Semi-leptonic B and B_s meson decays

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Lattice meets Continuum
Siegen, September 18, 2017

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introduction

Processes of interest

- Initial state is a pseudoscalar B or B_s meson
  $B^+ = (u\bar{b})$, $B^- = (\bar{u}b)$, $B^0 = (d\bar{b})$ and $\overline{B}^0 = (\bar{d}b)$ with mass ∼5280 GeV
  $B_s^0 = (s\bar{b})$ and $\overline{B_s}^0 = (\bar{s}b)$ with mass ∼5367 GeV

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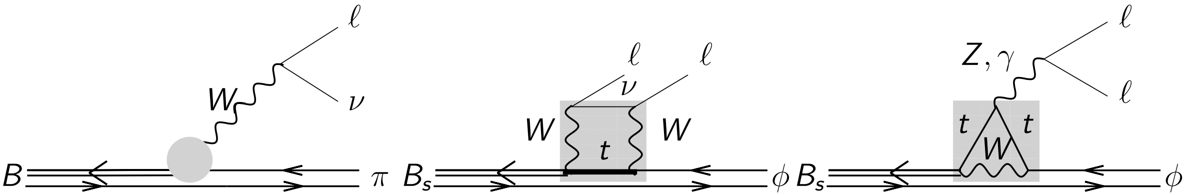
- Weak decays of the b-quark
  → Charged flavor changing currents mediated by $W^\pm$ (tree-level)
  → Flavor changing neutral currents (loop-level)



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- Weak decays of the b-quark
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  - → Flavor changing neutral currents (loop-level)
- Suppressed in the Standard Model
  - → CKM suppressed
  - → GIM suppressed (no FCNC)

$$\begin{bmatrix} d' \\ s' \\ b' \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} d \\ s \\ b \end{bmatrix} \quad \text{with} \quad \begin{bmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{bmatrix} = \begin{bmatrix} 0.97434 & 0.22506 & 0.00357 \\ 0.22492 & 0.97351 & 0.0411 \\ 0.00875 & 0.0403 & 0.99915 \end{bmatrix}$$

[PDG 2016]

Processes of interest

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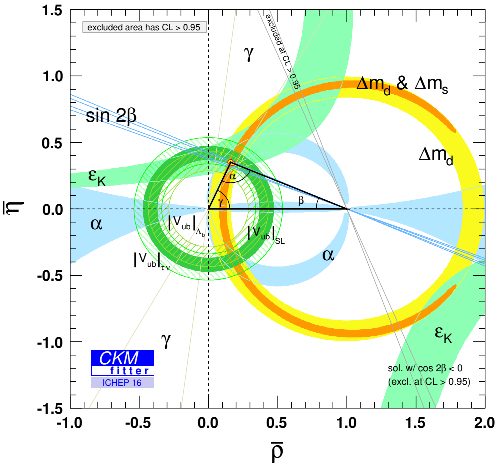
- Nonperturbative calculation of form factors
  → Exclusive semi-leptonic decays with one hadronic final state
  → Pseudoscalar or vector (narrow width approximation) final states
  → Only short distance contributions

Why are we interested in rare B decays?

- Charged current decays allow to determine CKM matrix elements $|V_{cb}|$ and $|V_{ub}|$
  - → Test unitarity of the CKM matrix
  - → Precision tests of the Standard Model
- Searches for / constraints on new physics
- Test of lepton flavor universality violation

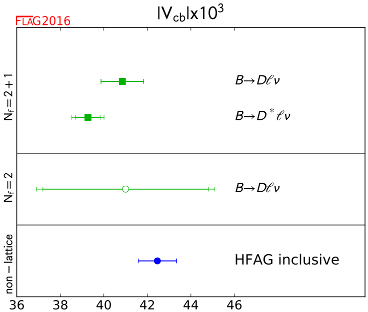


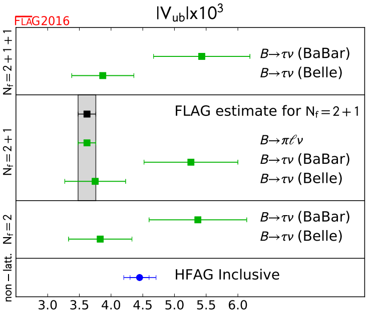
[http://ckmfitter.in2p3.fr]

Determination of $|V_{cb}|$ and $|V_{ub}|$

- Commonly $|V_{cb}|$ extracted from $B \to D^{(*)}\ell\nu$ and $|V_{ub}|$ extracted from $B \to \pi\ell\nu$
- Long standing tension between exclusive and inclusive determinations
  - → Revisit HQET constraints entering z-parametrizations

    [Bigi, Gambino PRD94 (2016) 094008][Bigi, Gambino, Schacht PLB769 (2017) 441-445]





[FLAG2016]

[Fermilab/MILC PRD92 (2015) 034506] [HPQCD PRD92 (2015) 054510]
[Fermilab/MILC PRD89 (2014)114504]
[Atoui et al. EPJC74 (2014) 2861]

[HPQCD PRD75 (2006)119906]
[Fermilab/MILC PRD92 (2015) 014024]
[RBC-UKQCD PRD91 (2015) 074510]

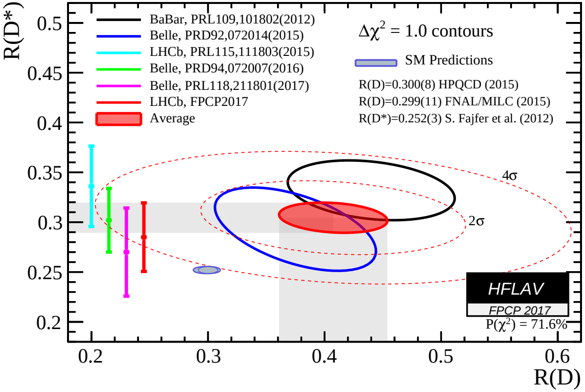
Searches for new physics (tree-level)

- Lepton flavor universality violations in $\mathcal{R}_D$ ratios

$$\mathcal{R}^{\tau/\mu}_{D^{(*)}} \equiv \frac{d\Gamma(B \to D^{(*)}\tau\nu_\tau)/d^2_q}{d\Gamma(B \to D^{(*)}\mu\nu_\mu)/d^2_q}$$

- Input: form factors over full q^2 range
  - → $B \to D\ell\nu$ [HPQCD PRD92 (2015) 054510] [Fermilab/MILC PRD92 (2015) 035606]
  - → $B \to D^*\ell\nu$ [Fajfer, Kamenik, Nisandzic PRD85 (2012) 094025]
  - → Shown theoretical uncertainty on $B \to D^*\ell\nu$ is suspiciously small

[HFLAV website]



Searches for new physics (loop-level)

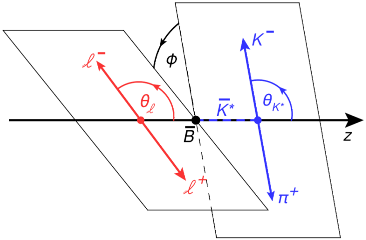
- $b \to s\ell^+\ell^-$ processes e.g. $B \to \pi\ell^+\ell^-, B \to K^{(*)}\ell^+\ell^-$, $B_s \to \phi\ell^+\ell^-$
- Decomposition into angular variables e.g. $B \to K^*\ell^+\ell^-$

$$\frac{1}{d(\Gamma+\bar{\Gamma})/dq^2}\frac{d^4q(\Gamma+\bar{\Gamma})}{dq^2 d\vec{\Omega}} = \frac{9}{32\pi}\Big[\frac{3}{4}(1-F_L)\sin^2\theta_K + F_L\cos^2\theta_K$$

$$+\frac{1}{4}(1-F_L)\sin^2\theta_K\cos 2\theta_l - F_L\cos^2\theta_K\cos 2\theta_l + S_3\sin^2\theta_l\cos 2\phi$$

$$+ S_4\sin 2\theta_K\sin 2\theta_l\cos\phi + S_5\sin 2\theta_K\sin\theta_l\cos\phi + \frac{4}{3}A_{FB}\sin^2\theta_K\cos\theta_l$$

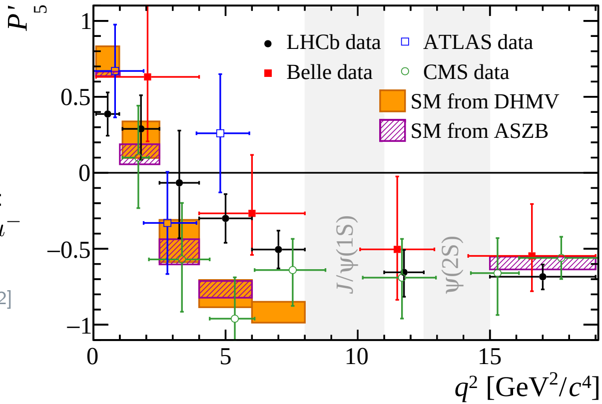
$$+ S_7\sin 2\theta_K\sin\theta_l\sin\phi + S_8\sin 2\theta_K\sin 2\theta_l\sin\phi + S_9\sin^2\theta_K\sin^2\theta_l\sin 2\phi\Big]$$



- F_L, A_{FB}, S_i are functions of Wilson coefficients ⇒ sensitive to new physics
- To reduce hadronic uncertainties, introduce $P'_{i=4,5,6,8} = -\frac{S_{j=4,5,7.8}}{\sqrt{F_L(1-F_L)}}$

Searches for new physics (loop-level)

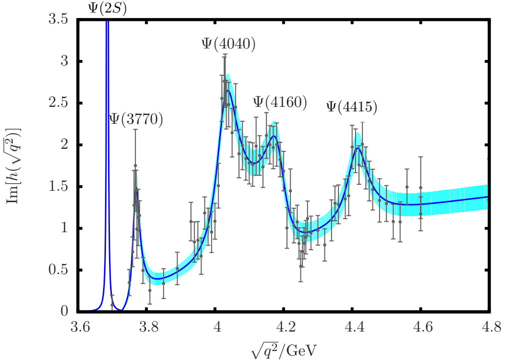
- Few sigma deviations from SM expectations seen for branching fractions and angular observables $3\,\mathrm{GeV}^2 \lesssim q^2 \lesssim 8\,\mathrm{GeV}^2$
- LHCb reported deviations for different processes: $B^0 \to K^0\mu^+\mu^-$, $B^+ \to K^+\mu^+\mu^-$, $B^0 \to K^*\mu^+\mu^-$ $B_s \to \phi\mu^+\mu^-$, $\Lambda_b \to \Lambda\mu^+\mu^-$
  [JHEP 06 (2014) 133][JHEP 11 (2016) 047][JHEP 04 (2017) 142]
  [JHEP 06 (2015) 115][JHEP 09 (2015) 179]
- Deviations seen by ATLAS, CMS, LHCb, Belle
- Hinting at new physics in Wilson coefficient C_9?
- Near J/Ψ resonance:
  $\to$ Are hadronic uncertainties under control?

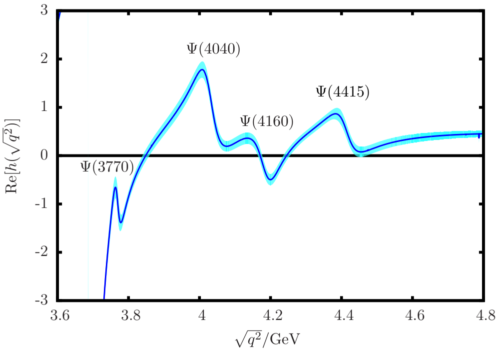


[LHCb JHEP 02 (2016) 104] [Belle PRL118 (2017) 111801]
[ATLAS-CONF-2017-023] [CMS-PAS-BPH-15-008]
[DHMV JHEP 12 (2014) 125, JHEP 10 (2016) 075]
[ASZB JHEP 08 (2016) 098, EPJC 75 (2015) 382]
plot: [Gershon arXiv:1707.05290]

Searches for new physics (loop-level)

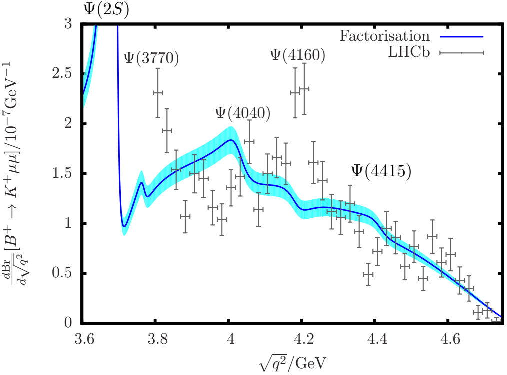
- Hadronic uncertainties: charm resonances [Lyon and Zwicky, arXiv:1406.0566]
  - → SM predictions rely on factorization approximation (FA)
  - → In the FA, charm-resonance contributions equal charm vacuum polarization
  - → Extract charm vacuum polarization via dispersion relation from BESII-data ($e^+e^- \to$ hadrons)





Searches for new physics (loop-level)

- Hadronic uncertainties: charm resonances [Lyon and Zwicky, arXiv:1406.0566]
  - → Derived SM prediction for $B \to K\ell\ell$ based on FA and lattice QCD [HPQCD PRD88 (2013) 054509] disagrees with LHCb data

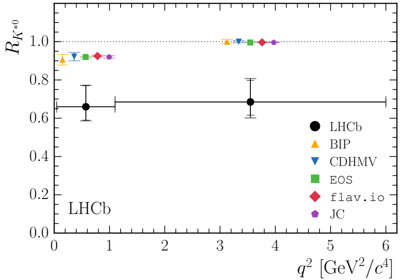


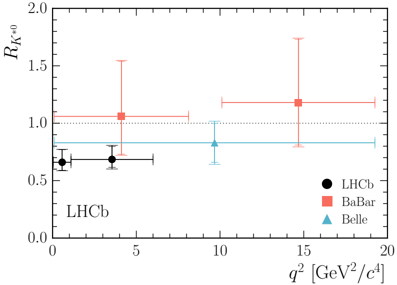
- Explore experimentally e.g. phase difference between short- and long-distance amplitude [LHCb EPJC77 (2017) 161]

Searches for new physics (loop-level)

- Lepton flavor universality violations in $\mathcal{R}_K$ ratios

$$\mathcal{R}_{K^*}^{\mu/e} \equiv \frac{d\Gamma(B^0 \to K^{*0}\mu^+\mu^-)/d_q^2}{d\Gamma(B^0 \to K^{*0}e^+e^-)/d_q^2}$$





[BIP EPJC76 (2016) 440]
[CDHMV JHEP 10 (2016) 075]
[EOS PRD95 (2017) 035029]
[flav.io JHEP08 (2016) 098]
[JC PRD93 (2016) 014028]

[BaBar PRD86 (2012) 032012]
[Belle PRL103 (2009) 171801]
[LHCb JHEP 08 (2017) 055]

plots: [LHCb JHEP 08 (2017) 055]

b-quarks

An additional challenge

- Masses: *b*-quark 4.18 GeV whereas *d*-quark 4.7 MeV
  - $\Rightarrow$ *b*-quark $\sim$ 1000 times heavy than *d*-quark
  - $\Rightarrow$ Mass of *b*-quark larger than cutoff (a^{-1})
- Simulate *b*-quark with effective action
  - $\rightarrow$ Requires renormalization of mixed action
  - $\rightarrow$ Fermilab-action/RHQ, NRQCD, HQET
- Extrapolate to physical *b*-quark
  - $\rightarrow$ allows for full nonperturbative renormalization
  - $\rightarrow$ ETMC ratio method, heavy HISQ, heavy DWF
- Similar considerations for *c*-quark (1.28 GeV)

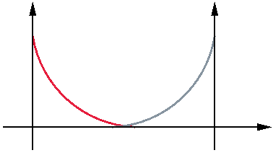
RHQ action

- Relativistic Heavy Quark action developed by Christ, Li, and Lin
  [Christ et al. PRD 76 (2007) 074505], [Lin, Christ PRD 76 (2007) 074506]
- Builds upon Fermilab approach [El-Khadra et al. PRD 55 (1997) 3933]
- Closely related to the Tsukuba formulation [S. Aoki et al. PTP 109 (2003) 383]

- Allows to tune the three parameters ($m_0 a$, c_P, ζ) nonperturbatively [PRD 86 (2012) 116003]
- Heavy quark mass is treated to all orders in $(m_b a)^n$
- Expand in powers of the spatial momentum through $O(\vec{p}a)$
  - → Resulting errors will be of $O(\vec{p}^2 a^2)$
  - → Allows computation of heavy-light quantities with discretization errors of the same size as in light-light quantities
- Applies for all values of the quark mass and as a smooth continuum limit

- Recently re-tuned to account for updated values of a^{-1}

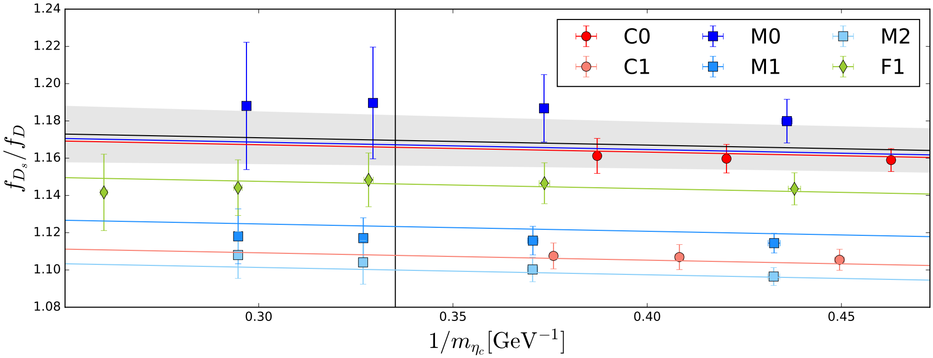
Heavy Möbius domain-wall fermions



- Domain-wall fermions [Kaplan PLB 288 (1992) 342] [Shamir NPB 406 (1993) 90]
  - → 5 dimensional formulation
  - → Perfect chirality for $L_s \to \infty$
  - → Residual chiral symmetry breaking: m_{res}
- Möbius domain-wall fermions [Brower, Neff Orginos, arXiv:1206.5214]: smaller m_{res} for same L_s
- Möbius DWF optimized for heavy quarks [Boyle et al. JHEP 1604 (2016) 037]
  - → Discretization errors well under control for $am_c \lesssim 0.4$
  - → Small, benign extrapolation of 3 charm-like masses for $a^{-1} = 1.784$ GeV
  - → Safe interpolations for $a^{-1} = 2.383$ GeV and 2.774 GeV
- Smeared Möbius domain-wall fermions [Suzuki et al. PoS LATTICE2015 (2016) 337]]
  - → Allows to reach larger am_c values

Heavy Möbius domain-wall fermions

- Example for extra-/interpolation: f_{D_s}/f_D [Boyle et al., arXiv:1701.02644]



Our set-up

- RBC-UKQCD's 2+1 flavor domain-wall fermion and Iwasaki gauge action ensembles
  - → Three lattice spacings $a \sim 0.11$ fm, 0.08 fm, 0.07 fm
    [PRD 78 (2008) 114509][PRD 83 (2011) 074508][PRD 93 (2016) 074505][arXiv:1701.02644]
- Unitary and partially quenched domain-wall up/down quarks
  [Kaplan PLB 288 (1992) 342], [Shamir NPB 406 (1993) 90]
  - → Domain-wall strange quarks at/near the physical value
  - → One ensemble with physical pions
- Charm: Möbius domain-wall fermions optimized for heavy quarks [Boyle et al. JHEP 1604 (2016) 037]
  - → Simulate 3 or 2 charm-like masses then extrapolate/interpolate
- Effective relativistic heavy quark (RHQ) action for bottom quarks
  [Christ et al. PRD 76 (2007) 074505], [Lin and Christ PRD 76 (2007) 074506]

2+1 Flavor Domain-Wall Iwasaki ensembles

L	a^{-1}(GeV)	am_l	am_s	M_π(MeV)	# configs.	#sources	
24	1.784	0.005	0.040	338	1636	1	[PRD 78 (2008) 114509]
24	1.784	0.010	0.040	434	1419	1	[PRD 78 (2008) 114509]
32	2.383	0.004	0.030	301	628	2	[PRD 83 (2011) 074508]
32	2.383	0.006	0.030	362	889	2	[PRD 83 (2011) 074508]
32	2.383	0.008	0.030	411	544	2	[PRD 83 (2011) 074508]
48	1.730	0.00078	0.0362	139	40	81/1⋆	[PRD 93 (2016) 074505]
64	2.359	0.000678	0.02661	139	—	—	[PRD 93 (2016) 074505]
48	2.774	0.002144	0.02144	234	70	24	[arXiv:1701.02644]

⋆ All mode averaging: 81 "sloppy" and 1 "exact" solve [Blum et al. PRD 88 (2012) 094503]

- Lattice spacing determined from combined analysis [Blum et al. PRD 93 (2016) 074505]
- a: ~ 0.11 fm, ~ 0.08 fm, ~ 0.07 fm

Target quantities

- Decay constants f_B and f_{B_s}
- $B^0 - \overline{B}^0$ mixing matrix elements
- Semi-leptonic form factors with charged and neutral flavor changing currents

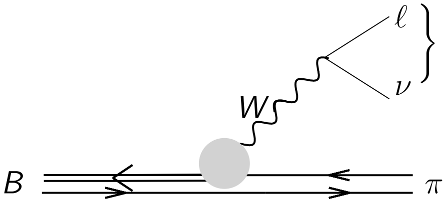
$$B \to \pi\ell\nu,\ B_s \to K\ell\nu,\ B \to D^{(*)}\ell\nu,\ B_s \to D_s^{(*)}\ell\nu,\ \ldots$$

$$B \to K^{(*)}\ell^+\ell^-,\ B_s \to \phi\ell^+\ell^-,\ \ldots$$

$\to$ Ratios $R(D^{(*)})$, $R(K^{(*)})$, ...

charged current decays

$|V_{ub}|$ from exclusive semi-leptonic $B \to \pi\ell\nu$ decay



$$q^2 = M_B^2 + M_\pi^2 - 2M_B E_\pi$$

- Conventionally parametrized by (neglecting term $\propto m_\ell^2 f_0^2$)

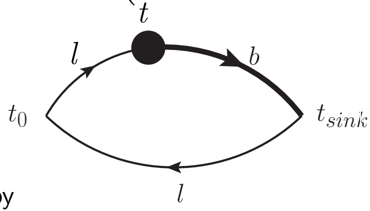
$$\frac{d\Gamma(B\to\pi\ell\nu)}{dq^2} = \frac{G_F^2}{192\pi^3 M_B^3}\left[\left(M_B^2 + M_\pi^2 - q^2\right)^2 - 4M_B^2 M_\pi^2\right]^{3/2} \times |f_+(q^2)|^2 \times |V_{ub}|^2$$

experiment known nonperturbative input CKM

$B \to \pi\ell\nu$ form factors

- Parametrize the hadronic matrix element for the flavor changing vector current in terms of the form factors $f_+(q^2)$ and $f_0(q^2)$

$$\langle\pi(k)|\bar{u}\gamma^\mu b|B(p)\rangle = f_+(q^2)\left(p^\mu + k^\mu - \frac{M_B^2 - M_\pi^2}{q^2}q^\mu\right) + f_0(q^2)\frac{M_B^2 - M_\pi^2}{q^2}q^\mu$$



- Calculate 3-point function by
  - → Inserting a quark source for a "light" propagator at t_0
  - → Allow it to propagate to t_{sink}, turn it into a sequential source for a b quark
  - → Use another "light" quark propagating from t_0 and contract both at t

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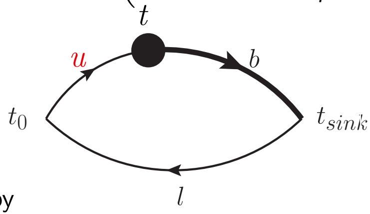
$$\langle \pi(k) | \bar{u} \gamma^\mu b | B(p) \rangle = f_+(q^2) \left(p^\mu + k^\mu - \frac{M_B^2 - M_\pi^2}{q^2} q^\mu \right) + f_0(q^2) \frac{M_B^2 - M_\pi^2}{q^2} q^\mu$$



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  - → Allow it to propagate to t_{sink}, turn it into a sequential source for a b quark
  - → Use another "light" quark propagating from t_0 and contract both at t
- On the lattice u and d quarks are degenerate (l); physically the daughter quark is a u-quark

Relating form factors f_+ and f_0 to $f_\parallel$ and $f_\perp$

- On the lattice we prefer using the B-meson rest frame and compute

$$f_\parallel(E_\pi) = \langle\pi|V^0|B\rangle/\sqrt{2M_B} \quad \text{and} \quad f_\perp(E_\pi)p_\pi^i = \langle\pi|V^i|B\rangle/\sqrt{2M_B}$$

- Both are related by

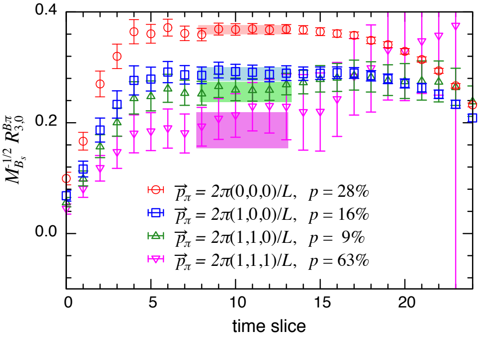
$$f_0(q^2) = \frac{\sqrt{2M_B}}{M_B^2 - M_\pi^2}\left[(M_B - E_\pi)f_\parallel(E_\pi) + (E_\pi^2 - M_\pi^2)f_\perp(E_\pi)\right]$$

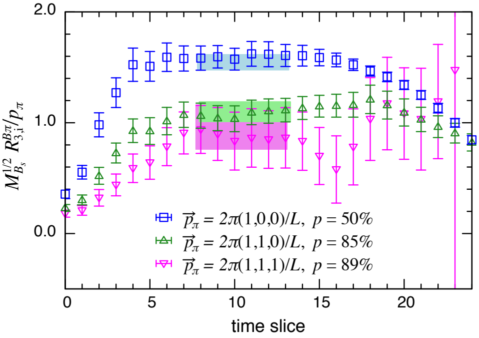
$$f_+(q^2) = \frac{1}{\sqrt{2M_B}}\left[f_\parallel(E_\pi) + (M_B - E_\pi)f_\perp(E_\pi)\right]$$

Lattice results for form factors $f_{\|}$ and $f_{\perp}$ [PRD 91 (2015) 074510]

$$f_{\|} = \lim_{t,T\to\infty} R_0^{B\to\pi}(t,T) \qquad f_{\perp} = \lim_{t,T\to\infty} \frac{1}{p_\pi^i} R_i^{B\to\pi}(t,T)$$

$$R_\mu^{B\to\pi}(t,T) = \frac{C_{3,\mu}^{B\to\pi}(t,T)}{C_2^\pi(t)C_2^B(T-t)}\sqrt{\frac{2E_\pi}{e^{-E_\pi t}e^{-M_B(T-t)}}}$$



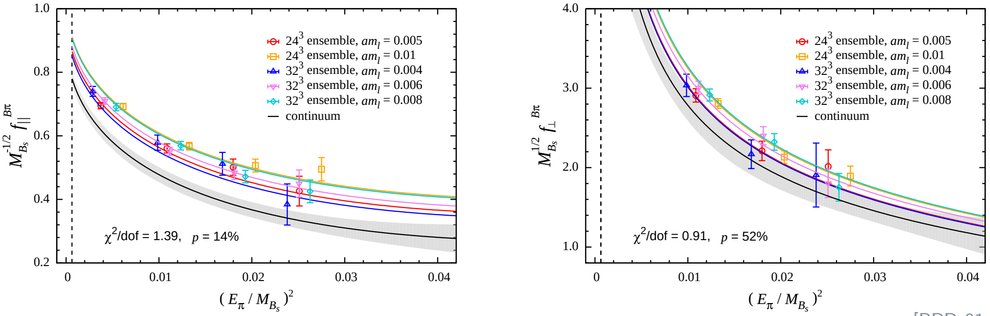


Chiral-continuum extrapolation using SU(2) hard-pion χPT

$$f_{\parallel}(M_\pi, E_\pi, a^2) = c_{\parallel}^{(1)} \left[1 + \left(\frac{\delta f_{\parallel}}{(4\pi f)^2} + c_{\parallel}^{(2)} \frac{M_\pi^2}{\Lambda^2} + c_{\parallel}^{(3)} \frac{E_\pi}{\Lambda} + c_{\parallel}^{(4)} \frac{E_\pi^2}{\Lambda^2} + c_{\parallel}^{(5)} \frac{a^2}{\Lambda^2 a_{32}^4}\right)\right]$$

$$f_{\perp}(M_\pi, E_\pi, a^2) = \frac{1}{E_\pi + \Delta} c_{\perp}^{(1)} \left[1 + \left(\frac{\delta f_{\perp}}{(4\pi f)^2} + c_{\perp}^{(2)} \frac{M_\pi^2}{\Lambda^2} + c_{\perp}^{(3)} \frac{E_\pi}{\Lambda} + c_{\perp}^{(4)} \frac{E_\pi^2}{\Lambda^2} + c_{\perp}^{(5)} \frac{a^2}{\Lambda^2 a_{32}^4}\right)\right]$$

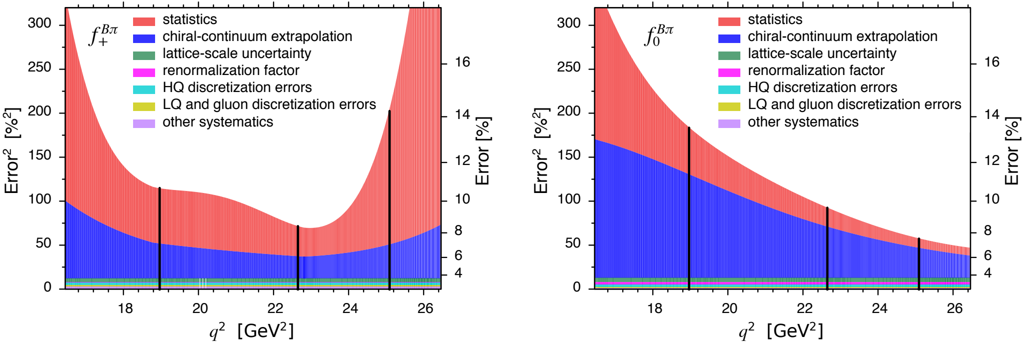
with δf non-analytic logs of the pion mass and hard-pion limit is taken by $\frac{M_\pi}{E_\pi} \to 0$



[PRD 91 (2015) 074510]

Obtaining form factors f_+ and f_0 [PRD 91 (2015) 074510]

- Extract $f_\parallel$ and $f_\perp$ for three different q^2 values (synthetic data points)
- Estimate all systematic errors and them add in quadrature
- Convert results to f_+ and f_0



z-expansion [PRD 91 (2015) 074510]

- Use the model-independent z-expansion fit to extrapolate lattice results to the full kinematic range [Boyd, Grinstein, Lebed, PRL 74 (1995) 4603] [Bourrely, Caprini, Lellouch, PRD 79 (2009) 013008]

$$z(q^2, t_0) = \frac{\sqrt{1-q^2/t_+}-\sqrt{1-t_0/t_+}}{\sqrt{1-q^2/t_+}+\sqrt{1-t_0/t_+}}$$

with $t_\pm = (M_B \pm M_\pi)^2$ and $t_0 \equiv t_{\text{opt}} = (M_B + M_\pi)(\sqrt{M_B} - \sqrt{M_\pi})^2$

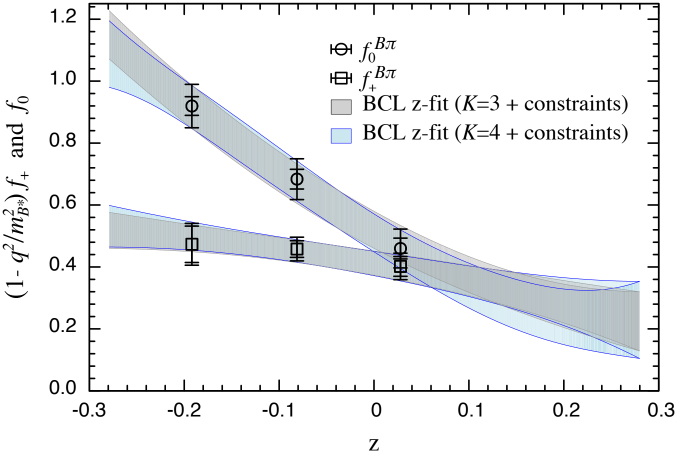
→ Minimizes the magnitude of z in the semi-leptonic region: $|z| \leq 0.279$

→ $f_0(q^2)$ is analytic in the semi-leptonic region except at the B^* pole

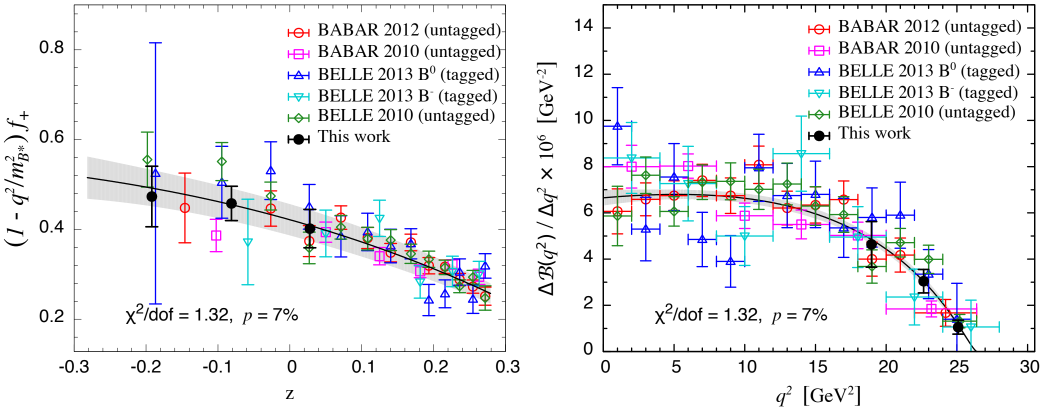
- Express $f_+(q^2)$ as convergent power series $f_+(q^2) = \frac{1}{1-q^2/M_{B^*}^2} \sum_{k=0}^{K-1} b_+^{(k)} \left[z^k - (-1)^{k-K} \frac{k}{K} z^k\right]$
- Use functional form for $f_0(q^2) = \sum_{k=0}^{K-1} b_0^{(k)} z^k$

- Exploit the kinematic constraint $f_+(q^2 = 0) = f_0(q^2 = 0)$
- Use HQ power counting to constrain size of the f_+ coefficients

z-expansion fit [PRD 91 (2015) 074510]



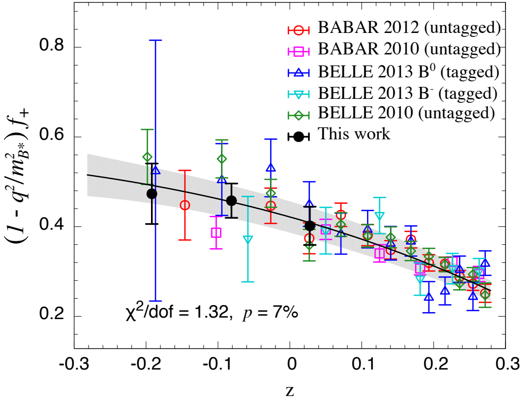
Combine with experimental data to determine $|V_{ub}|$ [PRD 91 (2015) 074510]

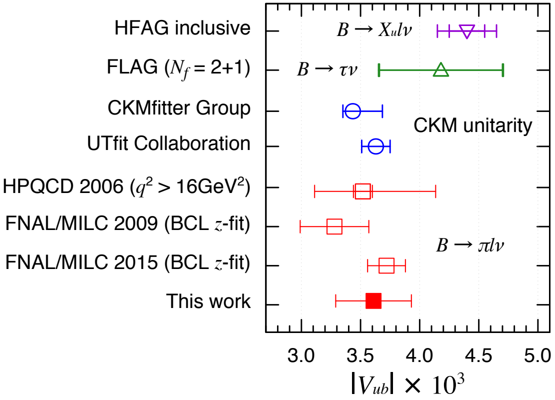


▶ Result: $|V_{ub}| = 3.61(32) \cdot 10^{-3}$

Comparison with other determinations

[PRD 91 (2015) 074510]





- Result: $|V_{ub}| = 3.61(32) \cdot 10^{-3}$
- Exhibits 2σ tension to inclusive results

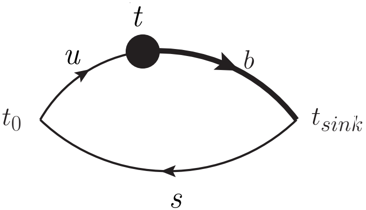
Alternative determinations of $|V_{cb}|$ and $|V_{ub}|$

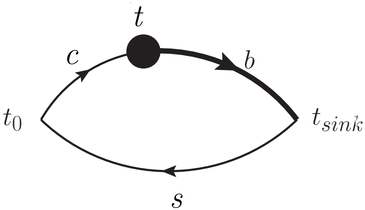
- $|V_{ub}|$ from $B \to \tau\nu$: errors too large
- Use lattice techniques to compute inclusive decays (→ Talk by Shoji Hashimoto)
  [Hashimoto PTEP 2017 (2017) 053B03] [Hansen, Meyer, Robaina arXiv:1704.08993]
- $|V_{cb}|/|V_{ub}|$ from exclusive baryonic decays: $\Lambda_b \to \Lambda_c \ell\nu$ and $\Lambda_b \to p\ell\nu$ (→ Talk by Stefan Meinel)
  [Detmold, Lehner, Meinel, PRD92 (2015) 034503]
- $|V_{cb}|$ from $B_s \to D_s \ell\nu$ and $|V_{ub}|$ from $B_s \to K\ell\nu$
  → B-factories typically run at the $\Upsilon(4s)$ threshold i.e. B but no B_s mesons are produced
  → Not (yet) experimentally measured with sufficient precision
  → LHC energies are large enough to produce sufficient B_s mesons → LHCb
  → Absolute normalization is challenging; ratios are preferred: determine $|V_{cb}|/|V_{ub}|$

$B_s \to K\ell\nu$ and $B_s \to D_s\ell\nu$ form factors

- Parametrize the hadronic matrix element for the flavor changing vector current V^μ in terms of the form factors $f_+(q^2)$ and $f_0(q^2)$

$$\langle K|V^\mu|B_s\rangle = f_+(q^2)\left(p^\mu_{B_s} + p^\mu_K - \frac{M^2_{B_s} - M^2_K}{q^2} q^\mu\right) + f_0(q^2)\frac{M^2_{B_s} - M^2_K}{q^2} q^\mu$$



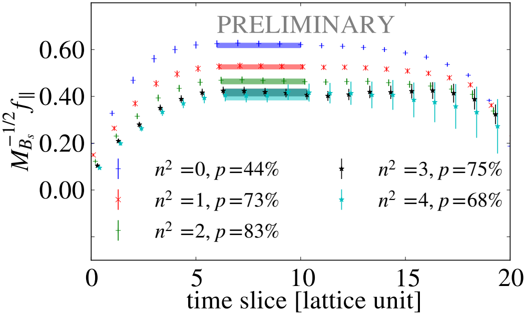


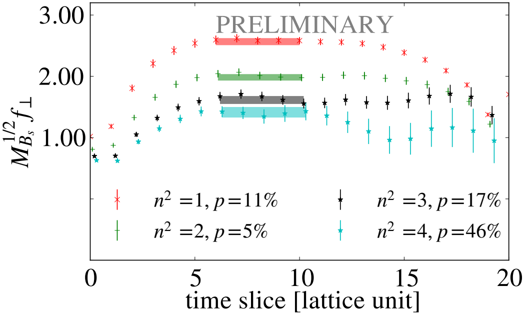
$$\langle D_s|V^\mu|B_s\rangle = f_+(q^2)\left(p^\mu_{B_s} + p^\mu_{D_s} - \frac{M^2_{B_s} - M^2_{D_s}}{q^2} q^\mu\right) + f_0(q^2)\frac{M^2_{B_s} - M^2_{D_s}}{q^2} q^\mu$$

Lattice results for form factors $f_\parallel$ and $f_\perp$ for $B_s \to K\ell\nu$

$$R_\mu^{B_s\to K}(t, t_{\text{sink}}) = \frac{C_{3,\mu}^{B_s\to K}(t, t_{\text{sink}})}{C_2^K(t)\, C_2^{B_s}(t_{\text{sink}} - t)} \sqrt{\frac{4 M_{B_s} E_K}{e^{-E_k t} e^{-M_{B_s}(t_{\text{sink}} - t)}}}$$

$$f_\parallel = \lim_{t, t_{\text{sink}}\to\infty} R_0^{B_s\to K}(t, t_{\text{sink}}) \qquad f_\perp = \lim_{t, t_{sink}\to\infty} \frac{1}{p_\pi^i} R_i^{B_s\to K}(t, t_{\text{sink}})$$



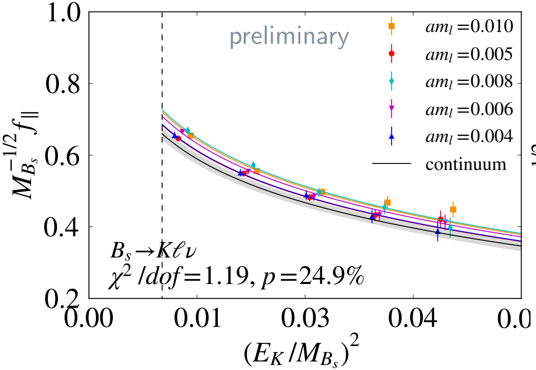


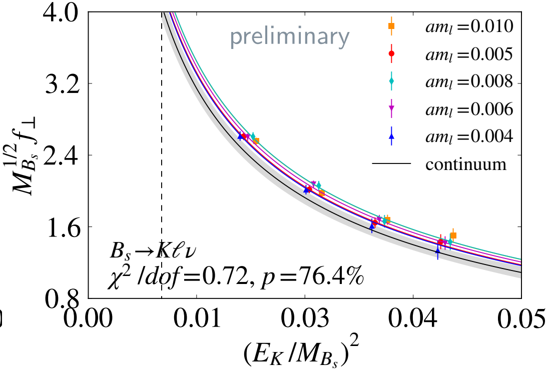
Chiral-continuum extrapolation using SU(2) hard-kaon χPT

$$f_\parallel(M_K, E_K, a^2) = \frac{1}{E_K+\Delta} c_\parallel^{(1)} \left[1 + \left(\frac{\delta f_\parallel}{(4\pi f)^2} + c_\parallel^{(2)} \frac{M_K^2}{\Lambda^2} + c_\parallel^{(3)} \frac{E_K}{\Lambda} + c_\parallel^{(4)} \frac{E_K^2}{\Lambda^2} + c_\parallel^{(5)} \frac{a^2}{\Lambda^2 a_{32}^4}\right)\right]$$

$$f_\perp(M_K, E_K, a^2) = \frac{1}{E_K+\Delta} c_\perp^{(1)} \left[1 + \left(\frac{\delta f_\perp}{(4\pi f)^2} + c_\perp^{(2)} \frac{M_K^2}{\Lambda^2} + c_\perp^{(3)} \frac{E_K}{\Lambda} + c_\perp^{(4)} \frac{E_K^2}{\Lambda^2} + c_\perp^{(5)} \frac{a^2}{\Lambda^2 a_{32}^4}\right)\right]$$

with δf non-analytic logs of the kaon mass and hard-kaon limit is taken by $M_K/E_K \to 0$





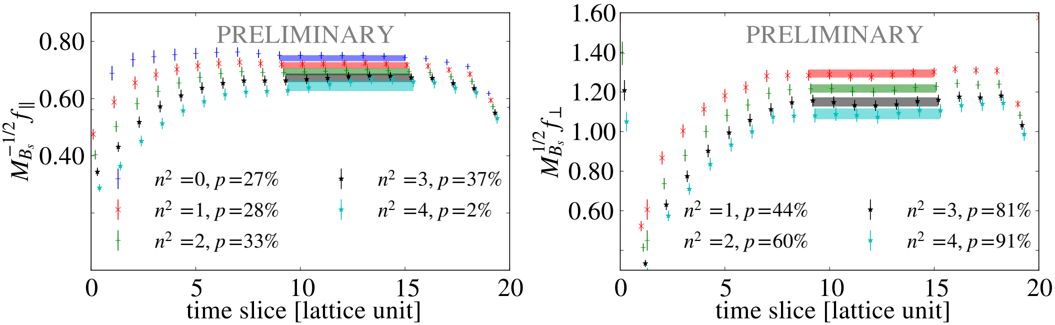
Next steps

- Analyze data at third, finer lattice spacing
- Estimate full systematic errors for three "synthetic" data points
- Perform z-expansion and polynomial fits
- Comparison with other result(s) [HPQCD PRD90 (2014) 054506]

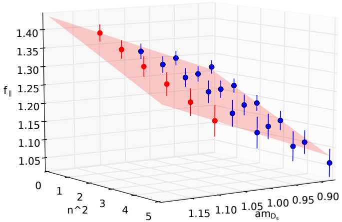
Lattice results for form factors $f_{\parallel}$ and $f_{\perp}$ for $B_s \to D_s \ell \nu$

$$R_{\mu}^{B_s \to D_s}(t, t_{\text{sink}}) = \frac{C_{3,\mu}^{B_s \to D_s}(t, t_{\text{sink}})}{C_2^{D_s}(t) C_2^{B_s}(t_{\text{sink}} - t)} \sqrt{\frac{4 M_{B_s} E_{D_s}}{e^{-E_{D_s} t} e^{-M_{B_s}(t_{\text{sink}} - t)}}}$$

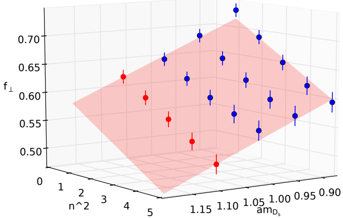
$$f_{\parallel} = \lim_{t, t_{\text{sink}} \to \infty} R_0^{B_s \to D_s}(t, t_{\text{sink}}) \qquad f_{\perp} = \lim_{t, t_{sink} \to \infty} \frac{1}{p_{\pi}^i} R_i^{B_s \to D_s}(t, t_{\text{sink}})$$



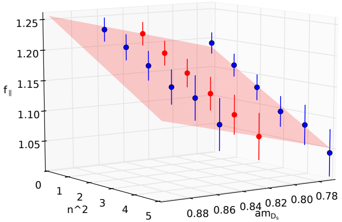
Charm extra-/interpolation for $B_s \to D_s \ell \nu$



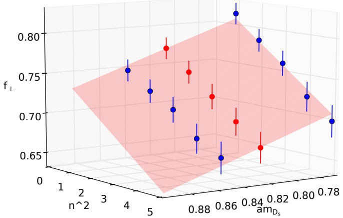
PRELIMINARY



PRELIMINARY



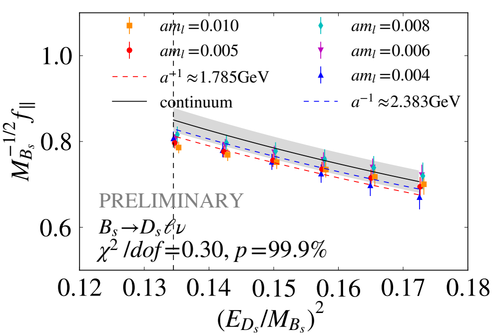
PRELIMINARY

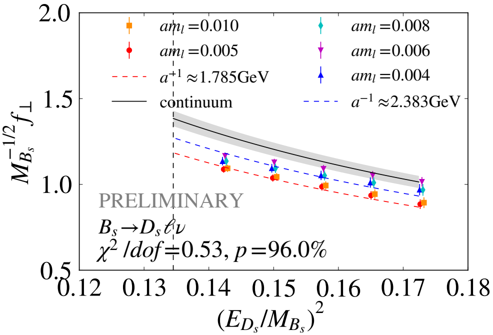


PRELIMINARY

Chiral-continuum extrapolation for $B_s \to D_s \ell \nu$

$$f(q,a) = \frac{c_0 + c_1(\Lambda_{QCD}a)^2}{1 + c_2(q/M_{B_c})^2}$$





Next steps

- Analyze data at third, finer lattice spacing
- Estimate full systematic errors for three "synthetic" data points
- Perform z-expansion and polynomial fits
- Comparison with other result(s) [HPQCD 2017]
- Explore advantageous ratios

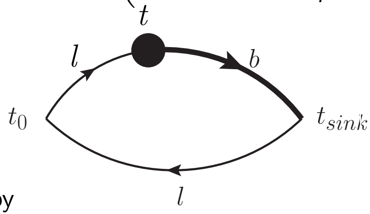
neutral current decays

(short distance contributions only)

Coming back to $B \to \pi\ell\nu$ form factors

- Parametrize the hadronic matrix element for the flavor changing vector current in terms of the form factors $f_+(q^2)$ and $f_0(q^2)$

$$\langle\pi(k)|\bar{u}\gamma^\mu b|B(p)\rangle = f_+(q^2)\left(p^\mu + k^\mu - \frac{M_B^2-M_\pi^2}{q^2}q^\mu\right) + f_0(q^2)\frac{M_B^2-M_\pi^2}{q^2}q^\mu$$

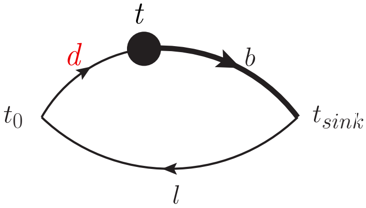


- Calculate 3-point function by
  - → Inserting a quark source for a “light” propagator at t_0
  - → Allow it to propagate to t_{sink}, turn it into a sequential source for a b quark
  - → Use another “light” quark propagating from t_0 and contract both at t

$B \to \pi \ell^+ \ell^-$ form factor

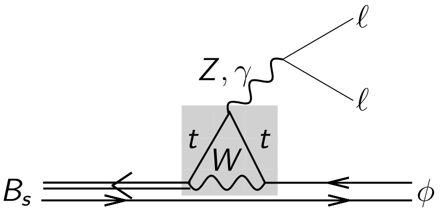
- If the daughter quark is a d-quark, we have a FCNC decay at loop-level
- Dominant contributions at short distance: f_0, f_+, and f_T

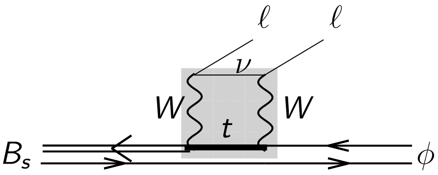
$$\langle \pi(k) | i\bar{d}\sigma^{\mu\nu} b(p) | B \rangle = 2\frac{p^\mu k^\nu - p^\nu k^\mu}{M_B + M_\pi} f_T(q^2)$$



- HPQCD
  - → Form factors f_0, f_+, and f_T for $B \to K\ell^+\ell^-$ [PRL111 (2013) 162002, Erratum: PRL112 (2014) 149902] [PRD88 (2013) 054509, Erratum: PRD88 (2013) 079901]
- Fermilab/MILC
  - → Tensor form factor f_T for $B \to \pi\ell^+\ell^-$ [PRL115 (2015) 152002]
  - → Form factors f_0, f_+, and f_T for $B \to K\ell^+\ell^-$ [PRD93 (2016) 025026]

$B_s \to \phi \ell^+ \ell^-$ form factors





- Vector final state treated in narrow width approximation
- Effective Hamiltonian

$$\mathcal{H}_{\text{eff}}^{b\to s} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_i^{10} C_i O_i^{(\prime)}$$

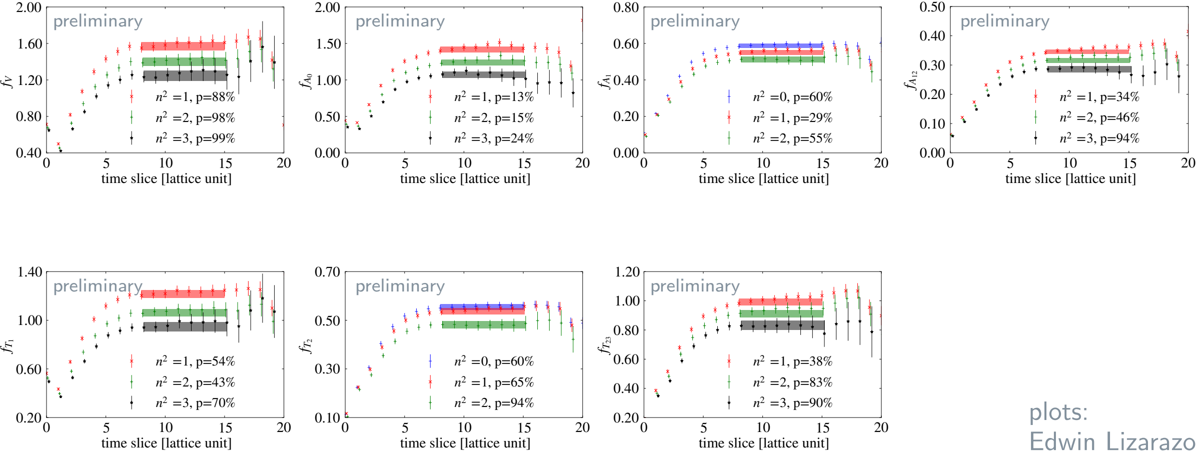
- Leading contributions at short distance

$$O_7^{(\prime)} = \frac{m_b e}{16\pi^2} \bar{s}\sigma^{\mu\nu} P_{R(L)} b F_{\mu\nu} \qquad O_9^{(\prime)} = \frac{e^2}{16\pi^2} \bar{s}\gamma^\mu P_{L(R)} b \bar{\ell}\gamma_\mu \ell \qquad O_{10}^{(\prime)} = \frac{e^2}{16\pi^2} \bar{s}\gamma^\mu P_{L(R)} b \bar{\ell}\gamma_\mu \gamma^5 \ell$$

Seven form factors

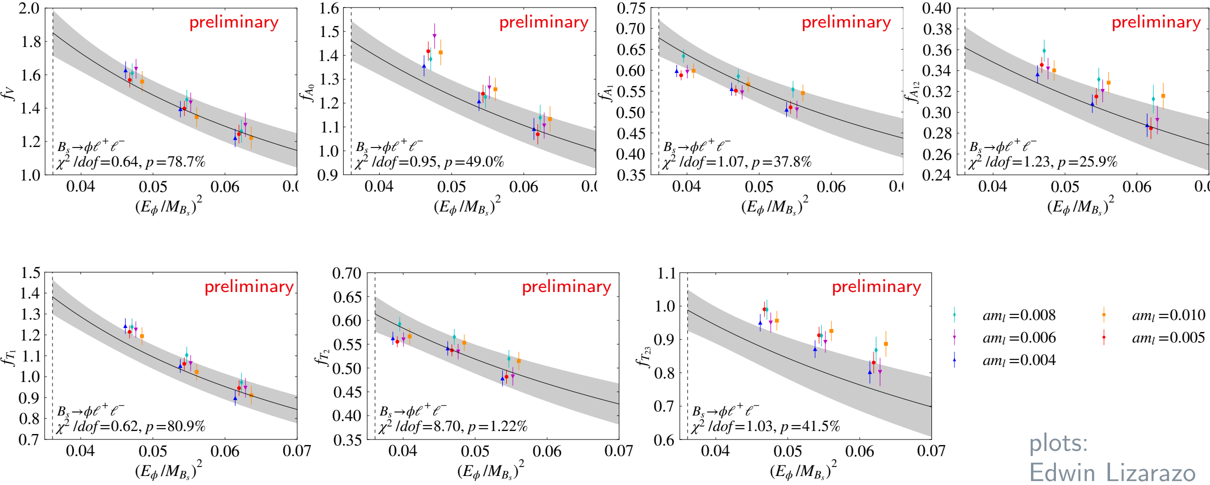
$$
\begin{aligned}
\langle\phi(k,\lambda)|\bar{s}\gamma^{\mu}b|B_s(p)\rangle =& f_V(q^2)\frac{2i\epsilon^{\mu\nu\rho\sigma}\varepsilon^*_{\nu}k_{\rho}p_{\sigma}}{M_{B_s}+M_{\phi}}\\
\langle\phi(k,\lambda)|\bar{s}\gamma^{\mu}\gamma_5 b|B_s(p)\rangle =& f_{A_0}(q^2)\frac{2M_{\phi}\varepsilon^*\cdot q}{q^2}q^{\mu}\\
&+ f_{A_1}(q^2)(M_{B_s}+M_{\phi})\left[\varepsilon^{*\mu}-\frac{\varepsilon^*\cdot q}{q^2}q^{\mu}\right]\\
&- f_{A_2}(q^2)\frac{\varepsilon^*\cdot q}{M_{B_s}+M_{\phi}}\left[k^{\mu}+p^{\mu}-\frac{M_{B_s}^2-M_{\phi}^2}{q^2}q^{\mu}\right]\\
q_{\nu}\langle\phi(k,\lambda)|\bar{s}\sigma^{\nu\mu}b|B_s(p)\rangle =& 2f_{T_1}(q^2)\epsilon^{\mu\rho\tau\sigma}\varepsilon^*_{\rho}k_{\tau}p_{\sigma}\ ,\\
q_{\nu}\langle\phi(k,\lambda)|\bar{s}\sigma^{\nu\mu}\gamma^5 b|B_s(p)\rangle =& i f_{T_2}(q^2)\left[\varepsilon^{*\mu}(M_{B_s}^2-M_{\phi}^2)-(\varepsilon^*\cdot q)(p+k)^{\mu}\right]\\
&+ i f_{T_3}(q^2)(\varepsilon^*\cdot q)\left[q^{\mu}-\frac{q^2}{M_{B_s}^2-M_{\phi}^2}(p+k)^{\mu}\right]
\end{aligned}
$$

$B_s \to \phi\ell\ell$: Seven form factors ($a^{-1} = 1.784$ GeV, $am_l^{sea} = 0.005$, $am_s = 0.03224$)



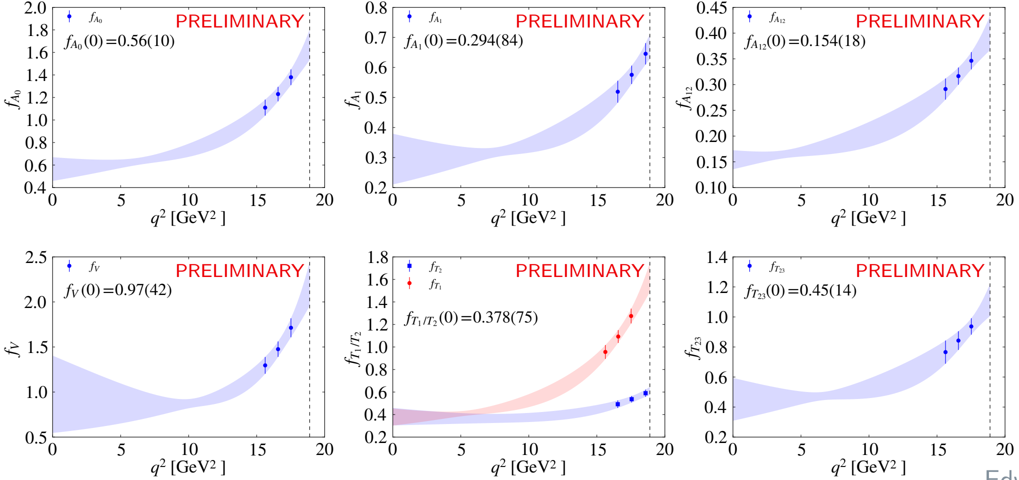
plots:
Edwin Lizarazo

$B_s \to \phi\ell\ell$: Seven form factors vs. q^2



plots:
Edwin Lizarazo

$B_s \to \phi \ell \ell$: First attempt to use z-parametrization



plots: Edwin Lizarazo

→ Ignoring any implications from resonances!

Next steps

- Analyze data at third, finer lattice spacing
- Estimate full systematic errors for three or four "synthetic" data points
- Re-do z-expansion and polynomial fits
- Compare to: Horgan, Liu, Meinel, and Wingate [PRD89 (2014) 090501][PoS Lattice2014 (2015) 372]
  $\rightarrow$ Angular analysis [PRL112 (2014) 212003]
- Consistent with LCSR results (based on same factorization approximation) at $q^2 = 0$?

Further challenges

- Vector final states are unstable in QCD → narrow width approximation
  - → Chiral perturbation theory cannot guide extrapolations of data at unphysically heavy pions
  - → Resonances in range of kinematic extrapolation (hadronic uncertainties)
  - → Pioneering work on $B \to K^* \to K\pi$ (cf. Leskovec, Meinel); new ideas [Hansen, Meyer, Robaina arXiv:1704.08993]
- Simulations with physical light and bottom quarks troubled by poor signal-to-noise ratio
  - → So far form factors only at q^2_{max} calculated [HPQCD PRD93 (2016) 034502]
- Long distance contributions

conclusion

Conclusion

- Semi-leptonic B and B_s decays allow many tests of the Standard Model and exhibit tantalizing signals
- Yet more data and improved theoretical predictions are needed
- About to complete calculation/update on $B_s \to D_s \ell\nu$ and $B_s \to K\ell\nu$
- Our general code is general i.e. we have already data for many processes of interest

  → $B \to \pi\ell\nu$, $B \to \pi\ell\ell$
  → $B \to K^*\ell\ell$
  → $B \to D^{(*)}\ell\nu$
  → $B_s \to K^{(*)}\ell\nu$, $B_s \to K^{(*)}\ell\ell$
  → $B_s \to D_s^{(*)}\ell\nu$
  → ...

Resources and Acknowledgments

USQCD: Ds, Bc, and pi0 cluster (Fermilab), qcd12s cluster (Jlab)
RBC qcdcl (RIKEN) and cuth (Columbia U)
UK: ARCHER, cirrus (EPCC) and DiRAC (UKQCD)