Beam emittance (size) diagnostic

Recent progress

T. Mitsuhashi

KEK
1. Review of X-ray interferometer
2. Possibility for bunch by bunch beam size measurement
   Undulator
   Wide aperture interferometer
   Inverse-contrast interferometer
3. Proposal for X-ray interferometer at ALBA
# Parameters of FCC-ee

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bending magnet length</td>
<td>24.585m</td>
</tr>
<tr>
<td>Bending radius</td>
<td>11590.8m</td>
</tr>
<tr>
<td>Magnetic field strength</td>
<td>0.0503T</td>
</tr>
<tr>
<td>Bending angle</td>
<td>2.144mrad</td>
</tr>
<tr>
<td>Beam energy and current</td>
<td>175GeV 6.6mA 45GeV 1500mA</td>
</tr>
<tr>
<td>Emittance</td>
<td>1.3pmrad</td>
</tr>
<tr>
<td>Estimated vertical beam size</td>
<td>$\sigma_y = 5.1\mu m$ / $\beta = 20m$ $= 0.05\mu rad / 100m$</td>
</tr>
</tbody>
</table>
175GeV
\( \rho = 11590.8 \text{m} \)
0.1nm

Divergence of beam
Order of \( 10^{-7} \text{rad} \)

Divergence of SR
Order of \( 10^{-5} \text{rad} \)
Expected spectrum from the bending magnet FCC-ee

Frequency integrated power

- 174W/20μrad for 175GeV
- 172W/20μrad for 45GeV
Character of bending magnet in FCC-ee

24.858m or 2.144 mrad

Bending radius 11590.8m

Bending angle of 2.144 mrad is 100 times larger than tail to tail opening of SR at 0.1nm (0.002 mrad).

So, this bend is classified as long magnet.

TR from magnet edge is week enough in X-ray region.
Extraction of hard X-rays from the ring

1. Light source
   use last bending magnet in Arc.
Geometrical condition for the extraction of SR from the last bending magnet

Enough separation between orbit and extraction structure of the vacuum duct is necessary to escape from corrective effect.

Some similar structure such as crotch absorber and branch optical beam line seems necessary to protect the crotch of the vacuum chamber from strong irradiation of SR.
X-ray interferometer
Simple double slit X-ray interferometer (Young type)

K-edge filter

Double slit $D=20$-few$100\mu$m, $a=8\mu$m

Be-window

100m

50-100m
175GeV
\( \rho = 11590.8 \text{m} \)

Double slit location
\( I_v / I_h = 0.016 \)

We do not need selection of polarization

\[ \theta \text{ in rad} \]
Spatial coherence vs. beam size  \( D=300\mu m, f=100m \)

\[ \lambda=0.10\text{nm} \]
Double slit of interferometer will not miss the beam size information.
Double slit interferometer (Young type)
with total reflection mirror

K-edge filter

Double slit  D=20-600μm, a=8μm

Totally reflection mirror

g-ray

Be-window

39m

101m
Expected interferogram for $\gamma=0.65$ (beam size of 5µm at 100m)

Double slit $\alpha=8\mu$m, $D=300\mu$m $f=100$m
Absorption of Krypton gas K-edge filter (1 atm, 100 mm pass).

Krypton gas filter has a nice window around 10keV
Shift in two optical axis

Δλ/λ = 20%
Background subtraction problem

Log scale plot for the interference fringe with two diffraction envelopes of slit
Escape from the shift in two optical axis

Elliptically deformed total reflection mirror
Interferogram by elliptically deformed total reflection mirror
Possibility for bunch by bunch beam size observation

Rough estimation from experience for interferometry using visible light in the ATF
ATF has 1nc in the single bunch.

Our experience in before, light intensity of 50 bunch is necessary for beam size measurement.

Still many parameters are not precise to make reliable estimation, but roughly speaking about 1-2 order more intensity will necessary for bunch by bunch beam size observation.

Wide aperture optics (so-called coded aperture is using in the super B factory in the KEK)
Increase flux density of the SR by inserting some undulator (an assumption)

**Parameters of Undulator**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnetic field</td>
<td>1T</td>
</tr>
<tr>
<td>period</td>
<td>20</td>
</tr>
<tr>
<td>$\lambda u$</td>
<td>100mm</td>
</tr>
</tbody>
</table>
Spectrum from Undulator (not including energy spread of the beam)

- 0.275nm at 45GeV
- 0.018nm at 175GeV
Problem

Simple fixed period undulator cannot cover the energy range from 45GeV to 175GeV.

Questions

Double slit can accept high flux density?

Is there any space for undulator(3-4m)?
Wide aperture interferometer
Fresnel transfer

Fourier transfer
a=10mm

D=30mm

a=38mm

D=30mm
Diffraction treatment for interferometer

\[ E_1(y) = u\left(2a, -\frac{D}{2}\right) \]

\[ E_2(y) = u\left(2a, +\frac{D}{2}\right) \]

\[ E(y) = E_1(y) + E_2(y) = u\left(2a, -\frac{D}{2}\right) + u\left(2a, +\frac{D}{2}\right) \]
Diffraction by this aperture is given by Fourier transfer of \( E \)

\[
f(\omega) = \mathcal{F}(E)
\]

\[
= \frac{1}{2} \left( e^{i \frac{D}{2} \pi \omega} + e^{-i \frac{D}{2} \pi \omega} \right) \sin c \left( \frac{\omega}{2a} \right)
\]

\[
= \cos \left( \frac{D}{2} \pi \omega \right) \sin c \left( \frac{\omega}{2a} \right)
\]

Then the intensity is given by square of \( E \)

\[
I(\omega) = \cos^2 \left( \frac{D}{2} \pi \omega \right) \sin^2 \left( \frac{\omega}{2a} \right)
\]

\[
= \frac{1}{2} \left( 1 + \cos(D \pi \omega) \right) \sin^2 \left( \frac{\omega}{2a} \right)
\]
This treatment is assuming the spatial coherence will be 1. When the spatial coherence is not equal to 1 as

\[ \gamma \approx \bar{\gamma} = \int_{-a+D/2}^{a+D/2} \gamma(D) \, dD \]
This large aperture interferometer can use under following condition.

\[
\gamma \approx \bar{\gamma} = \int_{-a+\frac{D}{2}}^{a+\frac{D}{2}} \gamma(D) dD
\]

Since we can enlarge the slit width 10-50 times more, this method is very useful for increase the intensity.
Inverse-contras interferometer
Using Babinet’s principle, Synthesize of two mask is given by,

\[ u_0(Q) = u_{ob}(Q) + u_{ap}(Q) \]

Then obstacle mask is given by

\[ u_{ob}(Q) = u_0(Q) - u_{ap}(Q) \]

Double slit mask is given by,

\[ u_{ap}(Q) = u_{ap1}(Q) + u_{ap2}(Q) \]

Then double obstacle mask is given by

\[ u_{ob}(Q) = u_0(Q) - (u_{ap1}(Q) + u_{ap2}(Q)) \]
The intensity is given by square of \( u \),

\[
I_{ob}(Q) = \left[ u_0(Q) - (u_{ap1}(Q) + u_{ap2}(Q)) \right]^2
\]

\[
= u^2_0(Q) - 2u_0(Q)(u_{ap1}(Q) + u_{ap2}(Q)) + (u_{ap1}(Q) + u_{ap2}(Q))^2
\]

\[
= I_0 - 2u_0u_{ap1} - 2u_0u_{ap2}
\]

\[
+ I_{ap1} + I_{ap2} + 2u_{ap1}u_{ap2}
\]

\( I_0 \) is intensity of aperture having no double obstacle, \( I_{ap1}, I_{ap2} \) are intensities double slit corresponding to double obstacle.
\( u_0u_{ap1}, u_0u_{ap2} \) are interference between disturbance of aperture having no obstacle and double slit corresponding to double obstacle. \( u_{ap1}u_{ap2} \) is interference between double slit corresponding to double obstacle.

Observing the intensity in infinite distance (very far away), the first 3 terms are localized in center (width of order of full aperture diffraction), and last 3 terms gives same intensity distribution ob double slite interferometer
When we observe the intensity distribution at diffraction width of single slit is almost same as opening of $I_0, u_0u_{ap1}, u_0u_{ap2}$ those are interference between disturbance of aperture having no obstacle and double slit corresponding to double obstacle will appear, and its intensity is enhanced by $u_0$.

This can contribute intensity increase in observation.

*But we need check experimentally.*
Proposal for test of X-ray interferometer

at

Long range beam line

ALBA

T. Mitsuhashi and Ubaldo Irizo
Source point to double slit 33.5m
10m will available for interferometer
### Light source: In vacuum Undurator

<table>
<thead>
<tr>
<th>Material</th>
<th>Sm$<em>5$Co$</em>{17}$ Pure Permanent Magnet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period</td>
<td>21.3 mm</td>
</tr>
<tr>
<td>Number of periods</td>
<td>92</td>
</tr>
<tr>
<td>K at minimum gap</td>
<td>1.6</td>
</tr>
<tr>
<td>source size (FWHM)</td>
<td>309 x 18 µm$^2$ (HxV)</td>
</tr>
<tr>
<td>source divergence (FWHM)</td>
<td>112 x 28-22 µrad$^2$ (HxV)</td>
</tr>
</tbody>
</table>
Spectrum of undulator radiation
7\textsuperscript{th} harmonics at 11.06\text{KeV}
Angular distribution of 7\textsuperscript{th} harmonics
Simple double slit X-ray interferometer (Young type)

Double slit \( D = 10-100 \mu m, 2 a = 8 \mu m \)

\[ 33.5m \quad 7-10m \]
Spatial coherence of X-ray with beam size of 5.5μm

$\lambda = 0.1\text{nm}$
$L = 33.5\text{m}$
Diffraction from single opening of the interferometer

Intensity of diffraction is given by Fresnel transform of pupil function $F$ of the single opening of the double slit.

$$I(x, y) = \left| \iint F(x_0, y_0) \exp \left\{ \frac{ik}{2z} \left[ (x_0 - x)^2 + (y_0 - y)^2 \right] \right\} d\xi d\eta \right|^2$$
\( \tau = \frac{f}{(a^2/\lambda)} < 1 \) \n
Fresnel like region

\( \tau = \frac{f}{(a^2/\lambda)} > 3 \) \n
Fraunhofer like region

In the case of ALBA, \( \lambda = 0.1 \text{nm}, a = 8 \mu\text{m} \)

Fraunhofer region > 1.92m
We need to use smaller than 18\(\mu m\)

**Fraunhofer region**
Interferogram wide view with monochromatic ray

$D=50\mu m$, $\lambda=0.1\text{nm}$, $\sigma=5.5\mu m$
20% K-edge filter at 0.1nm
In Comparison
Phase 1

*Measure single slit pattern in
*Measure single slit diffraction pattern with slit + ribbon
*Measure interference fringe with white beam
*Apply a metal K-edge filter in the beam line
*Observe floor vibration issue
*Lattice regulation for obtain small beam size
Status

*Double slit

Will be design in ALBA (may be one Doctor student available)

*Guarder for setting the double slit

Using the guarder in beamline

*Positioner stage for double slit

Using positioner stage in beamline in ALBA
*Image sensing

Use a YAG screen (ALBA) spatial resolution is about 1\(\mu\)m

With microscope + fast gated II camera (ALBA or KEK)