Heavy-to-Light Form Factors from LCSRs: Status and Perspectives

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FOR 1873

Outline

- Status of LCSR's for the form factors:
  - $B_{(s)} \to P \quad (P = \pi, K)$
    $\oplus$ New estimate of higher twist effects
  - $B \to V \quad (V = \rho, \omega, K^*)$ within approximation $\Gamma_V = 0$
  - $B \to \pi\pi$
- Current accuracy and perspectives

See also the talks by

* O. Witzel, S. Meinel, on Lattice QCD results on form factors
* P. Gambino, M. Wingate, on heavy-to-heavy transitions form factors
* Y.-M. Wang, on baryon form factors from LCSR

Outline of the LCSR method

- Design an appropriate correlation function
- Determine a momenta region with the light-cone (LC) dominance
- LC OPE for the correlation function
  in form of convolution of the hard-scattering kernels with the LC-distribution amplitudes (LCDAs)
- Hadronic dispersion relation
  to isolate the needed form factor and to relate it with the OPE result
- Quark-hadron duality
  to approximate the contribution of the excited states and continuum
- Borel Transformation
  to suppress the contribution beyond the effective threshold
- Final result for the relevant form factor (at large recoil)

  Two approaches:

  1. With light-meson LCDAs
  2. With HQET B-meson LCDAs

$B_{(s)} \to P$ form factors
The form factors definition

$$\langle P(p)|\bar{q}\gamma^\mu b|B(p+q)\rangle = f^+_{BP}(q^2)\left[2p^\mu + \left(1 - \frac{m_B^2 - m_P^2}{q^2}\right)q^\mu\right] + f^0_{BP}(q^2)\,\frac{m_B^2 - m_P^2}{q^2}\,q^\mu$$

$$\langle P(p)|\bar{q}\sigma^{\mu\nu}q_\nu b|B(p+q)\rangle = \frac{i f^T_{BP}(q^2)}{m_B + m_P}\left[2q^2 p^\mu + \left(q^2 - \left(m_B^2 - m_P^2\right)\right)q^\mu\right]$$

$B = B, B_s, \quad P = \pi, K, \quad q = u, s, d$

- Involved in the semileptonic $B \to (\pi, K)\ell\nu_\ell$ and $B_s \to K\ell\nu_\ell$ and in FCNC $B \to (\pi, K)\ell^+\ell^-$ and $B_s \to K\ell^+\ell^-$ decays
- Needed in non-leptonic decays (within QCDF approach)

$B_{(s)} \to P$ form factors

Correlation function

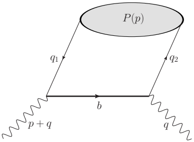
With light meson distribution amplitude:

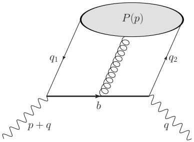
$$F^{\mu}_{BP}(p,q) = i\int d^4x\, e^{iqx}\langle P(p)|\,T\{\bar{q}_1(x)\Gamma^{\mu}b(x), m_b\bar{b}(0)i\gamma_5 q_2(0)\}|0\rangle$$

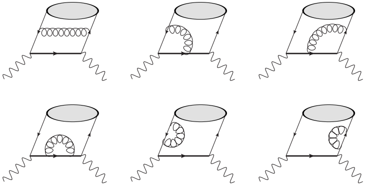
$$= \begin{cases} F_{BP}(q^2,(p+q)^2)p^{\mu} + \tilde{F}_{BP}(q^2,(p+q)^2)q^{\mu}, & \Gamma^{\mu} = \gamma^{\mu} \\ F^{T}_{BP}(q^2,(p+q)^2)\left[q^2p^{\mu} - (q\cdot p)q^{\mu}\right], & \Gamma^{\mu} = -i\sigma^{\mu\nu}q_{\nu} \end{cases}$$

$B^+ \to K^+ :\quad q_1 = u,\ q_2 = s,\qquad B^+ \to \pi^+ :\quad q_1 = u,\ q_2 = d$

$B^0_s \to K^+ :\quad q_1 = s,\ q_2 = u,\qquad B^0_s \to \bar{K}^0 :\quad q_1 = s,\ q_2 = d$







$B_{(s)} \to P$ form factors

The current accuracy of the OPE result

$$F_{BP}(q^2) \Rightarrow \mathrm{OPE} \Rightarrow f^+_{BP}(q^2) \qquad \text{(similar for } f^0_{BP}(q^2), f^T_{BP}(q^2))$$

$$\mathrm{OPE} = \left(T_0^{(2)} + (\alpha_s/\pi)T_1^{(2)} + (\alpha_s/\pi)^2 T_2^{(2)}\right) \otimes \varphi_P^{(2)}$$

$$+\frac{\mu_P}{m_b}\left(T_0^{(3)} + (\alpha_s/\pi)T_1^{(3)}\right) \otimes \varphi_P^{(3)} + T_0^{(4)} \otimes \varphi_P^{(4)} + \langle \bar{q}q \rangle \left(T_0^{(5)} \otimes \varphi_P^{(2)} + \frac{\mu_P}{m_b} T_0^{(6)} \otimes \varphi_P^{(3)}\right)$$

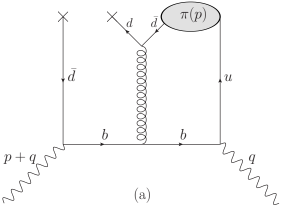
$$\mu_P = \frac{m_P^2}{m_{q_1} + m_{q_2}}, \qquad \varphi_P^{(k)} = \text{ AF + non - asympt. corrections (conformal spin)}$$

- LO twist 2, 3, 4 $q\bar{q}$ and $\bar{q}qG$ terms
  [V.Belyaev, A.Khodjamirian, R.Rückl (1993); V.Braun, V.Belyaev, A.Khodjamirian, R.Rückl (1996)]
- NLO $O(\alpha_s)$ twist 2 (collinear factorization)
  [A.Khodjamirian, R.Rückl, S.Weinzierl, O. Yakovlev (1997); E.Bagan, P.Ball, V.Braun (1997)]
- NLO $O(\alpha_s)$ twist 3 (coll. factorization for asympt. DA)
  [P. Ball, R. Zwicky (2001); G.Duplancic, A.Khodjamirian, B.Melic, Th.Mannel, N.Offen (2007)]
- Part of NNLO $O(\alpha_s^2\beta_0)$ twist 2 [A. Bharucha (2012)]
- LO twist 5 and twist 6 in factorization approximation [A.V. Rusov (2017)]

Factorizable twist-5 and twist-6 contributions

Diagrams

A.V. Rusov, 1705.01929 [hep-ph]



- An estimate of the twist-5 and twist-6 contributions is obtained in the factorization approximation
  [V.M. Braun, A. Khodjamirian, M. Maul (2000)], LCSR for $F_\pi(Q^2)$;
  [S.S. Agaev, V.M. Braun, N. Offen, F.A. Porkert (2011)], LCSR for $F_{\pi^0\gamma^*\gamma}(Q^2)$
- One needs the massive quark propagator expanded in the external gluonic field near the light cone

LC-expansion of the massive quark propagator

Final expression

K_0, K_1 – Bessel functions

$$
\begin{aligned}
S(x,0) &= -i\langle 0|T\{\psi(x),\bar\psi(0)\}|0\rangle \\
&= S^{(0)}(x) - \frac{ig_s}{16\pi^2}\int_0^1 du\Big[mK_0(m\sqrt{-x^2})(G(ux)\cdot\sigma) + \\
&+ \frac{im}{\sqrt{-x^2}}K_1(m\sqrt{-x^2})\,[\bar u \not{x}(G(ux)\cdot\sigma) + u\,(G(ux)\cdot\sigma)\not{x}] + \\
&+ 2u\bar u\left(imK_0(m\sqrt{-x^2}) - \frac{m\not{x}}{\sqrt{-x^2}}K_1(m\sqrt{-x^2})\right)x_\nu D_\mu G^{\nu\mu}(ux) + \\
&+ 2K_0(m\sqrt{-x^2})\left(u\bar u - \frac{1}{2}\right) D_\nu G^{\nu\mu}(ux)\gamma_\mu + \\
&+ u\bar u(1-2u)K_0(m\sqrt{-x^2})x_\mu \not{D} D_\nu G^{\mu\nu}(ux) - \\
&- iu\bar u K_0(m\sqrt{-x^2})\epsilon_{\sigma\mu\nu\rho}x^\sigma D^\nu D_\alpha G^{\alpha\rho}(ux)\gamma^\mu\gamma_5 + \\
&+ u\bar u\sqrt{-x^2}K_1(m\sqrt{-x^2})D_\nu D_\mu G^{\mu\rho}(ux)\sigma_\rho{}^\nu + \text{ higher derivatives}\Big]
\end{aligned}
$$

LC-expansion of the massive quark propagator

Massless case

Expansion of the Bessel functions:

$$K_0(m\sqrt{-x^2})\Big|_{m\to 0} \sim -\gamma_E - \ln\left(\frac{m}{2}\right) - \frac{1}{2}\ln\left(-x^2\right)$$

$$\sqrt{-x^2}K_1(m\sqrt{-x^2})\Big|_{m\to 0} \sim \frac{1}{m}$$

Reproduce results of [I.I. Balitsky, V.M. Braun (1989)] in massless case

$$S(x,0)|_{m\to 0} = \frac{\not{x}}{2\pi^2 x^4} - \frac{g_s}{16\pi^2 x^2}\int_0^1 du\,[\bar{u}\not{x}(G\cdot\sigma) + u(G\cdot\sigma)\not{x} + 2iu\bar{u}\not{x}x_\nu D_\mu G^{\nu\mu}]$$

$$+ \frac{g_s \ln(-x^2)}{16\pi^2}\int_0^1 du\left[i\left(u\bar{u} - \frac{1}{2}\right)D_\nu G^{\nu\mu}\gamma_\mu + \frac{i}{2}u\bar{u}(1-2u)x_\mu \not{D} D_\nu G^{\mu\nu}\right.$$

$$\left. + \frac{1}{2}u\bar{u}\epsilon_{\sigma\mu\nu\rho}x^\sigma D^\nu D_\alpha G^{\alpha\rho}\gamma^\mu\gamma_5 + \text{ higher derivatives}\right]$$

Factorizable twist-5 and twist-6 contributions

Outline of calculation

- Substitution of the propagator $S(x,0)$ (only terms $D_\mu G^{\mu\nu}(ux)$ needed for twist 5 and twist 6)
- Applying the equation of motion for the gluon-field strength:

$$D_\mu G^{\mu\nu}(ux) = -g_s \sum_q \bar{q}(ux)\gamma^\nu \frac{\lambda^a}{2} q(ux)\frac{\lambda^a}{2}$$

- Factorization

$$\langle \pi(p)|\bar{u}(x)q(ux)\bar{q}(ux)d(0)|0\rangle \to$$

$$\to \langle \bar{q}q\rangle \times (\langle \pi(p)|\bar{u}(ux)d(0)|0\rangle + \langle \pi(p)|\bar{u}(x)d(ux)|0\rangle)$$

- Both twist-5 $\to \langle \bar{q}q\rangle \varphi_\pi^{(2)}$ and twist-6 $\to \langle \bar{q}q\rangle \varphi_\pi^{(3)}$

Factorizable twist-5 and twist-6 contributions

Standard derivation of LCSR

- OPE result in the quasi-dispersion form:

$$F^{(\mathrm{OPE})}_{\mathrm{tw}5,6}(q^2,(p+q)^2) = \alpha_s \langle \bar{q} q \rangle \frac{C_F}{N_c} \pi m_b f_\pi \int_{m_b^2}^{\infty} ds \sum_{n=2,3,4} \frac{g_n(q^2,s)}{(s-(p+q)^2)^n}$$

- Dispersion relation in variable $(p+q)^2$

$$F^{(\mathrm{OPE})}_{\mathrm{tw}5,6}(q^2,(p+q)^2) = \frac{1}{\pi} \int_{m_b^2}^{\infty} ds \frac{\mathrm{Im} F^{(\mathrm{OPE})}_{\mathrm{tw}5,6}(q^2,s)}{s-(p+q)^2}$$

- Using quark-hadron duality

$$\text{excited states} + \text{continuum} = \frac{1}{\pi} \int_{s_0^B}^{\infty} ds \frac{\mathrm{Im} F^{(\mathrm{OPE})}_{\mathrm{tw}5,6}(q^2,s)}{s-(p+q)^2}$$

- After Borel Transformation $((p+q)^2 \to M^2)$

$$[f^+_{B\pi}(q^2)]_{\mathrm{tw}5,6} = \frac{e^{m_B^2/M^2}}{2\pi m_B^2 f_B} \int_{m_b^2}^{s_0^B} ds\, e^{-s/M^2}\, \mathrm{Im} F^{(\mathrm{OPE})}_{\mathrm{tw}5,6}(q^2,s)$$

Factorizable twist-5 and twist-6 contributions

Final result

Twist-5 and twist-6 corrections to the $B \to \pi$ form factor

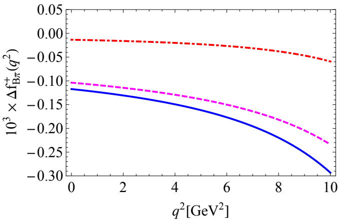
$$\left[f^+_{B\pi}(q^2)\right]_{\rm tw5,6} = \left(\frac{e^{m_B^2/M^2}}{2m_B^2 f_B}\right) \alpha_s \langle \bar{q} q \rangle \frac{C_F}{N_c} \pi m_b f_\pi \int\limits_{m_b^2}^{\infty} ds \sum_{n=2,3,4} \rho_n(q^2, s; s_0^B, M^2)$$

with auxiliary functions

$$\rho_n(q^2, s; s_0^B, M^2) = \frac{(-1)^{n-1}}{(n-1)!} g_n(q^2, s) \frac{d^{n-1}}{ds^{n-1}} \left[\theta(s_0^B - s) e^{-s/M^2}\right]$$

Factorizable twist-5 and twist-6 contributions

Numerical result



$$f^+_{B\pi}(0) = 0.301 \pm 0.023$$

[I.S. Imsong, A. Khodjamirian, Th. Mannel, D. van Dyk (2014)]
[A. Khodjamirian, A.V. Rusov (2017)]

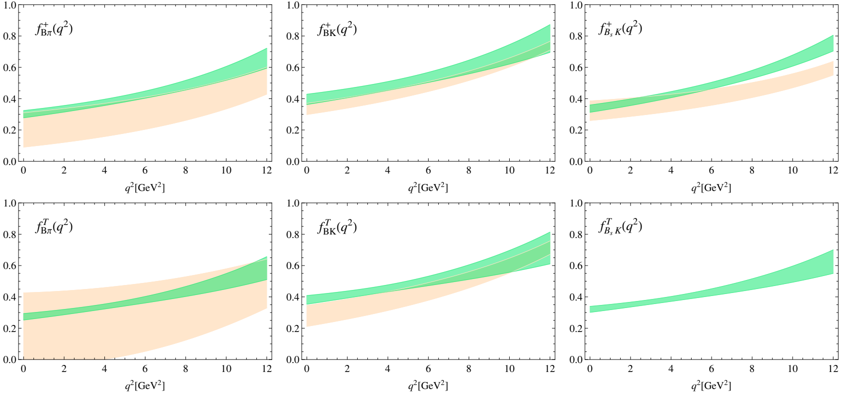
	$q^2 = 0$	$q^2 = 10$ GeV^2
$f^+_{B\pi}(q^2)$	0.301	0.562
Tw2 LO	47.5%	48.2%
Tw2 NLO	6.9%	5.9%
Tw3 LO	50.0%	54.2%
Tw3 NLO	−4.6%	−7.5%
Tw4 LO	0.2%	−0.8%
Tw5 LO-fact	−0.034%	−0.042%
Tw6 LO-fact	−0.004%	−0.011%

- Higher twist contributions are strongly suppressed!
- Expansion of LCDA ($a_2, a_4, \ldots$) is also under control (non-asymptotic corrections $\sim 10 - 15\%$)

$B \to \pi, K$ and $B_s \to K$ form factors

Results

A. Khodjamirian, A.V. Rusov, 1703.04765 [hep-ph]



Lattice QCD results: HPQCD (2014, 2016), Fermilab Lattice/MILC (2015)

$B_{(s)} \to V$ form factors

In zero-width approximation: $\Gamma_V = 0$
Correlation function with vector meson LCDA

$$F^{\mu}_{BV}(p,q) = i \int d^4x e^{iqx} \langle V(p)| T\{\bar{q}_1(x)\Gamma^{\mu} b(x), m_b \bar{b}(0) i\gamma_5 q_2(0)\}|0\rangle$$

$\Gamma^{\mu} = \gamma^{\mu}(1-\gamma_5),\ q_{\nu}\sigma^{\mu\nu}(1-\gamma_5)$

Current accuracy of the relevant OPE

- LO twist 2, 3, 4 terms
  [P. Ball, V.M. Braun (1998)]
- NLO $O(\alpha_s)$ twist 2
  [P. Ball, V.M. Braun (1998)]
- NLO $O(\alpha_s)$ twist 3
  [P. Ball, R. Zwicky (2005)]

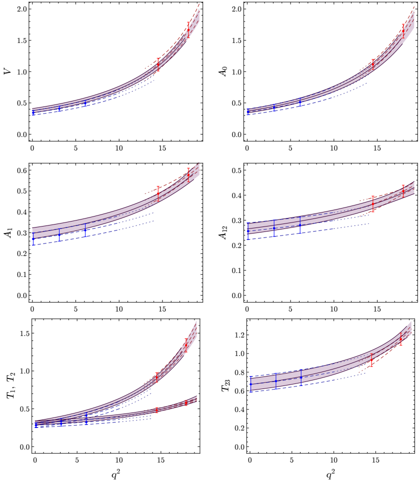
$B_{(s)} \to V$ form factors
Results

[A. Bharucha, D.M. Straub, R. Zwicky (2016)]

Results for $B \to K^*$ form factors (also $B \to \rho, \omega$ and $B_s \to \phi, \bar{K}^*$ [not shown here])

Lattice QCD results from

[R.R. Horgan, Z. Liu, S. Meinel, M. Wingate (2015)]



$B \to \pi\pi$ form factors

- Beyond narrow ρ-meson approximation in $B \to \pi\pi\ell\nu_\ell$
- Determine the rich set of observables in $B \to \pi\pi\ell\nu_\ell$
  [S. Faller, T. Feldmann, A. Khodjamirian, T. Mannel and D. van Dyk, 1310.6660 [hep-ph]]
- Provide factorizable parts of $B \to 3\pi$ nonleptonic amplitudes
  [S. Kränkl, T. Mannel and J. Virto, 1505.04111 [hep-ph]]
  [R. Klein, T. Mannel, J. Virto and K. K. Vos, 1708.02047 [hep-ph]]
- Not yet accessible in the lattice QCD?
- In QCD-based effective theories:
  - Heavy meson ChPT $\oplus$ dispersion relations
    [X. W. Kang, B. Kubis, C. Hanhart and U. G. Meißner, 1312.1193 [hep-ph]]
  - QCD factorization
    [P. Böer, T. Feldmann and D. van Dyk, 1608.07127 [hep-ph]]
- $B \to \pi\pi$ from LCSR with dipion LCDA
  [Ch. Hambrock, A. Khodjamirian, 1511.02509 [hep-ph]]

$B \to \pi\pi$ form factors

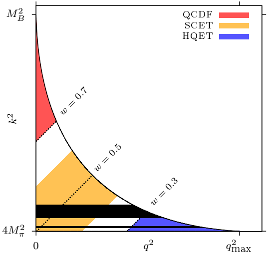
Definition of the $B \to \pi\pi$ form factors ($p = q + k,\ k = k_1 + k_2,\ \bar{k} = k_1 - k_2$):

$$i\langle\pi^+(k_1)\pi^0(k_2)|\bar{u}\gamma^\mu(1-\gamma_5)b|\bar{B}^0(p)\rangle = -F_\perp(q^2,k^2,\zeta)\,\frac{4}{\sqrt{k^2\lambda_B}}\,i\epsilon^{\mu\alpha\beta\gamma}\,q_\alpha\,k_{1\beta}\,k_{2\gamma}$$
$$+F_\parallel(q^2,k^2,\zeta)\,\frac{1}{\sqrt{k^2}}\left(\bar{k}^\mu - \frac{4(q\cdot k)(q\cdot\bar{k})}{\lambda_B}\,k^\mu + \frac{4k^2(q\cdot\bar{k})}{\lambda_B}\,q^\mu\right)$$
$$+F_t(q^2,k^2,\zeta)\,\frac{q^\mu}{\sqrt{q^2}} + F_0(q^2,k^2,\zeta)\,\frac{2\sqrt{q^2}}{\sqrt{\lambda_B}}\left(k^\mu - \frac{k\cdot q}{q^2}q^\mu\right)$$

The region of LCSR applicability:

$k^2 \lesssim 1\,\mathrm{GeV}^2$

$0 \leq q^2 \leq 12 - 14\mathrm{GeV}^2$



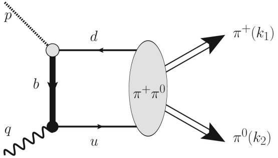
$B \to \pi\pi$ form factors
Correlation function with dipion LCDA

Ch. Hambrock, A. Khodjamirian, 1511.02509 [hep-ph]

- The correlation function: $k = k_1 + k_2, \quad \bar{k} = k_1 - k_2, \quad p = k + q$

$$\Pi_\mu(q, k_1, k_2) = i \int d^4x\, e^{iqx} \langle \pi^+(k_1)\pi^0(k_2)| T\{\bar{u}(x)\gamma_\mu(1-\gamma_5)b(x), im_b\bar{b}(0)\gamma_5 d(0)\}|0\rangle$$
$$= i\epsilon_{\mu\alpha\beta\rho} q^\alpha k_1^\beta k_2^\rho\, \Pi^{(V)} + q_\mu \Pi^{(A,q)} + k_\mu \Pi^{(A,k)} + \bar{k}_\mu \Pi^{(A,\bar{k})}$$

- Light-cone dominance region $x^2 \sim 0$:
  $k^2 \lesssim 1\,\mathrm{GeV}^2$
  $0 \le q^2 \le 12 - 14\mathrm{GeV}^2$



- At LO in α_s in twist-2 2π-LCDAs approximation (the only available ones)

The structure of dipion DAs

[M. V. Polyakov (1999)]

- Double expansion in Legendre and Gegenbauer polynomials:

$$\Phi_\perp(u,\zeta,k^2) = \frac{6u(1-u)}{f_{2\pi}^\perp} \sum_{n=0,2,..}^{\infty} \sum_{\ell=1,3,..}^{n+1} B_{n\ell}^\perp(k^2) C_n^{3/2}(2u-1) \beta_\pi P_\ell^{(0)}\left(\frac{2\zeta-1}{\beta_\pi}\right)$$

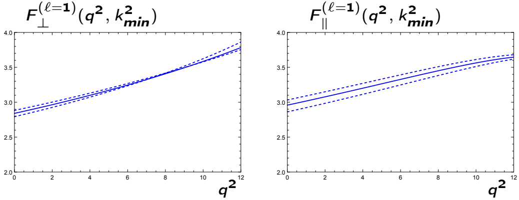
$$\Phi_\parallel(u,\zeta,k^2) = 6u(1-u) \sum_{n=0,2,..}^{\infty} \sum_{\ell=1,3,..}^{n+1} B_{n\ell}^\parallel(k^2) C_n^{3/2}(2u-1) \beta_\pi P_\ell^{(0)}\left(\frac{2\zeta-1}{\beta_\pi}\right)$$

- Gegenbauer moments (multiplicativey renormalizable)
  * $B_{n\ell}^{\perp,\parallel}(k^2)$ - complex functions at $k^2 > 4m_\pi^2$
  * $B_{01}^\parallel(k^2) = F_\pi(k^2)$ — pion timelike form factor
- Instanton vacuum model for the coefficients,
  $n = 0, 2, 4$, valid at small $k^2 \sim 4m_\pi^2$ [M.V. Polyakov, C. Weiss (1999)]

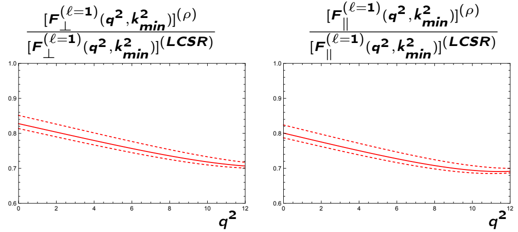
$$B_{01}^\perp(k^2) = 1 + \frac{k^2}{12M_0^2}, \quad B_{21}^\perp(k^2) = \frac{7}{36}\left(1 - \frac{k^2}{30M_0^2}\right), \quad B_{23}^\perp(k^2) = \frac{7}{36}\left(1 + \frac{k^2}{30M_0^2}\right)$$

$f_{2\pi}^\perp = 4\pi^2 f_\pi^2/3M_0 \simeq 650$ MeV, where $f_\pi = 132$ MeV is the pion decay constant.

- P-wave form factors (only twist-2, $k^2_{min} = 4m^2_\pi$):

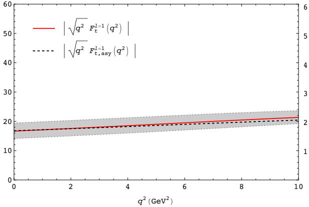


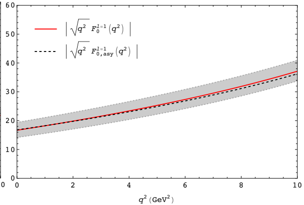
- ρ-meson contribution

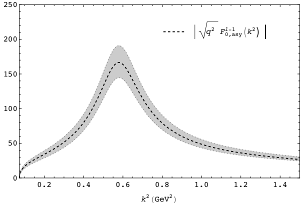


Using correlator with pseudoscalar interpolating current

[S. Cheng, A. Khodjamirian, J. Virto, 1709.00173 [hep-ph]]







- $F_t \sim \Phi_{\parallel}^{I=1}$, $B_{01}^{\parallel}(k^2) = F_\pi(k^2)$
- F_0 related with F_t and $F_{\parallel}$ (at twist-2 approximation)
- Belle data on $F_\pi(k^2)$ used as an input

Form factors from LCSR with B-meson LCDA

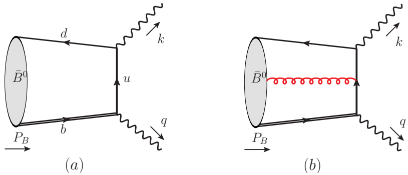
LCSR with B-meson distribution amplitudes:

- At tree level in QCD for $B \to P, V$ (two- and three-particle LCDAs)
  [A. Khodjamirian, T. Mannel and N. Offen (2007)]

- SCET LCSRs for $B \to P, V$
  [F. De Fazio, T. Feldmann and T. Hurth (2008)]

- The correlation function with B-meson LCDA:

$$F_{\mu\nu}(k,q) = i \int d^4x\, e^{ik\cdot x} \langle 0|\mathrm{T}\{\bar{d}(x)\Gamma_\mu u(x), \bar{u}(0)\gamma_\nu(1-\gamma_5)b(0)\}|\bar{B}^0(q+k)\rangle$$



- Flexible (used for calculation of $B \to P, V, \pi\pi, \ldots$ ($\Gamma_\mu = \gamma_\mu, \gamma_\mu\gamma_5$))

Form factors from LCSR with B-meson LCDA

- B-meson LCDA defined in HQET (see also talk by V.M. Braun):

$$\langle 0|\bar{d}_\alpha(x)[x,0]h_{v\beta}(0)|\bar{B}_v^0\rangle = -\frac{if_B m_B}{4}\int_0^\infty d\omega e^{-i\omega v\cdot x}\Big\{(1+\not{v}) \times \left(\phi_+^B(\omega) + \frac{\phi_-^B(\omega)-\phi_+^B(\omega)}{2v\cdot x}\not{x}\right)\gamma_5\Big\}_{\beta\alpha}$$

- Higher-twist B-meson Distribution Amplitudes in HQET
  [V.M. Braun, Y. Ji, A.N. Manashov (2017)]

- Key parameter: inverse moment

$$\frac{1}{\lambda_B} = \int_0^\infty d\omega \frac{\phi_+^B(\omega)}{\omega}$$

Two-point QCD SR in HQET: $\lambda_B = (460 \pm 110)\,\mathrm{MeV}$

[V.M. Braun, D.Yu Ivanov, G.P. Korchemsky (2004)]

Form factors from LCSR with B-meson LCDA

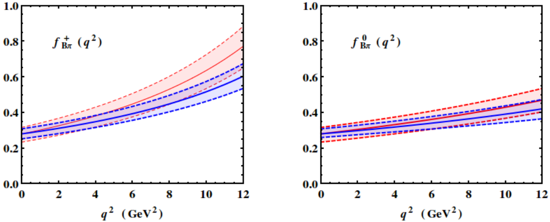
Some applications and results

- The most accurate determination of λ_B is possible form $B \to \gamma \ell \nu_\ell$

e.g. [M. Beneke, J. Pohrwild (2011)], [V.M. Braun, A. Khodjamirian (2012)]

- α_s corrections to $B \to \pi$ form factors (with two-particle DA)

[Y.-M. Wang, Y.-L. Shen (2015)]



- $B \to D^{(*)}$ at tree level

[S. Faller, A. Khodjamirian, Ch. Klein, Th. Mannel (2009)]

- Perturbative corrections to $B \to D$ form factors

[Y.-M. Wang, Y.-B. Wei, Y.-L. Shen, C.-D. Lü (2017)]

Form factors from LCSR with B-meson LCDA

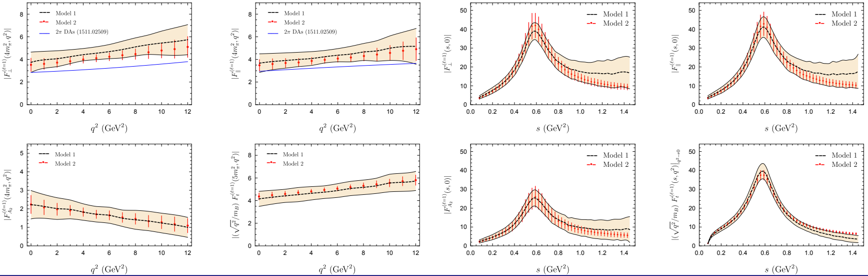
Results on $B \to \pi\pi$ form factors

S. Cheng, A. Khodjamirian, J. Virto, 1701.01633 [hep-ph]

$$\int_{4m_\pi^2}^{s_0^{2\pi}} ds\, e^{-s/M^2} \kappa_\pi(s) F_\pi^\star(s) F_\perp^{(l=1)}(s,q^2) = f_B m_B^2 \left[\int_0^{\sigma_0^{2\pi}} d\sigma\, e^{-s(\sigma,q^2)/M^2} \frac{\phi_+^B(\sigma m_B)}{m_B \bar{\sigma}} + \Delta V^{BV} \right]$$

Using two models for resonance contributions:

- Model 1: with two ρ- and ρ'-resonances
- Model 2: with three ρ-, ρ' and ρ''-resonances



Conclusion

- LCSR for heavy-to-light meson form factors at large recoil are complementary to LQCD
  (mutual extrapolations agree well)
- Accuracy of LCSR's with π, K DA's is comparable with LQCD
  (twist-5,6 smallness as a probe of LC expansion)
- Extending LCSR's to $B \to 2P,\ P = \pi, K$
  * First results with 2-pion DA's and B-meson DA's encouraging
  * More accuracy needed: LCDA, NLO corrections, etc.
- Future tasks/challenges:
  * More accurate LCDA's (light meson, dimeson, B-meson)
    (using LCSR's and data on pion FF's and $B \to \gamma \ell \nu_\ell$)
  * NLO twist-3 factorisation for non-asymptotic $B \to \pi, K$ DA's
  * Improving quark-hadron duality approximation
  * Global correlations between different QCD Sum Rules
    (see talk by D. van Dyk)

Backup

LC-expansion of the massive quark propagator

Final result in another form

An equivalent form

$$
\begin{aligned}
S(x,0) &= \int \frac{d^4p}{(2\pi)^4} e^{-ipx} \Bigg\{ \frac{\not{p} + m}{p^2 - m^2} \\
&- \frac{g_s}{(p^2-m^2)^3} \int\limits_0^1 du \Bigg[\frac{1}{2}(p^2 - m^2)\left(\bar{u}\not{p}(G(ux)\cdot\sigma) + u(G(ux)\cdot\sigma)\not{p}\right) + \\
&+ \frac{1}{2} m(p^2 - m^2)(G(ux)\cdot\sigma) - 4u\bar{u}(\not{p} + m)p_\nu D_\mu G^{\nu\mu}(ux) - \\
&- \frac{1}{2}(p^2 - m^2) D_\nu G^{\nu\mu}(ux)\gamma_\mu + 2iu\bar{u}(1-2u)p_\rho \not{D} D_\nu G^{\rho\nu}(ux) + \\
&+ 2u\bar{u}\epsilon_{\sigma\mu\nu\rho} p^\sigma D^\nu D_\alpha G^{\alpha\rho}(ux)\gamma^\mu\gamma_5 - 2mu\bar{u} D_\nu D_\mu G^{\mu\rho}(ux)\sigma_\rho{}^\nu \Bigg] \Bigg\}
\end{aligned}
$$