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Relation between beam energies and centre-of-mass energy

T. Tydecks, J. Wenninger

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Motivation

- energy calibration for Z operation mode (45.6 GeV) to accuracy in order 10^{-6}
  $\Rightarrow$ this corresponds to an uncertainty of 100 keV
- we typically measure average beam energy using resonant spin depolarization to high accuracy (comp. A. Blondel Tue 13:30)
- relation between beam energy and centre of mass (cm) energy
  - effect of synchrotron radiation / sawtoothing on cm energy
  - effect of RF phase jitter
  - effect of spurious dispersion

Beam energy / centre of mass energy

- for physics processes, centre of mass energy E_{cm} is relevant quantity instead of local beam energy $E_{1,2}$

  Relation between E_{cm} and momentum $P_i = (E_i, \vec{p}_i)$:

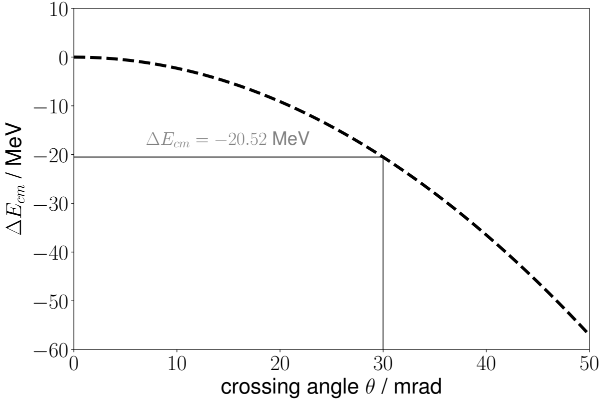
$$E_{cm} = \sqrt{(P_1 + P_2)^2} = \sqrt{(E_1 + E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2 c^2}$$

- for head on collision: $\vec{p}_1 + \vec{p}_2 = 0$ and $E_{cm} = E_1 + E_2$
- for crossing angle (assuming $|\vec{p}_1| = |\vec{p}_2| = p_0$):

$$E_{cm} = \sqrt{(P_1 + P_2)^2} = \sqrt{(E_1 + E_2)^2 - 4p_0^2(1 - \cos\theta)c^2}$$

Effect of crossing angle on cm energy

$$E_{cm} = \sqrt{(P_1 + P_2)^2} = \sqrt{(E_1 + E_2)^2 - 4p_0^2(1 - \cos\theta)c^2}$$



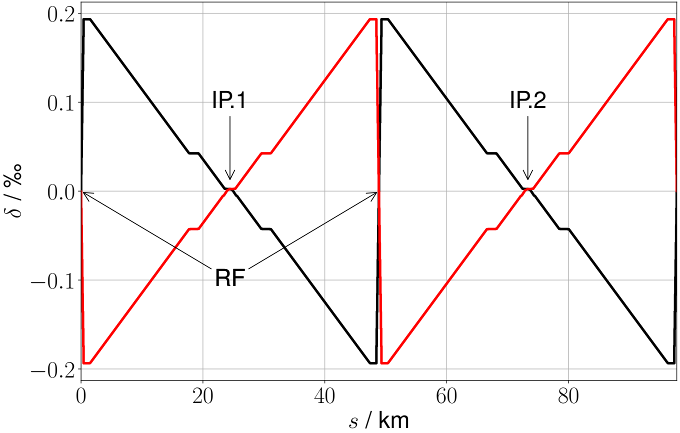
- in 5 min, crossing angle can be recorded to statistical precision of 0.3 µrad from angular distribution of dimuon events: $e^+e^- \rightarrow \mu^+\mu^-(\gamma)$

  Patrick Janot, *Determination from $e^+e^- \rightarrow \mu^+\mu^-(\gamma)$ events*, CDR

$\Rightarrow$ resulting uncertainty: $\sigma_{E_{cm}} \approx 1$ keV

Effect of synchrotron radiation (I)

- energy loss due to synchrotron radiation is restored in rf-cavities
- rf is centered in two straight sections leading to energy sawtooth



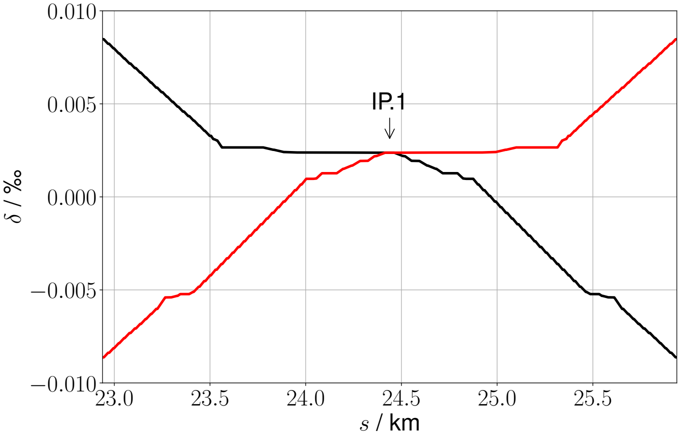
Effect of synchrotron radiation (II)

- due to assymmetry at IP to avoid hard synchrotron radiation in detectors

$\Rightarrow$ IP is no symmetry point regarding beam energy

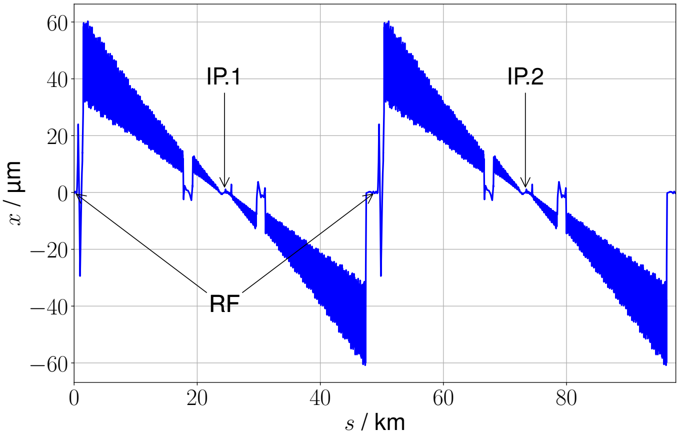
$\Rightarrow$ $E_{\mathrm{IP}} \neq E_{\mathrm{RF}}$

- $\Delta E_{cm} = 216\,\mathrm{keV}$
- precise model of beam energy along circumference needed to determine cm energy from ”average” beam energy measured by resonant spin depolarization



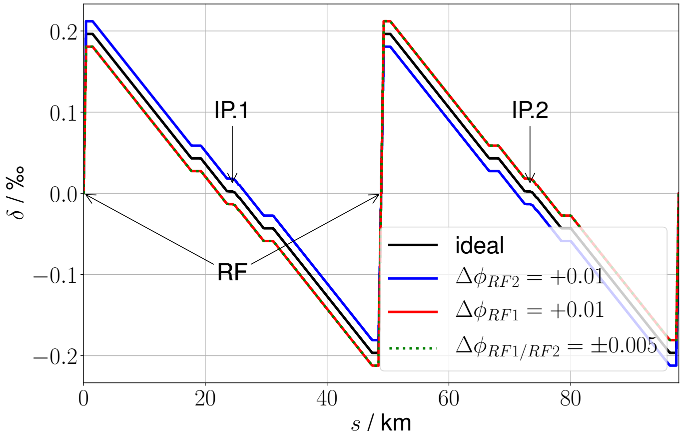
Effect of synchrotron radiation (III)

- precise model of beam energy requires precise model of magnetic induction along circumference
- energy sawtooth can be obtained from orbit if machine is not tapered and magnetic induction varies little between dipoles
- energy sawtooth can help improve machine model



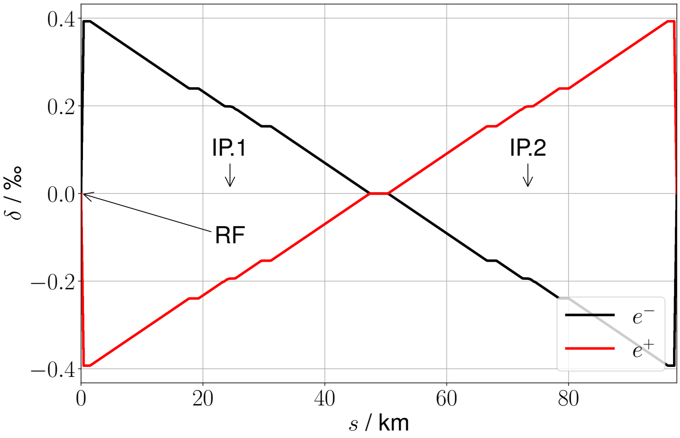
The problem with two rf systems

- up to now: assumed perfect rf
- what happens if two rf systems are out of phase?
  - for all displayed conditions: average energy measured by resonant spin depolarization is the same!
  - beam energy will vary asymmetrically regarding azimuthal position & particle species
- asymmetry between beam energies at IP's will be measured by μ pairs to great precision



The case for one rf system

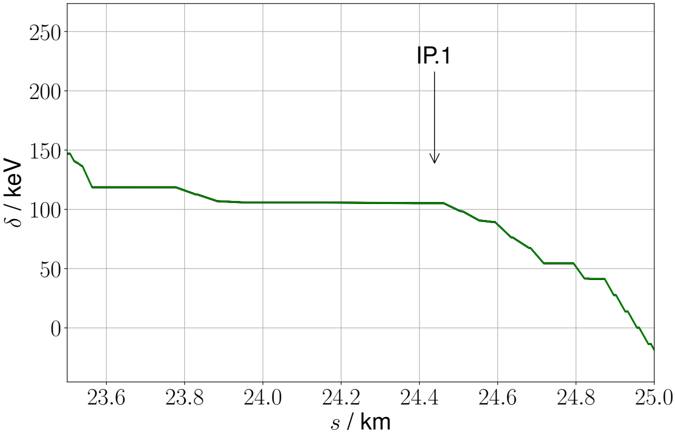
- in the case of one rf:
  - no detuning with respect to other rf-straight
  - energy determined by rf-frequency only
- however, grid would need to supply 100 MW in one point



RF phase jitter

- two rf-straights: each containing 20 rf cavities.
- random phase errors for all cavities

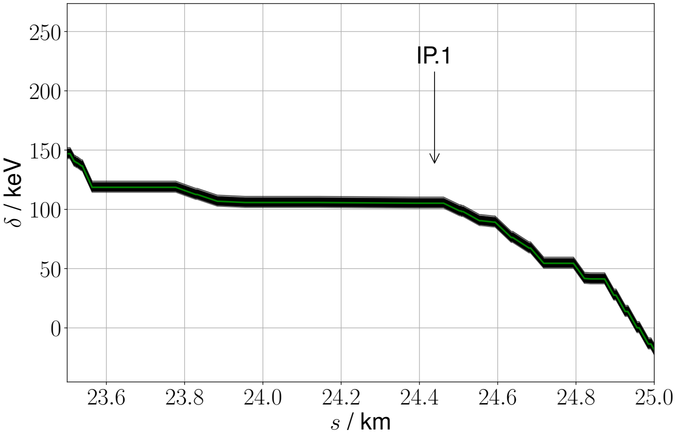
$\sigma_\phi = 10^{-5}$



RF phase jitter

- two rf-straights: each containing 20 rf cavities.
- random phase errors for all cavities

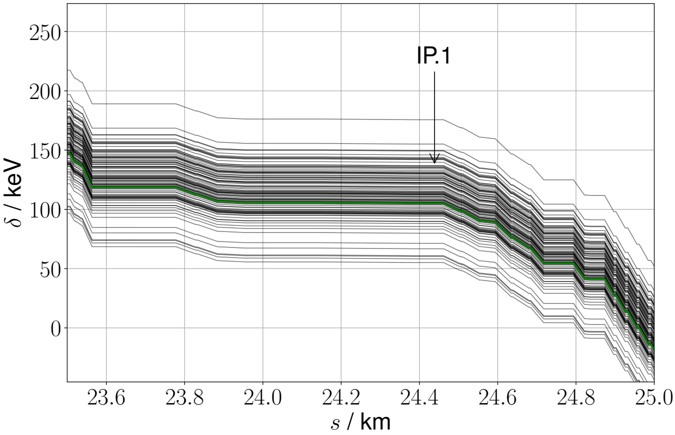
$\sigma_\phi = 10^{-4}$



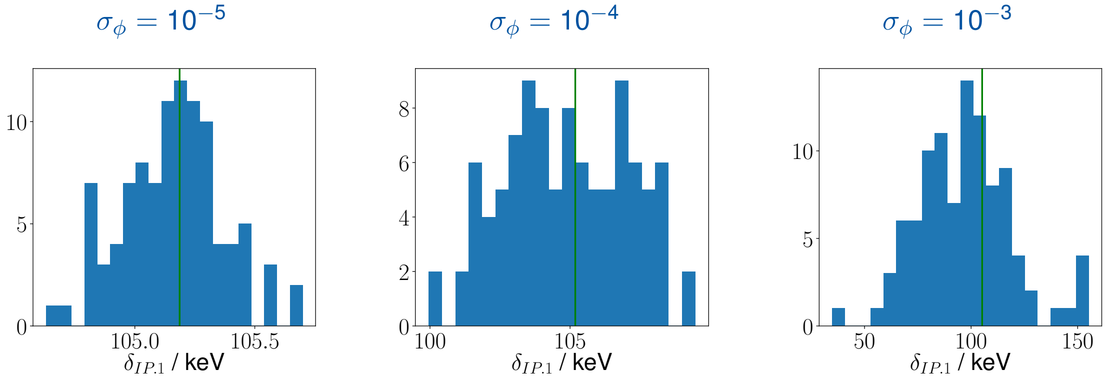
RF phase jitter

- two rf-straights: each containing 20 rf cavities.
- random phase errors for all cavities

$\sigma_\phi = 10^{-3}$



RF phase jitter



- usually σ_ϕ in the order of 1×10^{-4}

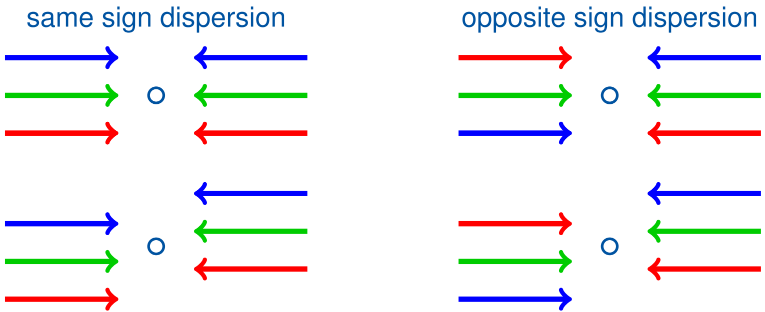
$\Rightarrow$ effect in the order of 5 keV on beam energy at IP

Spurious dispersion (I)

- in the case of spurious dispersion at the IP

$\Rightarrow$ particles are sorted according to their energy

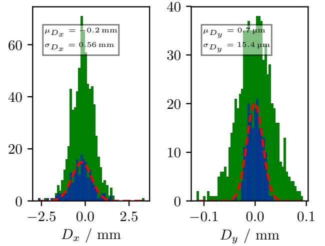
- even well corrected machine will have some dispersion left at IP
- depending on sign of dispersion per beam, different effects arise:



Spurious dispersion (II)

- depending on the sign of the dispersion, this leads to
  - reduction / increase in cm energy spread
  - shift of cm energy if beams do not collide head on
- a difference in dispersion ΔD leads to shift of cm energy[1]:

$$\Delta E_{cm} = -u_0 \frac{\sigma_E^2 \Delta D}{E_0 \sigma_u^2}$$



assuming: $\sigma_x = 6.4\,\mu m, \sigma_y = 28\,nm, \sigma_{D_x} = 0.1\,mm, \sigma_{D_y} = 1.0\,\mu m$

$\frac{u_0}{\sigma_u}$	0.1	0.5	1.0
$\Delta E_{cm}(D_x)$ / MeV	0.12	0.59	1.18
$\Delta E_{cm}(D_y)$ / MeV	0.28	1.42	2.84

(1) J. Jowett et al, Influence of Dispersion and Collision Offsets on the Center-of-mass Energy at LEP, CERN SL/ Note 95-46 (OP)

Summary & Outlook

- energy calibration for FCC-ee with a final uncertainty in the order of 100 keV will require an excellent machine model
- knowledge of magnetic induction along the circumference to high precision is mandatory
- sawtooth orbit would be an additional option to calibrate the model
- online monitoring will be necessary for
  - dispersion
  - beam overlap
  - crossing angle
- not covered here but also important: longitudinal impedance

Thanks for your attention...

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