A lattice study of decoupling of the charm quark

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Outline and References

Outline

- Motivation
- Effective theory of decoupling
- Model calculations with two heavy quarks
- Results from lattice simulations
- Conclusions and outlook

References

FK, T. Korzec, B. Leder and G. Moir, *Power corrections from decoupling of the charm quark*, `1706.04982`

M. Bruno, J. Finkenrath, FK, B. Leder and R. Sommer, *Effects of Heavy Sea Quarks at Low Energies*, PRL **114**, 102001 (2015)

A. Athenodorou, M. Bruno, J. Finkenrath, FK, B. Leder, M. Marinkovic and R. Sommer, *The decoupling of heavy sea quarks*, in preparation

Motivation

Do we need to simulate a dynamical charm quark?

- At energies $E \ll M_{\rm charm} \equiv M_{\rm c}$ the charm quark decouples. QCD_4 with up, down, strange and charm quarks can be described by an effective theory which at leading order is QCD_3 without the charm quark
- Simulations of QCD_3 are cheaper and simpler than QCD_4
- The gauge couplings of QCD_4 and QCD_3 can be matched in perturbation theory. This is used in the determination of α_s from QCD_3 simulations by the ALPHA Collaboration [Bruno et al., 1706.03821]
- We need non-perturbative tests:
  - How well does perturbation theory work for charm?
  - Can we neglect power corrections in $1/M_{\rm c}$?

Effective field theory (EFT)

Expansion in $(E/M)^n$: EFT for $E \ll M$ [Weinberg, Phys. Lett. B91 (1980)]

- Only virtual effects of heavy quark with mass M

  No states with explicit heavy quark (the HQET part)

- Effective Lagrangian (here for $N_\mathrm{f} \to N_\mathrm{f} - 1$)

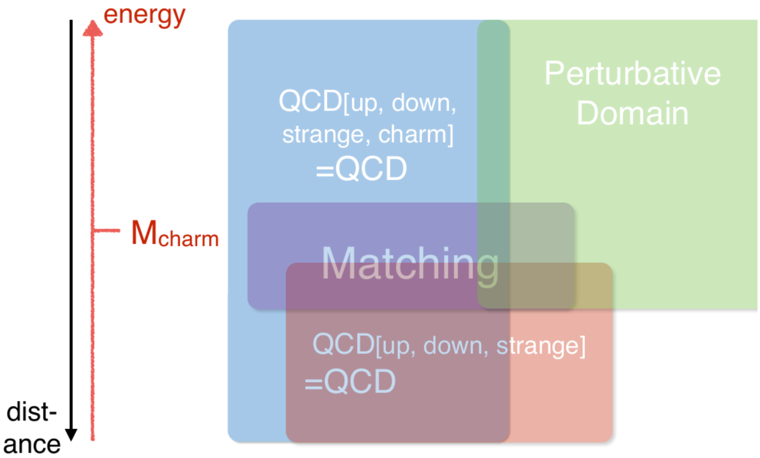
$$
\begin{aligned}
\mathscr{L}_\mathrm{QCD}^{(N_\mathrm{f})} &= \mathscr{L}_\mathrm{QCD}^{(N_\mathrm{f}-1)}(\psi_\mathrm{light}, \bar{\psi}_\mathrm{light}, A_\mu; g_-(M), m_-(M)) \\
&\quad + \frac{1}{M^2}\mathscr{L}_6 \\
\mathscr{L}_6 &= \bar{\psi}_\mathrm{light}\sigma_{\mu\nu}F_{\mu\nu}m_\mathrm{light}\psi_\mathrm{light} + \ldots
\end{aligned}
$$

  Due to gauge and chiral symmetry no dimension 5 operator

- Effective theory couplings $g_- = g_\mathrm{light}$, $m_- = m_\mathrm{light}$ determined by “matching” $\leftrightarrow$ decouping of the heavy quark

- Notation: Λ is the Lambda parameter in the $\overline{\mathrm{MS}}$ scheme; M is the renormalization group invariant quark mass

Charm quark ($M_c \gg \Lambda$)



Matching

- light quark masses are ignored (unimportant)
- QCD_4 has 2 parameters: charm mass M_c and coupling $\bar{g}_4(\mu)$; they are fixed by experiment
- QCD_3 has 1 parameter: coupling $\bar{g}_3(\mu)$
- Matching:

$$\text{observables}_4 \;=\; \text{observables}_3 + (1/M_\mathrm{c})^2$$

$$\begin{aligned}
\bar{g}_3^2(m_*) &= \bar{g}_4^2(m_*) \times [1 + c_2 g_4^4(m_*) + c_3 g_4^6(m_*) + c_4 g_4^8(m_*) + \ldots] \\
& c_2 = \frac{11}{72}\,(4\pi^2)^{-2} \\
& c_3 = \left[\frac{564731}{124416} - \frac{82043}{27648}\,\zeta_3 - 3\,\frac{2633}{31104}\right]\,(4\pi^2)^{-3} \\
& c_4 = \ldots
\end{aligned}$$

$\overline{m}(m_*) = m_*$ [Weinberg, 1980; Bernreuther and Wetzel, 1982; ...
Chetyrkin, Kühn and Sturm, 2006; Schröder and Steinhauser, 2006]

Matching (continued)

- Matching QCD_3 to QCD_4 leads to relation

$$\bar{g}_3 \;=\; f(\bar{g}_4, M)\,, \quad M = M_c$$

- In general:

$$\bar{g}^2_{N_f}(\mu) \;=\; \phi_{N_f}(\mu/\Lambda_{N_f}) \overset{\mu\to\infty}{\sim} \frac{1}{2b_0 \log(\mu/\Lambda_{N_f})}$$

- There is a relation

$$\Lambda_3 \;=\; \Lambda_{\rm dec}(M, \Lambda_4) = P_{3,4}(M/\Lambda_4)\,\Lambda_4$$

Mass dependence

Definition of η^{M}

Definition of mass dependence function ($P \equiv P_{3,4}$)

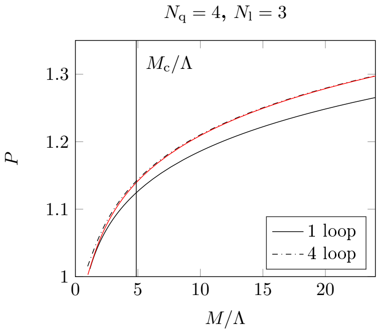
$$\eta^{\mathrm{M}}(M) \equiv \frac{M}{P}\,\frac{\partial P}{\partial M}\bigg|_{\Lambda_4} = \frac{M}{\Lambda_4}\frac{P'}{P}$$

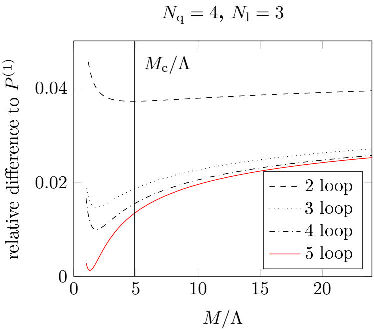
Perturbative expansion for large M ($\bar{g}_4 \equiv \bar{g}_4(m_*)$)

$$\begin{aligned} \eta^{\mathrm{M}} \quad = \quad & \eta_0 + \eta_1^{\mathrm{M}}\bar{g}_4^2 + \eta_2^{\mathrm{M}}\bar{g}_4^4 + \eta_3^{\mathrm{M}}\bar{g}_4^6 + \eta_4^{\mathrm{M}}\bar{g}_4^8 \ldots \\ & \eta_0 = 1 - \frac{b_0(4)}{b_0(3)} = 2/27 > 0 \\ & \eta_1 = (\eta_0 - 1)\left[\tilde{b}_1(4) - \tilde{b}_1(3)\right], \quad \tilde{b}_1(n) = b_1(n)/b_0(n) \end{aligned}$$

with $b_0(n) = (11-2n/3)/(4\pi)^2$, $b_1(n) = (102-38n/3)/(4\pi)^4$

The factor $P_{3,4}$ in perturbation theory





One-loop “approximation” $P_{3,4}^{(1)} = (M/\Lambda)^{2/27}$

Corrections $(P - P^{(1)})/P^{(1)}$ at $M = M_c$: 2% at three loop, higher loops yield smaller changes

Power corrections

Ratios

Consider two light hadronic scales, $m^{\mathrm{had},1}(M)$ and $m^{\mathrm{had},2}(M)$. After matching

$$m^{\mathrm{had,i}}\big|_{N_\mathrm{f}=4} = m^{\mathrm{had,i}}\big|_{N_\mathrm{f}=3} \quad \text{up to} \quad \mathrm{O}(\Lambda_4^2/M^2)$$

A consequence is

$$R(M) \;=\; \left.\frac{m^{\mathrm{had},1}(M)}{m^{\mathrm{had},2}(M)}\right|_{N_\mathrm{f}=4} = \left.\frac{m^{\mathrm{had},1}}{m^{\mathrm{had},2}}\right|_{N_\mathrm{f}=3} + \mathrm{O}(\Lambda_4^2/M^2)$$

Note that

$$\left.\frac{m^{\mathrm{had},1}}{m^{\mathrm{had},2}}\right|_{N_\mathrm{f}=3} = \frac{m^{\mathrm{had},1}/\Lambda_3}{m^{\mathrm{had},2}/\Lambda_3} = \frac{c_1}{c_2} = \text{pure number}$$

Power corrections

Ratios contd

Matching of the couplings is irrelevant for ratios R. We have direct access to power corrections:

$$R(M) \;=\; R(\infty) + k\Lambda^2/M^2$$

k is a number which depends on the ratio

Motivation

Limits of perturbative matching

- By itself perturbation theory for decoupling of the charm quark seems to work well. Is there an *independent* confirmation?
- In perturbation theory power corrections in $1/M$ are neglected. What is their size?

Non-perturbative test in a model

- To avoid a multi-scale problem and control the continuum limit we study a model, QCD_2 with $N_\mathrm{f} = 2$ degenerate quarks of mass M
- Effective theory for $E \ll M$ is a Yang–Mills (YM) theory ($N_\mathrm{f} = 0$, $M = \infty$) at leading order

Decoupling for QCD_2

EFT: $$\mathcal{L}_{\mathrm{dec}} = \mathcal{L}_{\mathrm{YM}} + \frac{1}{M^2}\mathscr{L}_6 + \mathrm{O}\left(\frac{\Lambda^4}{M^4}\right)$$

$$\mathscr{L}_6 = \omega_1 \mathrm{tr}\left\{D_\mu F_{\nu\rho} D_\mu F_{\nu\rho}\right\} + \omega_2 \mathrm{tr}\left\{D_\mu F_{\mu\rho} D_\nu F_{\nu\rho}\right\}$$

Matching of the couplings

Matching $\Leftrightarrow$ relation of Λ parameters:

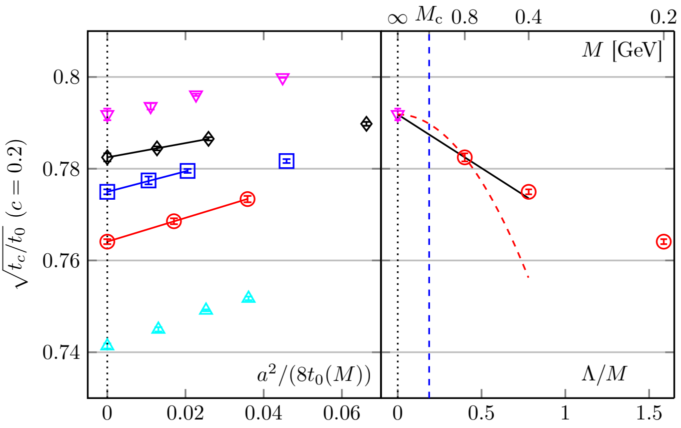
$$\Lambda_{\mathrm{YM}}(M,\Lambda) = P(M/\Lambda)\,\Lambda\,, \quad \Lambda \equiv \Lambda^{(N_\mathrm{f}=2)}$$

Power corrections in ratios R

Ratios of low energy hadronic scales. After matching:

$$R(M) = \frac{m^{\mathrm{had},1}(M)}{m^{\mathrm{had},2}(M)} = \underbrace{\frac{m^{\mathrm{had},1}_{\mathrm{YM}}}{m^{\mathrm{had},2}_{\mathrm{YM}}}}_{\text{pure number}} + \mathrm{O}(\Lambda^2/M^2)$$

$R = \sqrt{t_c/t_0}$ from [Bruno et al., 1410.8374]



Masses up to $M_c/2$, looks like linear behavior $R(M) - R(\infty) \propto \Lambda/M$. Masses are not large enough to see M^{-2} (dashed line) expected from EFT.

Simulations

The data of [Bruno et al., 1410.8374] were produced from simulations of $N_\mathrm{f} = 2$ O(a) improved Wilson quarks with plaquette gauge action

New ensembles

- We use $N_\mathrm{f} = 2$ twisted mass Wilson quarks at maximal twist with clover term and plaquette gauge action. We simulated new masses $M_\mathrm{c}/\Lambda = 4.8700$ (charm) and $M/\Lambda = 5.7781$ and also quenched ensembles ($M = \infty$)
- We use open boundary conditions and the `openQCD` simulation package
- We measure

$$R = \sqrt{t_c/t_0}\,,\ \sqrt{t_0}/w_0\,,\ r_0/\sqrt{t_0}$$

Ensembles at $M = M_c\,,\ 1.2M_c$ and quenched ($M = \infty$)

β	a [fm]	A	BC	$T \times L^3$	$M/\Lambda_{\overline{MS}}$	t_0/a^2	kMDU	τ_{exp} [kMDU]
5.6	≈ 0.042	tm	o	192×48^3	4.8700	6.609(15)	2.0	0.08
				192×48^3	5.7781	6.181(11)	2.1	0.08
5.7	≈ 0.036	tm	o	120×32^3	4.8703	9.104(36)	17.2	0.14
				192×48^3	5.7781	8.565(31)	2.7	0.12
5.88	≈ 0.028	tm	o	192×48^3	4.8700	14.622(62)	23.1	0.24
				120×32^3	5.7781	14.916(93)	59.9	0.23
6.0	≈ 0.023	tm	o	192×48^3	4.8700	22.39(12)	22.4	0.36
6.100	0.0778	–	o	120×32^3	∞	4.4329(32)	64.0	0.05
6.340	0.0545	–	o	120×32^3	∞	9.034(29)	20.1	0.13
$*$				120×24^3	∞	9.002(31)	60.9	0.13
6.672	0.0350	–	o	192×48^3	∞	21.924(81)	73.9	0.35
6.900	0.0261	–	o	192×64^3	∞	39.41(15)	160.2	0.65

Lattice spacing for $N_f = 2$ is determined from the scale L_1 [Blossier et al., 1203.6516, Fritzsch et al., 1205.5380]; for $N_f = 0$ from r_0

$Lm_{PS} \gg 4$; $L/\sqrt{t_0} \geq 8$; * finite volume study

Hadronic scales

Scales t_0, t_c and w_0 from the Wilson flow

Scale t_0 [Lüscher, 1006.4518]

$$\mathcal{E}(t_0) = 0.3\,, \quad \mathcal{E}(t) = t^2 \langle E(x,t) \rangle\,, \quad E = \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu}$$

Similarly

$$\mathcal{E}(t_c) = 0.2$$

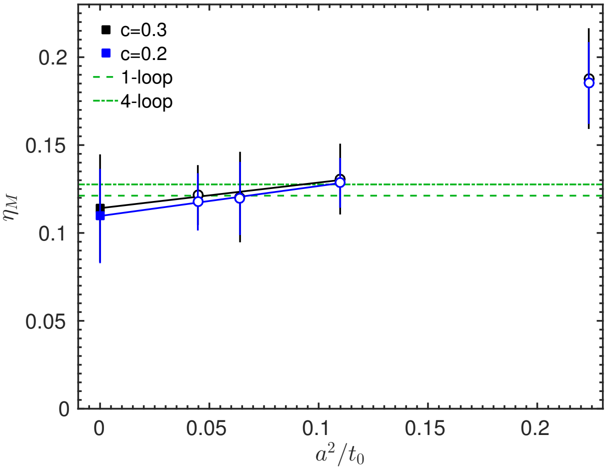
Scale w_0 [Borsanyi et al., 1203.4469]

$$w_0^2 \mathcal{E}'(w_0^2) = 0.3\,, \quad \mathcal{E}'(t) = \frac{\mathrm{d}}{\mathrm{d}t}\mathcal{E}(t)$$

Scale r_0 from the static force [Sommer, hep-lat/9310022]

$$r^2 F(r)\big|_{r=r_0} = 1.65\,, \quad F(r) = V'(r)$$

Mass-dependence function η^{M}



Mass-dependence function η^{M} at $M = M_{\mathrm{c}}$ can be computed from mass-derivative of t_0: $-\frac{\mu}{2t_0}\frac{\partial t_0}{\partial \mu} = \eta^{\mathrm{M}}(M) + \mathrm{O}(1/M^2)$

Ratios R of two hadronic scales

Fits of lattice artifacts

- From Symanzik's theory we expect O(a^2) cut-off effects. We fit our data to

$$R(a, M/\Lambda, \mathrm{A}) \;=\; R^{\mathrm{cont}}(M/\Lambda) + \frac{a^2}{t_0} c(M/\Lambda, \mathrm{A})$$

  where A is the action, “W” for Wilson, “tm” for twisted mass and “q” for quenched
- We apply a cut, $a^2/t_0(M) < 0.32$, to the fits
- For a check we also do a global fit

$$R(a, M/\Lambda, \mathrm{A}) \;=\; R^{\mathrm{cont}}(M/\Lambda) + \frac{a^2}{8t_0}\left[c(\mathrm{A}) + \alpha(\mathrm{A})\frac{M}{\Lambda} + \beta(\mathrm{A})\frac{M^2}{\Lambda^2}\right]$$

  with M-dependent slopes

Power corrections

Fits

Effective theory prediction

$$R(M) = R(\infty) + k\Lambda^2/M^2$$

with fit parameter k

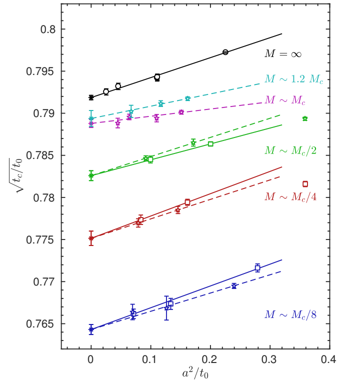
Linear fit

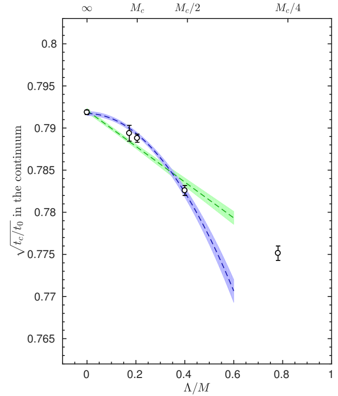
$$R(M) = R(\infty) + \tilde{k}\Lambda/M$$

possible when masses are too small to apply the effective theory

We compare fit ranges $M \in [M_c/2, \infty]$ and $M \in [M_c/4, \infty]$

$R = \sqrt{t_c/t_0}$





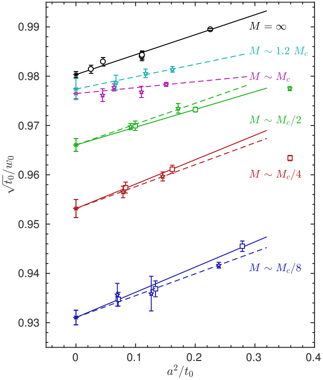
Charm sea-effects:

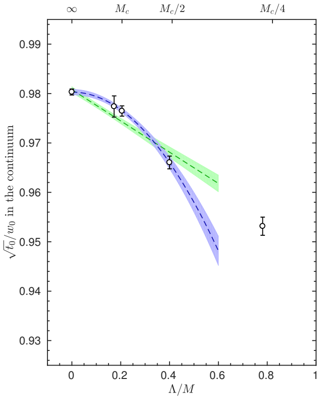
$1/2[R^{\rm cont}(M_{\rm c}) - R^{\rm cont}(\infty)]/R^{\rm cont}(\infty) = -0.00196(37)$

dashed lines are fits in mass range $M \in [M_{\rm c}/2, \infty]$:

$k = -0.058(04)(16)\,,\ \chi^2_{\rm quad}/{\rm dof} = 1.75/2$; $\chi^2_{\rm lin}/{\rm dof} = 9.55/2$

$R = \sqrt{t_0}/w_0$

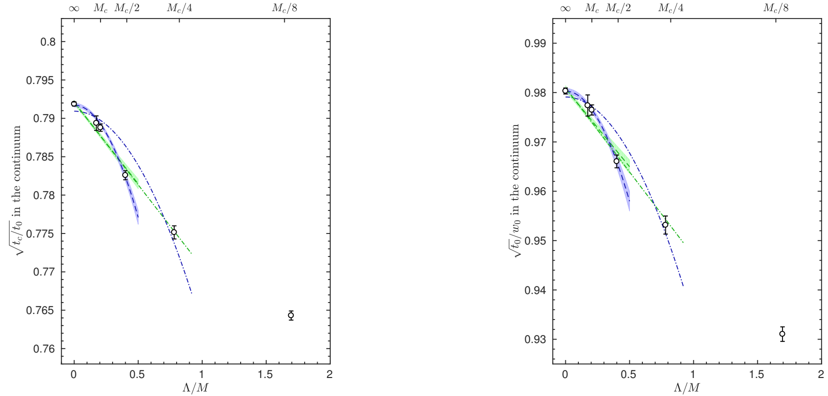




Charm sea-effects:
$1/2[R^{\rm cont}(M_c) - R^{\rm cont}(\infty)]/R^{\rm cont}(\infty) = -0.00194(59)$

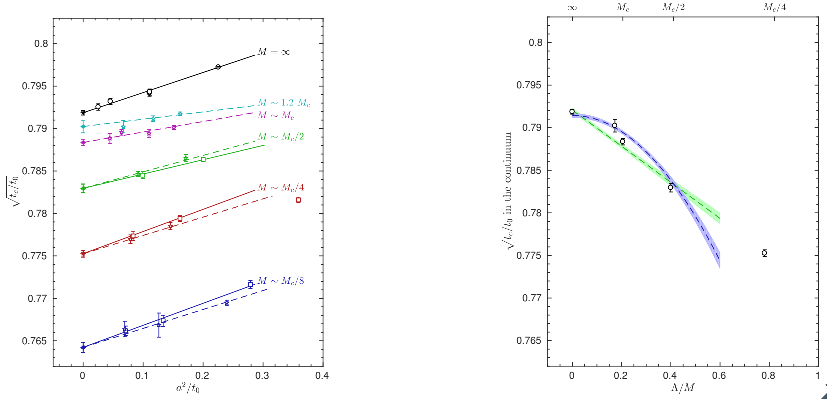
dashed lines are fits in mass range $M \in [M_c/2, \infty]$:
$k = -0.089(09)(03)\ ,\ \chi^2_{\rm quad}/{\rm dof} = 0.02/2$; $\chi^2_{\rm lin}/{\rm dof} = 8.54/2$

Dependence on the mass range



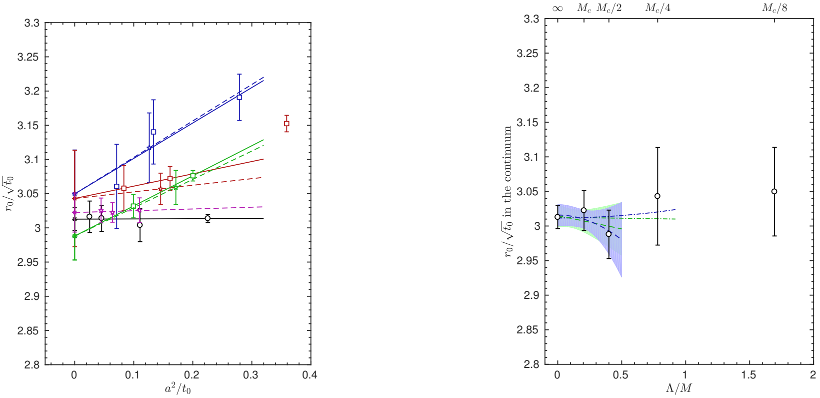
dashed-dotted lines are fits in mass range $M \in [M_\mathrm{c}/4, \infty]$:
quadratic fits are excluded; linear fits have much worse χ^2/dof than the quadratic fits down to $M_\mathrm{c}/2$

$R = \sqrt{t_c/t_0}$ (global fit)



The global fit yields continuum values consistent with the non-global fit and favors M^{-2} behavior

$R = r_0/\sqrt{t_0}$



Despite a state of the art determination of r_0 the precision of $r_0/\sqrt{t_0}$ is not enough to resolve the power corrections

Conclusions

- We studied the decoupling of the charm quark non-perturbatively in QCD$_2$ with $N_\mathrm{f} = 2$ heavy quarks of mass M around the charm mass M_c.
- Ratios of hadronic flow scales differ from their values in the Yang–Mills theory by power corrections in $1/M$ due to charm sea-effects. We are able to measure these tiny effects and find that their relative size is 2 permille at $M = M_\mathrm{c}$.
- Our data can be very well fitted by the EFT prediction for the power corrections $\propto M^{-2}$ down to $M = M_\mathrm{c}/2$.

Outlook

- η^M
- charm sea-effects in the charm mass, charmonium, f_{D_s}, strong coupling from the static force, ...