Precision calculations for the Z line shape at the FCC-ee

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Deriving mass and width of the Z-boson

- **LEP:** Collected $\simeq 17 \times 10^6$ decays (a few years of collecting data) [PDG 2017]

  $\Gamma_Z = 2495.2 \pm \Delta\Gamma_Z^{LEP}$, $\quad \Delta\Gamma_Z^{LEP} = 2.3$ MeV
- **FCC-ee:** Expected 10^{12} Z-boson decays [1] $\quad \Delta\Gamma_Z^{FCC} \simeq 0.1$ MeV
- Other EWPOs are R_l, R_b, $\sin^2\theta^l_{\rm eff}$, $\sin^2\theta^b_{\rm eff}$, e.g.: $\Delta R_l^{LEP} = 250 \cdot 10^{-4}$

  $\Delta R_l^{FCC} \simeq 2 \div 20 \cdot 10^{-4}$
- Huge statistics and precise systematics and beam energy
  $\longrightarrow$ Fine theoretical tests of the Standard Model or its extensions
- **Need for SM corrections at 2,3, and 4 loops.**

Unfolding QED effects and higher order resummation

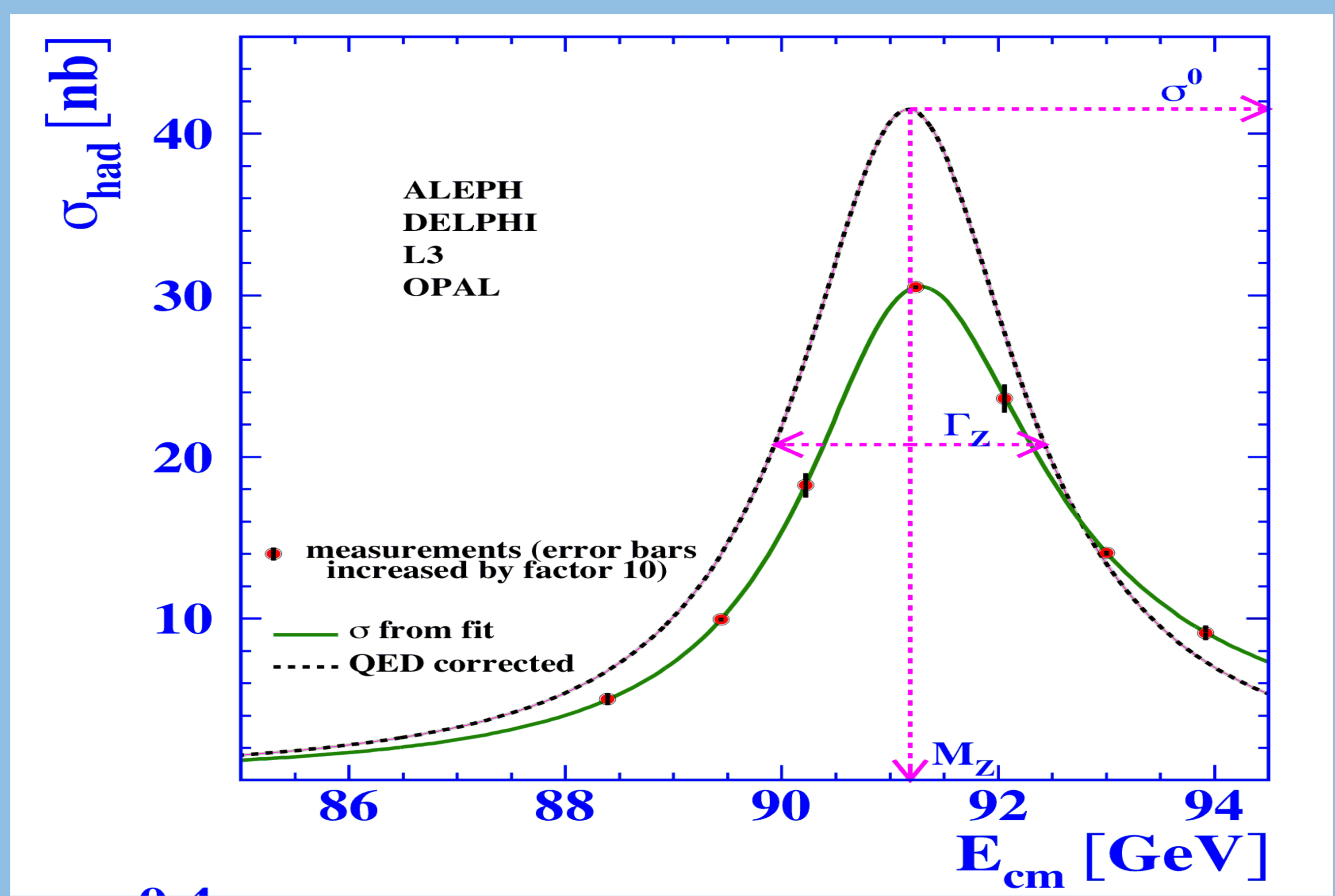


Fig.: S. Schael et al., Phys. Rept. 427 (2006) 257

Needs for substantially improved theoretical analysis software:

- QED Monte Carlo code of the KKMC-type [S. Jadach et al.]
- Unfolding code of the SMATASY type [M. Grünewald et al.]
- Electroweak library of the ZFITTER type [T. Riemann et al.]

$$\sigma^{meas} \xrightarrow{\text{KKMC},\cdots} \sigma^{real} \xrightarrow[\text{ZFITTER},\cdots]{\text{SMATASY},\cdots} \begin{cases} \sigma_0 \equiv \sigma^{eff,f} \\ M_Z, \Gamma_Z, \Gamma_f \\ A_{FB}^{eff,f}, A_{LR}^{eff,f} \\ R_b, R_\ell, R_{had} \\ \cdots \end{cases}$$

Electroweak pseudo-observables [EWPOs]

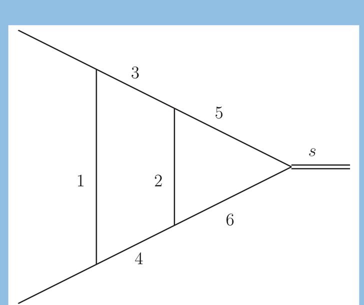
$$\begin{Bmatrix} \sigma_0 \equiv \sigma^{eff,f} \\ M_Z, \Gamma_Z, \Gamma_f \\ A_{FB}^{eff,f}, A_{LR}^{eff,f}, \\ R_b, R_\ell, R_{had} \\ \cdots \end{Bmatrix} \xrightarrow{\text{ZFITTER},\ldots} \begin{Bmatrix} sin^2\theta_W^{eff,f} \\ \Gamma_f \end{Bmatrix} \leftrightarrow \{ v_f^{th}, a_f^{th} \}$$

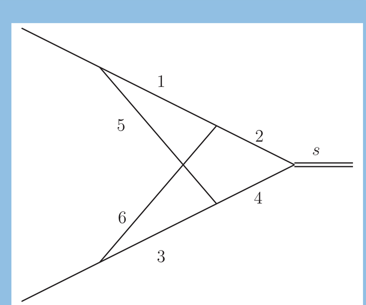
Most complicated 2-loop vertex: $Zb\bar{b}$ [PLB 2016, [2,3]]:

$$V_\mu^{Zb\bar{b}} = \gamma_\mu [v_b^{th} - a_b^{th}\gamma_5] =$$

fermionic, bosonic

$$\cdots + \quad + \quad + \cdots$$





planar, non-planar

Completing 2-loops: bosonic corrections [2,3]

Γ_i [MeV]	$\Gamma_e,\Gamma_\mu,\Gamma_\tau$	$\Gamma_{\nu_e},\Gamma_{\nu_\mu},\Gamma_{\nu_\tau}$	Γ_d,Γ_s	Γ_u,Γ_c	Γ_b	Γ_Z
$\mathcal{O}(\alpha)$	2.273	6.174	9.717	5.799	3.857	60.22
$\mathcal{O}(\alpha\alpha_s)$	0.288	0.458	1.276	1.156	2.006	9.11
$\mathcal{O}(N_f^2\alpha^2)$	0.244	0.416	0.698	0.528	0.694	5.13
$\mathcal{O}(N_f\alpha^2)$	0.120	0.185	0.493	0.494	0.144	3.04
$\mathcal{O}(\alpha^2_{\rm bos})$	**0.017**	**0.019**	**0.058**	**0.057**	**0.167**	**0.505**
$\mathcal{O}(\alpha_t\alpha_s^2,\ \alpha_t\alpha_s^3,\ \alpha_t^2\alpha_s,\ \alpha_t^3)$	0.038	0.059	0.191	0.170	**0.190**	1.20

Table 1: Weak 2-loop and QCD 3-loop corrections for various Γ_f
Red entries are preliminary, unpublished (March 2018) [3].

Three-loop corrections needed: theory estimations [3]

	δ_1 :	δ_2 :	δ_3 :	δ_4 :	δ_5 :	$\delta\Gamma_Z$ [MeV]
	$\mathcal{O}(\alpha^3)$	$\mathcal{O}(\alpha^2\alpha_s)$	$\mathcal{O}(\alpha\alpha_s^2)$	$\mathcal{O}(\alpha\alpha_s^3)$	$\mathcal{O}(\alpha^2_{bos})$	$\sqrt{\sum_{i=1}^5 \delta_i^2}$
TH1	0.26	0.3	0.23	0.035	0.1	0.5
TH2	0.13	0.15	0.11	0.017	10^{-4}	$\sqrt{\sum_{i=1}^5 (\delta_i/2)^2} \sim 0.2$
TH3	0.026	0.03	0.023	0.0035	10^{-4}	$\sqrt{\sum_{i=1}^5 (\delta_i/10)^2} \sim 0.05$

Table 2: At FCC-ee: $\Delta\Gamma_Z \sim 0.1$ MeV.

TH1 = 0.5 MeV (2016): Estimate of residual uncertainty of theoretical errors for Γ_Z [4]. Does not match the FCC-ee demand.

TH2 = 0.2 MeV: Value derives from TH1 by assuming the uncertainty ("no-go") to be solved ("how-to") by calculating the unknowns at an accuracy of 50% (1 digit). Would be not sufficent.

TH3 = 0.05 MeV: Like TH2, but assuming an accuracy of 10% (corresponding to a knowledge of 2 relevant digits) for the so far unknown weak 3-loops and QCD 4-loops. Matches the demand.

Term δ_5 was unknown in TH1 and was determined in [3] with 4 relevant digits. The δ_5 is 5 times bigger than its assumed uncertainty in TH1!

Next decade: complete 3-loop calculations [3]

$Z \to e^+ e^-, \ldots$			
Number of topologies	1 loop	2 loops	3 loops
	1	14 $\to^{(A)}$ 7 $\to^{(B)}$ **5**	211 $\to^{(A)}$ 84 $\to^{(B)}$ **51**
Number of diagrams	14	2012 $\to^{(A,B)}$ **880**	397690 $\to^{(A,B)}$ **91472**
Fermionic loops	0	**114**	**13104**
Bosonic loops	14	**766**	**78368**
Planar / Non-planar	14 / 0	**782 / 98**	**65487 / 25985**
QCD / EW	0 / 14	**0 / 880**	**144 / 91328**

Table 3: Presents the number of Z decay Feynman diagrams needed to be calculated for TH3 of Table 2. Tadpoles, products of lower loop diagrams (A) and symmetrical diagrams (B) are not included.

A first tackle might concentrate on the 13,104 electroweak 2-loop diagrams with closed internal fermionic loops, to be determined with a net accuracy of two relevant digits.

References

[1] J. Wenninger *et al. Future Circular Collider Study Lepton Collider Parameters* FCC-ACC-SPC-0003. https://fcc.web.cern.ch
[2] I. Dubovyk, A. Freitas, J. Gluza, T. Riemann, J. Usovitsch, Phys. Lett. B762 (2016) 184.
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[4] A. Freitas, Prog. Part. Nucl. Phys. 90 (2016) 201 doi:10.1016/j.ppnp.2016.06.004 [arXiv:1604.00406 [hep-ph]].

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