Form factors and HQS in $B \to D^* l\nu$ (20 years later)

1703.06124 & 1707.09509 with D.Bigi and S. Schacht

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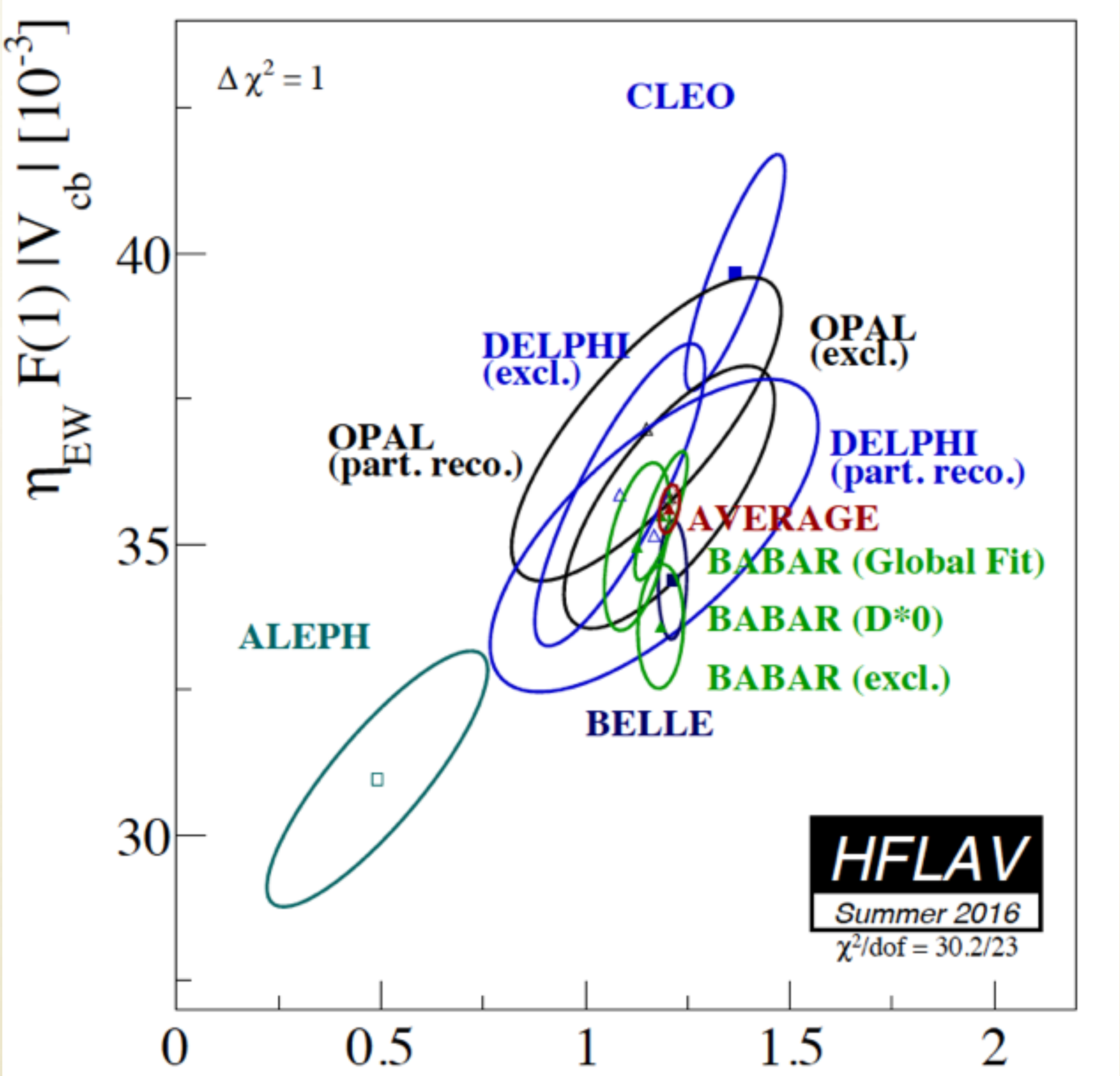
Lattice meets continuum 2017
Siegen, 18 Sept 2017

Since almost 20 years experimental & theory analyses of B->D$^{(*)}$lv are based on the CLN (Caprini,Lellouch, Neubert, 1998) parametrization of the form factors.

In view of the long-standing discrepancy between inclusive and exclusive determinations of V_{cb}, Belle has released deconvoluted B->D$^{(*)}$lv spectra that can be analysed with other parametrizations



V_{cb} from $B \to D^* \ell \nu$

At zero recoil, w=1, where rate vanishes, the ff is

$$\mathcal{F}(1) = \eta_A \left[1 + O\left(\frac{1}{m_c^2}\right) + \ldots \right]$$

Thanks to measurement of slopes and shape parameters, exp error only ~1.3% when extrapolation to zero recoil with CLN parameterization

The ff F(1) has been computed in Lattice QCD. Only one unquenched Lattice calculation is published:

F(1) =0.906(13) ➠ $|V_{cb}| = 39.05(47)_{exp}(58)_{th}\ 10^{-3}$

Bailey et al 1403.0635 (FNAL/MILC) HFLAV 2016

1.9% error

~ 3σ or ~ 8% from inclusive determination 42.00(65) 10^{-3}

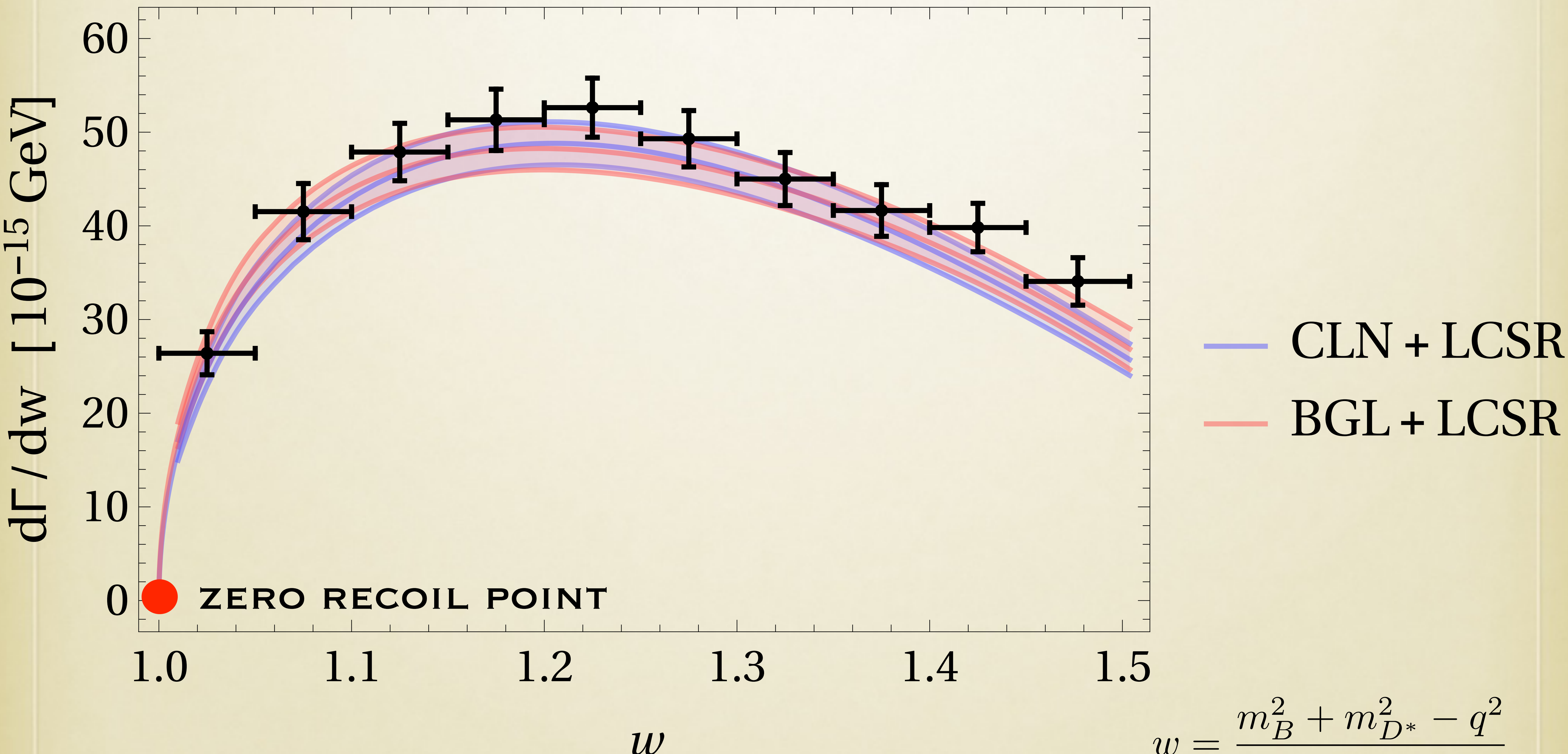
PG,Healey,Turczyk 2016

NEW HPQCD F(1)=0.857(41) preliminary, see M. Wingate talk

NB Heavy Quark Sum Rules estimate F(1)=0.86(2) PG, Mannel, Uraltsev 2012

New preliminary Belle analysis of $B \to D^* l \nu$ 1702.01521

for the first time w and angular deconvoluted distributions independent of parameterization. All previous analyses are CLN based.



$$w = \frac{m_B^2 + m_{D^*}^2 - q^2}{2 m_B m_{D^*}}$$

THE FITS 1703.06124

CLN

CLN Fit:	Data + lattice	Data + lattice + LCSR
χ^2/dof	34.3/36	34.8/39
$\|V_{cb}\|$	0.0382 (15)	0.0382 (14)
$\rho^2_{D^*}$	1.17 $\left(^{+15}_{-16}\right)$	1.16 (14)
$R_1(1)$	1.391 $\left(^{+92}_{-88}\right)$	1.372 (36)
$R_2(1)$	0.913 $\left(^{+73}_{-80}\right)$	0.916 $\left(^{+65}_{-70}\right)$
$h_{A_1}(1)$	0.906 (13)	0.906 (13)

reproduces Belle's deconvoluted results. Best CLN analysis V_{cb}=0.0374(13)

BGL (N=2)

BGL Fit:	Data + lattice	Data + lattice + LCSR
χ^2/dof	27.9/32	31.4/35
$\|V_{cb}\|$	0.0417 $\left(^{+20}_{-21}\right)$	0.0404 $\left(^{+16}_{-17}\right)$
a_0^f	0.01223(18)	0.01224(18)
a_1^f	−0.054 $\left(^{+58}_{-43}\right)$	−0.052 $\left(^{+27}_{-15}\right)$
a_2^f	0.2 $\left(^{+7}_{-12}\right)$	1.0 $\left(^{+0}_{-5}\right)$
$a_1^{\mathcal{F}_1}$	−0.0100 $\left(^{+61}_{-56}\right)$	−0.0070 $\left(^{+54}_{-52}\right)$
$a_2^{\mathcal{F}_1}$	0.12 (10)	0.089 $\left(^{+96}_{-100}\right)$
a_0^g	0.012 $\left(^{+11}_{-8}\right)$	0.0289 $\left(^{+57}_{-37}\right)$
a_1^g	0.7 $\left(^{+3}_{-4}\right)$	0.08 $\left(^{+8}_{-22}\right)$
a_2^g	0.8 $\left(^{+2}_{-17}\right)$	−1.0 $\left(^{+20}_{-0}\right)$

see also Grinstein & Kobach, 1703.08170
Jaiswal, Nandi, Kumar Patra, 1707.09977

9% and 6% (with LCSR) difference in V_{cb}

LCSR: Light Cone Sum Rule results from Faller et al, 0809.0222

$$h_{A_1}(w_{max}) = 0.65(18),$$
$$R_1(w_{max}) = 1.32(4), \quad R_2(w_{max}) = 0.91(17)$$

QUESTIONS

- **Why do CLN and BGL fits differ so much?** because BGL is more flexible: slight modifications to CLN lead to same V_{cb}
- **What's the basic difference between CLN and BGL?** They are based on the same dispersive (or unitarity) bounds, but CLN employs HQET relations to reduce number of parameters.
- **Are there theory uncertainties in the CLN approach?** The experimental analyses have systematically neglected the uncertainty estimated by CLN. We need to check that the assumptions made by CLN in 1998 are consistent with what we know now.

FUTURE SCENARIO

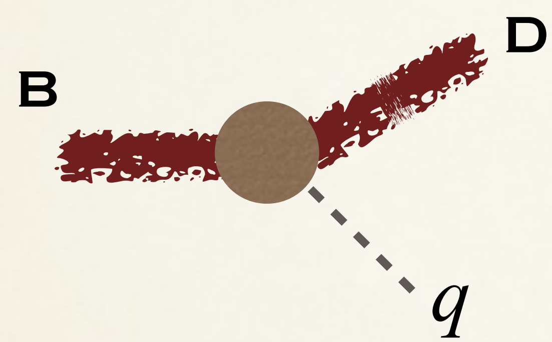
Future lattice fits	χ^2/dof	$\lvert V_{cb} \rvert$
CLN	56.4/37	0.0407 (12)
CLN+LCSR	59.3/40	0.0406 (12)
BGL	28.2/33	0.0409 (15)
BGL+LCSR	31.4/36	0.0404 (13)

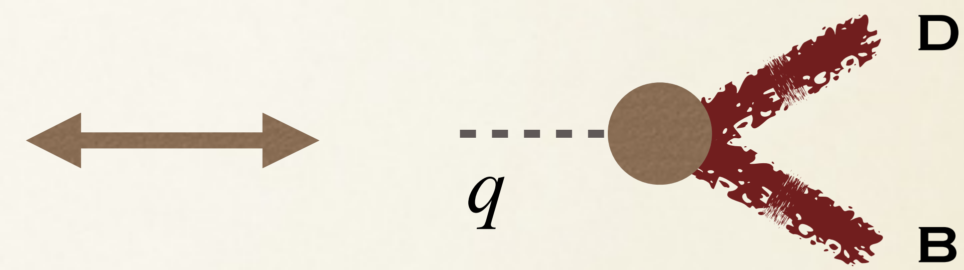
assuming Lattice QCD will provide an estimate of the slope with 5% accuracy

$$\frac{\partial \mathcal{F}}{\partial w}\Big|_{w=1} = -1.44 \pm 0.07$$

UNITARITY CONSTRAINTS

CROSSING + ANALITYCITY





PHYSICAL SEMILEPTONIC REGION

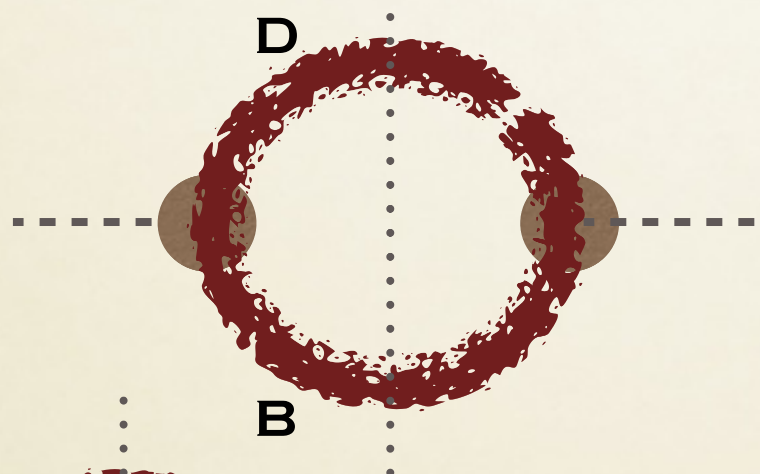
$m_\ell^2 \le q^2 \le (m_B - m_D)^2$

CUT FOR

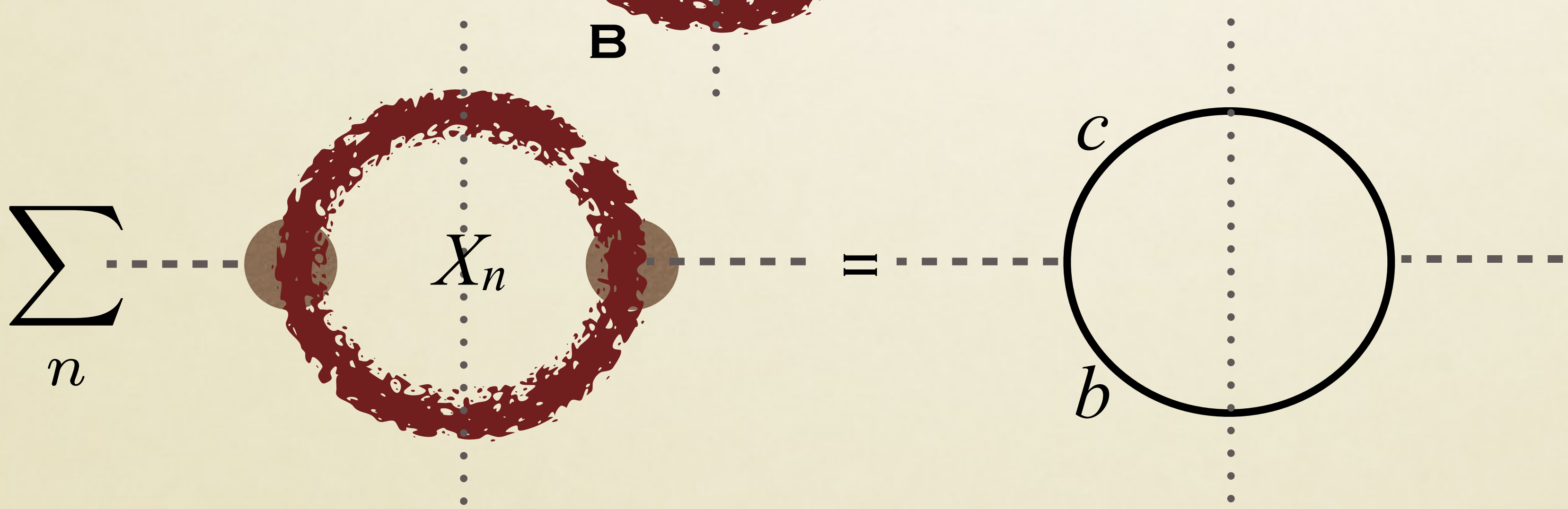
$q^2 \ge (m_B + m_D)^2$

POLES AT $q^2 = m^2_{Bc}$ ETC

$0 <$



$\propto |f_{+,0}|^2$



+ PERT CORR

USING QUARK-HADRON DUALITY. DISPERSION RELATIONS→ GLOBAL QHD

UNITARITY CONSTRAINTS

$$\left(-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}\right)\Pi^T(q^2)+\frac{q^\mu q^\nu}{q^2}\Pi^L(q^2)\equiv i\int d^4x\, e^{iqx}\langle 0|T J^\mu(x) J^{\dagger\nu}(0)|0\rangle$$

$$\chi^L(q^2)=\frac{\partial\Pi^L}{\partial q^2},\qquad \chi^T(q^2)=\frac{1}{2}\frac{\partial^2\Pi^T}{\partial(q^2)^2}$$

SATISFY UNSUBTRACTED DISP REL, PERT CALCULATION FOR $q^2=0$ Boyd, Grinstein, Lebed 1995

$$\chi_V^T(0)=[5.883+0.552_{\alpha_s}+0.050_{\alpha_s^2}]\;10^{-4}\,\mathrm{GeV}^{-2}=6.486(48)\;10^{-4}\,\mathrm{GeV}^{-2}$$
$$\chi_V^L(0)=[5.456+0.782_{\alpha_s}-0.034_{\alpha_s^2}]\;10^{-3}=6.204(81)\;10^{-3}$$

USING UP-TO-DATE QUARK MASSES AND 3LOOP CALCULATION Grigo et al 2012

$$\tilde{\chi}^T(0)=\chi^T(0)-\sum_{n=1,2}\frac{f_n^2(B_c^*)}{M_n^4(B_c^*)}$$

SUBTRACT BOUND STATE CONTRIBUTIONS

Type	Mass (GeV)	Decay constants (GeV)
1^-	6.329(3)	0.422(13)
1^-	6.920(20)	0.300(30)
1^-	7.020	
1^-	7.280	
0^+	6.716	
0^+	7.121	

UNITARITY CONSTRAINTS

$$z = \frac{\sqrt{1+w} - \sqrt{2}}{\sqrt{1+w} + \sqrt{2}} \qquad w = \frac{m_B^2 + m_{D^*}^2 - q^2}{2 m_B m_{D^*}} \qquad 0 < z < 0.056$$

$$f_i(z) = \frac{\sqrt{\chi_i}}{P_i(z)\phi_i(z)} \sum_{n=0}^{\infty} a_n^i z^n$$

BGL Boyd Grinstein Lebed 1997

TRUNCATED AT ORDER N

BLASCHKE FACTORS REMOVE POLES

PHASE SPACE FACTORS

$$\sum_{n=0}^{N} (a_n^i)^2 < 1$$

WEAK UNITARITY CONSTRAINTS

assume saturation by single hadron channel

For massless leptons only 3 form factors $A_{1,5}$ V_4 contribute to $B \to D^* l \nu$

$$\sum_{n=0}^{N} (a_n^{V_4})^2 < 1$$

vector current

$$\sum_{n=0}^{N} \left[(a_n^{A_1})^2 + (a_n^{A_5})^2 \right] < 1$$

axial vector current

STRONG UNITARITY CONSTRAINTS

Using information about the other channels the constraints become tighter. HQS implies that all $B^{(*)} \to D^{(*)}$ ff either vanish or are prop to the Isgur-Wise function

$$\sum_{i=1}^{H} \sum_{n=0}^{N} b_{in}^2 < 1$$

for *S, P, V, A* currents

Indeed, any ff F_j can be expressed in terms of the parameters of F_i using

$$F_j(z) = \left(\frac{F_j}{F_i}\right)_{\mathrm{HQET}} F_i(z)$$

which leads to (hyper)ellipsoids in the a_i space

CLN exploit NLO HQET relations between form factors to reduce to only 2 parameters for ff... **up to "less than 2%" (never included in exp analysis)**
In practice CLN fits employ

CAPRINI
LELLOUCH
NEUBERT
CLN
1998

$$h_{A1}(z) = h_{A1}(1)\left[1 - 8\rho^2 z + (53\rho^2 - 15)z^2 - (231\rho^2 - 91)z^3\right]$$

$$R_1(w) = R_1(1) - 0.12 w_1 + 0.05 w_1^2$$

$$R_2(w) = R_2(1) + 0.11 w_1 - 0.06 w_1^2$$

$$w_1 = w - 1$$

HQS BREAKING IN FF RELATIONS

HQET: $F_i(w) = \xi(w)\left[1 + c^i_{\alpha_s}\frac{\alpha_s}{\pi} + c^i_b\,\epsilon_b + c^i_c\,\epsilon_c + \ldots\right]$ $\quad \epsilon_{b,c} = \overline{\Lambda}/2m_{b,c}$

$$\eta(1) = 0.62 \pm 0.20\,, \qquad \eta'(1) = 0.0 \pm 0.2\,,$$
$$\hat\chi_2(1) = -0.06 \pm 0.02 \qquad \hat\chi_2'(1) = 0 \pm 0.02$$
$$\hat\chi_3'(1) = 0.04 \pm 0.02\,.$$

Subleading IW functions from QCD sumrules
Bernlochner et al 1703.05330

RATIOS $\dfrac{F_j(w)}{V_1(w)} = A_j\left[1 + B_j\,w_1 + C_j\,w_1^2 + D_j\,w_1^3 + \ldots\right]$ $\quad w_1 = w - 1$

Roughly $\quad \epsilon_c \sim 0.25, \qquad \epsilon_c^2 \sim 0.06 \qquad$ but coefficients?

$$\left.\frac{S_1(w)}{V_1(w)}\right|_{\rm LQCD} = 0.975(6) + 0.055(18)(w-1) + \ldots \qquad \left.\frac{S_1(w)}{V_1(w)}\right|_{\rm HQET} = 1.021(30) - 0.044(64)(w-1) + \ldots$$

$$\left.\frac{A_1(1)}{V_1(1)}\right|_{\rm LQCD} = 0.857(15), \qquad \left.\frac{A_1(1)}{V_1(1)}\right|_{\rm HQET} = 0.966(28)$$

$$\left.\frac{S_1(1)}{A_1(1)}\right|_{\rm LQCD} = 1.137(21). \qquad \left.\frac{S_1(1)}{A_1(1)}\right|_{\rm HQET} = 1.055(2),$$

5-13% differences, always > NLO correction

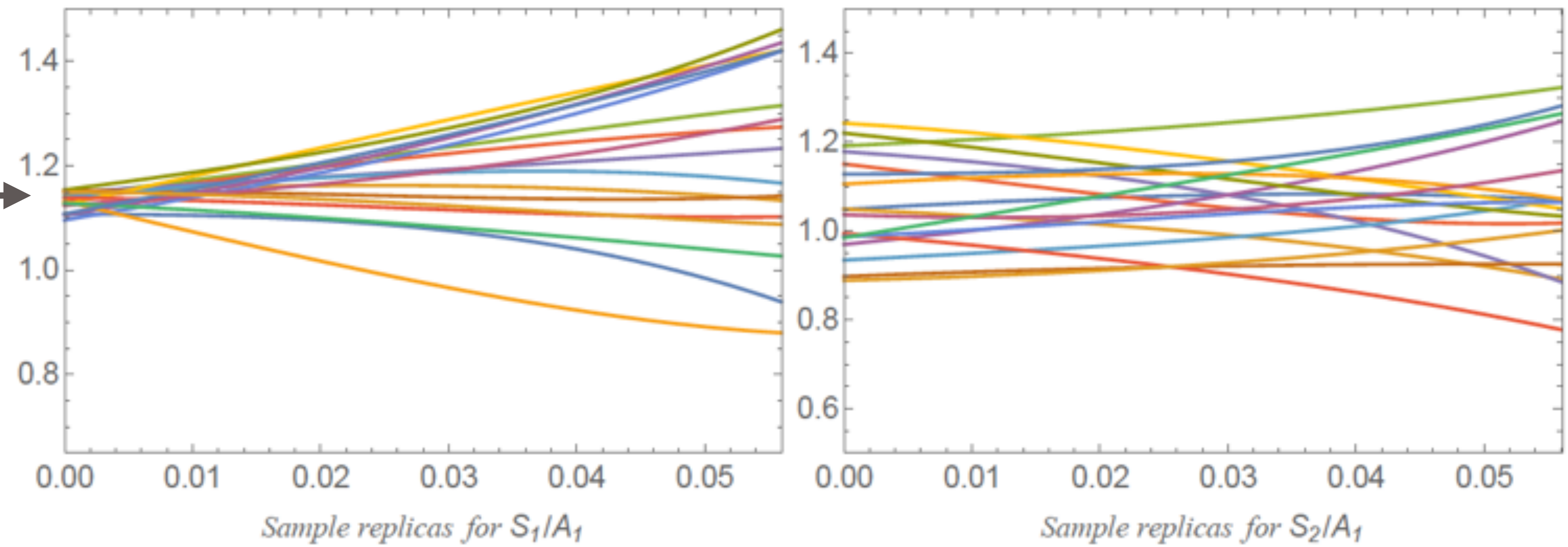
The size of NLO corrections varies strongly. Some ff are protected by Luke's theorem (no 1/m corrections at zero recoil), others are linked by kinematic relations to those protected.

NNLO corrections can be sizeable and are naturally O(10-20)%

$$\frac{F_j(w)}{V_1(w)} = A_j\left[1 + B_j\, w_1 + C_j\, w_1^2 + D_j\, w_1^3 + \dots\right]$$

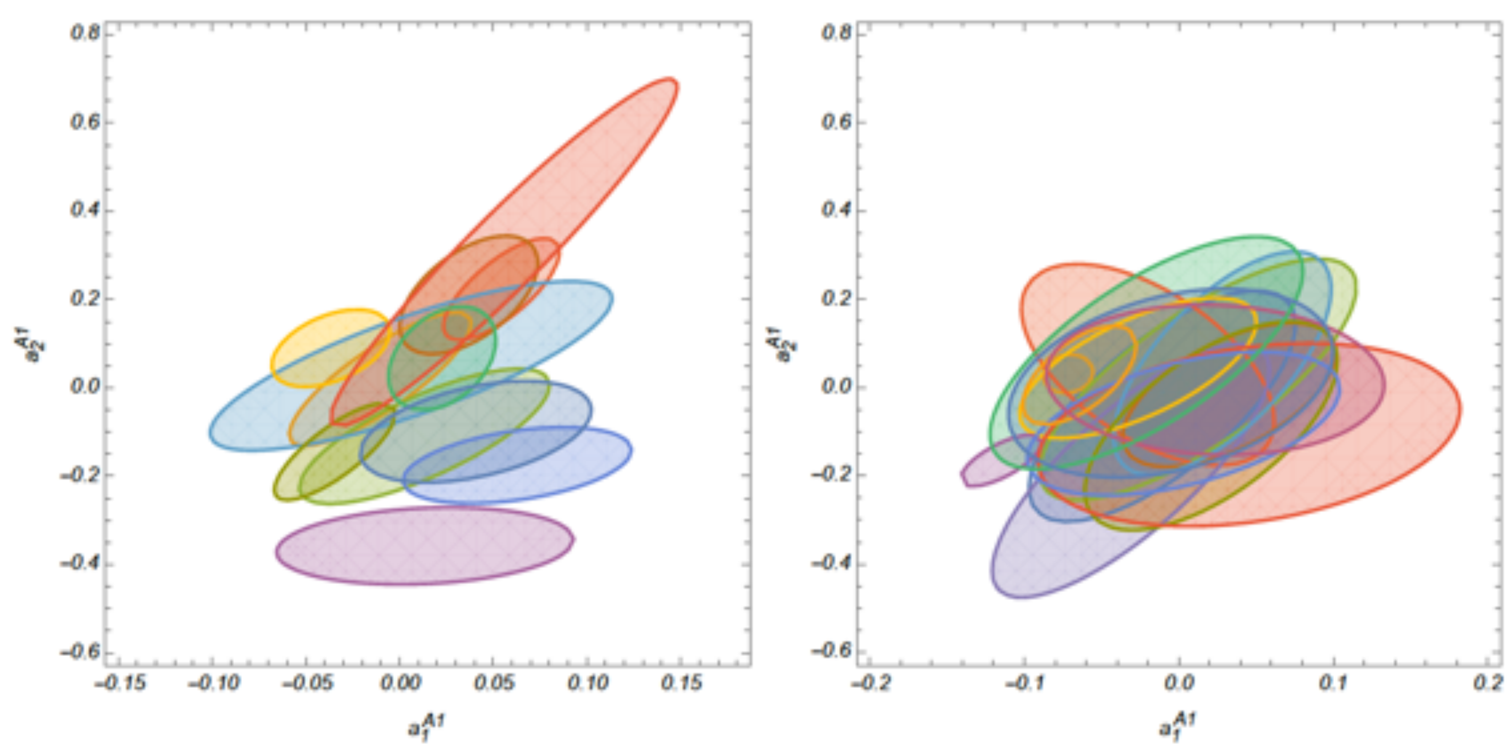
F_j	A_j	B_j	C_j	D_j
S_1	1.0208	−0.0436	0.0201	−0.0105
S_2	1.0208	−0.0749	−0.0846	0.0418
S_3	1.0208	0.0710	−0.1903	0.0947
P_1	1.2089	−0.2164	0.0026	−0.0007
P_2	0.8938	−0.0949	0.0034	−0.0009
P_3	1.0544	−0.2490	0.0030	−0.0008
V_1	1	0	0	0
V_2	1.0894	−0.2251	0.0000	0.0000
V_3	1.1777	−0.2651	0.0000	0.0000
V_4	1.2351	−0.1492	−0.0012	0.0003
V_5	1.0399	−0.0440	−0.0014	0.0004
V_6	1.5808	−0.1835	−0.0009	0.0003
V_7	1.3856	−0.1821	−0.0011	0.0003
A_1	0.9656	−0.0704	−0.0580	0.0276
A_2	0.9656	−0.0280	−0.0074	0.0023
A_3	0.9656	−0.0629	−0.0969	0.0470
A_4	0.9656	−0.0009	−0.1475	0.0723
A_5	0.9656	0.3488	−0.2944	0.1456
A_6	0.9656	−0.2548	0.0978	−0.0504
A_7	0.9656	−0.0528	−0.0942	0.0455

Lattice input

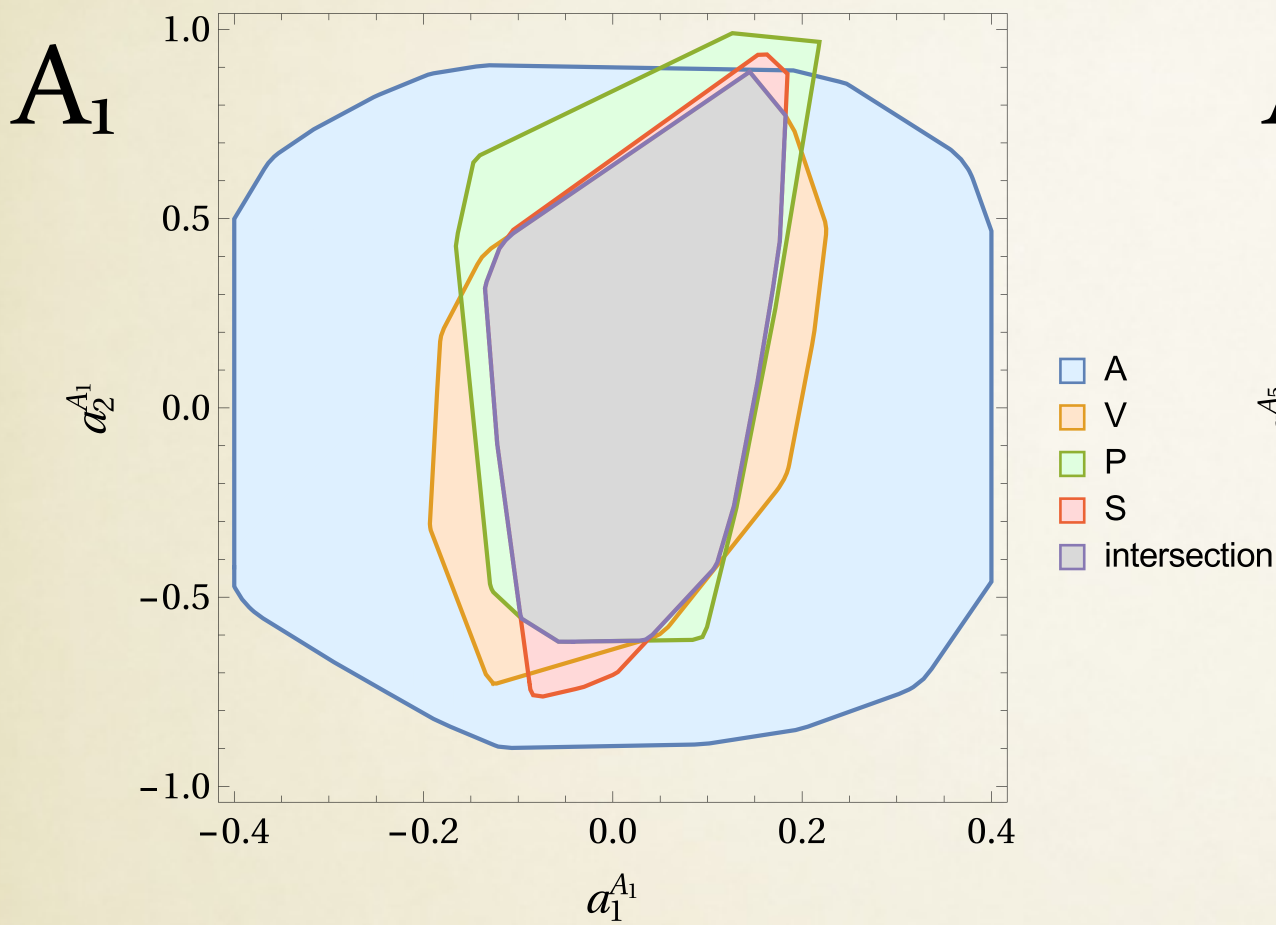


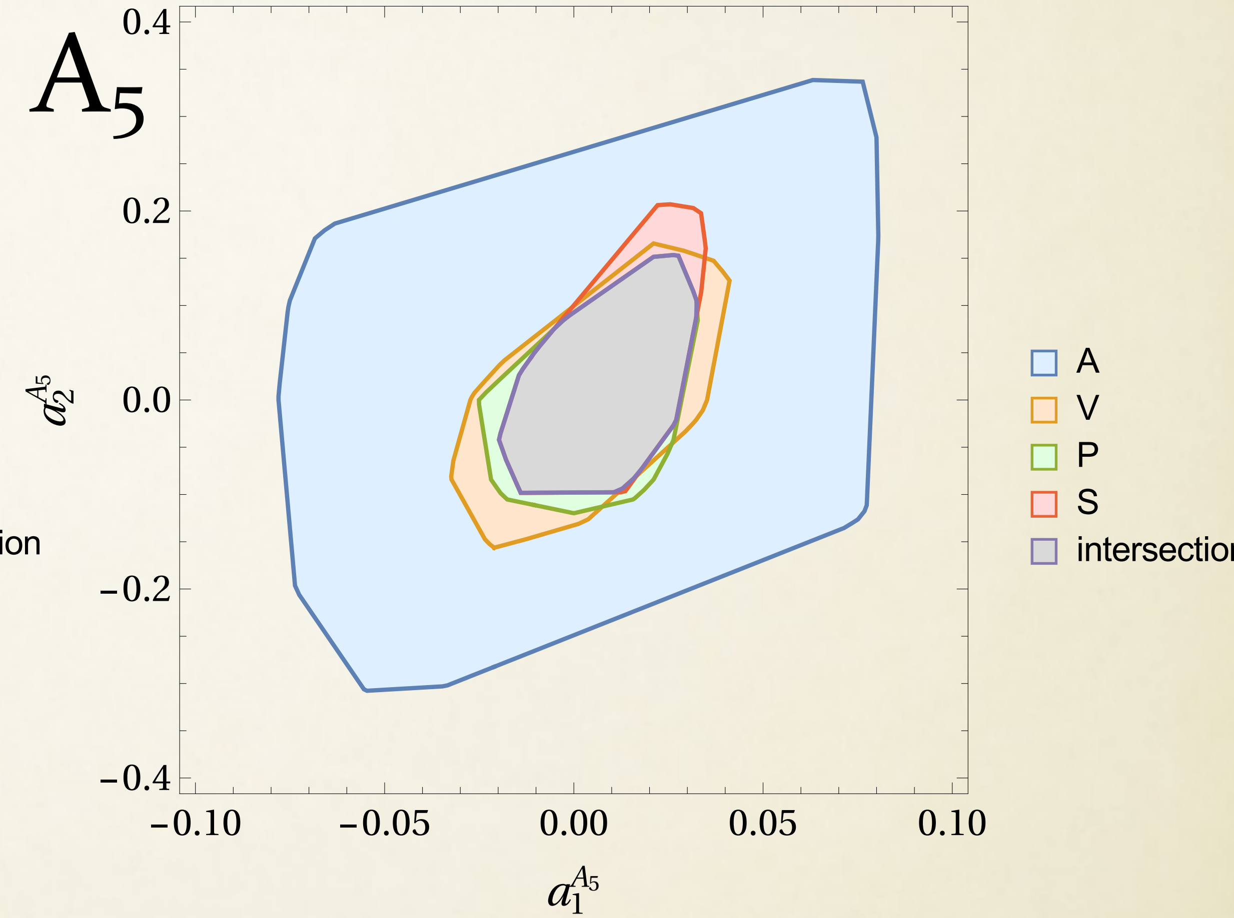
Each replica is a viable model of a f.f. complying with existing lattice and experimental results, and within a band centered in the HQET expectation: ~±25(30)% at zero (maximal) recoil.

a_0 generally fixed by LQCD: constraints are ellipses in (a_1, a_2) plane



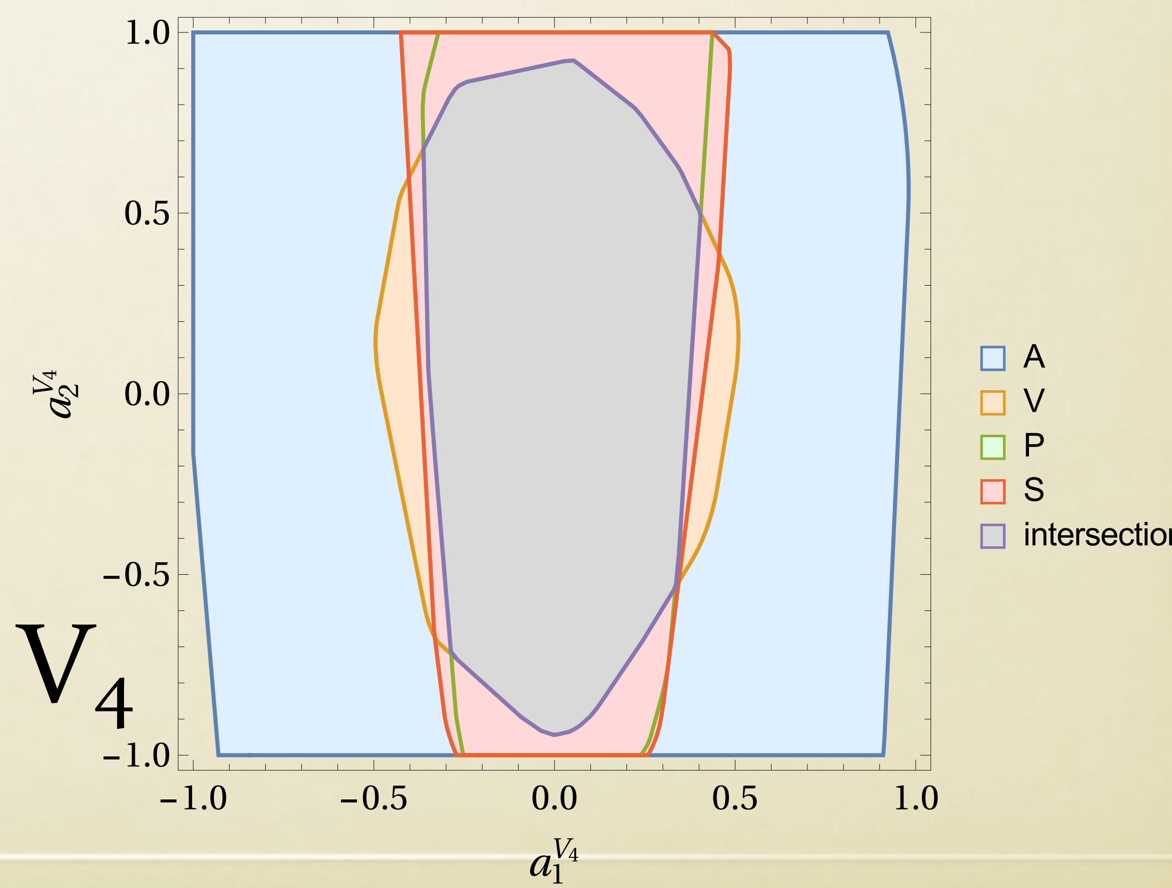
Constraints in the a_1-a_2 planes





Envelopes formed by a large number of ellipses represent allowed regions in *(a_1,a_2)* planes

One gets different (but consistent) constraints from the S,P,V,A channels: take intersection



Fit to new Belle's data + total branching ratio (world average)

with strong unitarity bounds

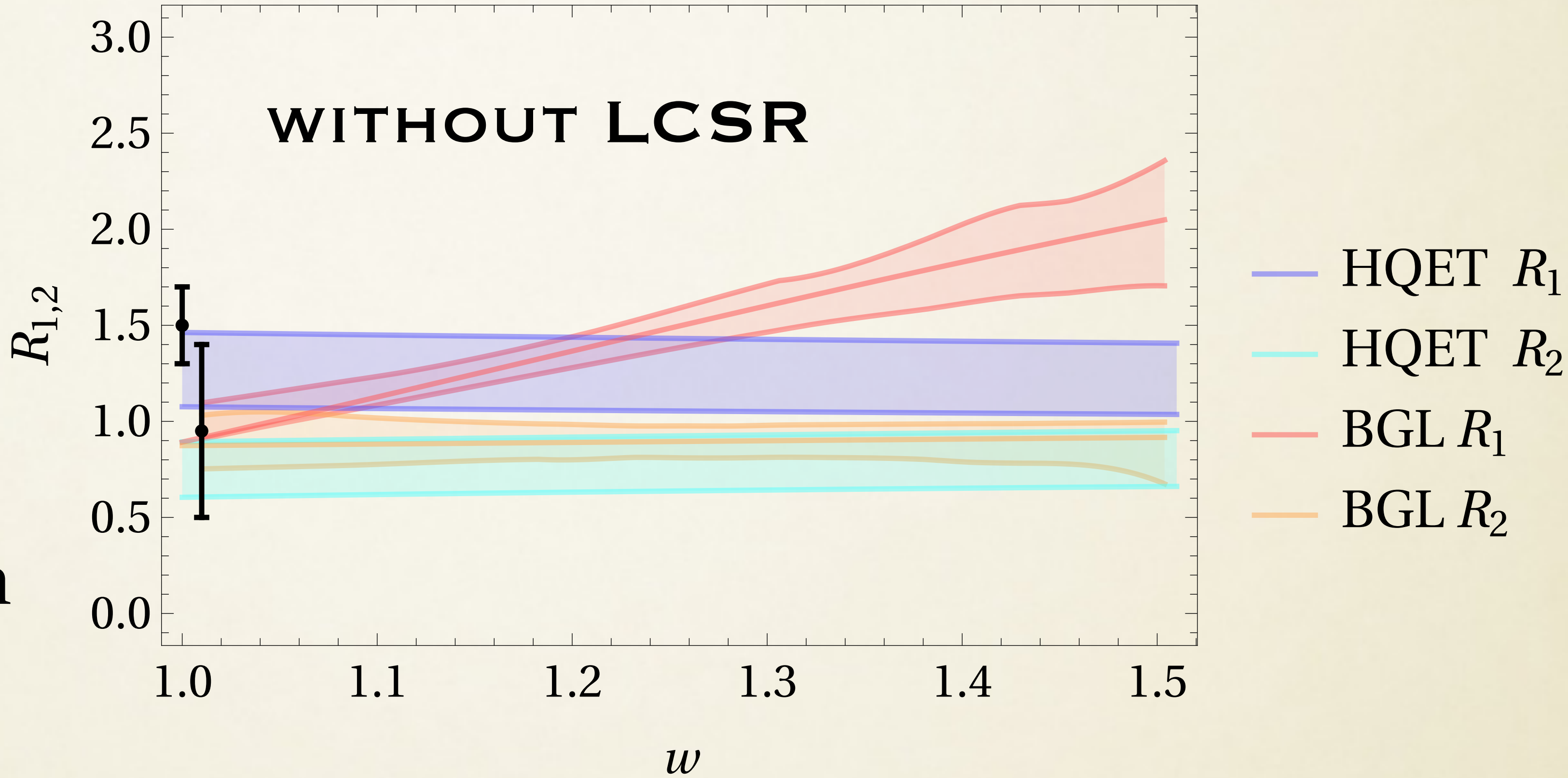
for reference CLN fit: $|V_{cb}|$=0.0392(12)

BGL Fit:	Data + lattice	Data + lattice + LCSR	Data + lattice	Data + lattice + LCSR
unitarity	weak	weak	strong	strong
χ^2/dof	28.2/33	32.0/36	29.6/33	33.1/36
$\lvert V_{cb}\rvert$	0.0424 (18)	0.0413 (14)	0.0415 (13)	$0.0406\ (^{+12}_{-13})$
$a_0^{A_1}$	0.01218(16)	0.01218(16)	0.01218(16)	0.01218(16)
$a_1^{A_1}$	$-0.053\ (^{+56}_{-44})$	$-0.052\ (^{+25}_{-14})$	$-0.046(^{+34}_{-18})$	$-0.029(^{+21}_{-13})$
$a_2^{A_1}$	$0.2\ (^{+8}_{-12})$	$0.99\ (^{+0}_{-46})$	$0.48(^{+2}_{-92})$	$0.5(^{+0}_{-3})$
$a_1^{A_5}$	$-0.0101\ (^{+59}_{-55})$	$-0.0072\ (^{+52}_{-50})$	$-0.0063(^{+36}_{-11})$	$-0.0051(^{+49}_{-13})$
$a_2^{A_5}$	0.12 (10)	$0.092\ (^{+92}_{-95})$	$0.062(^{+4}_{-64})$	$0.065\ (^{+9}_{-89})$
$a_0^{V_4}$	$0.011\ (^{+10}_{-8})$	$0.0286\ (^{+55}_{-36})$	$0.0209(^{+44}_{-0})$	$0.0299(^{+53}_{-35})$
$a_1^{V_4}$	$0.7\ (^{+3}_{-4})$	$0.08\ (^{+8}_{-22})$	$0.33(^{+4}_{-17})$	$0.04(^{+7}_{-20})$
$a_2^{V_4}$	$0.7\ (^{+2}_{-17})$	$-1.0\ (^{+20}_{-0})$	$0.6(^{+2}_{-13})$	$-0.9(^{+18}_{-0})$

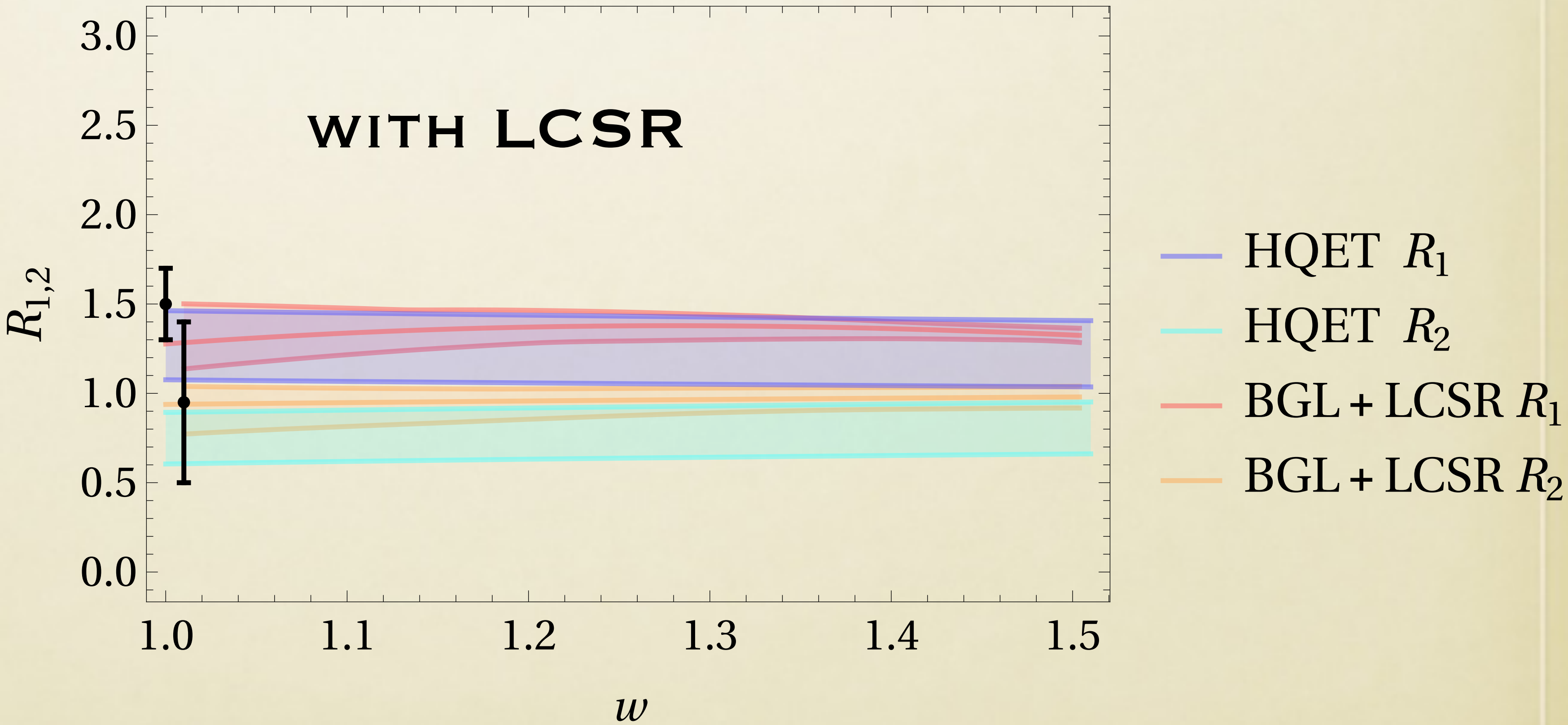
Using strong unitarity bounds brings BGL closer to CLN and reduce uncertainties but 3.5-5% difference persists

Consistency with HQS

Comparison of $R_{1,2}$ from BGL fit vs HQET+QCD sum rule predictions (with parametric + 15% th uncertainty)



black points from preliminary FNAL-MILC calculation according to Bernlochner et al 1708.07134



SEMITAUONIC ANOMALY

$$R(D^{(*)}) = \frac{\mathcal{B}(B \to D^{(*)}\tau\nu)}{\mathcal{B}(B \to D^{(*)}\mu\nu)}$$

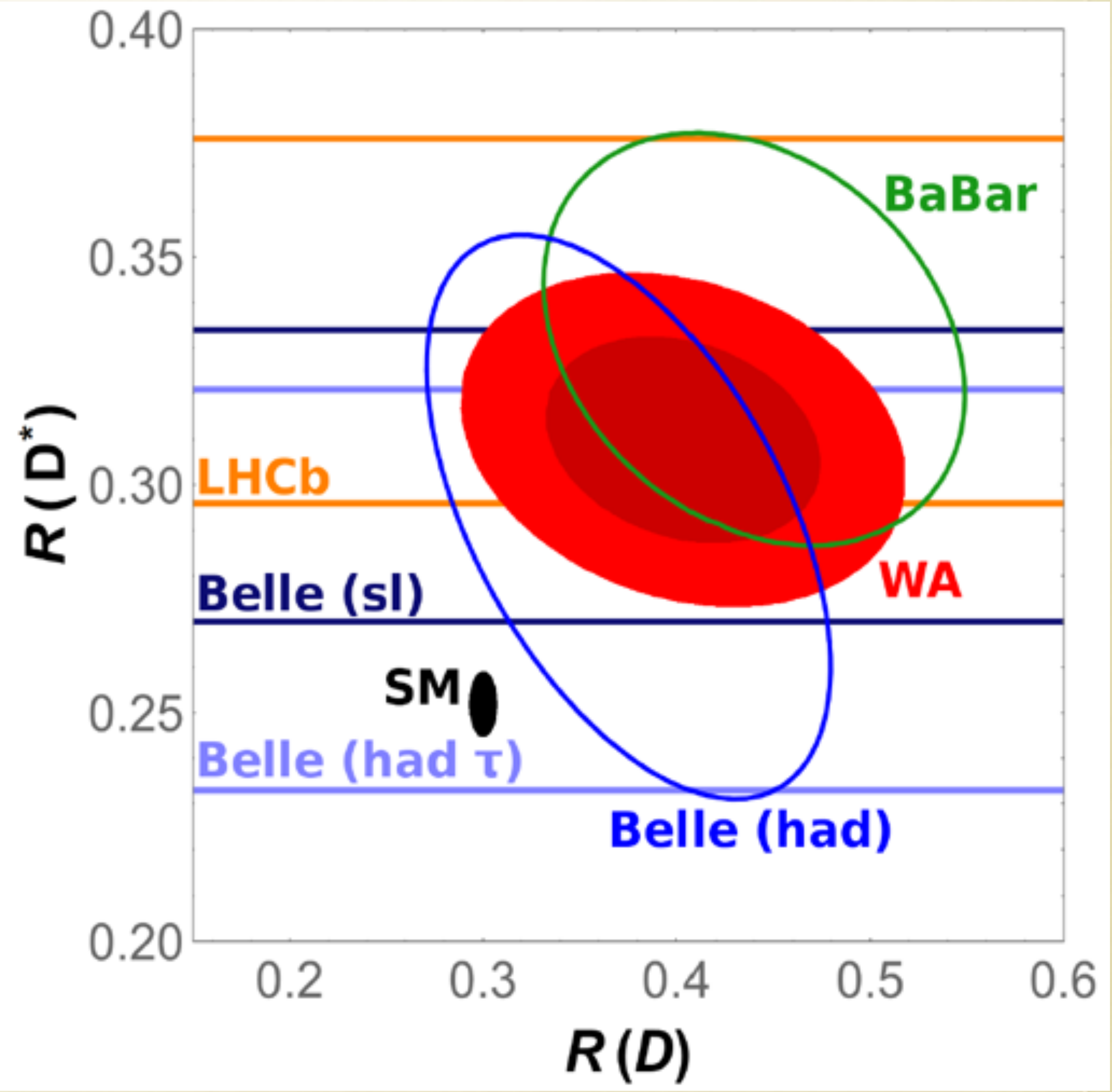
Combined discrepancy with SM 4.0σ

about 30% effect on tree-level process!

Lepton flavour universality violation: new scalars, leptoquarks, W'… possible connection with lepton flavour violation in $b \to sll$

Inconsistent with LEP inclusive measurement

SM predictions?



$$R(D^*)^{\rm exp} = 0.304 \pm 0.013 \pm 0.007$$

Celis et al., 1612.07757

Calculation of $R(D^*)$

$$\frac{d\Gamma_\tau}{dw} = \frac{d\Gamma_{\tau,1}}{dw} + \frac{d\Gamma_{\tau,2}}{dw} \quad \begin{cases} \frac{d\Gamma_{\tau,1}}{dw} = \left(1 - \frac{m_\tau^2}{q^2}\right)^2 \left(1 + \frac{m_\tau^2}{2q^2}\right) \frac{d\Gamma}{dw}, \\ \frac{d\Gamma_{\tau,2}}{dw} = k \frac{m_\tau^2 (m_\tau^2 - q^2)^2 r^3 (1+r)^2 (w^2-1)^{\frac{3}{2}} P_1(w)^2}{(q^2)^3} \end{cases}$$

±30%!!

$$R(D^*) = R_{\tau,1}(D^*) + R_{\tau,2}(D^*)$$

$$R_{\tau,1}(D^*) = \frac{\int_1^{w_{\tau,\max}} dw\, d\Gamma_{\tau,1}/dw}{\int_1^{w_{\max}} dw\, d\Gamma/dw}$$

$$R_{\tau,2}(D^*) = \frac{\int_1^{w_{\tau,\max}} dw\, d\Gamma_{\tau,2}/dw}{\int_1^{w_{\max}} dw\, d\Gamma/dw}$$

$$w_{\max} \approx 1.56, \qquad w_{\tau,\max} \approx 1.35$$

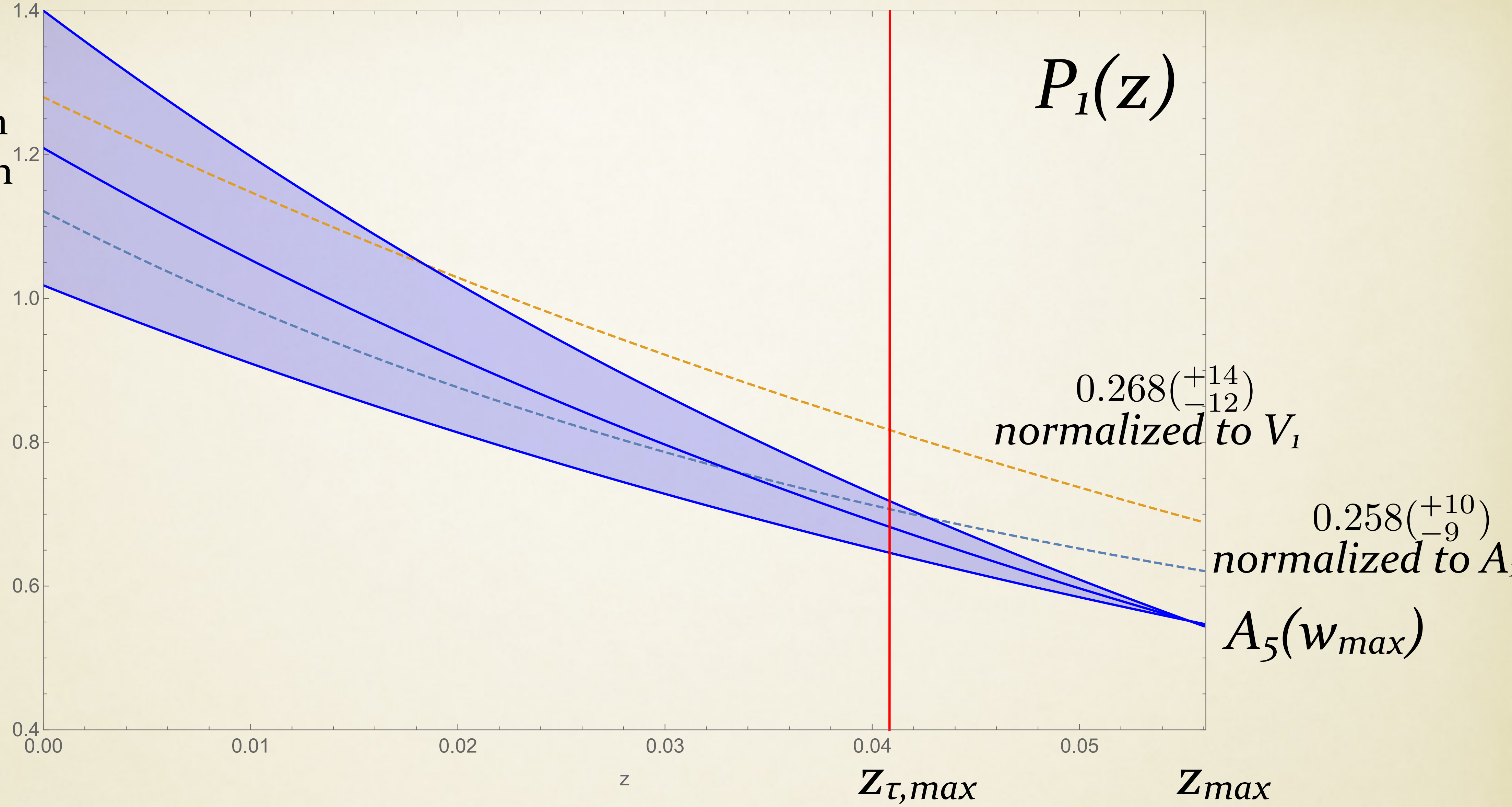
P_1 is a new ff, for which no lattice calculation is available yet but its contribution is only ~ 10%

$$R_{\tau,1} \sim 90\% R_\tau \qquad R_{\tau,2} \sim 10\% R_\tau$$

Again, normalize P_1 to one of the ff with proper uncertainties

$$P_1 = (P_1/V_1)_{\rm HQET} V_1^{exp} \qquad P_1 = (P_1/A_1)_{\rm HQET} A_1^{exp} \qquad P_1 = \xi(w)(1+\ldots)_{\rm HQET}$$

zero-recoil normalization to IW function ±15%



Important endpoint constraint

$$P_1(w_{max}) = A_5(w_{max}) = 0.545 \pm 0.025$$

$$R(D^*) = 0.260(5)(6) = 0.260(8)$$

2.6σ from exp

NB consistent but larger uncertainty than

0.252(3) Fejfer et al

0.257(3) Bernlochner et al

0.257(5) Jaiswal et al

Summary

- We revisited main ideas behind CLN, using LQCD & exp results and conservative theory uncertainties, and obtain *strong* unitarity bounds on BGL coefficients. We do *not* give a simplified parametrization. Our results provide the framework for future exp analyses.
- So is the V_{cb} puzzle resolved? not quite yet… but a few pieces start fitting together. The uncertainty of the $B \to D^* l\nu$ was underestimated. Most urgently, we now need a reanalysis of previous Babar and Belle data in the new framework.
- Lattice determinations of the zero recoil slope will settle the matter, but it will be impossible to combine them with the present HFLAV averages based on CLN. This is already the case for the $B \to D$ channel.
- Our SM prediction of $R(D^*)$ is a bit higher and more uncertain than people thought but the tau anomaly is still there. A LQCD determination of P_1 at zero recoil would almost halve the uncertainty.

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HQE parameters from unquenched lattice data on pseudoscalar and vector heavy-light meson masses

Based on arXiv:1704.06105

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HQE parameters from V and PS meson masses

We study the $1/\widetilde{m}_h$ dependence of two meson mass combinations:

Kronfeld, Simone 2000

$$\mathrm{M_{av}}(\widetilde{\mathrm{m}}_\mathrm{h}) \equiv \frac{\mathrm{M_{PS}}(\widetilde{\mathrm{m}}_\mathrm{h}) + 3\mathrm{M_V}(\widetilde{\mathrm{m}}_\mathrm{h})}{4}$$

(Spin averaged)

$$\Delta\mathrm{M}(\widetilde{\mathrm{m}}_\mathrm{h}) \equiv \mathrm{M_V}(\widetilde{\mathrm{m}}_\mathrm{h}) - \mathrm{M_{PS}}(\widetilde{\mathrm{m}}_\mathrm{h}).$$

(Hyperfine splitting)

Their HQET expansions read as

$$\frac{M_{av}(\widetilde{m}_h)}{\widetilde{m}_h} = 1 + \frac{\overline{\Lambda}|_\infty}{\widetilde{m}_h} + \frac{\mu_\pi^2|_\infty}{2\widetilde{m}_h^2} + \frac{(\rho_D^3 - \rho_{\pi\pi}^3 - \rho_S^3)|_\infty}{4\widetilde{m}_h^3} + \frac{\sigma^4|_\infty}{\widetilde{m}_h^4} \, ,$$

$$\widetilde{m}_h \Delta M(\widetilde{m}_h) = \frac{2}{3} c_G(\widetilde{m}_h, \widetilde{m}_b)\mu_G^2(\widetilde{m}_b)|_\infty + \frac{(\rho_{\pi G}^3 + \rho_A^3 - \rho_{LS}^3)|_\infty}{3\widetilde{m}_h} + \frac{\Delta\sigma^4|_\infty}{\widetilde{m}_h^2} \, .$$

$\overline{\Lambda}$: energy of the light quark and gluons,

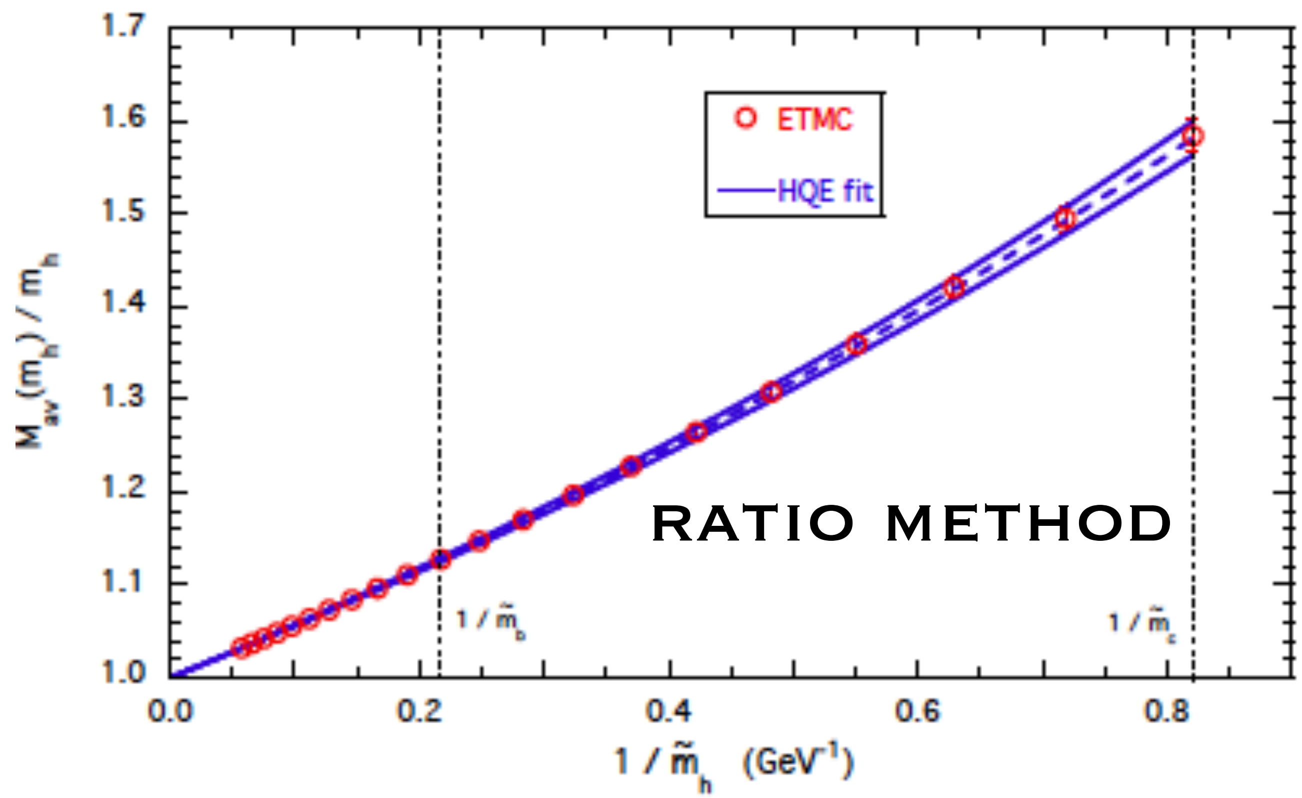
$\rho_{\pi\pi}^3, \rho_{\pi G}^3, \rho_S^3, \rho_A^3$: non local zero momentum transfer correlators,

$\widetilde{m}_h = m_h^{kin}(\mu_{soft})$: the heavy quark mass in the kinetic scheme at $\mu_{soft} = 1\mathrm{GeV}$.

HQE expansion parameters from $M_{av}(\widetilde{m}_h)$

We extend the chain equation beyond the b quark point, we choose $n \sim 20$ ($\widetilde{m}_h \simeq 4\widetilde{m}_b$),

$$\frac{M_{av}(\widetilde{m}_h^{(n)})}{\widetilde{m}_h^{(n)}} = \frac{M_{av}(\widetilde{m}_c)}{\widetilde{m}_c} \prod_{i=2}^{n} \overline{y}_M(\widetilde{m}_h^{(i)}, \lambda).$$



Correlated fit based on the HQE expansion

$$\frac{M_{av}(\widetilde{m}_h)}{\widetilde{m}_h} = 1 + \frac{\overline{\Lambda}}{\widetilde{m}_h} + \frac{\mu_\pi^2}{2\widetilde{m}_h^2} + \frac{\rho_D^3 - \rho_{\pi\pi}^3 - \rho_S^3}{4\widetilde{m}_h^3} + \frac{\sigma^4}{\widetilde{m}_h^4}$$

Results of the dim-6 fit $\widetilde{m}_h > \widetilde{m}_c$

$$\overline{\Lambda} = 0.551\ (13)_{stat}\ (2)_{syst}\ \text{GeV}$$
$$\mu_\pi^2 = 0.314\ (14)_{stat}\ (2)_{syst}\ \text{GeV}^2$$
$$\rho_D^3 - \rho_{\pi\pi}^3 - \rho_S^3 = 0.174\ (12)_{stat}\ (2)_{syst}\ \text{GeV}^3$$

Results of the dim-6 fit $\widetilde{m}_h > 2\widetilde{m}_c$

$$\overline{\Lambda} = 0.552\ (13)\ \text{GeV}$$
$$\mu_\pi^2 = 0.323\ (16)\ \text{GeV}^2$$
$$\rho_D^3 - \rho_{\pi\pi}^3 - \rho_S^3 = 0.153\ (24)\ \text{GeV}^3$$

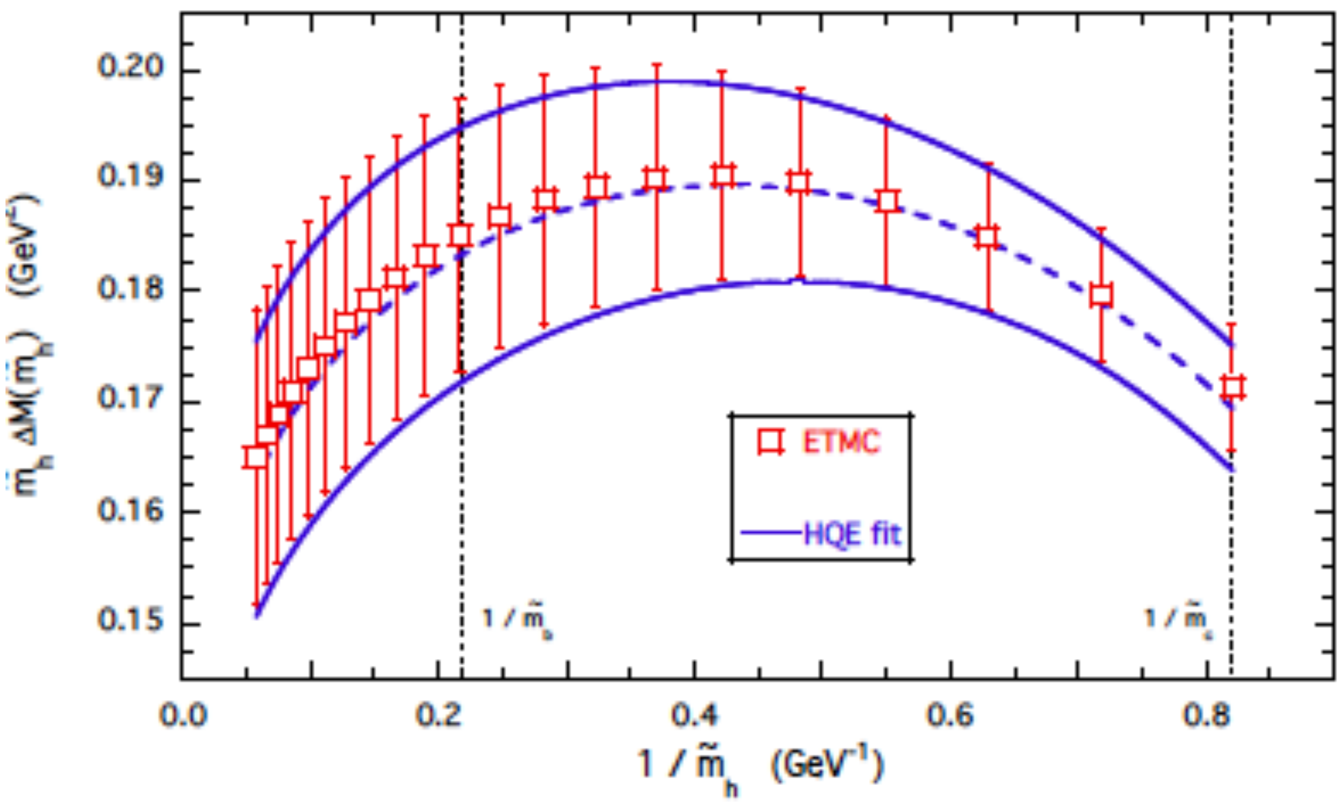
Results of the dim-7 fit

$$\overline{\Lambda} = 0.552\ (13)_{stat}\ (2)_{syst}\ \text{GeV}$$
$$\mu_\pi^2 = 0.325\ (17)_{stat}\ (3)_{syst}\ \text{GeV}^2$$
$$\rho_D^3 - \rho_{\pi\pi}^3 - \rho_S^3 = 0.133\ (34)_{stat}\ (6)_{syst}\ \text{GeV}^3$$
$$\sigma^4 = 0.0071\ (55)_{stat}\ (10)_{syst}\ \text{GeV}^4$$

HQE expansion parameters from $\Delta M(\widetilde{m}_h)$

The chain equation can be extended beyond the b quark point, we choose $n \sim 20$ ($\widetilde{m}_h \simeq 4\widetilde{m}_b$),

$$\widetilde{m}_h^{(n)} \frac{\Delta M(\widetilde{m}_h^{(n)})}{c_G(\widetilde{m}_h^{(n)}, \widetilde{m}_b)} = \widetilde{m}_c \frac{\Delta M(\widetilde{m}_c)}{c_G(\widetilde{m}_c, \widetilde{m}_b)} \prod_{i=2}^{n} \overline{y}_{\Delta M}(\widetilde{m}_h^{(i)}, \lambda) \, .$$



We apply a correlated fit based on the HQE expansion

$$\widetilde{m}_h \Delta M(\widetilde{m}_h) = \frac{2}{3} c_G(\widetilde{m}_h, \widetilde{m}_b) \mu_G^2(\widetilde{m}_b) + \frac{\rho_{\pi G}^3 + \rho_A^3 - \rho_{LS}^3}{3\widetilde{m}_h} + \frac{\Delta\sigma^4}{\widetilde{m}_h^4} \, .$$

Results of the dim-6 fit $\widetilde{m}_h > \widetilde{m}_c$

$$\mu_G^2(\widetilde{m}_b) = 0.250\,(18)_{\rm stat}\,(8)_{\rm syst}\ {\rm GeV}^2$$
$$\rho_{\pi G}^3 + \rho_A^3 - \rho_{LS}^3 = -0.143\,(57)_{\rm stat}\,(21)_{\rm syst}\ {\rm GeV}^3$$

Results of the dim-6 fit $\widetilde{m}_h > 2\widetilde{m}_c$

$$\mu_G^2(\widetilde{m}_b) = 0.254\,(22)\ {\rm GeV}^2$$
$$\rho_{\pi G}^3 + \rho_A^3 - \rho_{LS}^3 = -0.158\,(70)\ {\rm GeV}^3$$

Results of the dim-7 fit

$$\mu_G^2(\widetilde{m}_b) = 0.254\,(20)_{\rm stat}\,(9)_{\rm syst}\ {\rm GeV}^2$$
$$\rho_{\pi G}^3 + \rho_A^3 - \rho_{LS}^3 = -0.173\,(74)_{\rm stat}\,(25)_{\rm syst}\ {\rm GeV}^3$$
$$\Delta\sigma^4 = 0.0092\,(58)_{\rm stat}\,(14)_{\rm syst}\ {\rm GeV}^4$$

First Applications (Next, heavy quark sum rules)

In the BPS limit $\mu_\pi^2 = \mu_G^2$ and $\rho_D^3 + \rho_{LS}^3 = 0$,

- $(\mu_\pi^2 - \mu_G^2) = 0.064(19)\mathrm{GeV}^2$, deviation of $20 - 25\%$ from the BPS limit,
- $(\rho_D^3 + \rho_{LS}^3) \geq 0.317(65)\mathrm{GeV}^3$, $4.9 - 3.6\sigma$ from the BPS limit.

In this work μ_π^2 and $\mu_G^2(m_b)$ refer to asymptotic matrix elements, while semileptonic fits are sensitive to

$$\mu_\pi^2|_B = \mu_\pi^2|_\infty - \frac{\rho_{\pi\pi}^3 + \frac{1}{2}\rho_{\pi G}^3}{\widetilde{m}_b} + \mathcal{O}(1/\widetilde{m}_b^2) \, ,$$

$$\mu_G^2(m_b)|_B = \mu_G^2(m_b)|_\infty + \frac{\rho_S^3 + \rho_A^3 + \frac{1}{2}\rho_{\pi G}^3}{\widetilde{m}_b} + \mathcal{O}(1/\widetilde{m}_b^2) \, .$$

Recent determinations:[4] $\mu_\pi^2|_\mathrm{B} = 0.432(68)\mathrm{GeV}^2$, $\mu_\pi^2|_\mathrm{B} = 0.465(68)\mathrm{GeV}^2$
imposing $\mu_G^2|_\mathrm{B} = 0.35(7)\mathrm{GeV}^2$.

We can say,

$$\rho_{\pi\pi}^3 + \frac{1}{2}\rho_{\pi G}^3 = -0.51(35)\mathrm{GeV}^2, \quad \rho_S^3 + \rho_A^3 + \frac{1}{2}\rho_{\pi G}^3 + \overbrace{\frac{1}{2}\rho_{\pi G}^3 + \rho_{\pi\pi}^3} > 0, \quad \rho_S^3 + \rho_A^3 + \frac{1}{2}\rho_{\pi G}^3 > 0.51(35)\mathrm{GeV}^2$$

$$\Rightarrow \mu_G^2|_\mathrm{B} = \mu_G^2|_\infty + 0.11(8)\mathrm{GeV}^2 = 0.36(8)\mathrm{GeV}^2.$$

Thank you for your attention.

[4] [arXiv:1606.06174[hep-ph]],[arXiv:1411.6560[hep-ph]]

Importance of $|V_{xb}|$

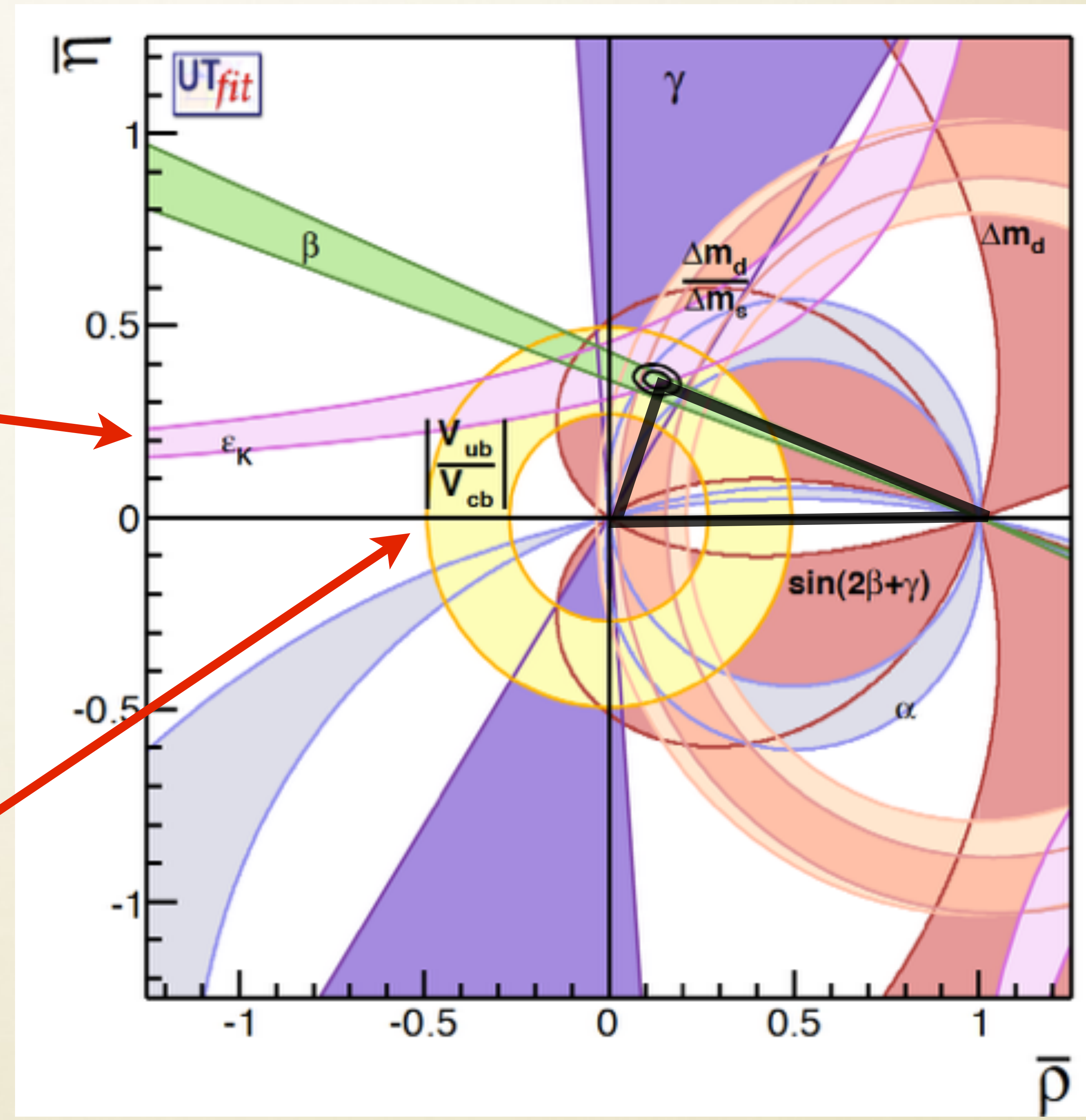
V_{cb} plays an important role in UT

$$\varepsilon_K \approx x|V_{cb}|^4 + \ldots$$

and in the prediction of FCNC:

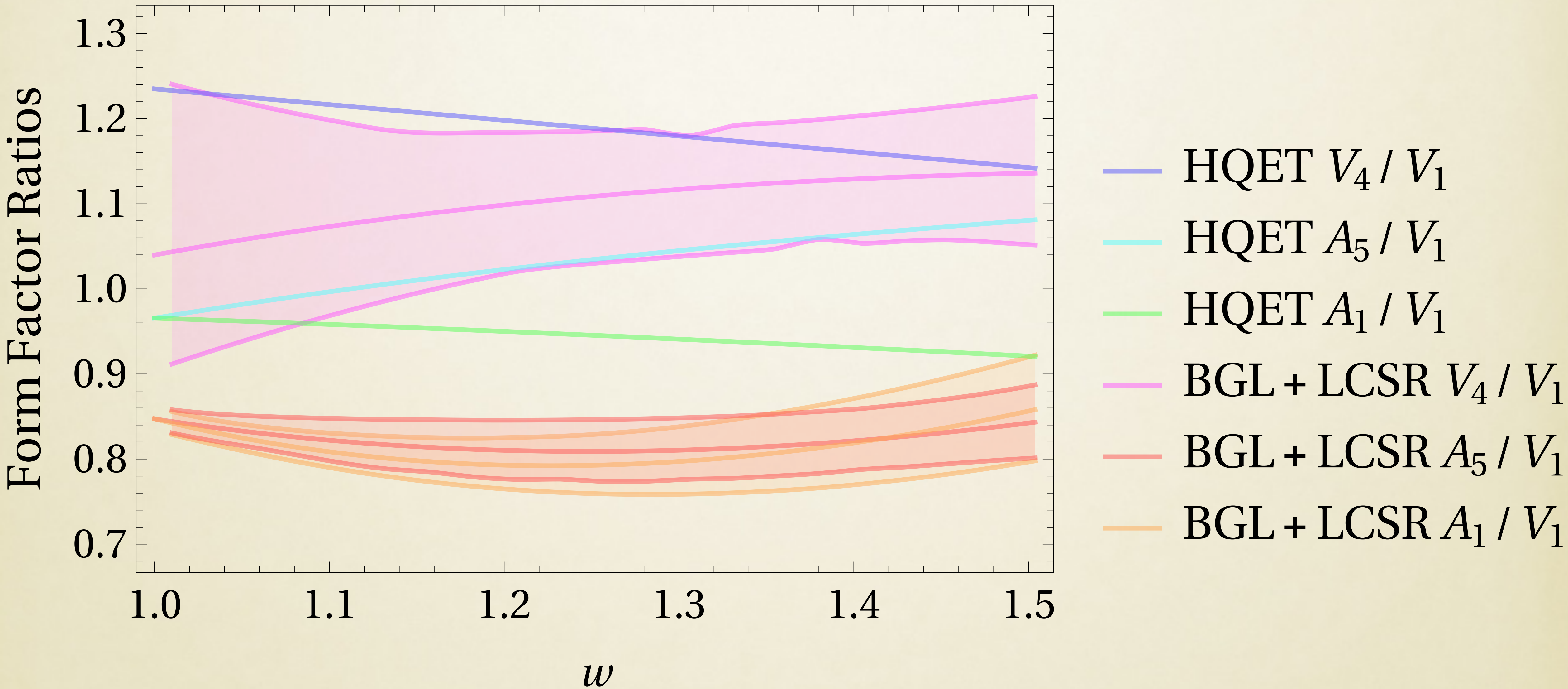
$$\propto |V_{tb}V_{ts}|^2 \simeq |V_{cb}|^2\left[1 + O(\lambda^2)\right]$$

where it often dominates the theoretical uncertainty.
V_{ub}/V_{cb} constrains directly the UT



Since several years, exclusive decays prefer smaller $|V_{ub}|$ and $|V_{cb}|$

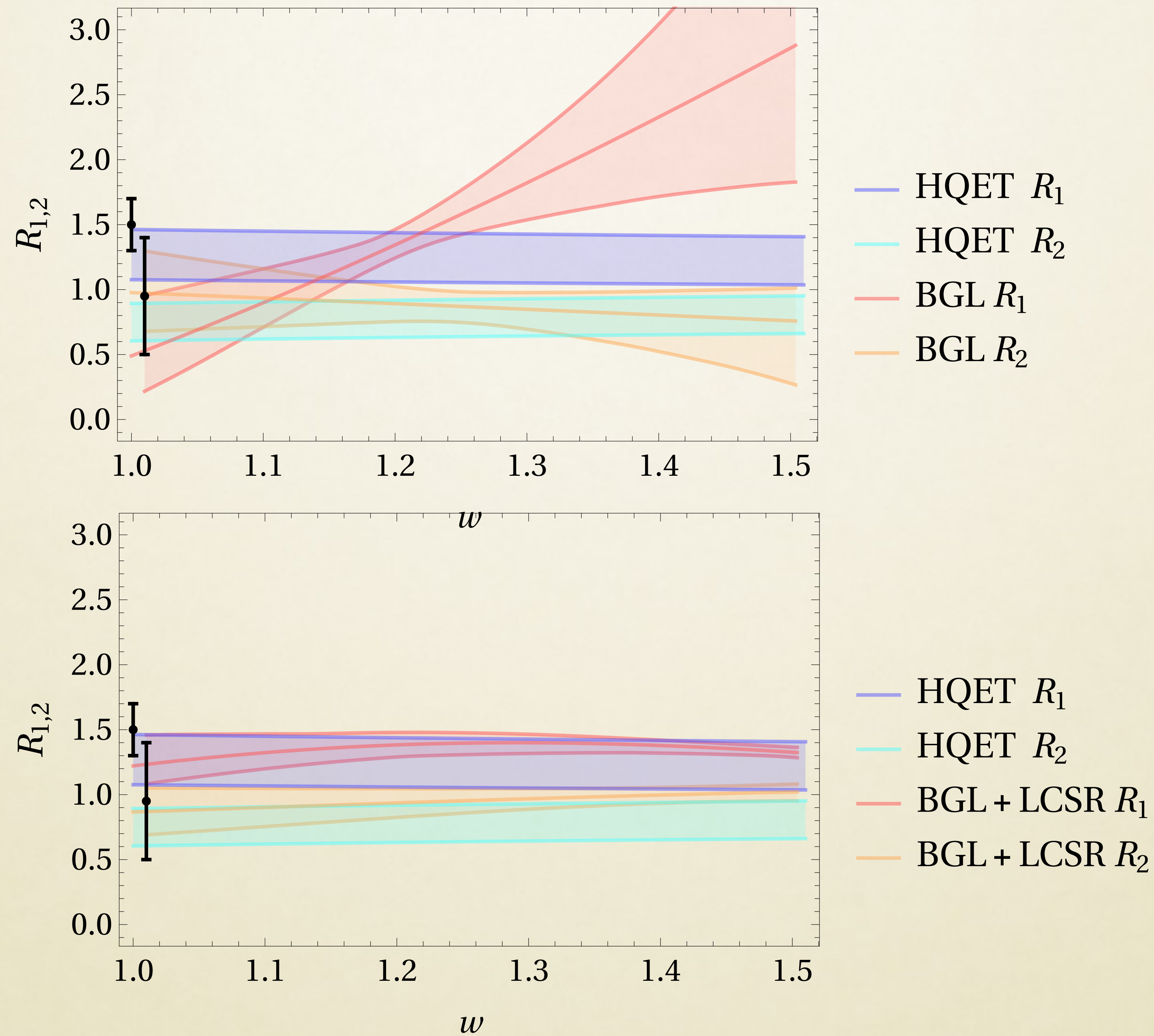
Comparing with HQET



HQET NLO predictions from Bernlochner et al (uncertainties not shown)

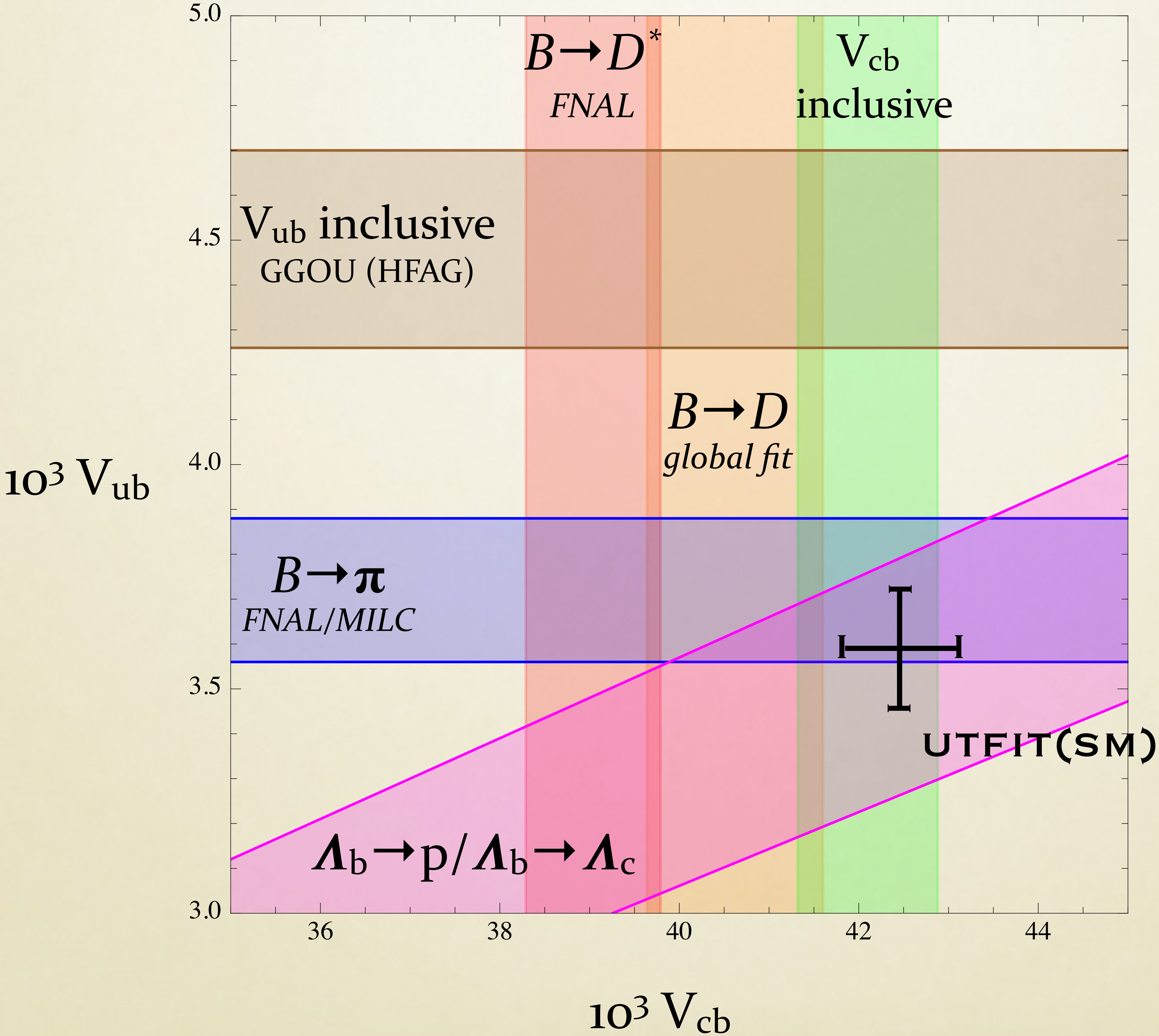
Bottom line: BGL fit compatible with HQET within uncertainties

Consistency with HQS (without strong unitarity)



V_{cb} and V_{ub} status

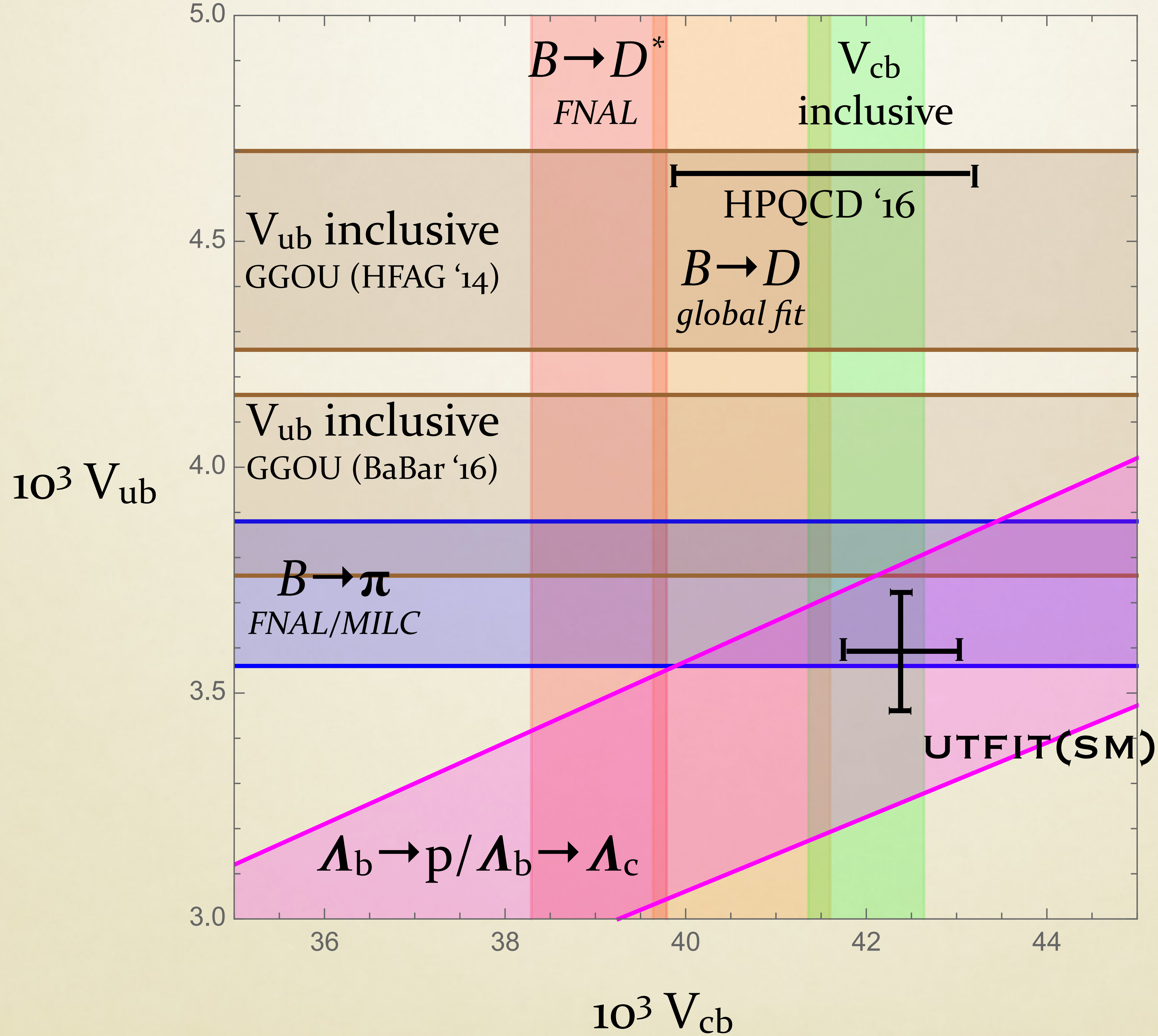
1σ ranges



reasonable consistency among exclusive channels

not all results at the same level

V_{cb} and V_{ub} updated status

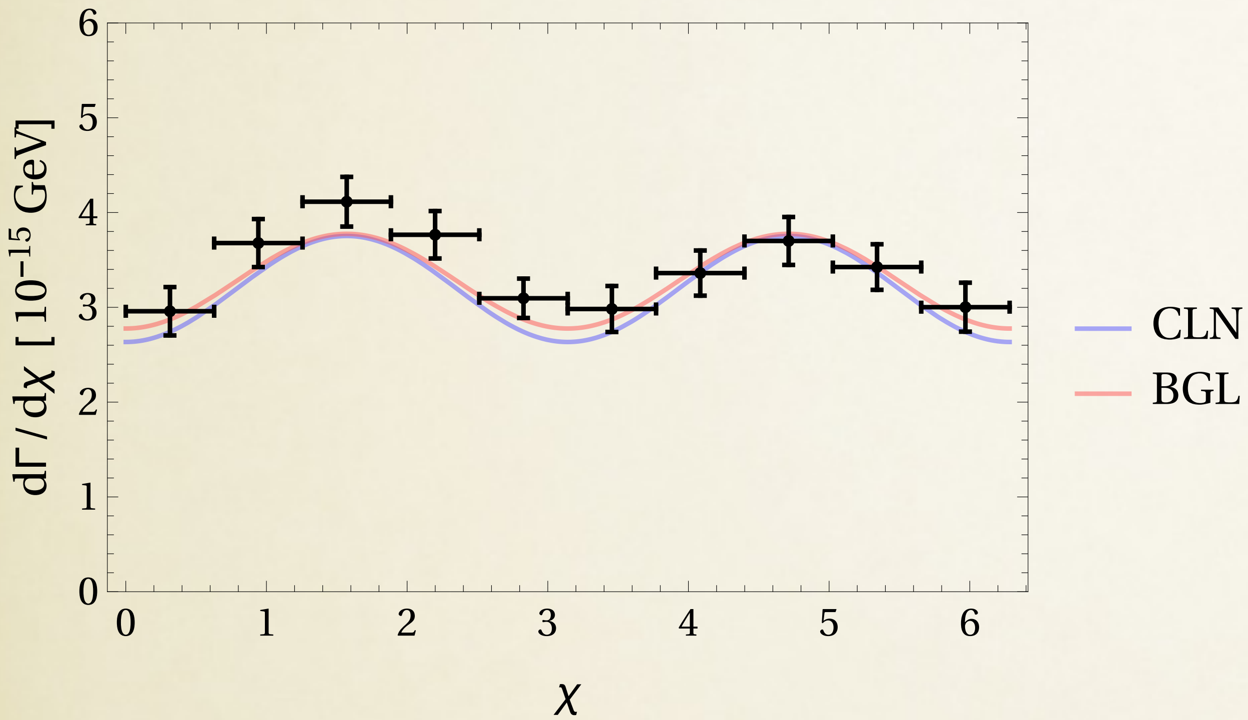


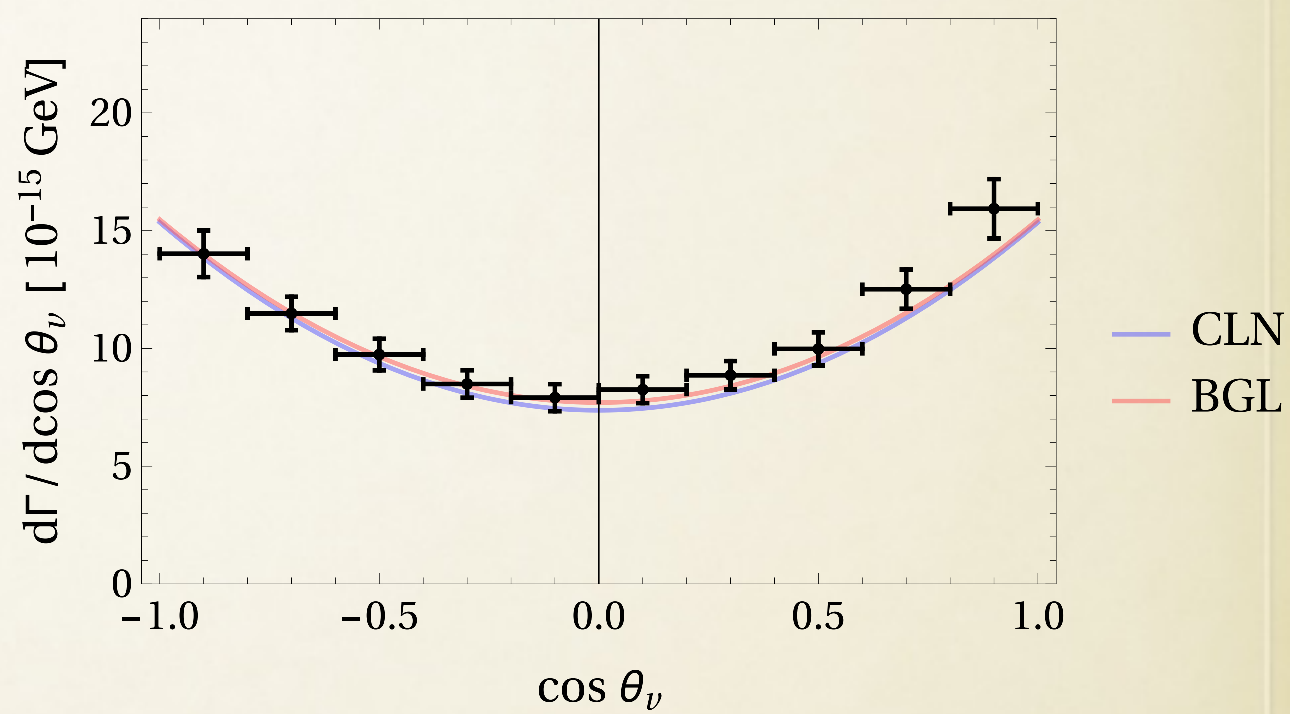
CKM 2016: NEW V_{ub} incl by Babar in agreement with exclusive

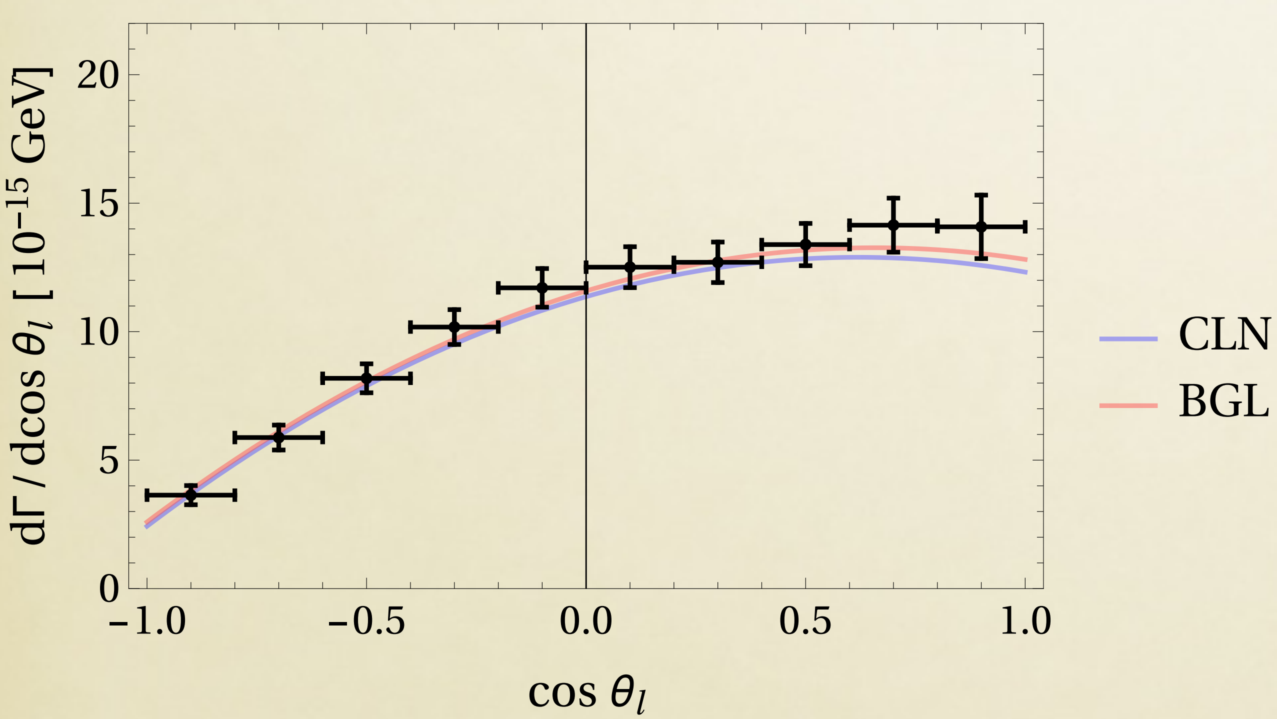
NEW HPQCD $B\to D^*$ result V_{cb}=41.5(1.7) 10^{-3}

NEW Belle $B\to D^*$ with FNAL V_{cb}=37.4(1.3) 10^{-3}

Angular dependence



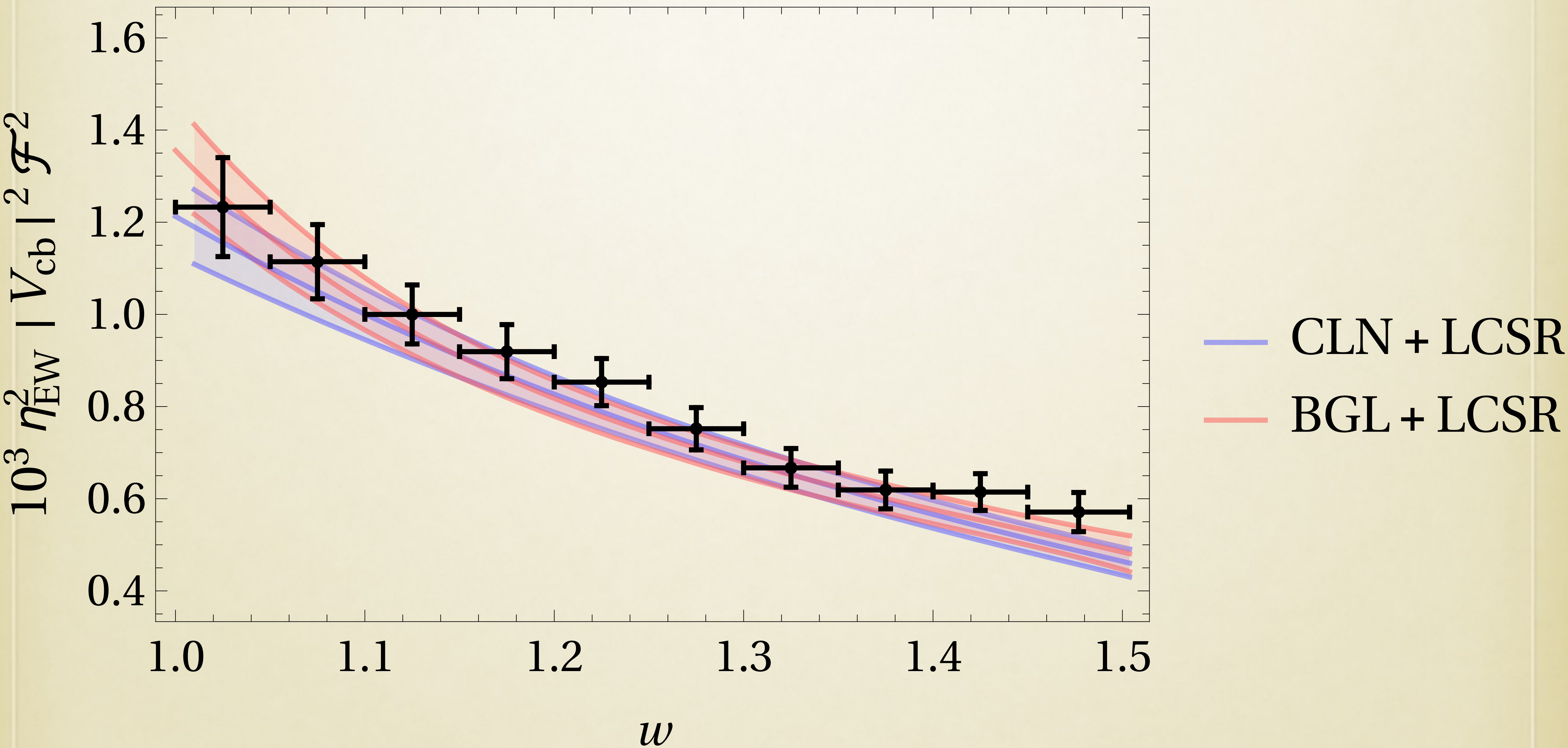


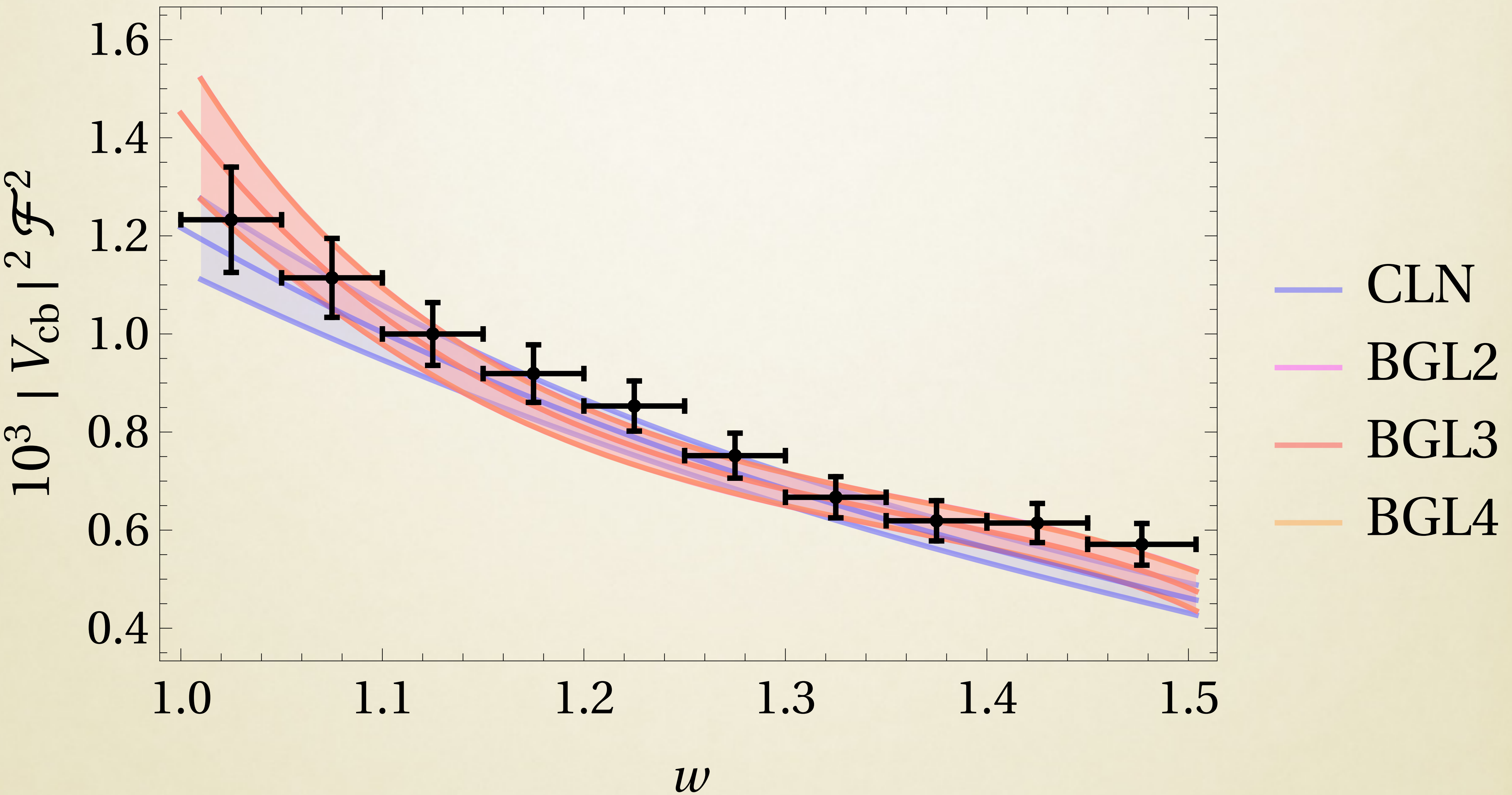


Angular bins are very little sensitive to the low recoil region. Effectively, they dilute the information of the first bins in the w spectrum

CLN fit *without* angular variables gives $|V_{cb}|$=*0.0409(16)*

WITH LCSR CONSTRAINTS





1.6
1.4
1.2
1.0
0.8
0.6
0.4
$10^3\,|V_{cb}|^2\,\mathcal{F}^2$
1.0
1.1
1.2
1.3
1.4
1.5
w
CLN
BGL2
BGL3
BGL4