Lattice QCD calculation of $B \rightarrow D^* \, l \, \nu$ form factor at zero recoil

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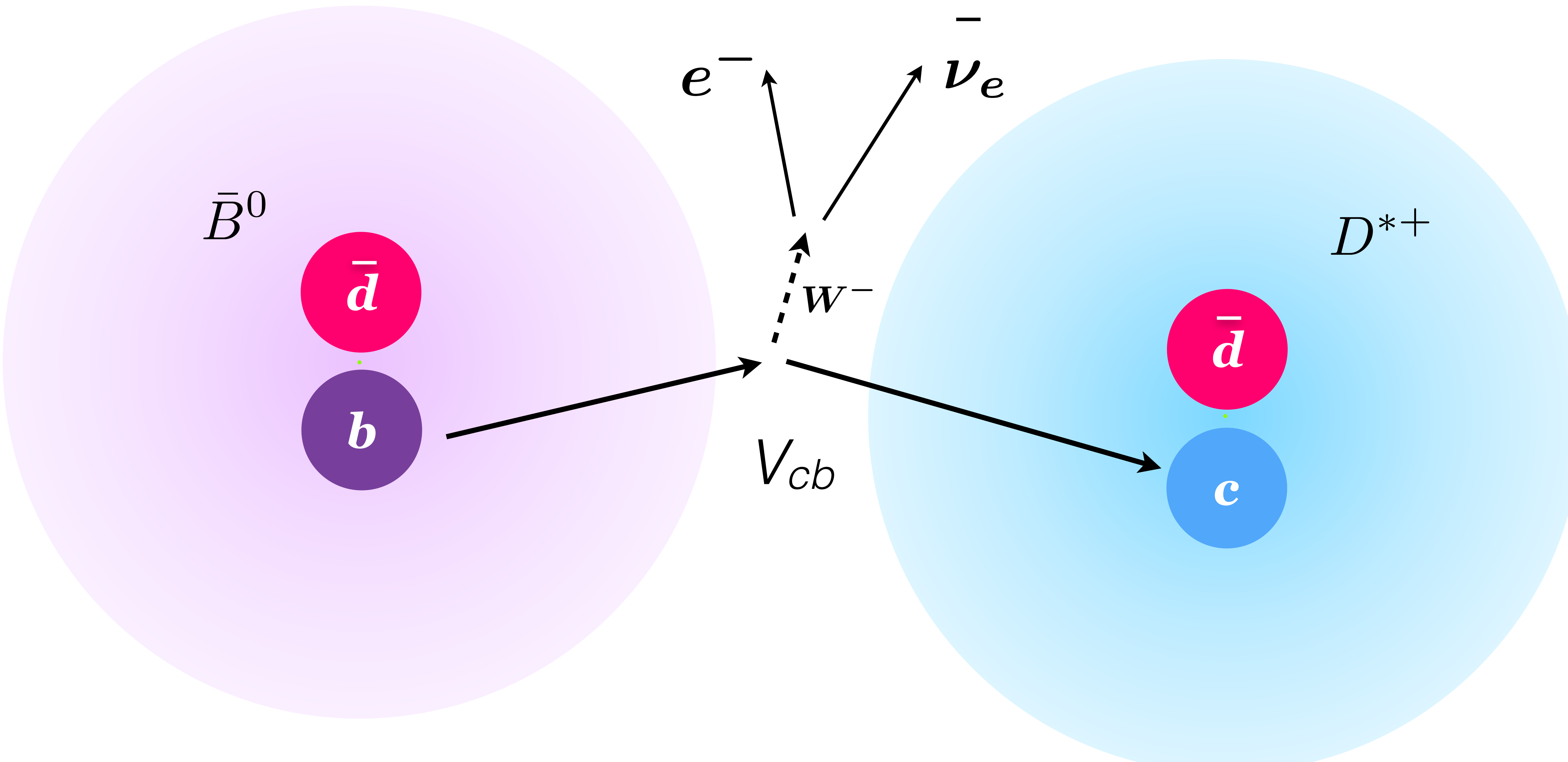
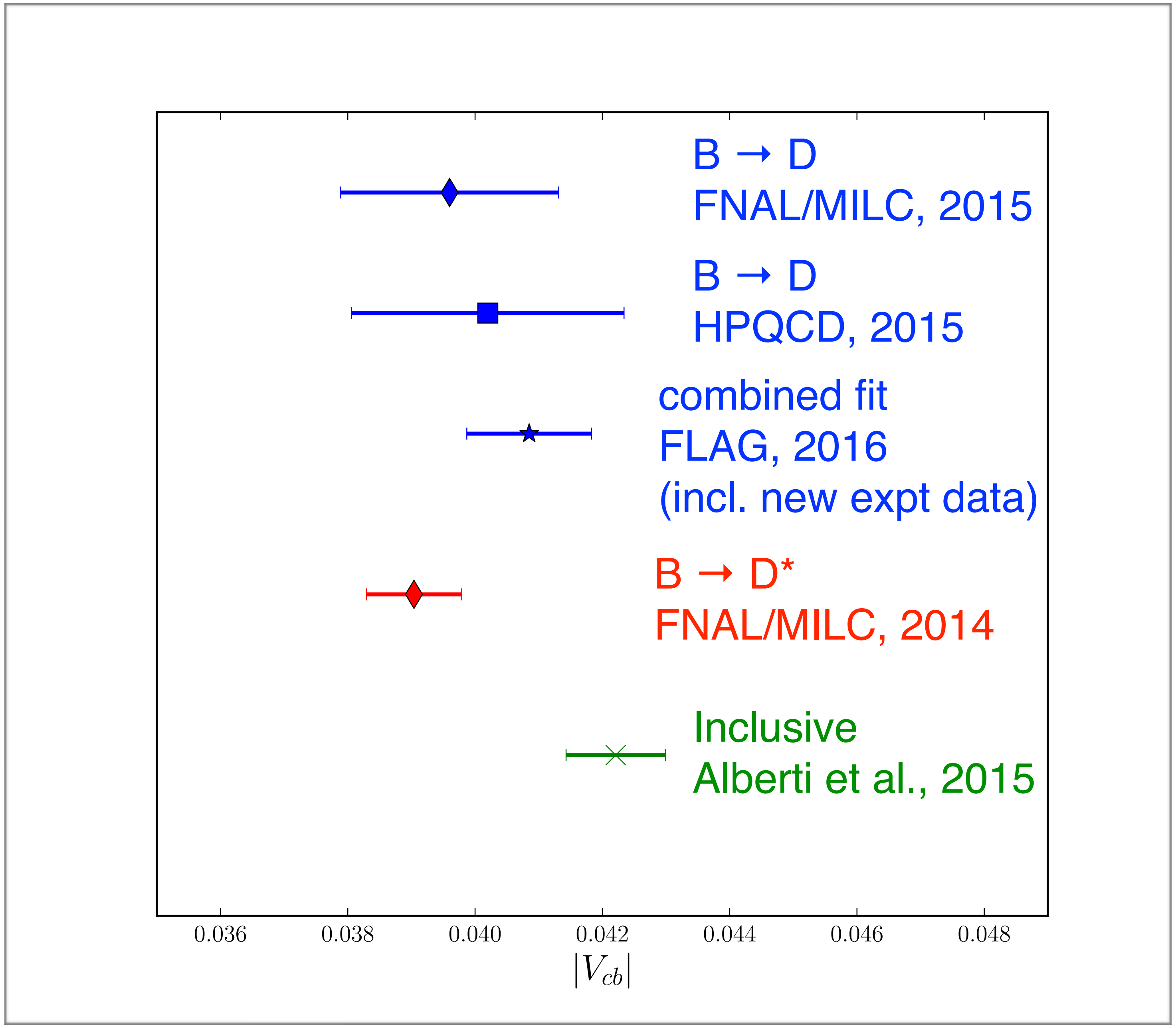
Outline

- Brief introduction
- Details of the calculation
- Results and implications

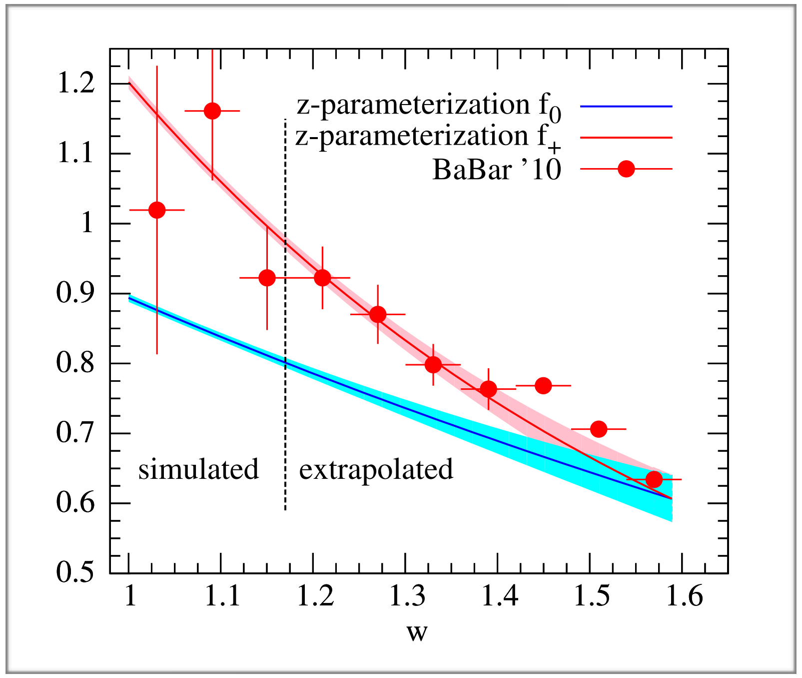


$|V_{cb}|$ (before 2/2017)



Published $B \to D$

Fermilab / MILC



Source	$f_+(\%)$	$f_0(\%)$
Statistics+matching+χPT cont. extrap.	1.2	1.1
(Statistics)	(0.7)	(0.7)
(Matching)	(0.7)	(0.7)
(χPT/cont. extrap.)	(0.6)	(0.5)
Heavy-quark discretization	0.4	0.4
Lattice scale r_1	0.2	0.2
Total error	1.2	1.1

Bailey et al. (FNAL/MILC), arXiv:1503.07237

HPQCD

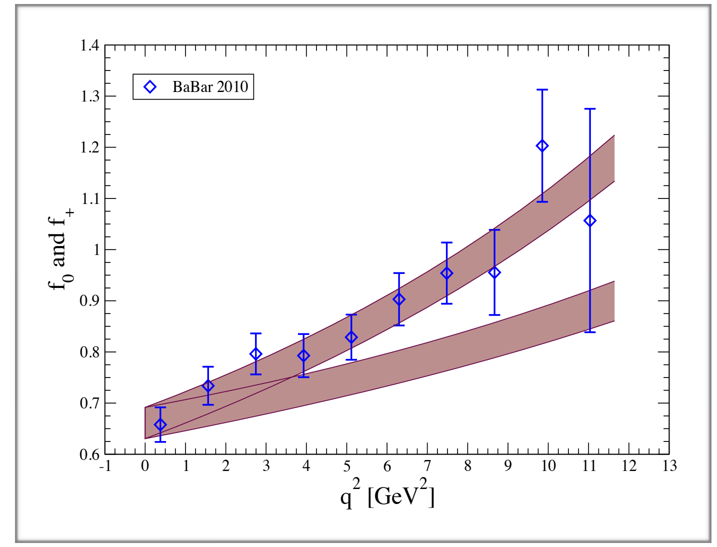


TABLE V. Error budget table for $|V_{cb}|$. The first three rows are from experiments, and the rest are from lattice simulations.

Type	Partial errors [%]
experimental statistics	1.55
experimental systematic	3.3
meson masses	0.01
lattice statistics	1.22
chiral extrapolation	1.14
discretization	2.59
kinematic	0.96
matching	2.11
electro-weak	0.48
finite size effect	0.1
total	5.34

Na et al. (HPQCD), arXiv:1505.03925

Published $B \to D^*$

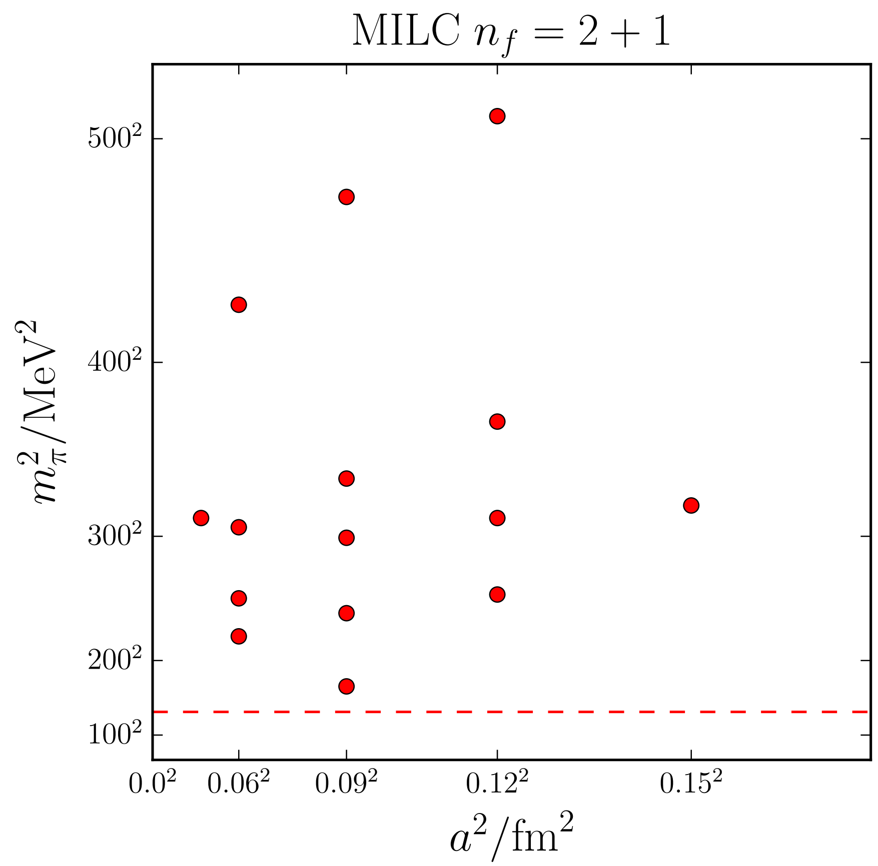
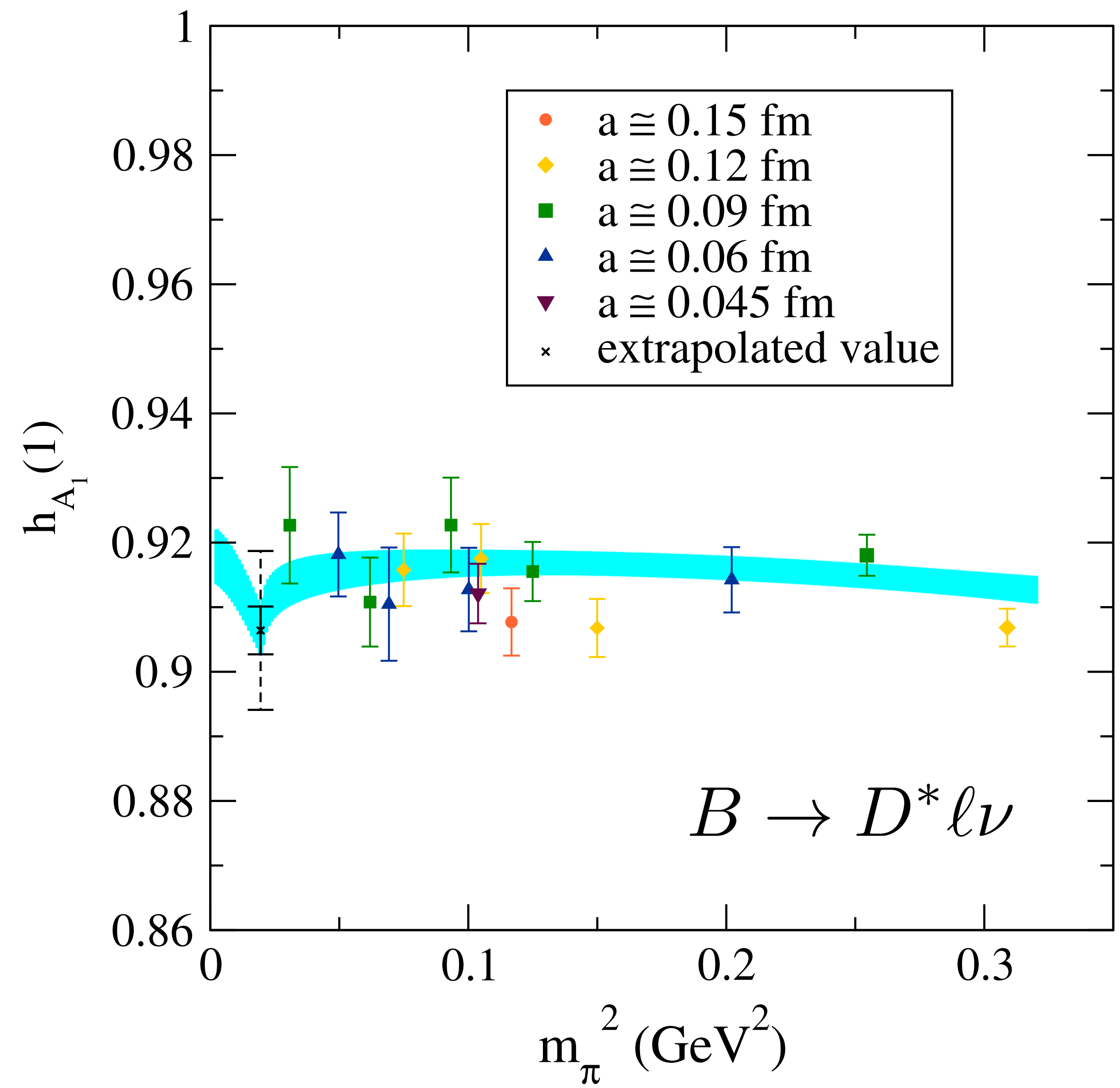


TABLE X. Final error budget for $h_{A_1}(1)$ where each error is discussed in the text. Systematic errors are added in quadrature and combined in quadrature with the statistical error to obtain the total error.

Uncertainty	$h_{A_1}(1)$
Statistics	0.4%
Scale (r_1) error	0.1%
χPT fits	0.5%
$g_{D^*D\pi}$	0.3%
Discretization errors	1.0%
Perturbation theory	0.4%
Isospin	0.1%
Total	1.4%

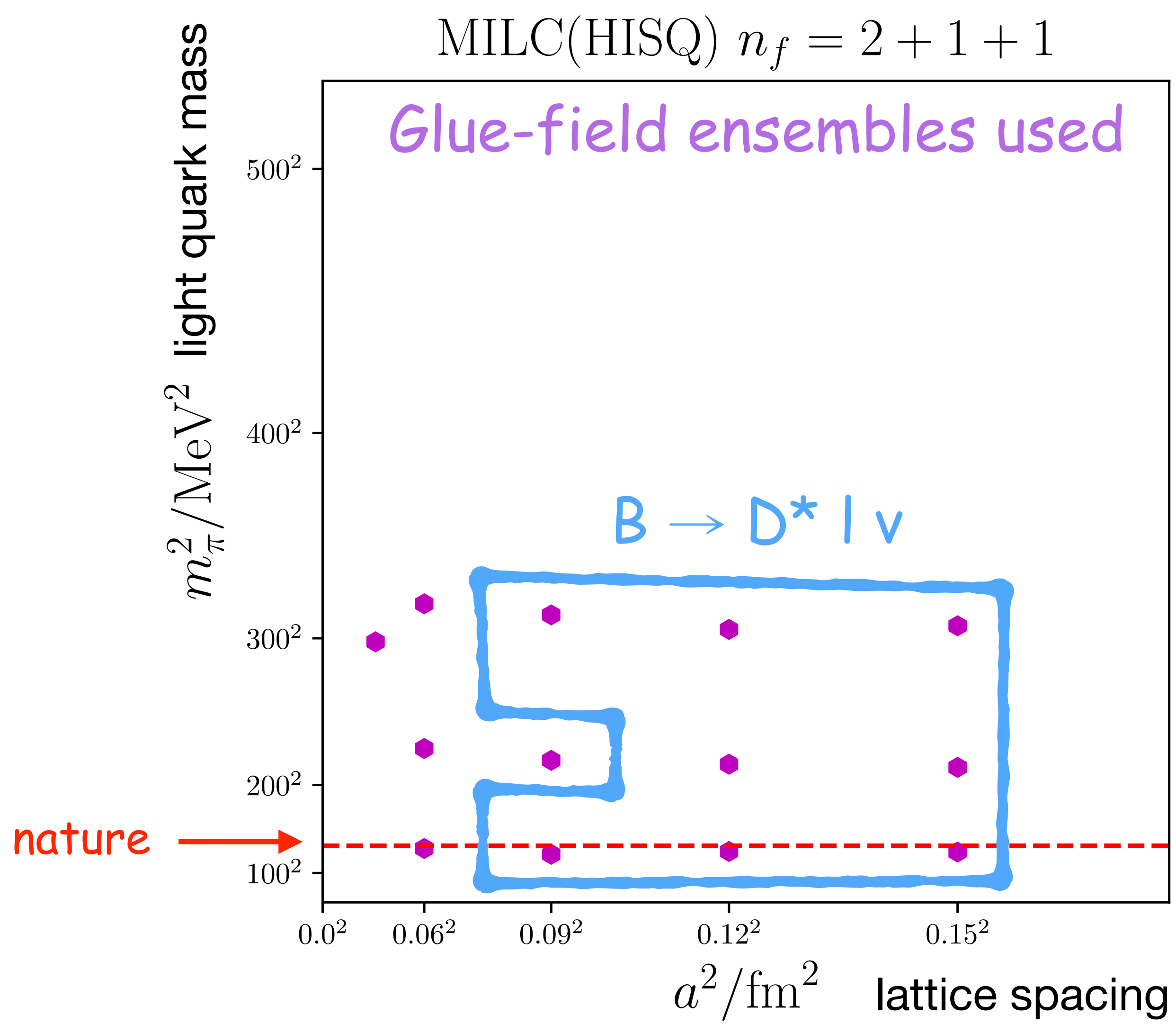
FNAL/MILC



Bailey et al. (FNAL/MILC), PRD89 (2014)

HPQCD calculation

Judd Harrison, Christine Davies, MBW (HPQCD)



- Statistically independent calculations from Fermilab/ MILC
- HISQ vs. AsqTad light/strange
- HISQ vs. FNAL charm
- NRQCD vs. FNAL bottom

Zero recoil

$$\frac{d\Gamma}{dw}(\bar{B}^0 \to D^{*+} l^- \bar{\nu}_l) = \frac{G_F^2 M_{D^*}^3 |\bar{\eta}_{EW} V_{cb}|^2}{4\pi^3} (M_B - M_{D^*})^2 \sqrt{w^2 - 1}\, \boxed{\chi(w) |\mathcal{F}(w)|^2}$$

$$\chi(1) = 1$$

$$\boxed{\mathcal{F}(1) = h_{A_1}(1)} = \frac{M_B + M_{D^*}}{2\sqrt{M_B M_{D^*}}} \boxed{A_1(q^2_{\max})}$$

$$\boxed{\langle D^*(p', \epsilon) | \bar{q} \gamma^\mu \gamma^5 Q | B(p) \rangle} = 2 M_{D^*} A_0(q^2) \frac{\epsilon^* \cdot q}{q^2} q^\mu + (M_B + M_{D^*}) \boxed{A_1(q^2)} \left[\epsilon^{*\mu} - \frac{\epsilon^* \cdot q}{q^2} q^\mu \right] - A_2(q^2) \frac{\epsilon^* \cdot q}{M_B + M_{D^*}} \left[p^\mu + p'^\mu - \frac{M_B^2 - M_{D^*}^2}{q^2} q^\mu \right].$$

$$\langle D^*(p', \epsilon) | \bar{q} \gamma^\mu Q | B(p) \rangle = \frac{2iV(q^2)}{M_B + M_{D^*}} \epsilon^{\mu\nu\rho\sigma} \epsilon^*_\nu p'_\rho p_\sigma$$

NRQCD matching

$$\langle \mathcal{J}^i \rangle = (1 + \alpha_s(\eta - \tau))\langle J^{(0)i}_{\rm latt} \rangle + \langle J^{(1)i}_{\rm latt} \rangle + e_4 \frac{\Lambda^2_{\rm QCD}}{m_b^2}$$

$$J^{(0)i}_{\rm latt}(x) = \bar{c}\gamma^i\gamma^5 Q$$
$$J^{(1)i}_{\rm latt}(x) = -\frac{1}{2am_b}\bar{c}\gamma^i\gamma^5\gamma\cdot\Delta Q$$

1-loop coefficients η & τ from Monahan, Shigemitsu, Horgan, PRD87 (2013)

Truncation errors enter at order: $\frac{\Lambda^2_{\rm QCD}}{m_b^2}$ included as Gaussian noise

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$\alpha_s \tau \langle J^{(0)}_{\mathrm{latt}} \rangle$	$\langle J^{(1)}_{\mathrm{latt}} \rangle$		
$0.00559(8)$	$0.0078(66)$	very coarse	& physical sea quark masses
$0.0064(1)$	$0.0055(48)$	coarse	
$0.0080(9)$	$0.0048(6)$	fine	

Cancellation expected from Luke's theorem

Chiral-continuum fit

Fit function:

Static limit

δ_a: disc errors

light quark mass

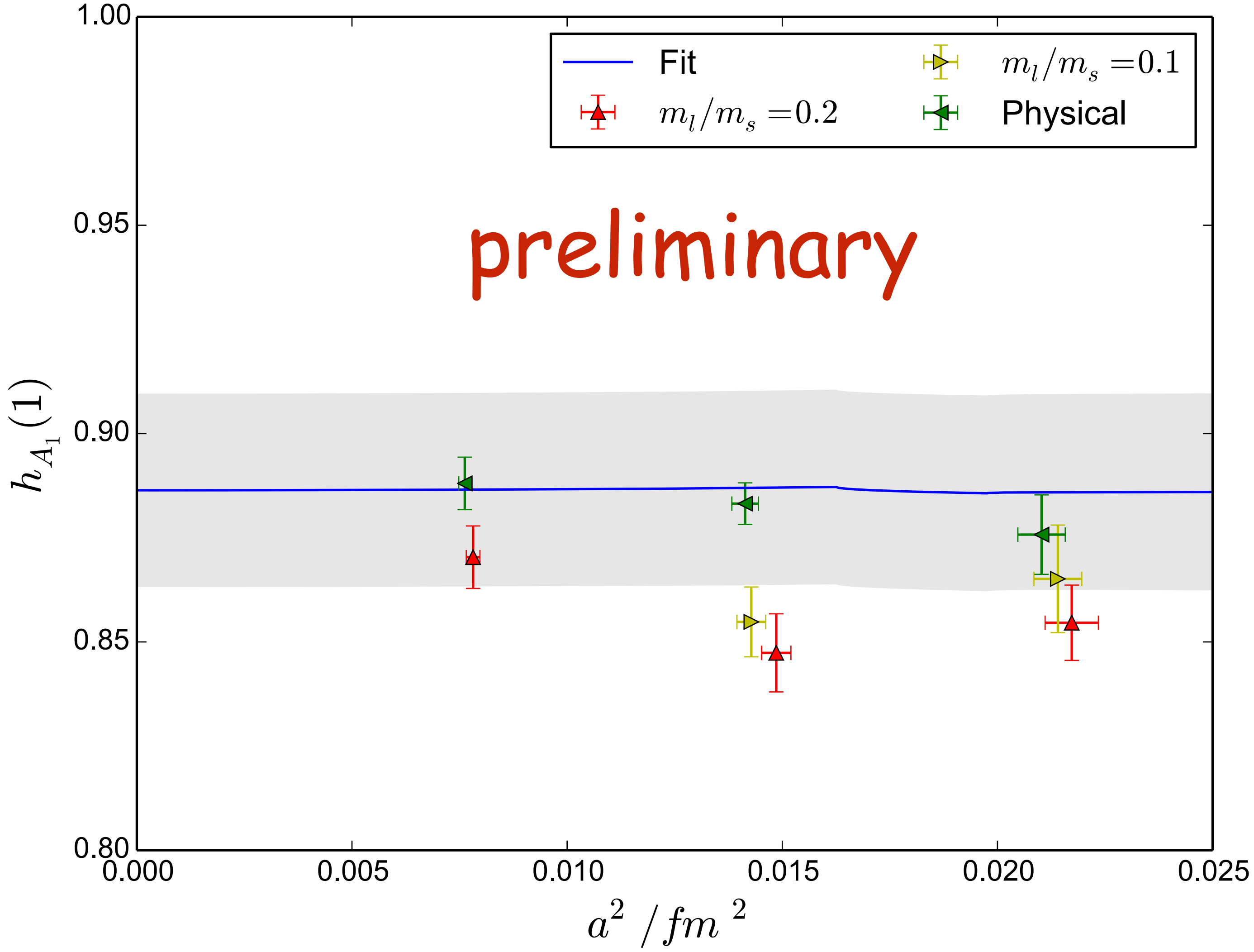
$$h_{A_1}(1) = (1+\delta_a^B)B + \delta_a^g \frac{g^2}{48\pi^2 f^2} \times \text{chiral logs} + C\frac{M_\pi^2}{\Lambda_\chi^2}$$
$$+ e_1\alpha_s^2\Big[1 + e_5(am_b - 2)/2 + e_6((am_b-2)/2)^2\Big]J_{\text{latt}}^{(0)}$$

2-loop matching error

with g^2 = 0.53(8)

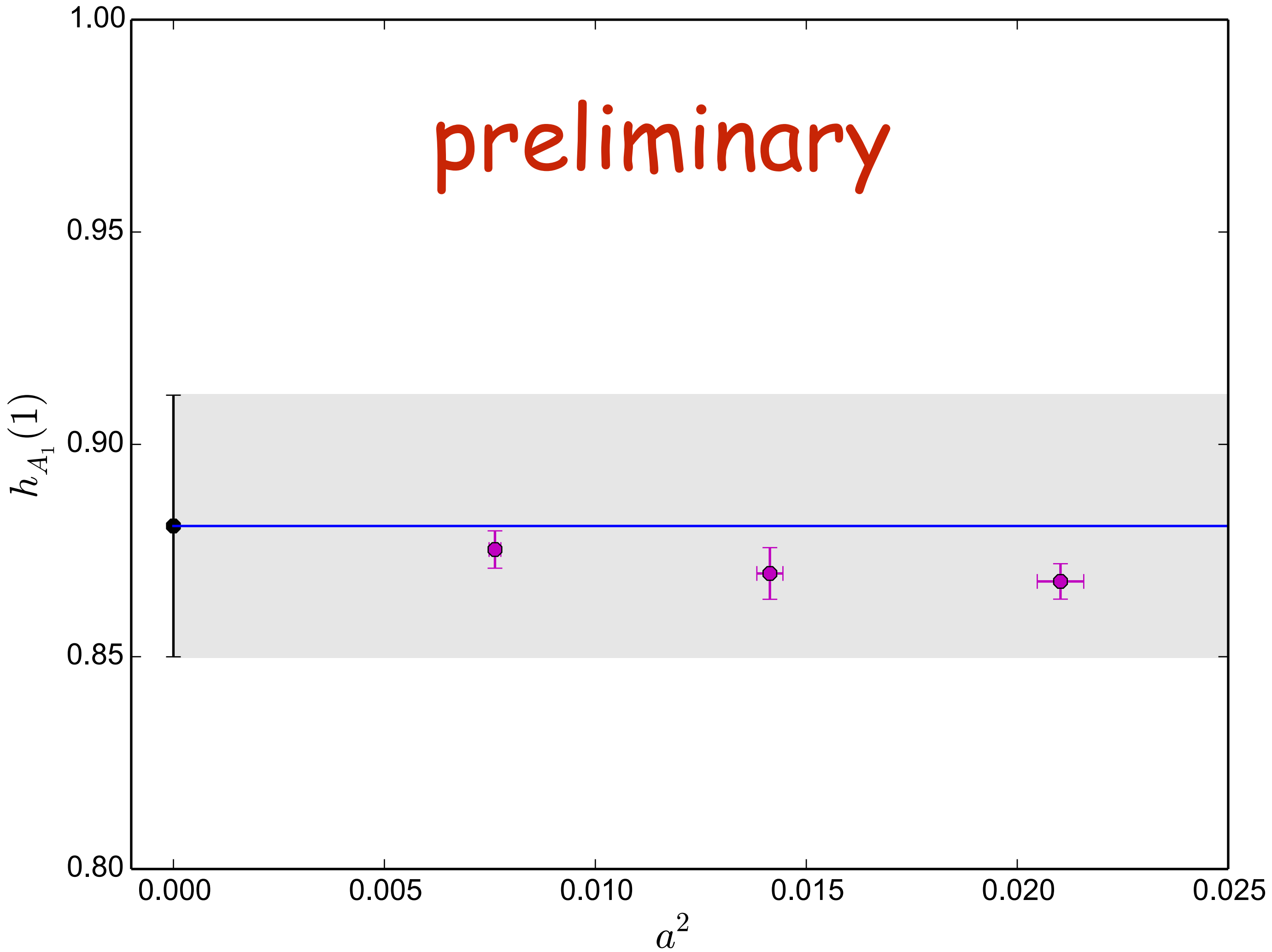
The α_s^2 uncertainty is the largest, by a factor of 2, compared to others

$B \to D^*$, lattice spacing



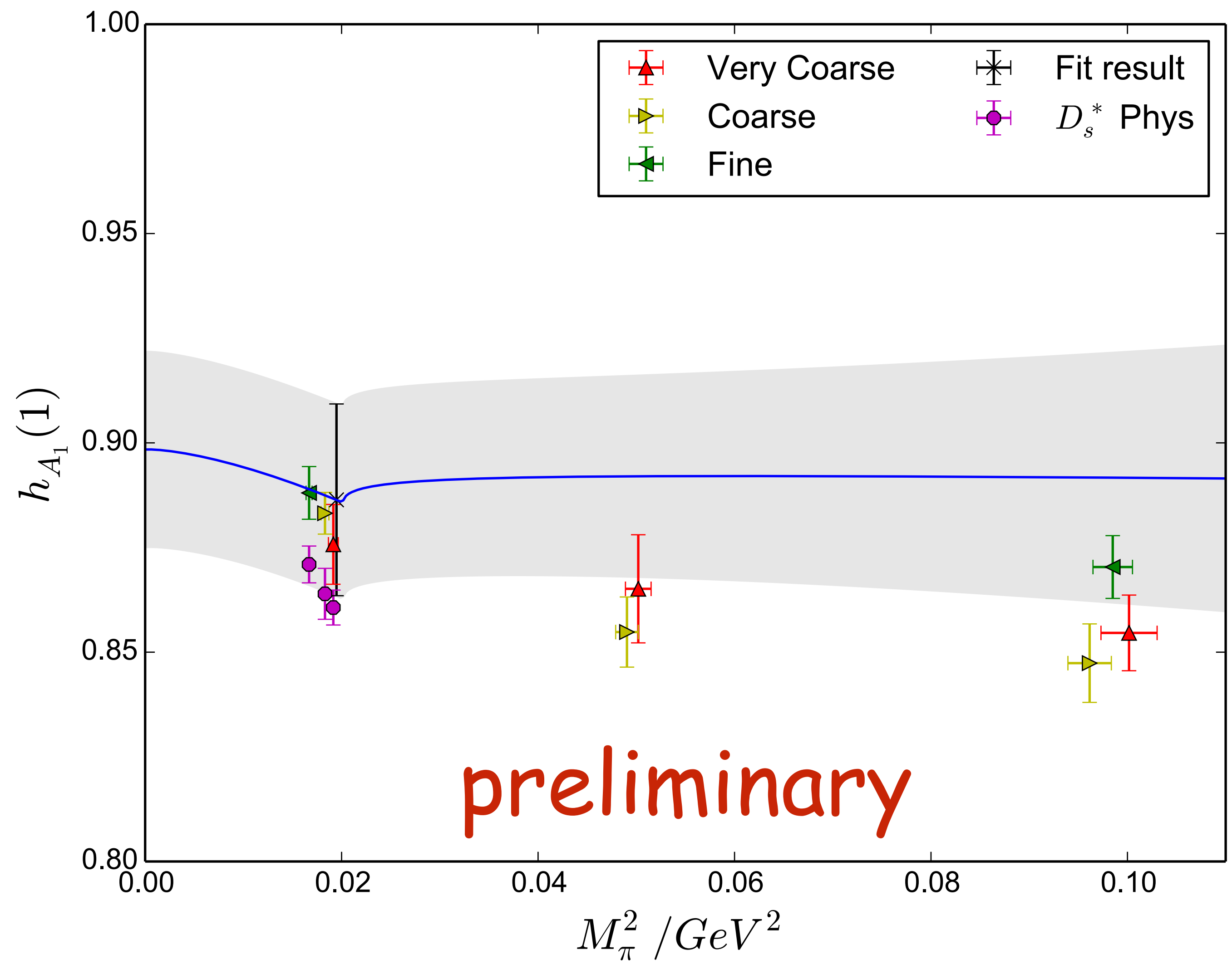
Continuum-physical mass curve

$B_s \rightarrow D_s^*$, lattice spacing



Continuum-physical mass curve

$B_{(s)} \rightarrow D_{(s)}{}^*$, quark mass



Continuum-physical mass curve

Preliminary results

$$\mathcal{F}^{B\to D^*}(1) = h_{A_1}(1) = 0.886(9)_{\rm stat}(21)_{\rm sys}$$

$$\mathcal{F}^{B_s\to D_s^*}(1) = h^s_{A_1}(1) = 0.881(12)_{\rm stat}(29)_{\rm sys}$$

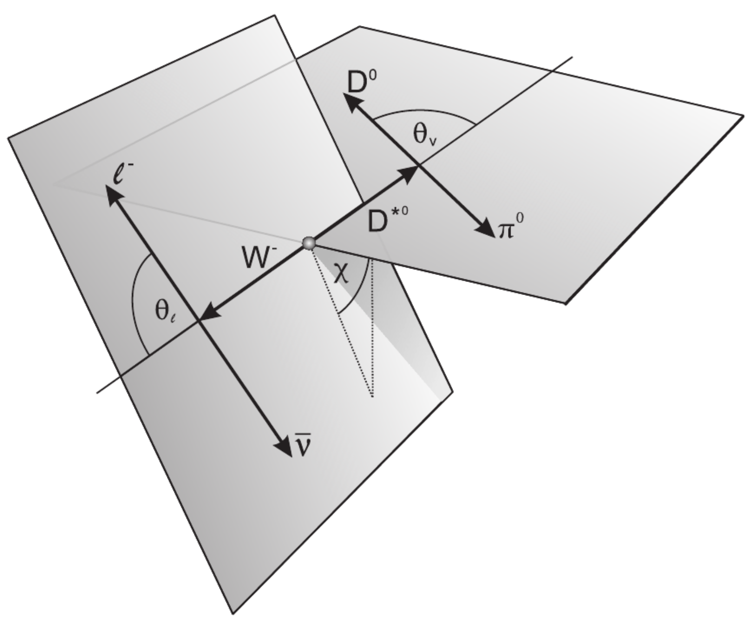
$$\frac{\mathcal{F}^{B\to D^*}(1)}{\mathcal{F}^{B_s\to D_s^*}(1)} = \frac{h_{A_1}(1)}{h^s_{A_1}(1)} = 1.006(18)_{\rm stat}(16)_{\rm sys}$$

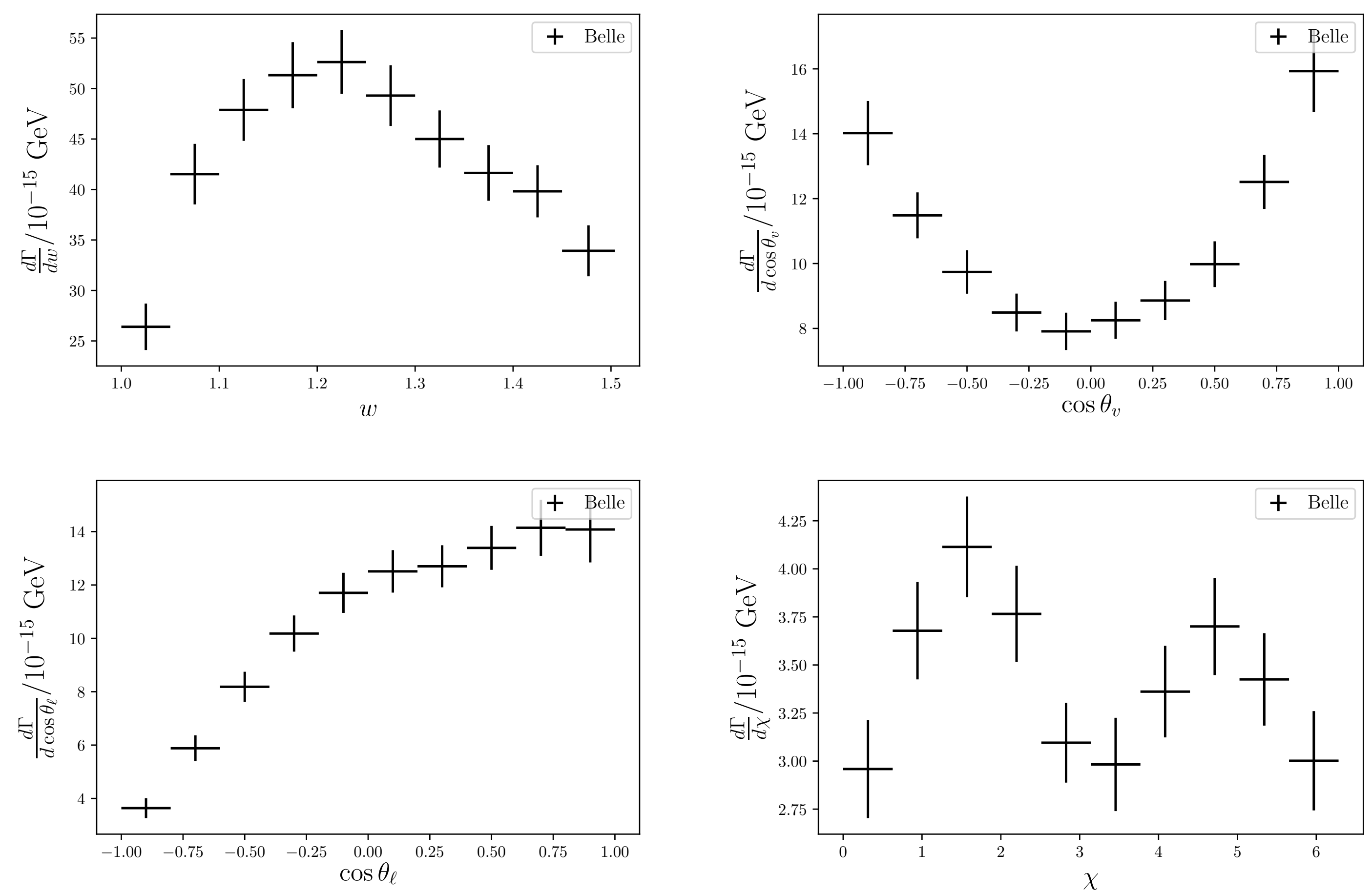
$h_{A_1}(1) = 0.906(4)_{\rm stat}(12)_{\rm sys}$ Fermilab/MILC, 2014

$h_{A_1}(1) = 0.901(11)$ weighted average

N.B. Change relative to Harrison et al (Lattice 2016) is due to better fits to very coarse, physical point data.

unfolded Belle data





Abdesselam et al., arXiv:1702.01521

CLN parametrization

Form factors entering helicity amplitudes (massless leptons)

$$h_{A_1}(w) = h_{A_1}(1)[1 - 8\rho^2 z + (r_{h2r}\rho^2 + r_{h2})z^2 + (r_{h3r}\rho^2 + r_{h3})z^3]$$
$$R_1(w) = R_1(1) + r_{11}(w-1) + r_{12}(w-1)^2$$
$$R_2(w) = R_2(1) + r_{21}(w-1) + r_{22}(w-1)^2$$

Fixed:

$$r_{h2r} = 53\,, r_{h2} = -15\,, r_{h3r} = -231\,, r_{h3} = 91$$
$$r_{11} = -0.12\,, r_{12} = 0.05\,, r_{21} = 0.11\,, r_{22} = -0.06$$

Using this “tight” CLN parametrization

$$I = |\bar{\eta}_{EW} V_{cb}| h_{A_1}(1)$$

$$I_{\text{Belle}} = 0.0348(12)$$ (unfolded)

$$I_{\text{HFLAV}} = 0.03561(11)(44)$$

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HQET uncertainties

$$h_{A_1}(w) = h_{A_1}(1)[1 - 8\rho^2 z + (r_{h2r}\rho^2 + r_{h2})z^2 + (r_{h3r}\rho^2 + r_{h3})z^3]$$
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Coefficients calculated through *Λ/m*

$$r_{h2r} = 53\,, r_{h2} = -15\,, r_{h3r} = -231\,, r_{h3} = 91$$ **BIG!**

$$r_{11} = -0.12\,, r_{12} = 0.05\,, r_{21} = 0.11\,, r_{22} = -0.06$$ small!

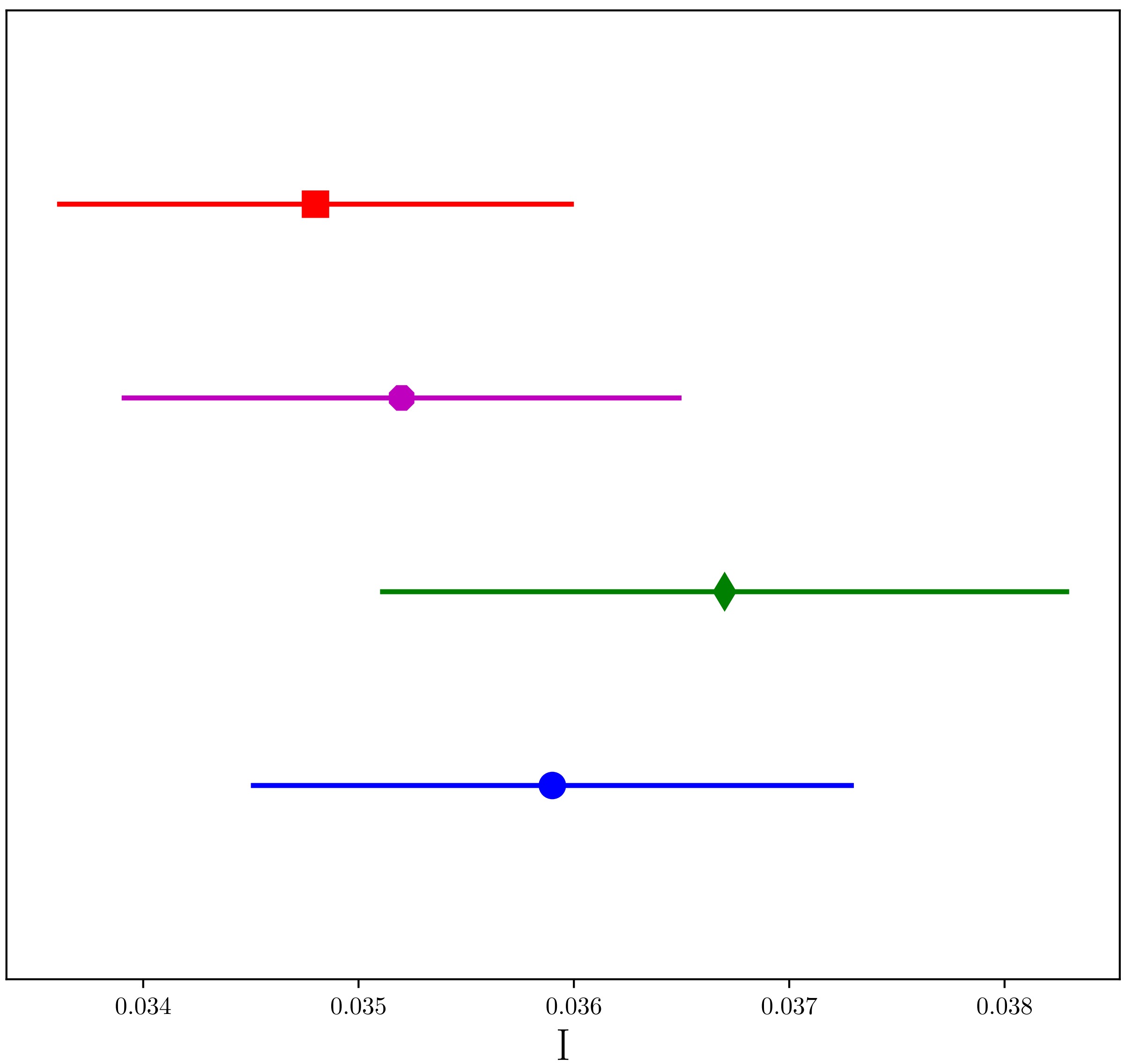
Ratios $V(q^2) = \frac{R_1(w)}{r'} h_{A_1}(w)$ $A_2(q^2) = \frac{R_2(w)}{r'} h_{A_1}(w)$

What are the uncertainties for the r's? 20%? 100%?

See P Gambino talk & papers by Bigi, Gambino, Schacht; Grinstein & Kobach; Bernlochner et al.; Jaiswal, et al.

Effects of uncertainties

CLN fits with errors for *r* ’s. Include *r* ’s as fit parameters with Gaussian priors



r_h	r_{ij}
0%	0%
20%	20%
100%	100%
10%	0 ± 1

BGL parametrization

$$F(t) = Q_F(t) \sum_{k=0}^{K_F-1} a_k^{(F)} z^k(t, t_0) \qquad Q_F(t) = \frac{1}{B_n(z)\phi_F(z)}$$

Blaschke factor

$$B_n(z) = \prod_{i=1}^{n} \frac{z - z_{P_i}}{1 - z z_{P_i}} \qquad z_{P_i} = z(M_{P_i}^2, t_-)$$

Unitarity bounds

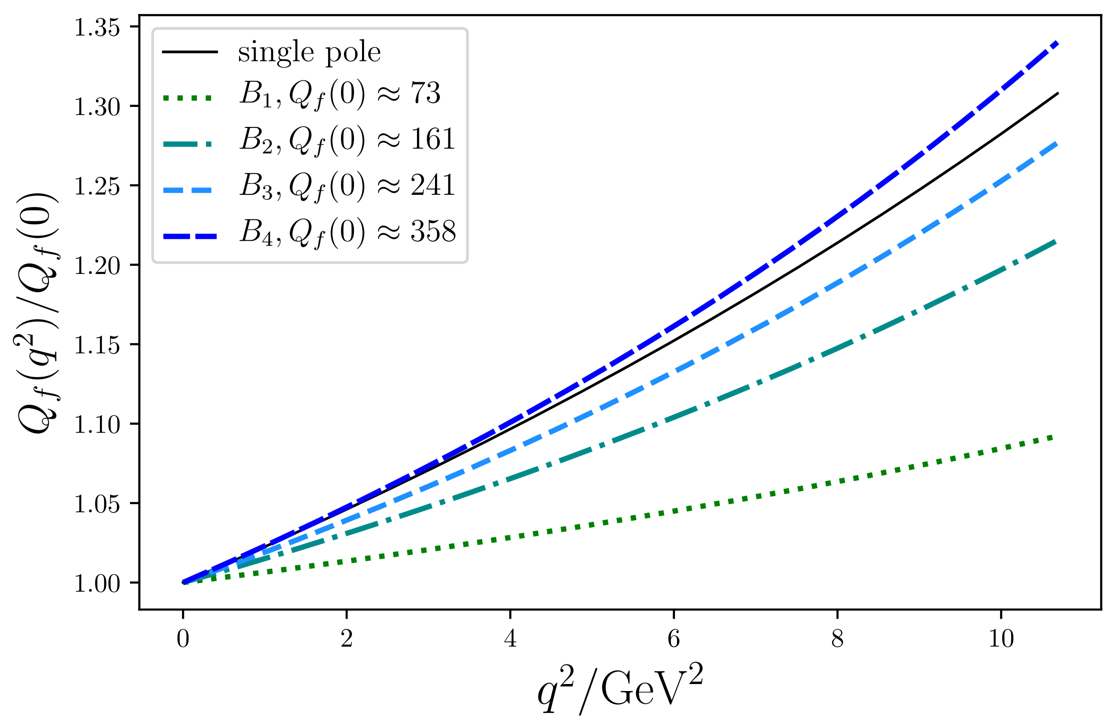
$$S_{fF} = \sum_{k=0}^{K_f-1} [(a_k^{(f)})^2 + (a_k^{(F_1)})^2] \leq 1 \qquad S_g = \sum_{k=0}^{K_g-1} (a_k^{(g)})^2 \leq 1$$

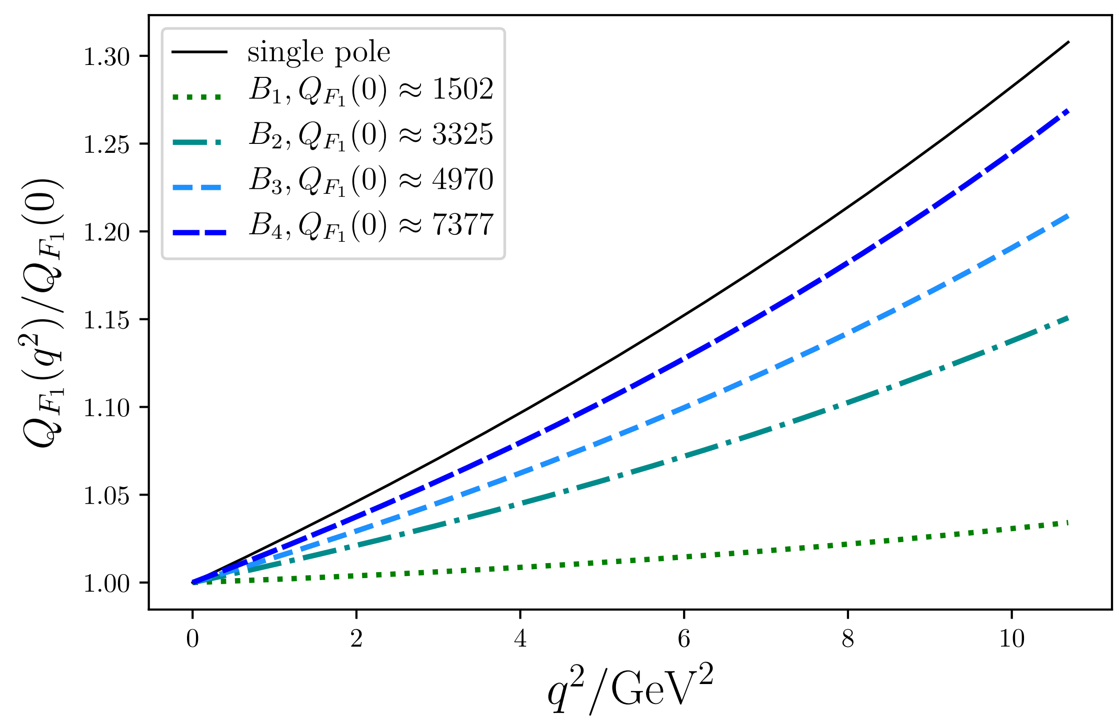
Predictions for B_c vector & axial vector resonances

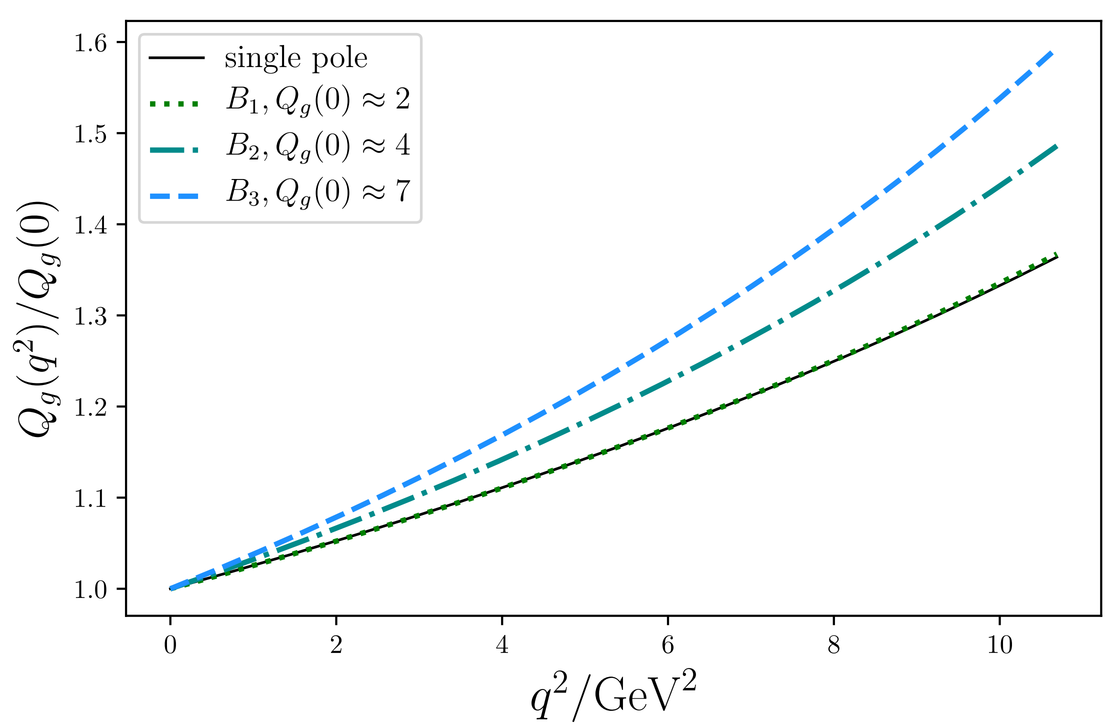
$$M_B + M_{D^*} = 7.290 \text{ GeV}$$

M_{1^-}/GeV	method	Ref.	M_{1^+}/GeV	method	Ref.
6.335(6)	lattice	[77]	6.745(14)	lattice	[77]
6.926(19)	lattice	[77]	6.75	model	[79, 80]
7.02	model	[79]	7.15	model	[79, 80]
7.28	model	[81]	7.15	model	[79, 80]

Dependence on n







BGL fits

fit	n_B^+	n_B^-	K	I	$a_0^{(f)}$	$a_1^{(f)}$	$a_0^{(F_1)}$	$a_1^{(F_1)}$	$a_0^{(g)}$	$a_1^{(g)}$	S_{fF}	S_g
BGL	2	2	2	0.0366(14)	0.02996(38)	−0.119(51)	0.005017(63)	−0.0146(40)	0.031(15)	0.88(50)	0.015(12)	0.78(89)
BGL	2	2	3	0.0376(16)	0.02996(38)	−0.147(62)	0.005016(63)	−0.030(13)	0.029(14)	0.98(50)	0.13(32)	0.97(98)
BGL	2	2	4	0.0376(16)	0.02996(38)	−0.147(62)	0.005016(63)	−0.030(13)	0.029(14)	0.98(50)	0.13(33)	0.97(98)
BGL	3	3	2	0.0368(15)	0.01908(24)	−0.069(36)	0.003195(40)	−0.0073(27)	0.0137(85)	0.63(30)	0.0051(49)	0.39(38)
BGL	3	3	3	0.0379(17)	0.01908(24)	−0.088(47)	0.003195(40)	−0.0180(85)	0.0125(82)	0.68(31)	0.06(21)	0.46(41)
BGL	3	3	4	0.0379(17)	0.01908(24)	−0.088(47)	0.003195(40)	−0.0180(87)	0.0125(82)	0.68(31)	0.06(21)	0.46(41)
BGL	4	3	2	0.0369(15)	0.01225(15)	−0.035(23)	0.002051(26)	−0.0032(18)	0.0137(84)	0.62(30)	0.0014(17)	0.39(37)
BGL	4	3	3	0.0380(17)	0.01225(15)	−0.049(36)	0.002051(26)	−0.0101(57)	0.0129(84)	0.66(32)	0.04(25)	0.43(42)
BGL	4	3	4	0.0380(17)	0.01225(15)	−0.049(36)	0.002051(26)	−0.0102(59)	0.0129(86)	0.66(33)	0.04(25)	0.43(43)

- Keeping only K=2 terms in z-expansion gives lower *I*
- Including more resonances does not affect *I*, but does lower S sums

BCL parametrization

Simple form which uses less theoretical information.

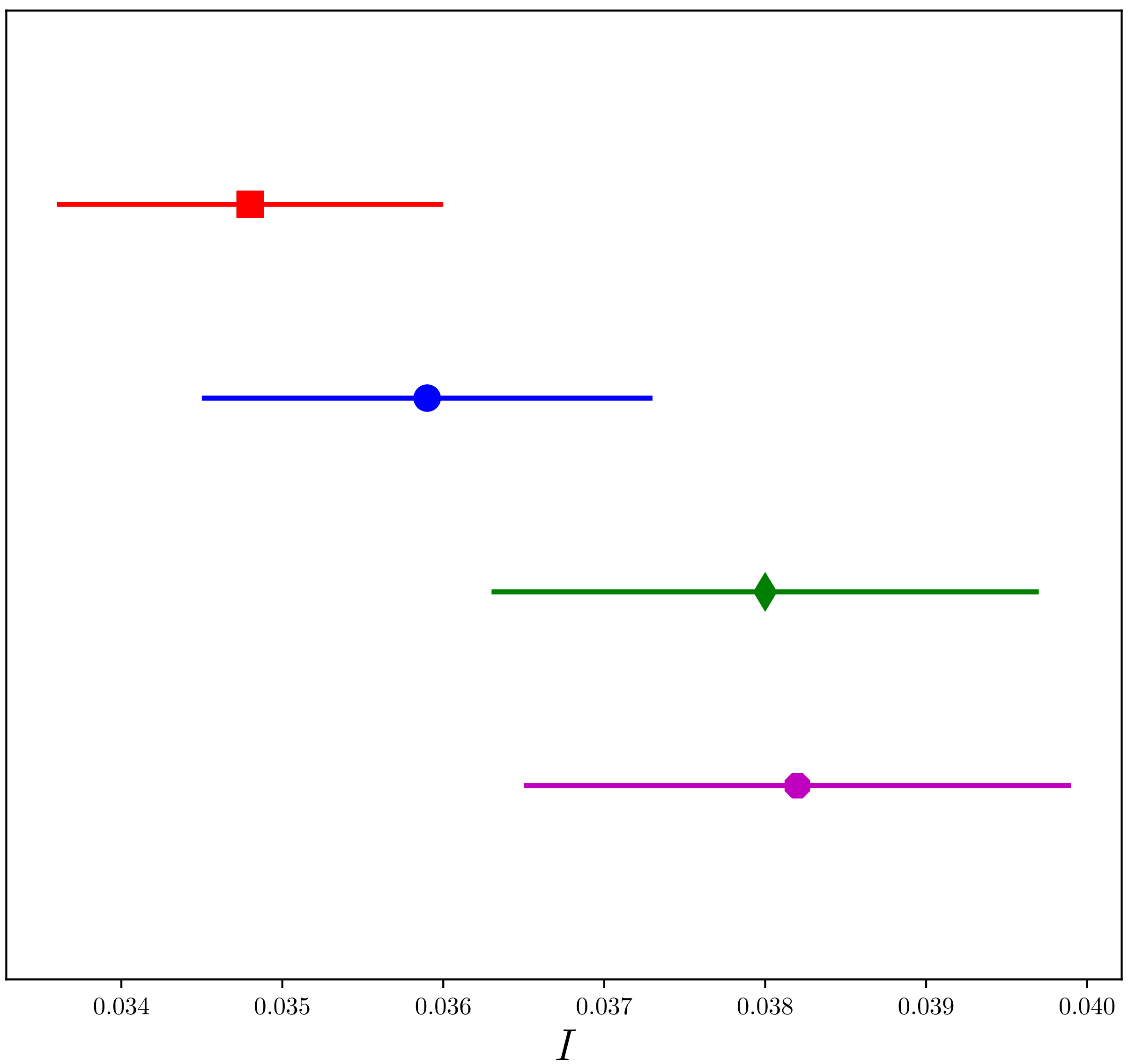
$$F(t) = Q_F(t) \sum_{k=0}^{K_F-1} a_k^{(F)} z^k(t, t_0) \qquad\qquad Q_F(t) = \frac{N_F}{1 - \frac{t}{M_P^2}}$$

Using BGL as a guide, choose N_f = 300, N_{F1} = 7000, N_g = 5

Clean baseline, against which affects of theoretical input (HQET, unitarity bounds) can be measured

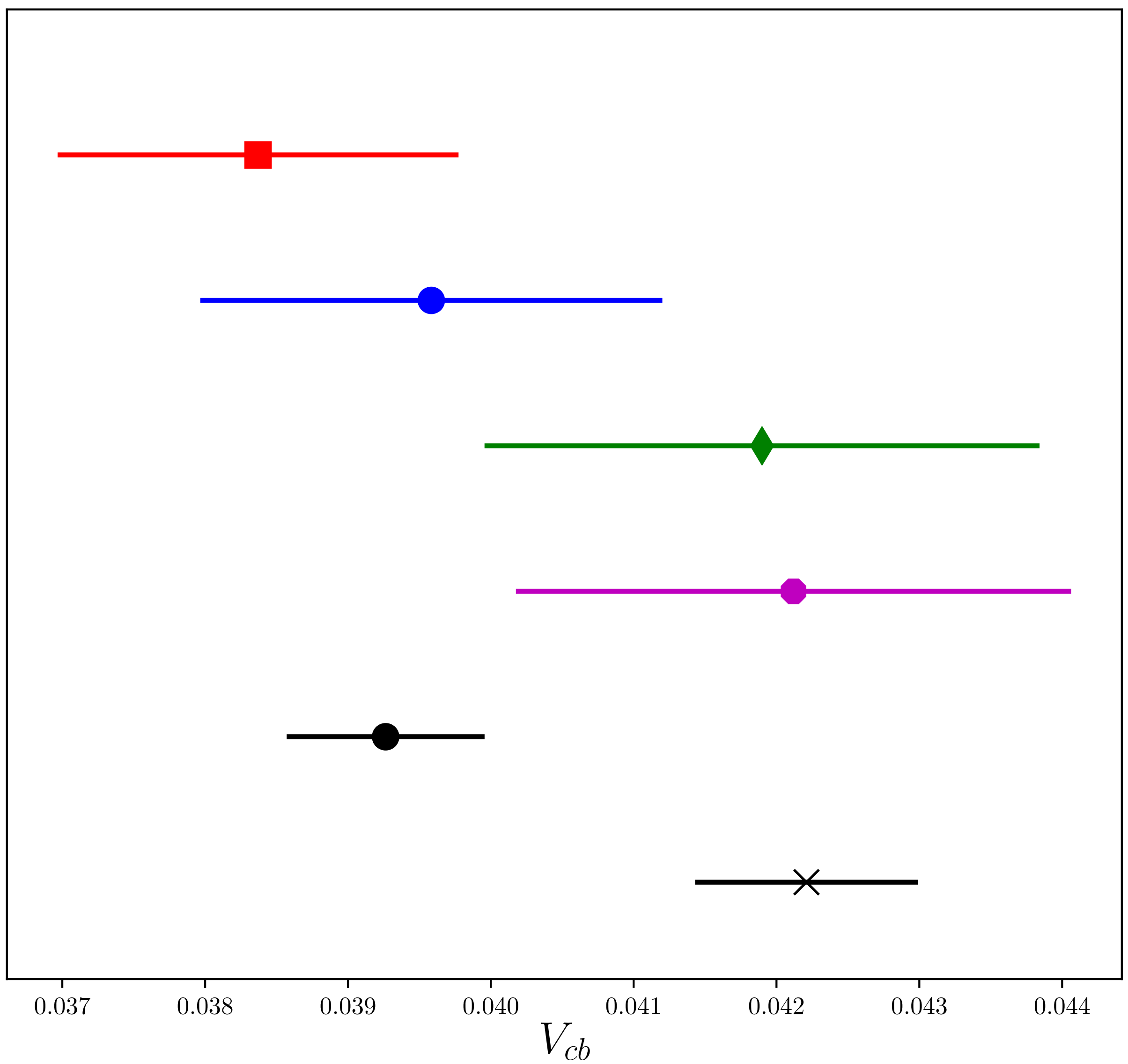
fit	n_B^+	n_B^-	K	I	$a_0^{(f)}$	$a_1^{(f)}$	$a_0^{(F_1)}$	$a_1^{(F_1)}$	$a_0^{(g)}$	$a_1^{(g)}$	S_{fF}	S_g
BCL	–	–	2	0.0367(15)	0.01496(19)	−0.047(27)	0.002935(37)	−0.0029(27)	0.027(13)	0.77(44)	0.0025(26)	0.60(69)
BCL	–	–	3	0.0378(17)	0.01496(19)	−0.065(40)	0.002935(37)	−0.0135(82)	0.026(13)	0.82(46)	0.08(38)	0.67(75)
BCL	–	–	4	0.0382(18)	0.01497(19)	−0.310(42)	0.002936(37)	−0.0151(83)	0.109(16)	−0.29(37)	0.143(67)	0.10(22)
BCL	–	–	5	0.0382(18)	0.01497(19)	−0.310(42)	0.002936(37)	−0.0151(83)	0.109(16)	−0.29(37)	0.143(67)	0.10(22)

Parametrization dependence



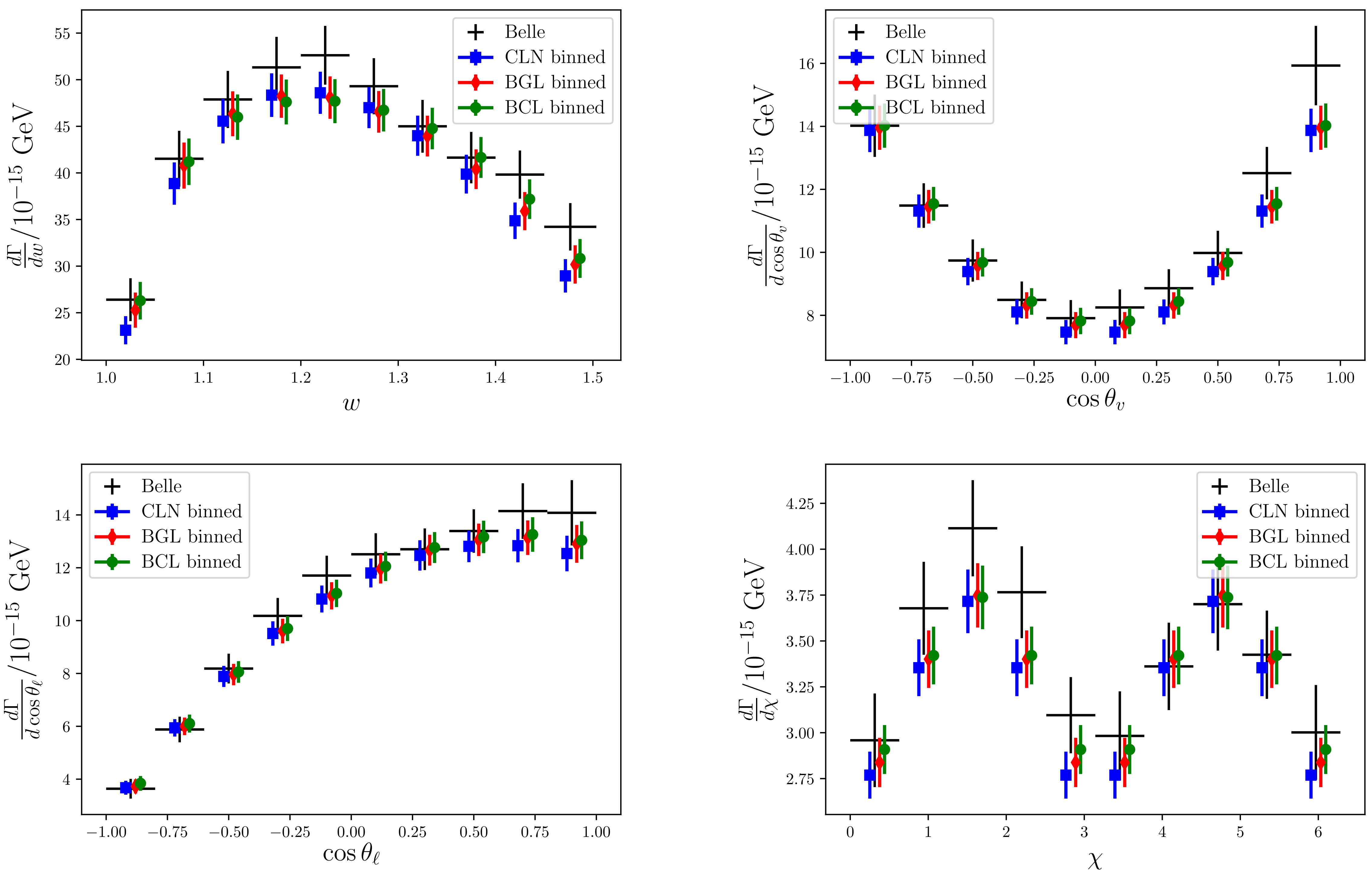
	r_h	r_{ij}
CLN	0%	0%
CLN	10%	0 ± 1
BGL		
BCL		

Parametrization dependence



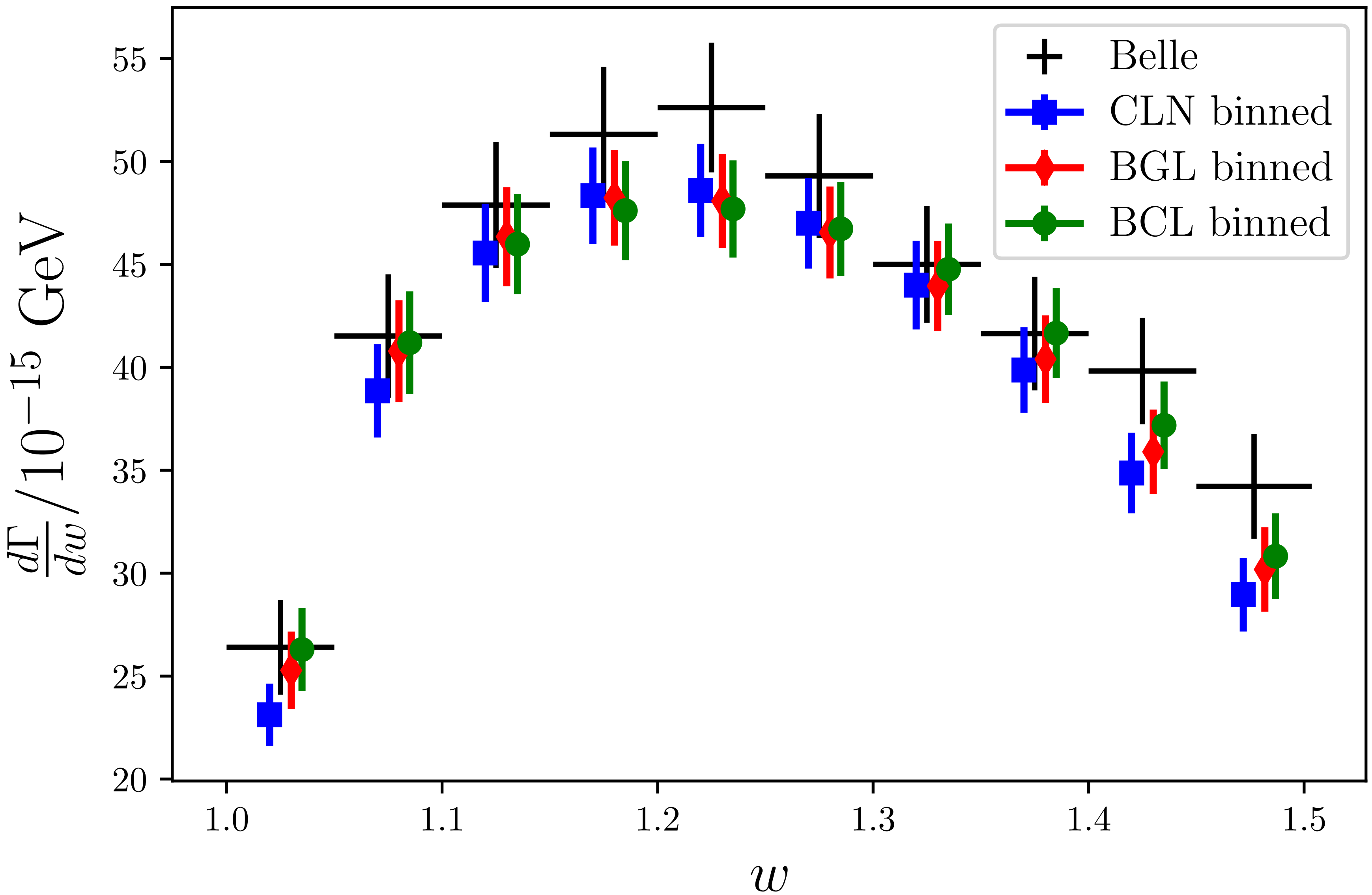
	r_h	r_{ij}
CLN	0%	0%
CLN	10%	0 ± 1
BGL		
BCL		
HFLAV /		
Inclusive		

Fits & data



Belle
CLN binned
BGL binned
BCL binned
$\frac{d\Gamma}{dw}/10^{-15}$ GeV
w
$\frac{d\Gamma}{d\cos\theta_v}/10^{-15}$ GeV
$\cos\theta_v$
$\frac{d\Gamma}{d\cos\theta_\ell}/10^{-15}$ GeV
$\cos\theta_\ell$
$\frac{d\Gamma}{d\chi}/10^{-15}$ GeV
χ

Fits & data



Belle
CLN binned
BGL binned
BCL binned
$\frac{d\Gamma}{dw}/10^{-15}$ GeV
w
55
50
45
40
35
30
25
20
1.0
1.1
1.2
1.3
1.4
1.5

Conclusions

- Independent calculation of zero recoil form factor, very soon to appear on arXiv
- Good agreement with Fermilab/MILC result
- Need to understand theoretical uncertainties inherent in parametrization
- Lattice QCD away from zero recoil can help