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HQET Operators of Dimension 8 and Above

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[Ayesha Gunawardna, GP JHEP **1707** 137 (2017)]

Outline

- Introduction
- A little bit of history
- HQET (and NRQCD) operators of dimension 8 and above
- Applications
- Conclusions

Introduction

A tale of two effective field theories

- HQET: Heavy Quark Effective Theory

$$\mathcal{L}_{HQET}^{kinetic} = \bar{h} i v \cdot D h$$

- NRQCD: Non Relativistic Quantum Chromodynamics
(NRQED: Non Relativistic Quantum Electrodynamics)

$$\mathcal{L}_{NRQCD}^{kinetic} = \psi^\dagger i D_t \psi + \psi^\dagger \frac{\mathbf{D}^2}{2M} \psi$$

- Different kinetic term and power counting
- Lagrangians can be related by
 - $h \rightarrow \psi$
 - Choosing $v = (1, 0, 0, 0)$

Aim higher

- Why look at higher dimensional operators?

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HQET Motivation

- Inclusive semileptonic B decays and $Q_{7\gamma} - \bar{Q}_{7\gamma}$ contribution to $B \rightarrow X_s \gamma$ can be described by local OPE

$$\Gamma = \sum_{n=0}^{\infty} \frac{1}{m_b^n} \sum_k c_{k,n} \langle O_{k,n} \rangle$$

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- $\langle O_{k,n} \rangle$ are matrix elements of HQET operators: non-perturbative
Often called HQET parameters
 $O_{k,n} \sim \bar{h} iD^{\mu_1} \dots iD^{\mu_n} (s^\lambda) h \cdot T_{\mu_1 \dots \mu_n \lambda}$, T is a Lorentz tensor

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- $|V_{cb}|$ extraction from inclusive B decays
uses dimension 7 and 8 HQET operators
[Gambino, Healey, Turczyk PLB **763**, 60 (2016)]

NRQED Motivation

- The proton radius puzzle
 - Muonic hydrogen [Pohl et al. Nature **466**, 213 (2010)]
 $r_E^p = 0.84184(67) \text{ fm}$
 - Regular hydrogen [Mohr et al. RMP **80**, 633 (2008)]
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- Use NRQED for proton structure effects in spectroscopy [Hill, GP, PRL **107** 160402 (2011)]

$$W_1(0, Q^2) = 2a_p(2+a_p) + \frac{Q^2}{m_p^2} \left\{ \frac{2m_p^3 \bar{\beta}}{\alpha} - a_p - \frac{2}{3} \left[(1+a_p)^2 m_p^2 (r_M^p)^2 - m_p^2 (r_E^p)^2 \right] \right\} + \dots$$

- Dimension 5 operator: $a_p = 1.793$
- Dimension 6 operator: $r_E^H = 0.8751(61) \text{ fm}$ or $r_E^{\mu H} = 0.84087(26)(29) \text{ fm}$
- Dimension 7 operators: $r_M = 0.776(34)(17) \text{ fm}$, $\bar{\beta} = 2.5(4) \times 10^{-4} \text{ fm}^3$

General Motivation

- What can we learn about the structure of effective field theories?
- For SM EFT the structure is simpler than expected
[Henning, Lu, Melia, Murayama, JHEP **1708**, 016 (2017)]
- What about HQET and NRQCD (NRQED)?

A little bit of history

Prehistory

$$D_t = \frac{\partial}{\partial t} + ieA^0, \quad \mathbf{D} = \nabla - ie\mathbf{A}$$

- Schrödinger equation: $iD_t + \frac{\mathbf{D}^2}{2M}$ (1926)
- Hydrogen Fine Structure:
 - Spin-Orbit: $\boldsymbol{\sigma} \cdot \mathbf{B}$ (1927)
 - Relativistic correction: \mathbf{D}^4 (1905?)
 - Darwin term: $\boldsymbol{\partial} \cdot \mathbf{E}$ (1928)

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 - Darwin term: $\boldsymbol{\partial} \cdot \mathbf{E}$ (1928)
- Organize operators in Lagrangian form
- The $\text{dim}=5,6$ were given in [Caswell, Lepage PLB **167**, 437 (1986)]

$$\mathcal{L}_{\text{NRQED}}^{\text{dim}=5,6} = \psi^\dagger \left\{ iD_t + \frac{\mathbf{D}^2}{2M} + \frac{\mathbf{D}^4}{8M^3} + c_{Fg} g \frac{\boldsymbol{\sigma} \cdot \mathbf{B}}{2M} + c_{Dg} g \frac{[\boldsymbol{\partial} \cdot \mathbf{E}]}{8M^2} \right. \\ \left. + ic_{Sg} g \frac{\boldsymbol{\sigma} \cdot (\mathbf{D} \times \mathbf{E} - \mathbf{E} \times \mathbf{D})}{8M^2} + c_{W1g} g \frac{\{\mathbf{D}^2, \boldsymbol{\sigma} \cdot \mathbf{B}\}}{8M^3} \right\} \psi$$

Dimension 5 HQET operators

- Dimension 5 HQET operators were considered in

$$\mathcal{L}_{\text{HQET}}^{\text{dim}=5} = \bar{h} i v \cdot D h + \frac{\bar{h} (iD)^2 h}{2M} + c_{FG} \frac{\bar{h} \sigma_{\mu\nu} G^{\mu\nu} h}{4M}$$

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- We can see the analogy between HQET and NRQED

| | NRQED (1920's-1980's) | HQET(1990's) |
|-------------|----------------------------------------|----------------------------------|
| Dimension 5 | \mathbf{D}^2 | $(iD)^2$ |
| | $\boldsymbol{\sigma} \cdot \mathbf{B}$ | $\sigma_{\mu\nu} G^{\mu\nu} / 2$ |

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- What about higher dimensional operators?

Higher dimensional operators

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The Curse of the Higher Dimensional Operators...

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hep-ph/9403249 → hep-ph/0611168 → arXiv:1009.4622

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"Two terms added to Lagrangian"
 - Paz et al. arXiv:1702.0890 v1 → v2
"discussion of operators with multiple color structures was added"

Dimension 5 and 6 HQET operators

- First systematic discussion of HQET parameters
[Mannel, PRD 50, 428 (1994)]

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- Between HQET fields $\bar{h} \dots h$ the Dirac basis reduces to $\{1, \sigma\} = \{1, s^\lambda\}$ with $v \cdot s = 0$
- $\bar{h} iD^{\mu_1} \dots iD^{\mu_n}(s^\lambda)h$ is the general operator
Since $iv \cdot Dh = 0$
 - $v_{\mu_1} \bar{h} iD^{\mu_1} \dots iD^{\mu_n}(s^\lambda)h = 0$
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- Dimension 3: $\bar{h}h$
- Dimension 4: $\bar{h}iD^\mu h = 0$
- Dimension 5: Two operators $\bar{h} iD^{\mu_1} iD^{\mu_2} h, \bar{h}iD^{\mu_1} iD^{\mu_2} s^\lambda h$
- Dimension 6: Two operators $\bar{h} iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} h, \bar{h}iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} s^\lambda h$

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- Dimension 5:

$$\langle B(v) | \bar{h}(iD_\alpha)(iD_\beta)h | B(v) \rangle = 2M_H [g_{\alpha\beta} - v_\alpha v_\beta] \frac{1}{3} \lambda_1$$

$$\langle B(v) | \bar{h}(iD_\alpha)(iD_\beta) s_\lambda h | B(v) \rangle = 2M_H d_H i \epsilon_{\nu\alpha\beta\lambda} v^\nu \frac{1}{6} \lambda_2$$

- Dimension 6:

$$\langle B(v) | \bar{h}(iD_\alpha)(iD_\mu)(iD_\beta) h_\nu | B(v) \rangle = 2M_H [g_{\alpha\beta} - v_\alpha v_\beta] v_\mu \frac{1}{3} \rho_1$$

$$\langle B(v) | \bar{h}(iD_\alpha)(iD_\mu)(iD_\beta) s_\lambda h | B(v) \rangle = 2M_H d_H i \epsilon_{\nu\alpha\beta\lambda} v^\nu v_\mu \frac{1}{6} \rho_2.$$

$d_H = 3$ pseudo scalar meson, $d_H = -1$ vector meson

Dimension 7 NRQCD/HQET operators

- The dimension 7 operators listed in [Manohar PRD **56**, 230 (1997)]

$$\begin{aligned}
 \mathcal{L}_{\text{NRQCD}}^{\text{dim}=7} = \psi^\dagger & \left\{ \frac{\mathbf{D}^4}{8M^3} + ic_M g \frac{\{\mathbf{D}^i, [\partial \times \mathbf{B}]^i\}}{8m_p^3} \right. \\
 & + c_{A1} g^2 \frac{(\mathbf{B}_a^i \mathbf{B}_b^i - \mathbf{E}_a^i \mathbf{E}_b^i) T^a T^b}{8M^3} - c_{A2} g^2 \frac{\mathbf{E}_a^i \mathbf{E}_b^i T^a T^b}{16M^3} \\
 & + c_{A3} g^2 \frac{(\mathbf{B}_a^i \mathbf{B}_b^i - \mathbf{E}_a^i \mathbf{E}_b^i) \delta^{ab}}{8M^3} - c_{A4} g^2 \frac{\mathbf{E}_a^i \mathbf{E}_b^i \delta^{ab}}{16M^3} \\
 & + c_{W1} g \frac{\{\mathbf{D}^2, \boldsymbol{\sigma} \cdot \mathbf{B}\}}{8M^3} - c_{W2} g \frac{D^i \boldsymbol{\sigma} \cdot \mathbf{B} D^i}{4m_p^3} + c_{p'p} g \frac{\boldsymbol{\sigma} \cdot \mathbf{D} \mathbf{B} \cdot \mathbf{D} + \mathbf{D} \cdot \mathbf{B} \boldsymbol{\sigma} \cdot \mathbf{D}}{8m_p^3} \\
 & \left. - c_{B1} g^2 \frac{\boldsymbol{\sigma} \cdot (\mathbf{B}_a \times \mathbf{B}_b - \mathbf{E}_a \times \mathbf{E}_b) f^{abc} T^c}{16M^3} + c_{B2} g^2 \frac{\boldsymbol{\sigma} \cdot (\mathbf{E}_a \times \mathbf{E}_b) f^{abc} T^c}{16M^3} \right\} \psi
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+ c_{W1} g \frac{\{D^2, \sigma \cdot B\}}{8M^3} - c_{W2} g \frac{D^i \sigma \cdot B D^i}{4m_p^3} + c_{P'P} g \frac{\sigma \cdot DB \cdot D + D \cdot B \sigma \cdot D}{8m_p^3} \\
\left. - c_{B1} g^2 \frac{\sigma \cdot (B_a \times B_b - E_a \times E_b) f^{abc} T^c}{16M^3} + c_{B2} g^2 \frac{\sigma \cdot (E_a \times E_b) f^{abc} T^c}{16M^3} \right\} \psi$$

- Comments:

- Explicit color structures are taken from [Gunawardna, GP JHEP **1707** 137 (2017)]
- Last line vanishes for NRQED but not for NRQCD
- Notice the operators with same Lorentz structure, but different color

Dimension 7 HQET operators

- How many HQET operators do we have at dimension 7?

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- Dimension 7 NRQCD Lagrangian
 - 6 Spin Independent (SI) operators
 - 5 Spin Dependent (SD) operators[Manohar PRD **56**, 230 (1997)]

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- Dimension 7 for inclusive semileptonic B decays need
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- Why the difference?
- No systematic derivation in either source

Dimension 8 HQET operators

- Dimension 8 corrections for inclusive semileptonic B decays need 7 SI operators and 11 SD operators
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- The **dim=8** NRQED Lagrangian was given in
[Hill, Lee, GP, Solon, PRD **87** 053017 (2013)]

$$\begin{aligned}
 \mathcal{L}_{\text{NRQED}}^{\text{dim}=8} = \psi^\dagger \left\{ & c_{X1} g^2 \frac{[D^2, \mathbf{D} \cdot \mathbf{E} + \mathbf{E} \cdot \mathbf{D}]}{M^4} + c_{X2} g^2 \frac{\{D^2, [\partial \cdot \mathbf{E}]\}}{M^4} \right. \\
 & + c_{X3} g^2 \frac{[\partial^2 \partial \cdot \mathbf{E}]}{M^4} + i c_{X4} g^2 \frac{\{D^i, [\mathbf{E} \times \mathbf{B}]^i\}}{M^4} \\
 & + i c_{X5} g^2 \frac{D^i \boldsymbol{\sigma} \cdot (\mathbf{D} \times \mathbf{E} - \mathbf{E} \times \mathbf{D}) D^i}{M^4} + i c_{X6} g^2 \frac{\epsilon^{ijk} \sigma^i D^j [\partial \cdot \mathbf{E}] D^k}{M^4} \\
 & + c_{X7} g^2 \frac{\boldsymbol{\sigma} \cdot \mathbf{B} [\partial \cdot \mathbf{E}]}{M^4} + c_{X8} g^2 \frac{[\mathbf{E} \cdot \partial \boldsymbol{\sigma} \cdot \mathbf{B}]}{M^4} + c_{X9} g^2 \frac{[\mathbf{B} \cdot \partial \boldsymbol{\sigma} \cdot \mathbf{E}]}{M^4} \\
 & \left. + c_{X10} g^2 \frac{[E^i \boldsymbol{\sigma} \cdot \partial B^i]}{M^4} + c_{X11} g^2 \frac{[B^i \boldsymbol{\sigma} \cdot \partial E^i]}{M^4} + c_{X12} g^2 \frac{\boldsymbol{\sigma} \cdot \mathbf{E} \times [\partial_t \mathbf{E} - \partial \times \mathbf{B}]}{M^4} \right\} \psi
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+ i c_{X5} g^2 \frac{D^i \boldsymbol{\sigma} \cdot (\mathbf{D} \times \mathbf{E} - \mathbf{E} \times \mathbf{D}) D^i}{M^4} + i c_{X6} g^2 \frac{\epsilon^{ijk} \sigma^i D^j [\partial \cdot \mathbf{E}] D^k}{M^4} \\
+ c_{X7} g^2 \frac{\boldsymbol{\sigma} \cdot \mathbf{B} [\partial \cdot \mathbf{E}]}{M^4} + c_{X8} g^2 \frac{[\mathbf{E} \cdot \partial \boldsymbol{\sigma} \cdot \mathbf{B}]}{M^4} + c_{X9} g^2 \frac{[\mathbf{B} \cdot \partial \boldsymbol{\sigma} \cdot \mathbf{E}]}{M^4} \\
\left. + c_{X10} g^2 \frac{[E^i \boldsymbol{\sigma} \cdot \partial B^i]}{M^4} + c_{X11} g^2 \frac{[B^i \boldsymbol{\sigma} \cdot \partial E^i]}{M^4} + c_{X12} g^2 \frac{\boldsymbol{\sigma} \cdot \mathbf{E} \times [\partial_t \mathbf{E} - \partial \times \mathbf{B}]}{M^4} \right\} \psi$$

- 4 SI operators and 8 SD operators
- Missing operators are presumably NRQCD operators
- No systematic derivation

Dimension 8 HQET operators

- The $\text{dim}=8$ NRQED Lagrangian was given in [Hill, Lee, GP, Solon, PRD **87** 053017 (2013)]
- Lagrangian can be constructed by considering all possible combinations of iD_t , $i\mathbf{D}$, \mathbf{E} , \mathbf{B} , and σ that are
 - Rotationally invariant
 - P and T even
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 - Rotationally invariant
 - P and T even
 - Hermitian
- Such construction of $\mathcal{L}_{\text{NRQED}}^{\text{dim} \leq 6}$ was demonstrated in [GP, Mod. Phys. Lett. A 30, 1550128 (2015)]
- For higher dimensional operators this becomes tedious:
 - Different choices for operators
 - Are operators linearly independent?
 - How many linearly independent operators?

Lesson from History

- Finding all of the HQET operators at a given dimension is not easy
- Is there a systematic way to do that?

HQET (and NRQCD) operators at dimension 8 and above: General Method

[Ayesh Gunawardna, GP JHEP **1707** 137 (2017)]

General method

- Like [Mannel, PRD 50, 428 (1994)]

we consider matrix elements of the form

$$\langle H | \bar{h} iD^{\mu_1} \dots iD^{\mu_n} h | H \rangle$$

$$\langle H | \bar{h} iD^{\mu_1} \dots iD^{\mu_n} s^\lambda h | H \rangle$$

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- We use various constraints, some of which were not used before
 - Orthogonality of v to μ_1, μ_n, λ
 - Parity and Time reversal symmetry
 - Hermitian conjugation
 - Four dimensions
 - Possible multiple color structures

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- We use various constraints, some of which were not used before
 - Orthogonality of v to μ_1, μ_n, λ
 - Parity and Time reversal symmetry
 - Hermitian conjugation
 - Four dimensions
 - Possible multiple color structures
- To decompose them in terms of the tensors
 - v^{μ_i}
 - $g^{\mu_i \mu_j}$
 - $\epsilon^{\rho \sigma \alpha \beta}$

General method: Orthogonality

- Consider matrix elements of the form

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- Since $iv \cdot Dh = 0$

- $v_{\mu_1} \bar{h} iD^{\mu_1} \dots iD^{\mu_n} (s^\lambda) h = 0$

- $v_{\mu_n} \bar{h} iD^{\mu_1} \dots iD^{\mu_n} (s^\lambda) h = 0$

- $v_\lambda \bar{h} iD^{\mu_1} \dots iD^{\mu_n} (s^\lambda) h = 0$

[Mannel, PRD 50, 428 (1994)]

- More accurately, the $1/M$ corrections to $iv \cdot Dh = 0$ give rise to higher dimensional operators. One can impose this order by order.

- Similarly for NRQCD (NRQED):

$$\psi^\dagger (iD_t O + O iD_t) \psi / M^n \text{ can be eliminated by } \psi \rightarrow \psi - O\psi / M^n$$

[GP, Mod. Phys. Lett. A 30, 1550128 (2015)]

General method: PT symmetry

- We consider matrix elements of the form

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- Parity and Time reversal are symmetries of HQET

In particular under PT:

- $p = (p^0, \vec{p}) \xrightarrow{PT} (p^0, \vec{p}) = p \Rightarrow v = p/m \xrightarrow{PT} v$
- $iD^\mu \xrightarrow{PT} iD^\mu$
- $\bar{h}h \xrightarrow{PT} \bar{h}h$
- $\bar{h}s^\lambda h \xrightarrow{PT} -\bar{h}s^\lambda h$

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$$- \bar{h} h \xrightarrow{PT} \bar{h} h$$

$$- \bar{h} s^\lambda h \xrightarrow{PT} -\bar{h} s^\lambda h$$

- Since T is anti-linear

$$\langle H | \bar{h} iD^{\mu_1} \dots iD^{\mu_n} h | H \rangle \xrightarrow{PT} \langle H | \bar{h} iD^{\mu_1} \dots iD^{\mu_n} h | H \rangle^*$$

$$\langle H | \bar{h} iD^{\mu_1} \dots iD^{\mu_n} s^\lambda h | H \rangle \xrightarrow{PT} -\langle H | \bar{h} iD^{\mu_1} \dots iD^{\mu_n} s^\lambda h | H \rangle^*$$

- SI matrix elements are real, SD matrix elements are imaginary

General method: Hermitian conjugation

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- $\bar{h}h$, $\bar{h}s^\lambda h$, iD^μ are hermitian using Hermitian conjugation

$$\begin{aligned} \langle H | \bar{h} iD^{\mu_1} \dots iD^{\mu_n} (s^\lambda) h | H \rangle &= \langle H | \left(\bar{h} iD^{\mu_1} \dots iD^{\mu_n} (s^\lambda) h \right)^\dagger | H \rangle^* \\ &= \langle H | \bar{h} iD^{\mu_n} \dots iD^{\mu_1} (s^\lambda) h | H \rangle^* \end{aligned}$$

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- Combining with the PT constraints
Under inversion of the indices:
 - SI matrix elements are symmetric
 - SD matrix elements are anti-symmetric

General method: Tensor decomposition

- We consider matrix elements of the form

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- B is a pseudo-scalar \Rightarrow matrix element can only depend on v^{μ_i} , $g^{\mu_i \mu_j}$, and $\epsilon^{\rho\sigma\alpha\beta}$

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- Alternatively following

[Mannel, Turczyk, Uraltsev JHEP **1011**, 109 (2010)]

Define $\Pi^{\mu\nu} = g^{\mu\nu} - v^\mu v^\nu$

For the standard choice of $v = (1, 0, 0, 0)$: $\Pi^{00} = 0$ and $\Pi^{ij} = -\delta^{ij}$

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- Four dimensions \Rightarrow only four independent directions

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- Not all tensors with more than four indices are independent

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- Not all tensors with more than four indices are independent
- Example: for dimension 7 SD HQET operators need $\Pi^{\mu\nu} \epsilon^{\rho\sigma\alpha\beta} v_\rho$: three indices are the same
Tensors obtained by permuting indices are not linearly independent

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Tensors obtained by permuting indices are not linearly independent
- Example: for dimension 11 SI HQET operators need $\Pi^{\mu_1\mu_2} \Pi^{\mu_3\mu_4} \Pi^{\mu_5\mu_6} \Pi^{\mu_7\mu_8}$: four indices are the same

General method: Color factors

- We consider matrix elements of the form

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- Starting at dimension 7 we can have multiple color factors

E.g. consider $\psi^\dagger E_a^i T^a E_b^j T^b \psi$ [Kobach, Pal PLB **772** 225 (2017)]

$$\{T^a, T^b\} = \frac{1}{3}\delta^{ab} + d^{abc} T^c \Rightarrow \text{two color structures}$$

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- Use basis $\{T^a, T^b\}$ and δ^{ab} :

- $\psi^\dagger E_a^i E_b^j \{T^a, T^b\} \psi$: generated by commutators and anti-commutators

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 \Rightarrow extra α_s suppression \Rightarrow not needed at $\mathcal{O}(\alpha_s^0)$

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- Use basis $\{T^a, T^b\}$ and δ^{ab} :
 - $\psi^\dagger E_a^i E_b^j \{T^a, T^b\} \psi$: generated by commutators and anti-commutators
 - $\psi^\dagger E_a^i E_b^j \delta^{ab} \psi$: generated by one-gluon exchange between ψ^\dagger and ψ
 \Rightarrow extra α_s suppression \Rightarrow not needed at $\mathcal{O}(\alpha_s^0)$
- Decomposition of $\langle H | \bar{h} iD^{\mu_1} \dots iD^{\mu_n} (s^\lambda) h | H \rangle$ does not distinguish $\{T^a, T^b\}$ from δ^{ab} . Need to be put “by hand”.

General method: Summary

- We consider matrix elements of the form

$$\langle H | \bar{h} iD^{\mu_1} \dots iD^{\mu_n} h | H \rangle$$

$$\langle H | \bar{h} iD^{\mu_1} \dots iD^{\mu_n} s^\lambda h | H \rangle$$

- We express them in terms of v^{μ_i} , $\Pi^{\mu_i \mu_j}$, and $\epsilon^{\rho\sigma\alpha\beta} v_\rho$ using
 - Orthogonality: $v_{\mu_1} = v_{\mu_n} = v_\lambda = 0$
 - P, T , and Hermitian conjugation:
SI (SD) matrix elements are sym. (anti-sym.) under inversion
 - Four dimensions:
not all tensors are linearly independent
 - Checking possible multiple color structures

HQET (and NRQCD) operators at dimension 8 and above: Applications

[Ayesh Gunawardna, GP JHEP **1707** 137 (2017)]

Results: SI Dimension 7 HQET operators

- We look at $\langle H | \bar{h} iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} h | H \rangle$
It can depend on v^{μ_i} , $\Pi^{\mu_i \mu_j}$,
- We can have $\Pi \Pi$:

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- We can have $\Pi \Pi$: $\Pi^{\mu_1 \mu_2} \Pi^{\mu_3 \mu_4}$

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- or $\Pi v v$:

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- or $\Pi v v$: $\Pi^{\mu_1 \mu_4} v^{\mu_2} v^{\mu_3}$

$$\frac{1}{2M_H} \langle H | \bar{h} iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} h | H \rangle = a_{12}^{(7)} \Pi^{\mu_1 \mu_2} \Pi^{\mu_3 \mu_4} + a_{13}^{(7)} \Pi^{\mu_1 \mu_3} \Pi^{\mu_2 \mu_4} + \\ + a_{14}^{(7)} \Pi^{\mu_1 \mu_4} \Pi^{\mu_2 \mu_3} + b^{(7)} \Pi^{\mu_1 \mu_4} v^{\mu_2} v^{\mu_3}$$

Notice that the tensors are symmetric under inversion of indices

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Notice that the tensors are symmetric under inversion of indices

- Multiple color structure arise from $\bar{h} \{ [iD^{\mu_i}, iD^{\mu_j}], [iD^{\mu_k}, iD^{\mu_l}] \} h$:

Results: SI Dimension 7 HQET operators

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- We can have $\Pi \Pi$: $\Pi^{\mu_1 \mu_2} \Pi^{\mu_3 \mu_4}$ $\Pi^{\mu_1 \mu_3} \Pi^{\mu_2 \mu_4}$ $\Pi^{\mu_1 \mu_4} \Pi^{\mu_2 \mu_3}$
- or $\Pi v v$: $\Pi^{\mu_1 \mu_4} v^{\mu_2} v^{\mu_3}$

$$\frac{1}{2M_H} \langle H | \bar{h} iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} h | H \rangle = a_{12}^{(7)} \Pi^{\mu_1 \mu_2} \Pi^{\mu_3 \mu_4} + a_{13}^{(7)} \Pi^{\mu_1 \mu_3} \Pi^{\mu_2 \mu_4} + a_{14}^{(7)} \Pi^{\mu_1 \mu_4} \Pi^{\mu_2 \mu_3} + b^{(7)} \Pi^{\mu_1 \mu_4} v^{\mu_2} v^{\mu_3}$$

Notice that the tensors are symmetric under inversion of indices

- Multiple color structure arise from $\bar{h} \{ [iD^{\mu_i}, iD^{\mu_j}], [iD^{\mu_k}, iD^{\mu_l}] \} h$:
 $\bar{h} \{ [iD^{\mu_1}, iD^{\mu_2}], [iD^{\mu_3}, iD^{\mu_4}] \} h$, $\bar{h} \{ [iD^{\mu_1}, iD^{\mu_3}], [iD^{\mu_2}, iD^{\mu_4}] \} h$, and
 $\bar{h} \{ [iD^{\mu_1}, iD^{\mu_4}], [iD^{\mu_2}, iD^{\mu_3}] \} h$

Results: SI Dimension 7 HQET operators

- We look at $\langle H | \bar{h} iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} h | H \rangle$

It can depend on v^{μ_i} , $\Pi^{\mu_i \mu_j}$,

- We can have $\Pi \Pi$: $\Pi^{\mu_1 \mu_2} \Pi^{\mu_3 \mu_4}$ $\Pi^{\mu_1 \mu_3} \Pi^{\mu_2 \mu_4}$ $\Pi^{\mu_1 \mu_4} \Pi^{\mu_2 \mu_3}$
- or $\Pi v v$: $\Pi^{\mu_1 \mu_4} v^{\mu_2} v^{\mu_3}$

$$\frac{1}{2M_H} \langle H | \bar{h} iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} h | H \rangle = a_{12}^{(7)} \Pi^{\mu_1 \mu_2} \Pi^{\mu_3 \mu_4} + a_{13}^{(7)} \Pi^{\mu_1 \mu_3} \Pi^{\mu_2 \mu_4} + \\ + a_{14}^{(7)} \Pi^{\mu_1 \mu_4} \Pi^{\mu_2 \mu_3} + b^{(7)} \Pi^{\mu_1 \mu_4} v^{\mu_2} v^{\mu_3}$$

Notice that the tensors are symmetric under inversion of indices

- Multiple color structure arise from $\bar{h} \{ [iD^{\mu_i}, iD^{\mu_j}], [iD^{\mu_k}, iD^{\mu_l}] \} h$:
 $\bar{h} \{ [iD^{\mu_1}, iD^{\mu_2}], [iD^{\mu_3}, iD^{\mu_4}] \} h$, $\bar{h} \{ [iD^{\mu_1}, iD^{\mu_3}], [iD^{\mu_2}, iD^{\mu_4}] \} h$, and
 $\bar{h} \{ [iD^{\mu_1}, iD^{\mu_4}], [iD^{\mu_2}, iD^{\mu_3}] \} h$

Contracting with tensors above: $a_{13}^{(7)} - a_{14}^{(7)}$ and $b^{(7)}$

\Rightarrow 2 op. with 2 color structures: 6 in total but only 4 at tree level

Results: SI Dimension 7 HQET operators

- We look at $\langle H | \bar{h} iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} h | H \rangle$

It can depend on v^{μ_i} , $\Pi^{\mu_i \mu_j}$,

- We can have $\Pi \Pi$: $\Pi^{\mu_1 \mu_2} \Pi^{\mu_3 \mu_4}$ $\Pi^{\mu_1 \mu_3} \Pi^{\mu_2 \mu_4}$ $\Pi^{\mu_1 \mu_4} \Pi^{\mu_2 \mu_3}$
- or $\Pi v v$: $\Pi^{\mu_1 \mu_4} v^{\mu_2} v^{\mu_3}$

$$\frac{1}{2M_H} \langle H | \bar{h} iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} h | H \rangle = a_{12}^{(7)} \Pi^{\mu_1 \mu_2} \Pi^{\mu_3 \mu_4} + a_{13}^{(7)} \Pi^{\mu_1 \mu_3} \Pi^{\mu_2 \mu_4} + \\ + a_{14}^{(7)} \Pi^{\mu_1 \mu_4} \Pi^{\mu_2 \mu_3} + b^{(7)} \Pi^{\mu_1 \mu_4} v^{\mu_2} v^{\mu_3}$$

Notice that the tensors are symmetric under inversion of indices

- Multiple color structure arise from $\bar{h} \{ [iD^{\mu_i}, iD^{\mu_j}], [iD^{\mu_k}, iD^{\mu_l}] \} h$:
 $\bar{h} \{ [iD^{\mu_1}, iD^{\mu_2}], [iD^{\mu_3}, iD^{\mu_4}] \} h$, $\bar{h} \{ [iD^{\mu_1}, iD^{\mu_3}], [iD^{\mu_2}, iD^{\mu_4}] \} h$, and
 $\bar{h} \{ [iD^{\mu_1}, iD^{\mu_4}], [iD^{\mu_2}, iD^{\mu_3}] \} h$

Contracting with tensors above: $a_{13}^{(7)} - a_{14}^{(7)}$ and $b^{(7)}$

\Rightarrow 2 op. with 2 color structures: 6 in total but only 4 at tree level

- Explains 4 HQET SI op. in [Mannel, Turczyk, Uraltsev JHEP **1011**, 109 (2010)] and 6 NRQCD SI in [Manohar PRD **56**, 230 (1997)]

Results: SD Dimension 7 HQET operators

- We look at $\langle H | \bar{h} iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} s^\lambda h | H \rangle$

By parity it must contain $\epsilon^{\rho\mu_k\mu_l\lambda} v_\rho$

The 2 other indices can be

Results: SD Dimension 7 HQET operators

- We look at $\langle H | \bar{h} iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} s^\lambda h | H \rangle$

By parity it must contain $\epsilon^{\rho\mu_k\mu_l\lambda} v_\rho$

The 2 other indices can be $\Pi^{\mu_i\mu_j}$ or

Results: SD Dimension 7 HQET operators

- We look at $\langle H | \bar{h} iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} s^\lambda h | H \rangle$

By parity it must contain $\epsilon^{\rho\mu_k\mu_l\lambda} v_\rho$

The 2 other indices can be $\Pi^{\mu_i\mu_j}$ or $v^{\mu_i} v^{\mu_j}$

The tensors must also be anti-symmetric under inversion of indices

Results: SD Dimension 7 HQET operators

- We look at $\langle H | \bar{h} iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} s^\lambda h | H \rangle$

By parity it must contain $\epsilon^{\rho\mu_k\mu_l\lambda} v_\rho$

The 2 other indices can be $\Pi^{\mu_i\mu_j}$ or $v^{\mu_i} v^{\mu_j}$

The tensors must also be anti-symmetric under inversion of indices

$$\begin{aligned} \frac{1}{2M_H} \langle H | \bar{h} iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} s^\lambda h | H \rangle &= i\tilde{a}_{12}^{(7)} (\Pi^{\mu_1\mu_2} \epsilon^{\rho\mu_3\mu_4\lambda} v_\rho - \Pi^{\mu_4\mu_3} \epsilon^{\rho\mu_2\mu_1\lambda} v_\rho) \\ &+ i\tilde{a}_{13}^{(7)} (\Pi^{\mu_1\mu_3} \epsilon^{\rho\mu_2\mu_4\lambda} v_\rho - \Pi^{\mu_4\mu_2} \epsilon^{\rho\mu_3\mu_1\lambda} v_\rho) + \\ &+ i\tilde{a}_{14}^{(7)} \Pi^{\mu_1\mu_4} \epsilon^{\rho\mu_2\mu_3\lambda} v_\rho + i\tilde{a}_{23}^{(7)} \Pi^{\mu_2\mu_3} \epsilon^{\rho\mu_1\mu_4\lambda} v_\rho + i\tilde{b}^{(7)} v^{\mu_2} v^{\mu_3} \epsilon^{\rho\mu_1\mu_4\lambda} v_\rho \end{aligned}$$

Results: SD Dimension 7 HQET operators

- We look at $\langle H | \bar{h} iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} s^\lambda h | H \rangle$

By parity it must contain $\epsilon^{\rho\mu_k\mu_l\lambda} v_\rho$

The 2 other indices can be $\Pi^{\mu_i\mu_j}$ or $v^{\mu_i} v^{\mu_j}$

The tensors must also be anti-symmetric under inversion of indices

$$\begin{aligned} \frac{1}{2M_H} \langle H | \bar{h} iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} s^\lambda h | H \rangle &= i\tilde{a}_{12}^{(7)} (\Pi^{\mu_1\mu_2} \epsilon^{\rho\mu_3\mu_4\lambda} v_\rho - \Pi^{\mu_4\mu_3} \epsilon^{\rho\mu_2\mu_1\lambda} v_\rho) \\ &+ i\tilde{a}_{13}^{(7)} (\Pi^{\mu_1\mu_3} \epsilon^{\rho\mu_2\mu_4\lambda} v_\rho - \Pi^{\mu_4\mu_2} \epsilon^{\rho\mu_3\mu_1\lambda} v_\rho) + \\ &+ i\tilde{a}_{14}^{(7)} \Pi^{\mu_1\mu_4} \epsilon^{\rho\mu_2\mu_3\lambda} v_\rho + i\tilde{a}_{23}^{(7)} \Pi^{\mu_2\mu_3} \epsilon^{\rho\mu_1\mu_4\lambda} v_\rho + i\tilde{b}^{(7)} v^{\mu_2} v^{\mu_3} \epsilon^{\rho\mu_1\mu_4\lambda} v_\rho \end{aligned}$$

- Multiple color structure arise from $\bar{h} \{ [iD^{\mu_i}, iD^{\mu_j}], [iD^{\mu_k}, iD^{\mu_l}] \} h$
Contractions with tensors above give *no* contribution

Results: SD Dimension 7 HQET operators

- We look at $\langle H | \bar{h} iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} s^\lambda h | H \rangle$

By parity it must contain $\epsilon^{\rho\mu_k\mu_l\lambda} v_\rho$

The 2 other indices can be $\Pi^{\mu_i\mu_j}$ or $v^{\mu_i} v^{\mu_j}$

The tensors must also be anti-symmetric under inversion of indices

$$\begin{aligned} \frac{1}{2M_H} \langle H | \bar{h} iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} s^\lambda h | H \rangle = & i\tilde{a}_{12}^{(7)} (\Pi^{\mu_1\mu_2} \epsilon^{\rho\mu_3\mu_4\lambda} v_\rho - \Pi^{\mu_4\mu_3} \epsilon^{\rho\mu_2\mu_1\lambda} v_\rho) \\ & + i\tilde{a}_{13}^{(7)} (\Pi^{\mu_1\mu_3} \epsilon^{\rho\mu_2\mu_4\lambda} v_\rho - \Pi^{\mu_4\mu_2} \epsilon^{\rho\mu_3\mu_1\lambda} v_\rho) + \\ & + i\tilde{a}_{14}^{(7)} \Pi^{\mu_1\mu_4} \epsilon^{\rho\mu_2\mu_3\lambda} v_\rho + i\tilde{a}_{23}^{(7)} \Pi^{\mu_2\mu_3} \epsilon^{\rho\mu_1\mu_4\lambda} v_\rho + \tilde{b}^{(7)} v^{\mu_2} v^{\mu_3} \epsilon^{\rho\mu_1\mu_4\lambda} v_\rho \end{aligned}$$

- Multiple color structure arise from $\bar{h} \{[iD^{\mu_i}, iD^{\mu_j}], [iD^{\mu_k}, iD^{\mu_l}]\} h$
Contractions with tensors above give *no* contribution
- Explains 5 HQET SD op. in [Mannel, Turczyk, Uraltsev JHEP **1011**, 109 (2010)] and 5 NRQCD SD in [Manohar PRD **56**, 230 (1997)]

Results: SI Dimension 8 HQET operators

- Using the general method

$$\begin{aligned} \frac{1}{2M_H} \langle H | \bar{h} iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} iD^{\mu_5} h | H \rangle = & a_{12}^{(8)} (\Pi^{\mu_1\mu_2} \Pi^{\mu_3\mu_5} v^{\mu_4} + \Pi^{\mu_1\mu_3} \Pi^{\mu_4\mu_5} v^{\mu_2}) + \\ & a_{13}^{(8)} (\Pi^{\mu_1\mu_3} \Pi^{\mu_2\mu_5} v^{\mu_4} + \Pi^{\mu_3\mu_5} \Pi^{\mu_1\mu_4} v^{\mu_2}) + a_{15}^{(8)} (\Pi^{\mu_1\mu_5} \Pi^{\mu_3\mu_4} v^{\mu_2} + \Pi^{\mu_1\mu_5} \Pi^{\mu_2\mu_3} v^{\mu_4}) + \\ & b_{12}^{(8)} \Pi^{\mu_1\mu_2} \Pi^{\mu_4\mu_5} v^{\mu_3} + b_{14}^{(8)} \Pi^{\mu_1\mu_4} \Pi^{\mu_2\mu_5} v^{\mu_3} + b_{15}^{(8)} \Pi^{\mu_1\mu_5} \Pi^{\mu_2\mu_4} v^{\mu_3} + \\ & c^{(8)} \Pi^{\mu_1\mu_5} v^{\mu_2} v^{\mu_3} v^{\mu_4} \end{aligned}$$

Results: SI Dimension 8 HQET operators

- Using the general method

$$\begin{aligned} \frac{1}{2M_H} \langle H | \bar{h} iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} iD^{\mu_5} h | H \rangle = & a_{12}^{(8)} (\Pi^{\mu_1\mu_2} \Pi^{\mu_3\mu_5} v^{\mu_4} + \Pi^{\mu_1\mu_3} \Pi^{\mu_4\mu_5} v^{\mu_2}) + \\ & a_{13}^{(8)} (\Pi^{\mu_1\mu_3} \Pi^{\mu_2\mu_5} v^{\mu_4} + \Pi^{\mu_3\mu_5} \Pi^{\mu_1\mu_4} v^{\mu_2}) + a_{15}^{(8)} (\Pi^{\mu_1\mu_5} \Pi^{\mu_3\mu_4} v^{\mu_2} + \Pi^{\mu_1\mu_5} \Pi^{\mu_2\mu_3} v^{\mu_4}) + \\ & b_{12}^{(8)} \Pi^{\mu_1\mu_2} \Pi^{\mu_4\mu_5} v^{\mu_3} + b_{14}^{(8)} \Pi^{\mu_1\mu_4} \Pi^{\mu_2\mu_5} v^{\mu_3} + b_{15}^{(8)} \Pi^{\mu_1\mu_5} \Pi^{\mu_2\mu_4} v^{\mu_3} + \\ & c^{(8)} \Pi^{\mu_1\mu_5} v^{\mu_2} v^{\mu_3} v^{\mu_4} \end{aligned}$$

- Multiple color structures arise from
 - $\bar{h} \{ [iD^{\mu_i}, iD^{\mu_j}], [iD^{\mu_k}, [iD^{\mu_l}, iD^{\mu_m}]] \} h$: 20 possibilities
 - $\bar{h} \{ iD^{\mu_m}, \{ [iD^{\mu_i}, iD^{\mu_j}], [iD^{\mu_k}, iD^{\mu_l}] \} \} h$: 15 possibilities

Results: SI Dimension 8 HQET operators

- Using the general method

$$\frac{1}{2M_H} \langle H | \bar{h} iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} iD^{\mu_5} h | H \rangle = a_{12}^{(8)} (\Pi^{\mu_1\mu_2} \Pi^{\mu_3\mu_5} v^{\mu_4} + \Pi^{\mu_1\mu_3} \Pi^{\mu_4\mu_5} v^{\mu_2}) + a_{13}^{(8)} (\Pi^{\mu_1\mu_3} \Pi^{\mu_2\mu_5} v^{\mu_4} + \Pi^{\mu_3\mu_5} \Pi^{\mu_1\mu_4} v^{\mu_2}) + a_{15}^{(8)} (\Pi^{\mu_1\mu_5} \Pi^{\mu_3\mu_4} v^{\mu_2} + \Pi^{\mu_1\mu_5} \Pi^{\mu_2\mu_3} v^{\mu_4}) + b_{12}^{(8)} \Pi^{\mu_1\mu_2} \Pi^{\mu_4\mu_5} v^{\mu_3} + b_{14}^{(8)} \Pi^{\mu_1\mu_4} \Pi^{\mu_2\mu_5} v^{\mu_3} + b_{15}^{(8)} \Pi^{\mu_1\mu_5} \Pi^{\mu_2\mu_4} v^{\mu_3} + c^{(8)} \Pi^{\mu_1\mu_5} v^{\mu_2} v^{\mu_3} v^{\mu_4}$$

- Multiple color structures arise from

- $\bar{h} \{ [iD^{\mu_i}, iD^{\mu_j}], [iD^{\mu_k}, [iD^{\mu_l}, iD^{\mu_m}]] \} h$: 20 possibilities
- $\bar{h} \{ iD^{\mu_m}, \{ [iD^{\mu_i}, iD^{\mu_j}], [iD^{\mu_k}, iD^{\mu_l}] \} \} h$: 15 possibilities

Contractions with tensors above give 1 contribution

\Rightarrow 1 op. with 2 color structures: 8 in total but only 7 at tree level

Results: SI Dimension 8 HQET operators

- Using the general method

$$\frac{1}{2M_H} \langle H | \bar{h} iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} iD^{\mu_5} h | H \rangle = a_{12}^{(8)} (\Pi^{\mu_1\mu_2} \Pi^{\mu_3\mu_5} v^{\mu_4} + \Pi^{\mu_1\mu_3} \Pi^{\mu_4\mu_5} v^{\mu_2}) + a_{13}^{(8)} (\Pi^{\mu_1\mu_3} \Pi^{\mu_2\mu_5} v^{\mu_4} + \Pi^{\mu_3\mu_5} \Pi^{\mu_1\mu_4} v^{\mu_2}) + a_{15}^{(8)} (\Pi^{\mu_1\mu_5} \Pi^{\mu_3\mu_4} v^{\mu_2} + \Pi^{\mu_1\mu_5} \Pi^{\mu_2\mu_3} v^{\mu_4}) + b_{12}^{(8)} \Pi^{\mu_1\mu_2} \Pi^{\mu_4\mu_5} v^{\mu_3} + b_{14}^{(8)} \Pi^{\mu_1\mu_4} \Pi^{\mu_2\mu_5} v^{\mu_3} + b_{15}^{(8)} \Pi^{\mu_1\mu_5} \Pi^{\mu_2\mu_4} v^{\mu_3} + c^{(8)} \Pi^{\mu_1\mu_5} v^{\mu_2} v^{\mu_3} v^{\mu_4}$$

- Multiple color structures arise from
 - $\bar{h} \{ [iD^{\mu_i}, iD^{\mu_j}], [iD^{\mu_k}, [iD^{\mu_l}, iD^{\mu_m}]] \} h$: 20 possibilities
 - $\bar{h} \{ iD^{\mu_m}, \{ [iD^{\mu_i}, iD^{\mu_j}], [iD^{\mu_k}, iD^{\mu_l}] \} \} h$: 15 possibilities
 Contractions with tensors above give 1 contribution
 \Rightarrow 1 op. with 2 color structures: 8 in total but only 7 at tree level
- Explains 7 HQET dimension 8 SI operators in [Mannel, Turczyk, Uraltsev JHEP **1011**, 109 (2010)]
 The new operator will be listed below

Results: SD Dimension 8 HQET operators

- Using the general method

$$\begin{aligned}
 & \frac{1}{2M_H} \langle H | \bar{h} iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} iD^{\mu_5} s^\lambda h | H \rangle = \\
 & i\tilde{a}_{12}^{(8)} \left(v^{\mu_3} \Pi^{\mu_1 \mu_2} \epsilon^{\rho \mu_4 \mu_5 \lambda} v_\rho - v^{\mu_3} \Pi^{\mu_4 \mu_5} \epsilon^{\rho \mu_2 \mu_1 \lambda} v_\rho \right) + i\tilde{a}_{14}^{(8)} \left(v^{\mu_3} \Pi^{\mu_1 \mu_4} \epsilon^{\rho \mu_2 \mu_5 \lambda} v_\rho - v^{\mu_3} \Pi^{\mu_5 \mu_2} \epsilon^{\rho \mu_4 \mu_1 \lambda} v_\rho \right) + \\
 & + i\tilde{a}_{15}^{(8)} v^{\mu_3} \Pi^{\mu_1 \mu_5} \epsilon^{\rho \mu_2 \mu_4 \lambda} v_\rho + i\tilde{a}_{24}^{(8)} v^{\mu_3} \Pi^{\mu_2 \mu_4} \epsilon^{\rho \mu_1 \mu_5 \lambda} v_\rho + \\
 & + i\tilde{b}_{13}^{(8)} \left(v^{\mu_2} \Pi^{\mu_1 \mu_3} \epsilon^{\rho \mu_4 \mu_5 \lambda} v_\rho - v^{\mu_4} \Pi^{\mu_5 \mu_3} \epsilon^{\rho \mu_2 \mu_1 \lambda} v_\rho \right) + i\tilde{b}_{14}^{(8)} \left(v^{\mu_2} \Pi^{\mu_1 \mu_4} \epsilon^{\rho \mu_3 \mu_5 \lambda} v_\rho - v^{\mu_4} \Pi^{\mu_5 \mu_2} \epsilon^{\rho \mu_3 \mu_1 \lambda} v_\rho \right) + \\
 & + i\tilde{b}_{15}^{(8)} \left(v^{\mu_2} \Pi^{\mu_1 \mu_5} \epsilon^{\rho \mu_3 \mu_4 \lambda} v_\rho - v^{\mu_4} \Pi^{\mu_1 \mu_5} \epsilon^{\rho \mu_3 \mu_2 \lambda} v_\rho \right) + i\tilde{b}_{34}^{(8)} \left(v^{\mu_2} \Pi^{\mu_3 \mu_4} \epsilon^{\rho \mu_1 \mu_5 \lambda} v_\rho - v^{\mu_4} \Pi^{\mu_3 \mu_2} \epsilon^{\rho \mu_5 \mu_1 \lambda} v_\rho \right) + \\
 & + i\tilde{b}_{35}^{(8)} \left(v^{\mu_2} \Pi^{\mu_3 \mu_5} \epsilon^{\rho \mu_1 \mu_4 \lambda} v_\rho - v^{\mu_4} \Pi^{\mu_3 \mu_1} \epsilon^{\rho \mu_5 \mu_2 \lambda} v_\rho \right) + i\tilde{b}_{45}^{(8)} \left(v^{\mu_2} \Pi^{\mu_4 \mu_5} \epsilon^{\rho \mu_1 \mu_3 \lambda} v_\rho - v^{\mu_4} \Pi^{\mu_2 \mu_1} \epsilon^{\rho \mu_5 \mu_3 \lambda} v_\rho \right) + \\
 & + i\tilde{c}^{(8)} v^{\mu_2} v^{\mu_3} v^{\mu_4} \epsilon^{\rho \mu_1 \mu_5 \lambda} v_\rho.
 \end{aligned}$$

Results: SD Dimension 8 HQET operators

- Using the general method

$$\begin{aligned}
 & \frac{1}{2M_H} \langle H | \bar{h} iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} iD^{\mu_5} s^\lambda h | H \rangle = \\
 & i\tilde{a}_{12}^{(8)} \left(v^{\mu_3} \Pi^{\mu_1 \mu_2} \epsilon^{\rho \mu_4 \mu_5 \lambda} v_\rho - v^{\mu_3} \Pi^{\mu_4 \mu_5} \epsilon^{\rho \mu_2 \mu_1 \lambda} v_\rho \right) + i\tilde{a}_{14}^{(8)} \left(v^{\mu_3} \Pi^{\mu_1 \mu_4} \epsilon^{\rho \mu_2 \mu_5 \lambda} v_\rho - v^{\mu_3} \Pi^{\mu_5 \mu_2} \epsilon^{\rho \mu_4 \mu_1 \lambda} v_\rho \right) + \\
 & + i\tilde{a}_{15}^{(8)} v^{\mu_3} \Pi^{\mu_1 \mu_5} \epsilon^{\rho \mu_2 \mu_4 \lambda} v_\rho + i\tilde{a}_{24}^{(8)} v^{\mu_3} \Pi^{\mu_2 \mu_4} \epsilon^{\rho \mu_1 \mu_5 \lambda} v_\rho + \\
 & + i\tilde{b}_{13}^{(8)} \left(v^{\mu_2} \Pi^{\mu_1 \mu_3} \epsilon^{\rho \mu_4 \mu_5 \lambda} v_\rho - v^{\mu_4} \Pi^{\mu_5 \mu_3} \epsilon^{\rho \mu_2 \mu_1 \lambda} v_\rho \right) + i\tilde{b}_{14}^{(8)} \left(v^{\mu_2} \Pi^{\mu_1 \mu_4} \epsilon^{\rho \mu_3 \mu_5 \lambda} v_\rho - v^{\mu_4} \Pi^{\mu_5 \mu_2} \epsilon^{\rho \mu_3 \mu_1 \lambda} v_\rho \right) + \\
 & + i\tilde{b}_{15}^{(8)} \left(v^{\mu_2} \Pi^{\mu_1 \mu_5} \epsilon^{\rho \mu_3 \mu_4 \lambda} v_\rho - v^{\mu_4} \Pi^{\mu_1 \mu_5} \epsilon^{\rho \mu_3 \mu_2 \lambda} v_\rho \right) + i\tilde{b}_{34}^{(8)} \left(v^{\mu_2} \Pi^{\mu_3 \mu_4} \epsilon^{\rho \mu_1 \mu_5 \lambda} v_\rho - v^{\mu_4} \Pi^{\mu_3 \mu_2} \epsilon^{\rho \mu_5 \mu_1 \lambda} v_\rho \right) + \\
 & + i\tilde{b}_{35}^{(8)} \left(v^{\mu_2} \Pi^{\mu_3 \mu_5} \epsilon^{\rho \mu_1 \mu_4 \lambda} v_\rho - v^{\mu_4} \Pi^{\mu_3 \mu_1} \epsilon^{\rho \mu_5 \mu_2 \lambda} v_\rho \right) + i\tilde{b}_{45}^{(8)} \left(v^{\mu_2} \Pi^{\mu_4 \mu_5} \epsilon^{\rho \mu_1 \mu_3 \lambda} v_\rho - v^{\mu_4} \Pi^{\mu_2 \mu_1} \epsilon^{\rho \mu_5 \mu_3 \lambda} v_\rho \right) + \\
 & + i\tilde{c}^{(8)} v^{\mu_2} v^{\mu_3} v^{\mu_4} \epsilon^{\rho \mu_1 \mu_5 \lambda} v_\rho.
 \end{aligned}$$

- Checking for multiple color structures as before
 Contractions with tensors above give 6 contributions
 \Rightarrow 6 op. with 2 color structures: 17 in total but only 11 at tree level

Results: SD Dimension 8 HQET operators

- Using the general method

$$\frac{1}{2M_H} \langle H | \bar{h} iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} iD^{\mu_5} s^\lambda h | H \rangle =$$

$$i\tilde{a}_{12}^{(8)} \left(v^{\mu_3} \Pi^{\mu_1 \mu_2} \epsilon^{\rho \mu_4 \mu_5 \lambda} v_\rho - v^{\mu_3} \Pi^{\mu_4 \mu_5} \epsilon^{\rho \mu_2 \mu_1 \lambda} v_\rho \right) + i\tilde{a}_{14}^{(8)} \left(v^{\mu_3} \Pi^{\mu_1 \mu_4} \epsilon^{\rho \mu_2 \mu_5 \lambda} v_\rho - v^{\mu_3} \Pi^{\mu_5 \mu_2} \epsilon^{\rho \mu_4 \mu_1 \lambda} v_\rho \right) +$$

$$+ i\tilde{a}_{15}^{(8)} v^{\mu_3} \Pi^{\mu_1 \mu_5} \epsilon^{\rho \mu_2 \mu_4 \lambda} v_\rho + i\tilde{a}_{24}^{(8)} v^{\mu_3} \Pi^{\mu_2 \mu_4} \epsilon^{\rho \mu_1 \mu_5 \lambda} v_\rho +$$

$$+ i\tilde{b}_{13}^{(8)} \left(v^{\mu_2} \Pi^{\mu_1 \mu_3} \epsilon^{\rho \mu_4 \mu_5 \lambda} v_\rho - v^{\mu_4} \Pi^{\mu_5 \mu_3} \epsilon^{\rho \mu_2 \mu_1 \lambda} v_\rho \right) + i\tilde{b}_{14}^{(8)} \left(v^{\mu_2} \Pi^{\mu_1 \mu_4} \epsilon^{\rho \mu_3 \mu_5 \lambda} v_\rho - v^{\mu_4} \Pi^{\mu_5 \mu_2} \epsilon^{\rho \mu_3 \mu_1 \lambda} v_\rho \right) +$$

$$+ i\tilde{b}_{15}^{(8)} \left(v^{\mu_2} \Pi^{\mu_1 \mu_5} \epsilon^{\rho \mu_3 \mu_4 \lambda} v_\rho - v^{\mu_4} \Pi^{\mu_1 \mu_5} \epsilon^{\rho \mu_3 \mu_2 \lambda} v_\rho \right) + i\tilde{b}_{34}^{(8)} \left(v^{\mu_2} \Pi^{\mu_3 \mu_4} \epsilon^{\rho \mu_1 \mu_5 \lambda} v_\rho - v^{\mu_4} \Pi^{\mu_3 \mu_2} \epsilon^{\rho \mu_5 \mu_1 \lambda} v_\rho \right) +$$

$$+ i\tilde{b}_{35}^{(8)} \left(v^{\mu_2} \Pi^{\mu_3 \mu_5} \epsilon^{\rho \mu_1 \mu_4 \lambda} v_\rho - v^{\mu_4} \Pi^{\mu_3 \mu_1} \epsilon^{\rho \mu_5 \mu_2 \lambda} v_\rho \right) + i\tilde{b}_{45}^{(8)} \left(v^{\mu_2} \Pi^{\mu_4 \mu_5} \epsilon^{\rho \mu_1 \mu_3 \lambda} v_\rho - v^{\mu_4} \Pi^{\mu_2 \mu_1} \epsilon^{\rho \mu_5 \mu_3 \lambda} v_\rho \right) +$$

$$+ i\tilde{c}^{(8)} v^{\mu_2} v^{\mu_3} v^{\mu_4} \epsilon^{\rho \mu_1 \mu_5 \lambda} v_\rho.$$

- Checking for multiple color structures as before
 Contractions with tensors above give 6 contributions
 \Rightarrow 6 op. with 2 color structures: 17 in total but only 11 at tree level
- Explains 11 HQET dimension 8 SD operators in
 [Mannel, Turczyk, Uraltsev JHEP **1011**, 109 (2010)]
 The new operators will be listed below

New Result: Dimension 8 NRQCD Lagrangian

- We can now list the dimension 8 NRQCD Lagrangian
[Gunawardna, GP JHEP **1707** 137 (2017), Kobach, Pal PLB **772** 225 (2017)]

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- We can now list the dimension 8 NRQCD Lagrangian

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$$\begin{aligned}
 \mathcal{L}_{\text{NRQCD}}^{\text{dim}=8} = & \psi^\dagger \left\{ \dots c_{X1g} \frac{[D^2, \{D^i, E^i\}]}{m_p^4} + c_{X2g} \frac{\{D^2, [D^i, E^i]\}}{m_p^4} + c_{X3g} \frac{[D^i, [D^i, [D^j, E^j]]]}{m_p^4} \right. \\
 & + ic_{X4a} g^2 \frac{\{D^i, \epsilon^{ijk} E_a^j B_b^k \{T^a, T^b\}\}}{2M^4} + ic_{X4b} g^2 \frac{\{D^i, \epsilon^{ijk} E_a^j B_b^k \delta^{ab}\}}{m_p^4} + ic_{X5g} \frac{D^i \sigma \cdot (D \times E - E \times D) D^i}{m_p^4} \\
 & + ic_{X6g} \frac{\epsilon^{ijk} \sigma^i D^j [D^l, E^l] D^k}{m_p^4} + c_{X7a} g^2 \frac{\{\sigma \cdot B_a T^a, [D^i, E^i]_b T^b\}}{2M^4} + c_{X7b} g^2 \frac{\sigma \cdot B_a [D^i, E^i]_a}{m_p^4} \\
 & + c_{X8a} g^2 \frac{\{E_a^j T^a, [D^i, \sigma \cdot B]_b T^b\}}{2M^4} + c_{X8b} g^2 \frac{E_a^j [D^i, \sigma \cdot B]_a}{m_p^4} + c_{X9a} g^2 \frac{\{B_a^i T^a, [D^i, \sigma \cdot E]_b T^b\}}{2M^4} \\
 & + c_{X9b} g^2 \frac{B_a^i [D^i, \sigma \cdot E]_a}{m_p^4} + c_{X10a} g^2 \frac{\{E_a^j T^a, [\sigma \cdot D, B^i]_b T^b\}}{2M^4} + c_{X10b} g^2 \frac{E_a^i [\sigma \cdot D, B^i]_a}{m_p^4} \\
 & + c_{X11a} g^2 \frac{\{B_a^i T^a, [\sigma \cdot D, E^i]_b T^b\}}{2M^4} + c_{X11b} g^2 \frac{B_a^i [\sigma \cdot D, E^i]_a}{m_p^4} + \tilde{c}_{X12a} g^2 \frac{\epsilon^{ijk} \sigma^i E_a^j [D_t, E^k]_b \{T^a, T^b\}}{2M^4} \\
 & + \tilde{c}_{X12b} g^2 \frac{\epsilon^{ijk} \sigma^i E_a^j [D_t, E^k]_a}{m_p^4} + ic_{X13g} \frac{[E^i, [D_t, E^i]]}{m_p^4} + ic_{X14g} \frac{[B^i, (D \times E + E \times D)^i]}{m_p^4} \\
 & \left. + ic_{X15g} \frac{[E^i, (D \times B + B \times D)^i]}{m_p^4} + c_{X16g} \frac{[\sigma \cdot B, \{D^i, E^i\}]}{m_p^4} + c_{X17g} \frac{[B^i, \{D^i, \sigma \cdot E\}]}{m_p^4} + c_{X18g} \frac{[E^i, \{\sigma \cdot D, B^i\}]}{m_p^4} \right\} \psi
 \end{aligned}$$

- 25 operators

- c_{Xib} start at $\mathcal{O}(\alpha_s)$

New Result: Dimension 9 HQET operators

- Using the general method: SI Dimension 9 HQET operators

New Result: Dimension 9 HQET operators

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$$\begin{aligned}
 \frac{1}{2M_H} \langle H | \bar{h} iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} iD^{\mu_5} iD^{\mu_6} h | H \rangle = & a_{12,34}^{(9)} \Pi^{\mu_1 \mu_2} \Pi^{\mu_3 \mu_4} \Pi^{\mu_5 \mu_6} + \\
 & + a_{12,35}^{(9)} (\Pi^{\mu_1 \mu_2} \Pi^{\mu_3 \mu_5} \Pi^{\mu_4 \mu_6} + \Pi^{\mu_1 \mu_3} \Pi^{\mu_2 \mu_4} \Pi^{\mu_5 \mu_6}) + a_{12,36}^{(9)} (\Pi^{\mu_1 \mu_2} \Pi^{\mu_3 \mu_6} \Pi^{\mu_4 \mu_5} + \Pi^{\mu_1 \mu_4} \Pi^{\mu_2 \mu_3} \Pi^{\mu_5 \mu_6}) + \\
 & + a_{13,25}^{(9)} \Pi^{\mu_1 \mu_3} \Pi^{\mu_2 \mu_5} \Pi^{\mu_4 \mu_6} + a_{13,26}^{(9)} (\Pi^{\mu_1 \mu_3} \Pi^{\mu_2 \mu_6} \Pi^{\mu_4 \mu_5} + \Pi^{\mu_1 \mu_5} \Pi^{\mu_2 \mu_3} \Pi^{\mu_4 \mu_6}) + a_{14,25}^{(9)} \Pi^{\mu_1 \mu_4} \Pi^{\mu_2 \mu_5} \Pi^{\mu_3 \mu_6} + \\
 & + a_{14,26}^{(9)} (\Pi^{\mu_1 \mu_4} \Pi^{\mu_2 \mu_6} \Pi^{\mu_3 \mu_5} + \Pi^{\mu_1 \mu_5} \Pi^{\mu_2 \mu_4} \Pi^{\mu_3 \mu_6}) + a_{15,26}^{(9)} \Pi^{\mu_1 \mu_5} \Pi^{\mu_2 \mu_6} \Pi^{\mu_3 \mu_4} + a_{16,23}^{(9)} \Pi^{\mu_1 \mu_6} \Pi^{\mu_2 \mu_3} \Pi^{\mu_4 \mu_5} + \\
 & + a_{16,24}^{(9)} \Pi^{\mu_1 \mu_6} \Pi^{\mu_2 \mu_4} \Pi^{\mu_3 \mu_5} + a_{16,25}^{(9)} \Pi^{\mu_1 \mu_6} \Pi^{\mu_2 \mu_5} \Pi^{\mu_3 \mu_4} + b_{12,36}^{(9)} (\Pi^{\mu_1 \mu_2} \Pi^{\mu_3 \mu_6} v^{\mu_4} v^{\mu_5} + \Pi^{\mu_1 \mu_4} \Pi^{\mu_5 \mu_6} v^{\mu_2} v^{\mu_3}) + \\
 & + b_{12,46}^{(9)} (\Pi^{\mu_1 \mu_2} \Pi^{\mu_4 \mu_6} v^{\mu_3} v^{\mu_5} + \Pi^{\mu_1 \mu_3} \Pi^{\mu_5 \mu_6} v^{\mu_2} v^{\mu_4}) + b_{12,56}^{(9)} \Pi^{\mu_1 \mu_2} \Pi^{\mu_5 \mu_6} v^{\mu_3} v^{\mu_4} + \\
 & + b_{13,26}^{(9)} (\Pi^{\mu_1 \mu_3} \Pi^{\mu_2 \mu_6} v^{\mu_4} v^{\mu_5} + \Pi^{\mu_1 \mu_5} \Pi^{\mu_4 \mu_6} v^{\mu_2} v^{\mu_3}) + b_{13,46}^{(9)} \Pi^{\mu_1 \mu_3} \Pi^{\mu_4 \mu_6} v^{\mu_2} v^{\mu_5} + \\
 & + b_{14,26}^{(9)} (\Pi^{\mu_1 \mu_4} \Pi^{\mu_2 \mu_6} v^{\mu_3} v^{\mu_5} + \Pi^{\mu_1 \mu_5} \Pi^{\mu_3 \mu_6} v^{\mu_2} v^{\mu_4}) + b_{14,36}^{(9)} \Pi^{\mu_1 \mu_4} \Pi^{\mu_3 \mu_6} v^{\mu_2} v^{\mu_5} + b_{15,26}^{(9)} \Pi^{\mu_1 \mu_5} \Pi^{\mu_2 \mu_6} v^{\mu_3} v^{\mu_4} + \\
 & + b_{16,23}^{(9)} (\Pi^{\mu_1 \mu_6} \Pi^{\mu_2 \mu_3} v^{\mu_4} v^{\mu_5} + \Pi^{\mu_1 \mu_6} \Pi^{\mu_4 \mu_5} v^{\mu_2} v^{\mu_3}) + b_{16,24}^{(9)} (\Pi^{\mu_1 \mu_6} \Pi^{\mu_2 \mu_4} v^{\mu_3} v^{\mu_5} + \Pi^{\mu_1 \mu_6} \Pi^{\mu_3 \mu_5} v^{\mu_2} v^{\mu_4}) + \\
 & + b_{16,25}^{(9)} \Pi^{\mu_1 \mu_6} \Pi^{\mu_2 \mu_5} v^{\mu_3} v^{\mu_4} + b_{16,34}^{(9)} \Pi^{\mu_1 \mu_6} \Pi^{\mu_3 \mu_4} v^{\mu_2} v^{\mu_5} + c^{(9)} \Pi^{\mu_1 \mu_6} v^{\mu_2} v^{\mu_3} v^{\mu_4} v^{\mu_5}
 \end{aligned}$$

New Result: Dimension 9 HQET operators

- Using the general method: SI Dimension 9 HQET operators

$$\begin{aligned}
 \frac{1}{2M_H} \langle H | \bar{h} iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} iD^{\mu_5} iD^{\mu_6} h | H \rangle = & a_{12,34}^{(9)} \Pi^{\mu_1 \mu_2} \Pi^{\mu_3 \mu_4} \Pi^{\mu_5 \mu_6} + \\
 & + a_{12,35}^{(9)} (\Pi^{\mu_1 \mu_2} \Pi^{\mu_3 \mu_5} \Pi^{\mu_4 \mu_6} + \Pi^{\mu_1 \mu_3} \Pi^{\mu_2 \mu_4} \Pi^{\mu_5 \mu_6}) + a_{12,36}^{(9)} (\Pi^{\mu_1 \mu_2} \Pi^{\mu_3 \mu_6} \Pi^{\mu_4 \mu_5} + \Pi^{\mu_1 \mu_4} \Pi^{\mu_2 \mu_3} \Pi^{\mu_5 \mu_6}) + \\
 & + a_{13,25}^{(9)} \Pi^{\mu_1 \mu_3} \Pi^{\mu_2 \mu_5} \Pi^{\mu_4 \mu_6} + a_{13,26}^{(9)} (\Pi^{\mu_1 \mu_3} \Pi^{\mu_2 \mu_6} \Pi^{\mu_4 \mu_5} + \Pi^{\mu_1 \mu_5} \Pi^{\mu_2 \mu_3} \Pi^{\mu_4 \mu_6}) + a_{14,25}^{(9)} \Pi^{\mu_1 \mu_4} \Pi^{\mu_2 \mu_5} \Pi^{\mu_3 \mu_6} + \\
 & + a_{14,26}^{(9)} (\Pi^{\mu_1 \mu_4} \Pi^{\mu_2 \mu_6} \Pi^{\mu_3 \mu_5} + \Pi^{\mu_1 \mu_5} \Pi^{\mu_2 \mu_4} \Pi^{\mu_3 \mu_6}) + a_{15,26}^{(9)} \Pi^{\mu_1 \mu_5} \Pi^{\mu_2 \mu_6} \Pi^{\mu_3 \mu_4} + a_{16,23}^{(9)} \Pi^{\mu_1 \mu_6} \Pi^{\mu_2 \mu_3} \Pi^{\mu_4 \mu_5} + \\
 & + a_{16,24}^{(9)} \Pi^{\mu_1 \mu_6} \Pi^{\mu_2 \mu_4} \Pi^{\mu_3 \mu_5} + a_{16,25}^{(9)} \Pi^{\mu_1 \mu_6} \Pi^{\mu_2 \mu_5} \Pi^{\mu_3 \mu_4} + b_{12,36}^{(9)} (\Pi^{\mu_1 \mu_2} \Pi^{\mu_3 \mu_6} v^{\mu_4} v^{\mu_5} + \Pi^{\mu_1 \mu_4} \Pi^{\mu_5 \mu_6} v^{\mu_2} v^{\mu_3}) + \\
 & + b_{12,46}^{(9)} (\Pi^{\mu_1 \mu_2} \Pi^{\mu_4 \mu_6} v^{\mu_3} v^{\mu_5} + \Pi^{\mu_1 \mu_3} \Pi^{\mu_5 \mu_6} v^{\mu_2} v^{\mu_4}) + b_{12,56}^{(9)} \Pi^{\mu_1 \mu_2} \Pi^{\mu_5 \mu_6} v^{\mu_3} v^{\mu_4} + \\
 & + b_{13,26}^{(9)} (\Pi^{\mu_1 \mu_3} \Pi^{\mu_2 \mu_6} v^{\mu_4} v^{\mu_5} + \Pi^{\mu_1 \mu_5} \Pi^{\mu_4 \mu_6} v^{\mu_2} v^{\mu_3}) + b_{13,46}^{(9)} \Pi^{\mu_1 \mu_3} \Pi^{\mu_4 \mu_6} v^{\mu_2} v^{\mu_5} + \\
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 & + b_{16,23}^{(9)} (\Pi^{\mu_1 \mu_6} \Pi^{\mu_2 \mu_3} v^{\mu_4} v^{\mu_5} + \Pi^{\mu_1 \mu_6} \Pi^{\mu_4 \mu_5} v^{\mu_2} v^{\mu_3}) + b_{16,24}^{(9)} (\Pi^{\mu_1 \mu_6} \Pi^{\mu_2 \mu_4} v^{\mu_3} v^{\mu_5} + \Pi^{\mu_1 \mu_6} \Pi^{\mu_3 \mu_5} v^{\mu_2} v^{\mu_4}) + \\
 & + b_{16,25}^{(9)} \Pi^{\mu_1 \mu_6} \Pi^{\mu_2 \mu_5} v^{\mu_3} v^{\mu_4} + b_{16,34}^{(9)} \Pi^{\mu_1 \mu_6} \Pi^{\mu_3 \mu_4} v^{\mu_2} v^{\mu_5} + c^{(9)} \Pi^{\mu_1 \mu_6} v^{\mu_2} v^{\mu_3} v^{\mu_4} v^{\mu_5}
 \end{aligned}$$

- There are also multiple color structures

Arise from combining pure color octets:

$$[iD^{\mu_i}, iD^{\mu_j}], \quad [iD^{\mu_i}, [iD^{\mu_j}, iD^{\mu_k}]], \quad [iD^{\mu_i}, [iD^{\mu_j}, [iD^{\mu_k}, iD^{\mu_l}]]]$$

For phenomenological applications at the current level of precision only $T^a T^b$ is needed

New Result: Moments of the leading power shape function

- The shape function is important for charmless inclusive B decays
Its moments are related to HQET parameters
The matrix elements decomposition makes their calculation easy

$$2M_B \int d\omega \omega^k S(\omega) = n_{\mu_1} \dots n_{\mu_k} \langle \bar{B}(v) | \bar{h} iD^{\mu_1} \dots iD^{\mu_k} h | \bar{B}(v) \rangle$$

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$$\int d\omega S(\omega) = 1, \quad \int d\omega \omega S(\omega) = 0, \quad \int d\omega \omega^2 S(\omega) = -a^{(5)} = -\lambda_1/3,$$

$$\int d\omega \omega^3 S(\omega) = -a^{(6)} = -\rho_1/3,$$

$$\int d\omega \omega^4 S(\omega) = a_{12}^{(7)} + a_{13}^{(7)} + a_{14}^{(7)} - b^{(7)} = m_1/5 - m_2/3,$$

$$\int d\omega \omega^5 S(\omega) = 2a_{12}^{(8)} + 2a_{13}^{(8)} + 2a_{15}^{(8)} + b_{12}^{(8)} + b_{14}^{(8)} + b_{15}^{(8)} - c^{(8)} = \\ = (-8r_1 + 2r_2 + 2r_3 + 2r_4 + r_5 + r_6 + r_7) / 15,$$

$$\int d\omega \omega^6 S(\omega) = -a_{12,34}^{(9)} - 2a_{12,35}^{(9)} - 2a_{12,36}^{(9)} - a_{13,25}^{(9)} - 2a_{13,26}^{(9)} - a_{14,25}^{(9)} - 2a_{14,26}^{(9)} - a_{15,26}^{(9)} \\ - a_{16,23}^{(9)} - a_{16,24}^{(9)} - a_{16,25}^{(9)} + 2b_{12,36}^{(9)} + 2b_{12,46}^{(9)} + b_{12,56}^{(9)} + 2b_{13,26}^{(9)} + b_{13,46}^{(9)} \\ + 2b_{14,26}^{(9)} + b_{14,36}^{(9)} + b_{15,26}^{(9)} + 2b_{16,23}^{(9)} + 2b_{16,24}^{(9)} + b_{16,25}^{(9)} + b_{16,34}^{(9)} - c^{(9)}$$

Comparison to Hilbert Series method

- Kobach and Pal have used the Hilbert series method to find the NRQED and NRQCD/HQET operators with $\dim \leq 8$
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- Still, a useful check

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SI (SD) matrix elements are sym. (anti-sym.) under inversion
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- Also, in the last 2 years we have learned that the structure of effective field theories is simpler than we think:
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Thank you!