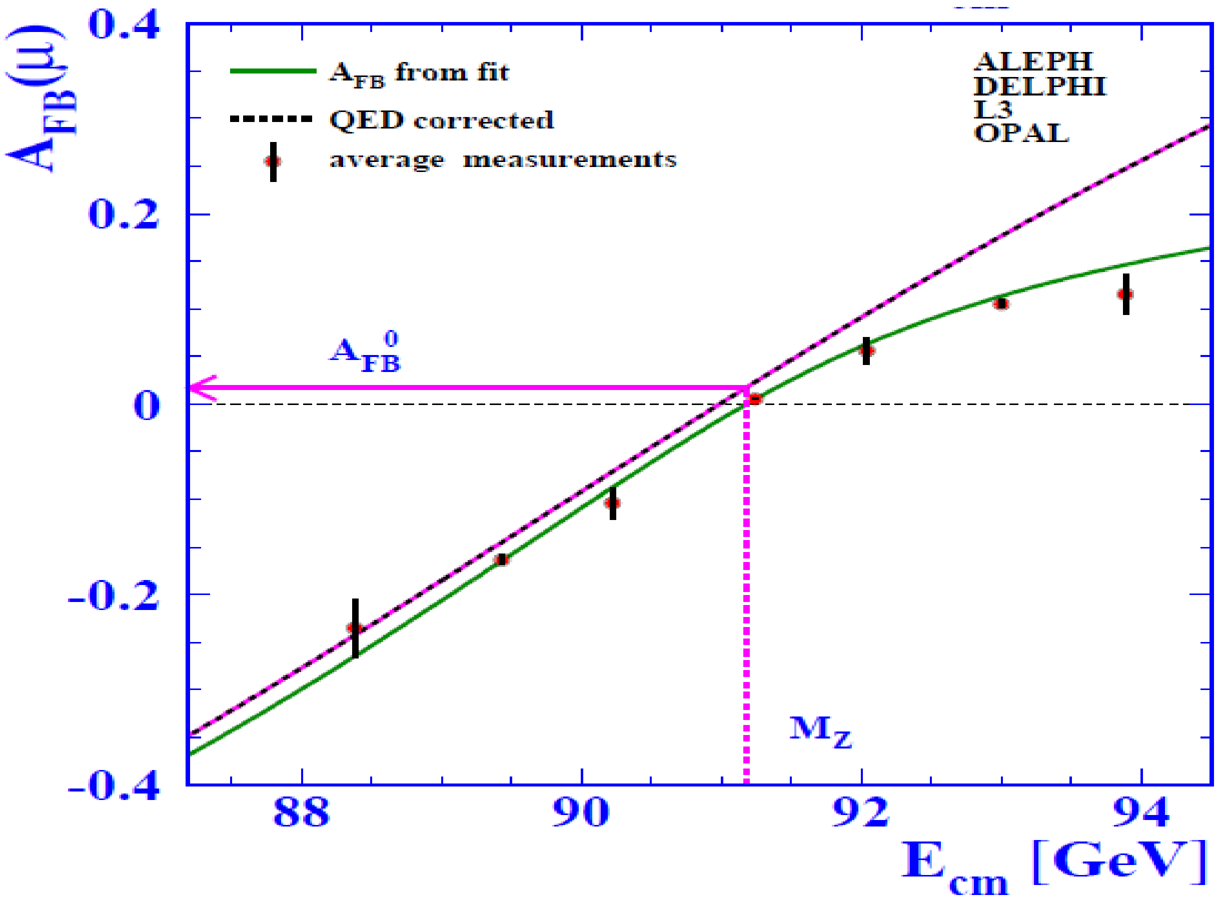
Verifies polarimeter with experimentally measured cross-section ratios

statistics

ΔA_{LR} = 0.0025 with about 10^6 Z^0 events,

ΔA_{LR} = 0.000045 with 5.10^{10} Z and 30% polarization in collisions.

$\Delta \sin^2\theta_W^{eff}$ (stat) = $O(2.10^{-6})$



$A_{FB}(\mu)$
0.4
0.2
0
-0.2
-0.4
A_{FB} from fit
QED corrected
average measurements
ALEPH
DELPHI
L3
OPAL
A_{FB}^{0}
M_Z
88
90
92
94
E_{cm} [GeV]

	$A_{FB}^{\mu\mu}$ @ FCC-ee		A_{LR} @ ILC	A_{LR} @ FCC-ee
visible Z decays	10^{12}	visible Z decays	10^{9}	5.10^{10}
muon pairs	10^{11}	beam polarization	90%	30%
$\Delta A_{FB}^{\mu\mu}$ (stat)	$3\ 10^{-6}$	ΔA_{LR} (stat)	$4.2\ 10^{-5}$	$4.5\ 10^{-5}$
$\Delta\ E_{cm}$ (MeV)	0.1		2.2	?
$\Delta A_{FB}^{\mu\mu}$ (E_{CM})	$9.2\ 10^{-6}$	ΔA_{LR} (E_{CM})	$4.1\ 10^{-5}$	
$\Delta A_{FB}^{\mu\mu}$	$1.0\ 10^{-5}$	ΔA_{LR}	$5.9\ 10^{-5}$	
$\Delta \sin^2\theta^{lept}_W$	$5.9\ 10^{-6}$		$7.5\ 10^{-6}$	$6\ 10^{-6}$ +?

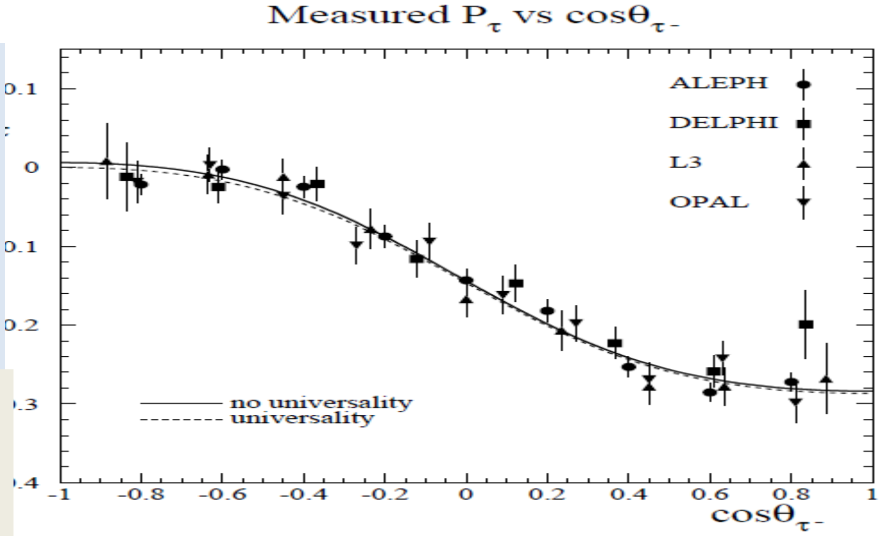
All exceeds the theoretical precision from $\Delta\alpha(m_Z)$ ($3 10^{-5}$) or the comparison with m_W (500keV)

But this precision on $\Delta \sin^2\theta^{\ell ept}_W$ can only be exploited at FCC-ee!

The forward backward tau polarization asymmetry is very clean.
Dependence on E_{CM} same as A_{LR} negl.
At FCC-ee

ALEPH data 160 pb^{-1} (80 s @ FCC-ee !)

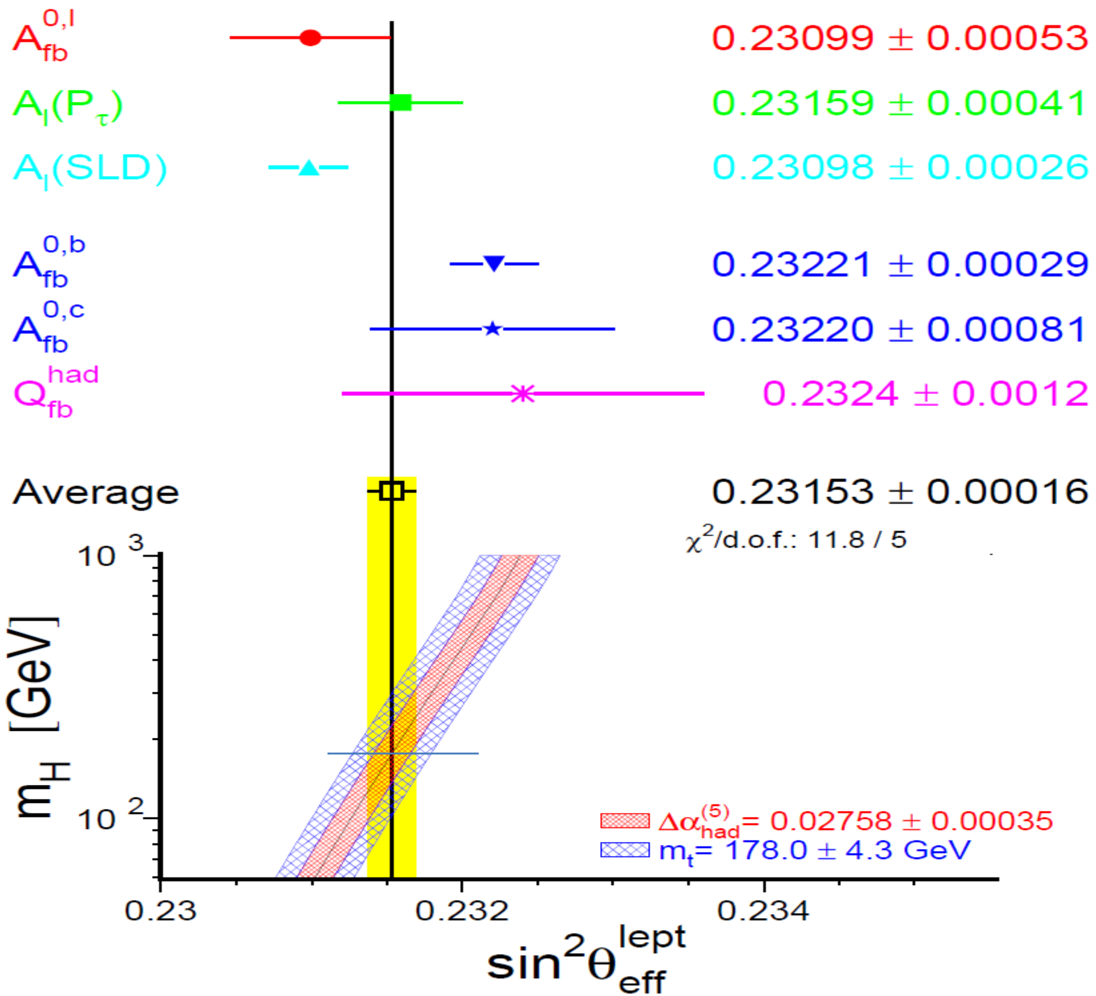
Already syst. level of 6 10^{-5} on $\sin^2\theta^{eff}_W$
much improvement possible
by using dedicated selection
e.g. tau→ π ν to avoid had. model



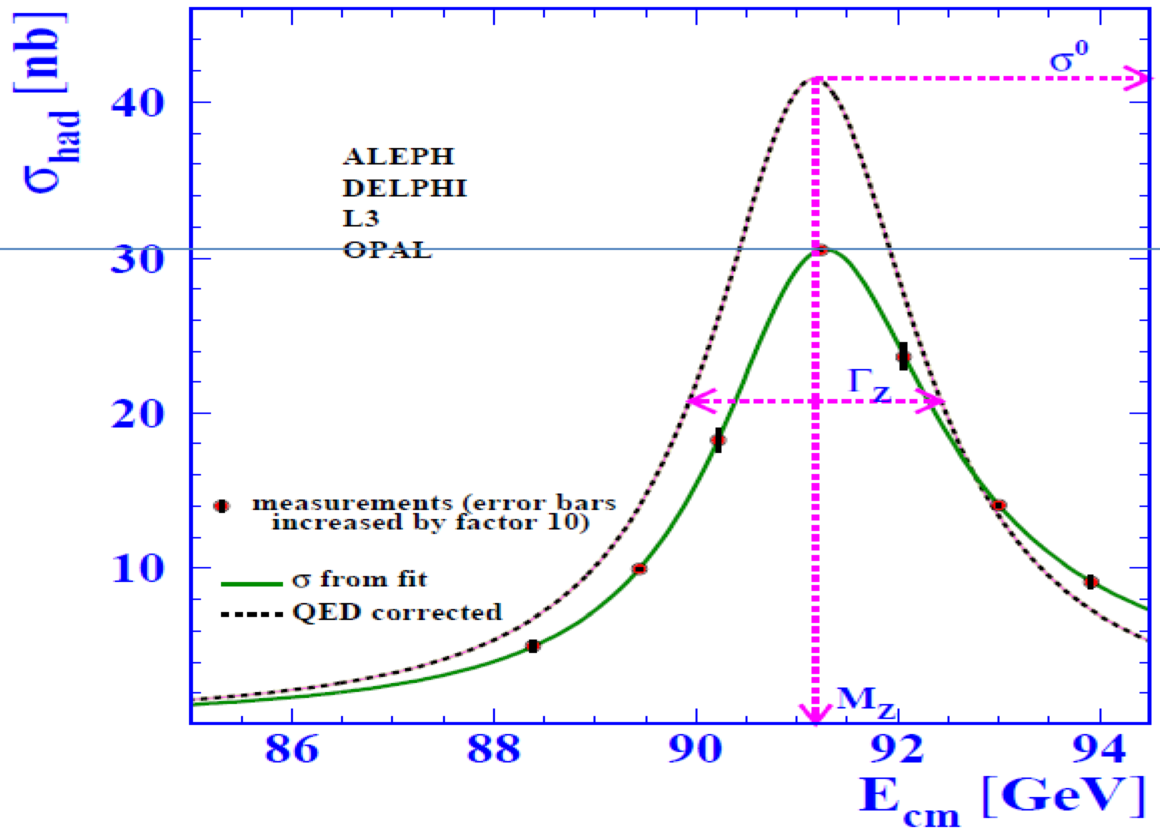
4.7: The values of $\mathcal{P}_\tau$ as a function of $\cos\theta_{\tau^-}$ as measured by each of the LEP ex- nts. Only the statistical errors are shown. The values are not corrected for radiation, ence or pure photon exchange. The solid curve overlays Equation 4.2 for the LEP values of $\mathcal{A}_\tau$ and $\mathcal{A}_e$. The dashed curve overlays Equation 4.2 under the assumption of lepton universality for the LEP value of $\mathcal{A}_\ell$.

	ALEPH		DELPHI		L3		OPAL	
	$\delta\mathcal{A}_\tau$	$\delta\mathcal{A}_e$	$\delta\mathcal{A}_\tau$	$\delta\mathcal{A}_e$	$\delta\mathcal{A}_\tau$	$\delta\mathcal{A}_e$	$\delta\mathcal{A}_\tau$	$\delta\mathcal{A}_e$
ZFITTER	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
τ branching fractions	0.0003	0.0000	0.0016	0.0000	0.0007	0.0012	0.0011	0.0003
two-photon bg	0.0000	0.0000	0.0005	0.0000	0.0007	0.0000	0.0000	0.0000
had. decay model	0.0012	0.0008	0.0010	0.0000	0.0010	0.0001	0.0025	0.0005

Table 4.2: The magnitude of the major common systematic errors on $\mathcal{A}_\tau$ and $\mathcal{A}_e$ by category for each of the LEP experiments.



$A_{fb}^{0,l}$ 0.23099 ± 0.00053
$A_l(P_\tau)$ 0.23159 ± 0.00041
A_l(SLD) 0.23098 ± 0.00026
$A_{fb}^{0,b}$ 0.23221 ± 0.00029
$A_{fb}^{0,c}$ 0.23220 ± 0.00081
Q_{fb}^{had} 0.2324 ± 0.0012
Average 0.23153 ± 0.00016
χ^2/d.o.f.: 11.8 / 5
10^3
10^2
m_H [GeV]
$\Delta\alpha_{had}^{(5)}$ = 0.02758 ± 0.00035
m_t = 178.0 ± 4.3 GeV
0.23
0.232
0.234
$\sin^2\theta_{eff}^{lept}$



σ_{had} [nb]
40
30
20
10
ALEPH
DELPHI
L3
OPAL
σ^0
Γ_Z
M_Z
measurements (error bars increased by factor 10)
σ from fit
QED corrected
86
88
90
92
94
E_{cm} [GeV]

Going through the observables

the weak mixing angle as **defined** by the relation

$$A_\ell = \frac{2 g^e_V \, g^e_A}{(g^e_V)^2 + (g^e_A)^2} = \frac{(g^e_L)^2 - (g^e_R)^2}{(g^e_L)^2 + (g^e_R)^2}$$

with $(g^e_L) = ½ - \sin^2\theta^{lept}_W$ and $(g^e_R) = -\sin^2\theta^{lept}_W$

$A_\ell \approx 8(1/4 - \sin^2\theta^{lept}_W)$ very sensitive to $\sin^2\theta^{lept}_W$!

Or

$A_{LR} = A_e$ measured from $(\sigma_{vis,L} - \sigma_{vis,R}) / (\sigma_{vis,L} - \sigma_{vis,R})$

(total visible cross-section had + $\mu\mu$ + $\tau\tau$, (35 nb) for 100% Left Polarization

$$g_{Vf} = \sqrt{\mathcal{R}_f}\,(T_3^f - 2 Q_f \mathcal{K}_f \sin^2\theta_W)$$

$$g_{Af} = \sqrt{\mathcal{R}_f}\, T_3^f \, .$$

$A_{FB}^{\mu\mu} = ¾ A_e A_\mu = ¾ A_\ell^2$

$$\begin{aligned} A_{FB}^{0,f} &= \frac{3}{4}\mathcal{A}_e\mathcal{A}_f \\ A_{LR}^0 &= \mathcal{A}_e \\ A_{LRFB}^0 &= \frac{3}{4}\mathcal{A}_f \\ \langle \mathcal{P}_\tau^0 \rangle &= -\mathcal{A}_\tau \\ A_{FB}^{pol,0} &= -\frac{3}{4}\mathcal{A}_e \, . \end{aligned}$$

$$\begin{aligned} A_{FB} &= \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} \\ A_{LR} &= \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}\frac{1}{\langle|\mathcal{P}_e|\rangle} \\ A_{LRFB} &= \frac{(\sigma_F - \sigma_B)_L - (\sigma_F - \sigma_B)_R}{(\sigma_F + \sigma_B)_L + (\sigma_F + \sigma_B)_R}\frac{1}{\langle|\mathcal{P}_e|\rangle} \, . \end{aligned}$$