The twist 2 pion DA from Euclidean correlation functions

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with

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Lattice meets continuum@Siegen

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Outline

- DAs: definitions and moments of DAs
- Lattice calculation of the 2nd pion DA[1]
- "Direct" lattice calculation of the pion DA[2]
- First position space results
- Outlook

1) RQCD: VM Braun, S Collins, M Göckeler, P Pérez-Rubio, A Schäfer, RW Schiel, A Sternbeck, 1503.03656; RQCD: GB, VM Braun, M Göckeler, M Gruber, F Hutzler, P Korcyl, B Lang, A Schäfer, 1705.10236

2) RQCD: GB, VM Braun, M Göckeler, M Gruber, F Hutzler, P Korcyl, B Lang, A Schäfer, P Wein, J-J Zhang, 1709.04325; + B Gläßle, in preparation

Not covered:

ρ DAs [RQCD: VM Braun et al, 1612.02955]

octet baryon DAs [RQCD: GB et al, 1512.02050]

What are distribution amplitudes?

Wavefunction of a hadron (here pion) near the infinite momentum frame, written as a superposition of different Fock states:

$$|\pi\rangle = c_1|\bar{q}q\rangle + c_2|\bar{q}Gq\rangle + c_3|\bar{q}q\bar{q}q\rangle + \dots$$

Light front wavefunction (Distribution amplitude, DA) describes the distribution of the longitudinal momentum among the partons.

Momentum fractions $0 \leq u_f \leq 1$, $\sum_{f\in\{q,\bar{q},G\}} u_f = 1$.

At leading twist (twist 2) only the valence quarks contribute:

$$u = u_q = 1 - u_{\bar{q}}, \quad \xi = u_q - u_{\bar{q}} = 2u - 1 \in [-1, 1].$$

In hard processes higher Fock states are power suppressed.

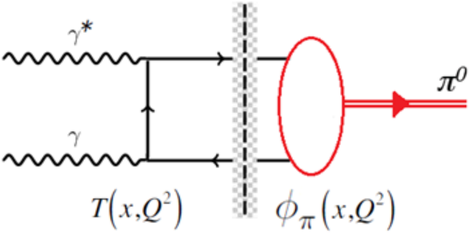
PDFs are (within the parton model) single particle probability densities and can **directly be extracted from fits to DIS and SIDIS data**.

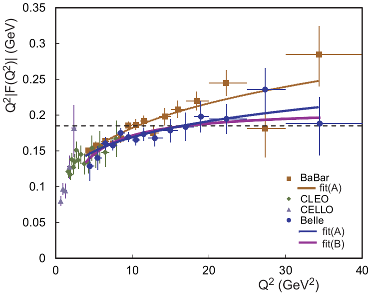
DAs are wavefunctions and only appear within convolutions in hard exclusive processes. Due to other hadronic uncertainties (and experimental techniques), it is much **harder to extract these reliably from experimental data**.

Distribution amplitudes II

DAs are needed for the theoretical description of hard exclusive processes.

Example: collinear factorization of the $\gamma\gamma^* \to \pi^0$ photoproduction formfactor ($Q^2 \gtrsim \mu^2 \gg \Lambda^2$) [Belle, 1205.3249]





$$F_{\pi\gamma}(Q^2) = \frac{2F_\pi}{3}\int_0^1 du\, \underbrace{T_{q\bar{q}\gamma}(u,\mu,Q^2)}_{\text{hard matching function}}\ \underbrace{\phi_\pi(u,\mu)}_{\text{soft factor (DA)}} + \underbrace{\text{higher twist}}_{F_\pi \mathcal{O}(1/Q^2)}.$$

μ is the factorization scale and we renormalize the hard coefficient function T at the scale $\mu_R^2 = Q^2$.

Definition of DAs

Non-local light front matrix element at a separation n ($n^2 = 0$):

$$\left\langle 0 \left| \bar{d}\left(\frac{n}{2}\right) \not{n}\gamma_5 \left[\frac{n}{2}, -\frac{n}{2}\right] u\left(-\frac{n}{2}\right) \right| \pi^+(p) \right\rangle$$
$$= iF_\pi \, n\cdot p \int_0^1 du \, \exp\left\{-\, i \underbrace{[-u + (1-u)]}_{=-\xi} \frac{(n\cdot p)}{2}\right\} \phi_\pi(u,\mu)$$

$[n/2, -n/2]$ above denotes a gauge covariant Schwinger line.

The DA is non-accessible in Euclidean spacetime but moments of DAs are:

$$\langle \xi^n \rangle = \int_0^1 du\,(2u-1)^n \phi_\pi(u,\mu)\,, \quad \langle \xi^0 \rangle = 1\,, \quad \langle \xi^1 \rangle = 0.$$

$\langle \xi^{0,2} \rangle$ can be extracted from local matrix elements $\langle 0 | O^{\pm}_{\mu\nu\rho} | \pi^+(p) \rangle$ with

$$O^{\pm}_{\mu\nu\rho} = \bar{d}\left\{\left[\overleftarrow{D}_{(\mu}\overleftarrow{D}_{\nu} \pm 2\overleftarrow{D}_{(\mu}\overrightarrow{D}_{\nu} + \overrightarrow{D}_{(\mu}\overrightarrow{D}_{\nu}\right]\gamma_{\rho)}\gamma_5\right\} u,$$

where $(\cdots)$ gives a traceless symmetrized expression, e.g.:
$A_{(\mu}B_{\nu)} = \frac{1}{2}\left(A_\mu B_\nu + A_\nu B_\mu\right) - \delta_{\mu\nu}\frac{A\cdot B}{4}$..

Gegenbauer moments

Gegenbauer expansion:

$$\phi_\pi(u,\mu) = 6u(1-u)\left[1 + \sum_{n\in\mathbb{N}} a_{2n}^\pi(\mu)\, C_{2n}^{3/2}(2u-1)\right]$$

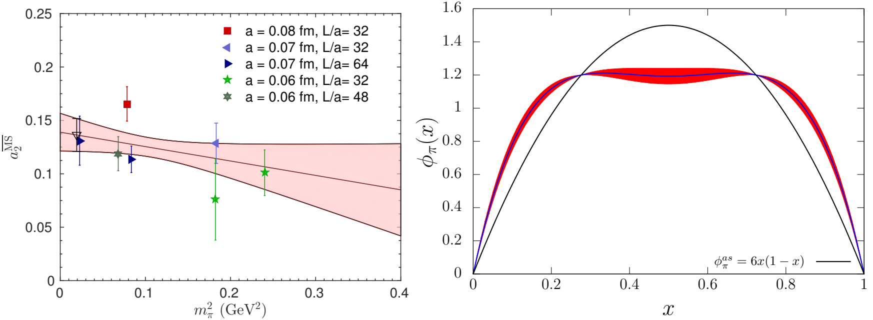
Collinear conformal symmetry: $C_n^{3/2}(\xi)$ in $SL(2,\mathbb{R})$ analogous to $Y_{\ell m}(\theta,\phi)$ in SO(3).
$\langle\xi^{2n}\rangle$ and a_{2n}^π are related by simple algebraic expressions:

$$a_2^\pi(\mu) = \frac{7}{12}\left(5\langle\xi^2\rangle - 1\right) = \frac{7}{12}\left(5\langle\xi^2\rangle - \langle\xi^0\rangle\right)$$

$a_{2n}^\pi(\mu) \to 0$ as $\mu \to \infty$: At large scales the lower moments will dominate.

Previous results ($\mu = 2$ GeV)

$N_f = 2$, $M_\pi = 150 - 490$ MeV, $LM_\pi = 3.4 - 6.7$.



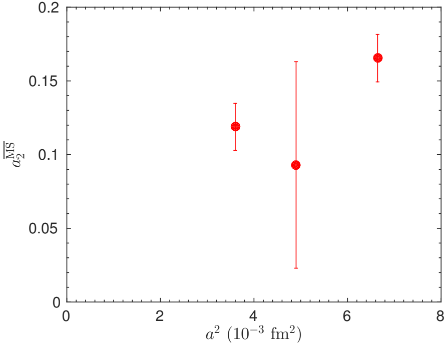
$a_2^{\overline{\text{MS}}}(2\,\text{GeV}) = 0.136(15)(15)(?)$
[RQCD: VM Braun et al, 1503.03656],

$a_2^{\overline{\text{MS}}}(2\,\text{GeV}) = 0.233(30)(60)$ ($N_f = 2 + 1$)
[RBC/UKQCD: R Arthur et al, 1011.5906]

$a_2^{\overline{\text{MS}}}(2\,\text{GeV}) = 0.211(114)$
[QCDSF/UKQCD: VM Braun et al, hep-lat/0606012]

Challenge: statistical errors

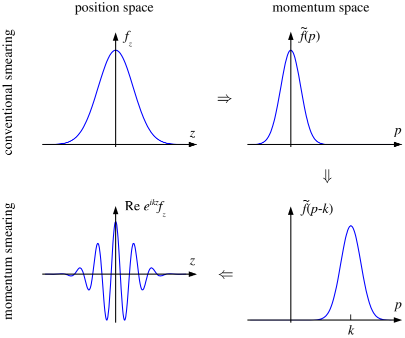
- Continuum extrapolation is not reliable due to big errors:

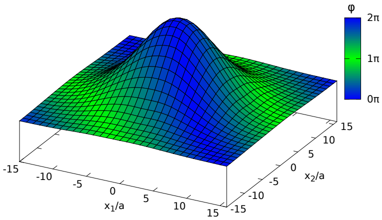


- ⚡⚡⚡ Second moment of pion DA requires at least two non-vanishing momentum components, e.g., $\vec{p} = (1,1,0)2\pi/L$
- ⚡⚡⚡ Moreover, employing two derivatives considerably deteriorates the signal-to-noise ratio.

Momentum smearing

Intuition: interpolating wavefunction used to create $|\pi^+(p)\rangle$ should acquire a phase for $\vec{p} \neq \vec{0} \Rightarrow$ momentum smearing.

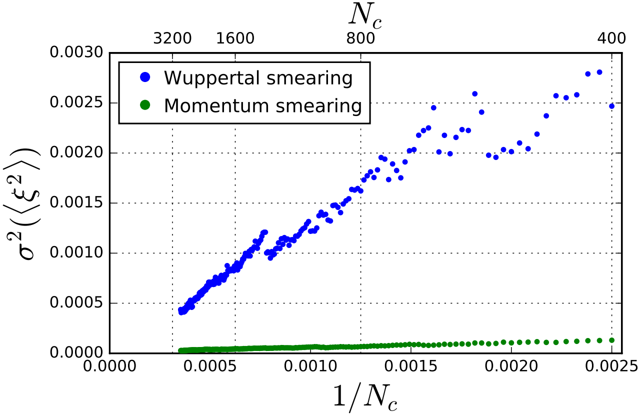




[RQCD: GB, B Lang, B Musch, A Schäfer, 1602.05525]

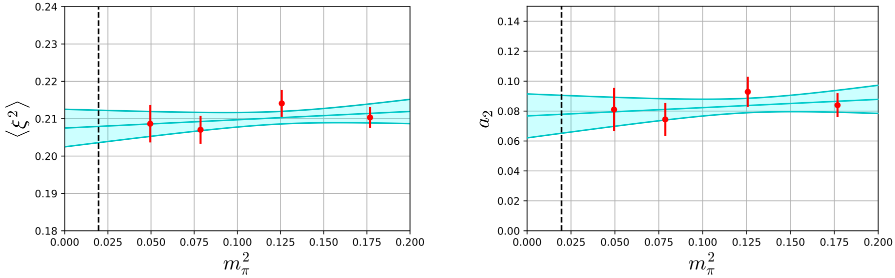
Squared error as a function of the statistics

- Ensemble "H105" for $\vec{n}_{\vec{p}} = (110), (101), (011)$
- Each momentum requires 2 inversions for the momentum smearing (6 inversions).
- Additional momenta only require additional Fourier sums for the Wuppertal smearing (1 inversion).



Chiral Extrapolation

Note that in one loop ChiPT $\nexists$ chiral logs in this DA moment.



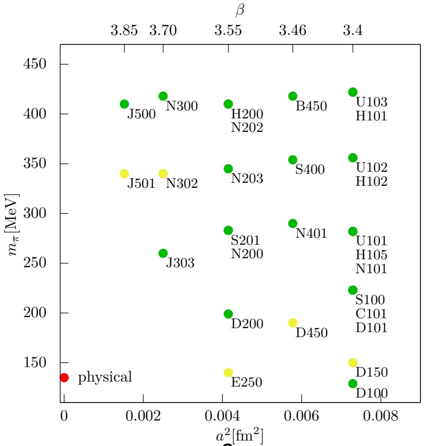
$N_f = 2 + 1$, where $m_s + 2m_{ud} = \text{const}$

[RQCD: GB, VM Braun, M Göckeler, M Gruber, F Hutzler, P Korcyl, B Lang, A Schäfer, 1705.10236]

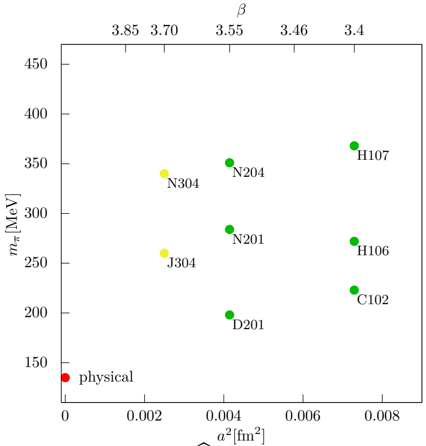
We explore higher order perturbative matching to the $\overline{\text{MS}}$ scheme.

This was at $a \approx 0.086\,\text{fm}$. Soon: the continuum limit.

CLS ensemble overview



$m_s + 2m_\ell = \text{const}$



$\widehat{m}_s \approx \text{const}$

E: $192 \cdot 96^3$, J: $192 \cdot 64^3$, D: $128 \cdot 64^3$, N: $128 \cdot 48^3$, C: $96 \cdot 48^3$,
S: $128 \cdot 32^3$, H: $96 \cdot 32^3$, B: $64 \cdot 32^3$, U: $128 \cdot 24^3$.

$\exists$ additional ensembles with $m_s = m_\ell$.

"Direct" determination of the DA

Large momentum effective theory (LaMET) [X Ji,1305.1539]: compute "quasi-distribution", in analogy to "quasi-PDF".

$$\tilde{\phi}_\pi(u, p_z) = \frac{i}{F_\pi} \int_{-\infty}^{\infty} \frac{dz}{2\pi} e^{-i(u-1)p_z z} \langle \pi(p) | \bar{\psi}(0) \gamma_z \gamma_5 [0, z] \psi(z) | 0 \rangle,$$

with quark fields separated along the spatial z direction ($(z^\mu) = (0, 0, 0, z)$).
Then match to pion DA (like [X Ji, 1506.00248] for PDFs):

$$\tilde{\phi}_\pi(u, a^{-1}, p_z) = \int_0^1 dv\, Z_\phi(u, v, a^{-1}, \mu, p_z) \phi_\pi(v, \mu) + \mathcal{O}\left(\frac{\Lambda^2}{p_z^2}, \frac{M_\pi^2}{p_z^2}\right).$$

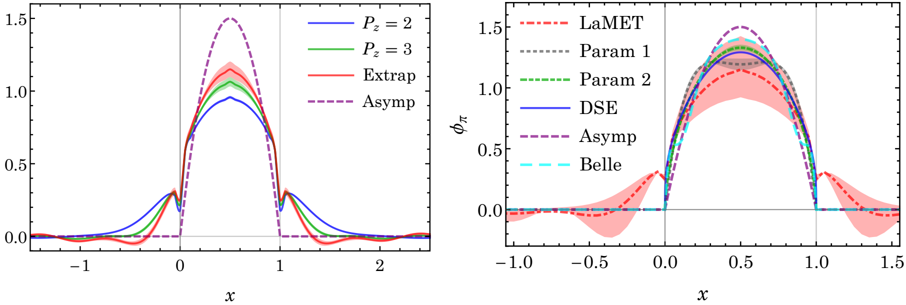
Obvious problems: Integration over all values of z.
Reliability of perturbative matching to the $\overline{\mathrm{MS}}$ scheme at large z.
Z_ϕ has many arguments and is power divergent.
However: [K Orginos, A Radyushkin, J Karpie, S Safeiropoulos, 1706.05373]

The pion DA is an interesting test case for the more involved nucleon (quasi)-PDFs.

First numerical study of the pion "quasi-DA"

$N_f \approx 2+1+1$, $M_\pi = 310\,\text{MeV}$, $a = 0.12\,\text{fm}$, $LM_\pi = 4.5$

[J-H Zhang, J-W Chen, X Ji, L Jin, H-W Lin, 1702.00008]



$x = u$

What do we do differently?

We will **directly** compute the DA in X-space. The method is related to "quasi-PDFs", however, there are two **essential** differences:

- We **do not** employ a Schwinger line $[z/2, -z/2]$, simplifying renormalization.
- Most importantly: we **do not** transform the DA to the longitudinal momentum fraction space.

We compute an object $(z = (0, \vec{z}) \Rightarrow z^2 = -\vec{z}^2 < 0\,, |p \cdot z| = |-\vec{p} \cdot \vec{z}|)$

$$\sqrt{2}\,T(p \cdot z, z^2) = \langle 0|\bar{d}(z/2)\mathbb{1}\, \overbrace{q(z/2)\ \bar{q}(-z/2)}\,\gamma_5 u(-z/2)|\pi^+(p)\rangle\,,$$

where q is an auxiliary field. (Other Γ-Γ-combinations are possible.)
$T(z^2)$ can then be factorized just like

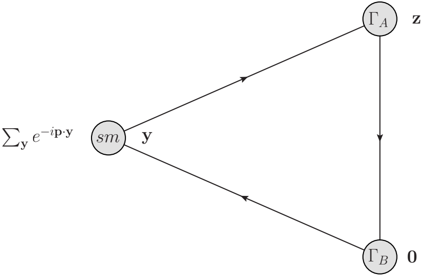
$$F_{\pi\gamma}(Q^2) = \frac{2F_\pi}{3}\int_0^1 du\, H_{q\bar{q}\gamma}(u, \mu, Q^2)\,\phi_\pi(u, \mu) \quad + \quad \text{higher twist.}$$

into a hard, perturbative function and the DA, as long as $2/|z| \gg \Lambda$.
We set $\mu_R = 2/|z|$, which "plays" the role of $|Q|$. ($|z| = \sqrt{-z^2}$)

$T(p\cdot z, z^2)$

Following [VM Braun, D Müller, 0709.1348], we use a light quark propagator for $q(z/2)\bar{q}(-z/2)$. Advantage: Renormalization factor of $\bar{q}\Gamma u$ known. Other suggestions:

- "Static" propagator: Large Energy Effective Theory (LEET) [MJ Dugan, B Grinstein, PLB255(91)583], Large Momentum Effective Theory (LaMET) [X Ji, 1305.1539],
- Scalar propagator: [U Aglietti et al, hep-ph/9806277]; [A Abada et al, hep-ph/0105221],
- Heavy quark propagator: [W Detmold, CJD Lin, hep-lat/0507007].



Tree level result:

$$T(p\cdot z, z^2) = F_\pi \frac{p\cdot z}{2\pi^2 z^4}\Phi_\pi(p\cdot z),$$

with the X-space DA

$$\Phi_\pi(p\cdot z) = \int_0^1 du\, e^{i(u-1/2)p\cdot z}\phi_\pi(u).$$

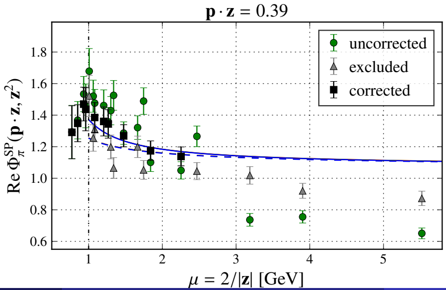
The DA in momentum and in X-space

In practice we compute for large t

$$\frac{T(p\cdot z, z^2)}{F_\pi} = \frac{Z_S Z_P}{Z_A} \frac{\langle 0|[\bar{d}\mathbb{1}q](0,\vec{z}/2)\ [\bar{q}\gamma_5 u](0,-\vec{z}/2)\ O_\pi^\dagger(-t,\vec{p})|0\rangle}{\langle 0|[\bar{d}\gamma_0\gamma_5 u](0,\vec{0})\ O_\pi^\dagger(-t,\vec{p})|0\rangle} iE(\vec{p}),$$

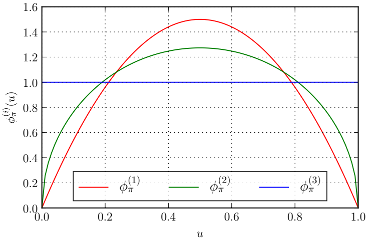
where the lattice currents are renormalized to the $\overline{\mathrm{MS}}$ scheme via $Z_S(\mu_R, g^2)$, $Z_P(\mu_R, g^2)$ and $Z_A(g^2)$ and $\mu_R = 2/|z|$.
We tree-level correct for $\vec{z}$-dependent lattice artefacts:

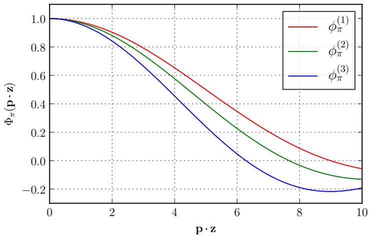
$$T(p\cdot z, z^2) \mapsto T(p\cdot z, z^2)\frac{\mathrm{tr}\,[\not{z}\, G^{\mathrm{tree}}_{\mathrm{cont}}(z)]}{\mathrm{tr}\,[\not{z}\, G^{\mathrm{tree}}_{\mathrm{latt}}(z,a)]} \quad \text{(for the chiral even part)}$$



Three illustrative models of the DA (taken at a scale $\mu_0 = 1\,\text{GeV}$):

$$\phi_\pi^{(1)}(u) = 6u(1-u), \quad \phi_\pi^{(2)}(u) = \frac{8}{\pi}\sqrt{u(1-u)}, \quad \phi_\pi^{(3)}(u) = 1$$





$|z| < 2/\text{GeV} \approx 0.4\,\text{fm}$ is **necessary** for factorization, $\overline{\text{MS}}$ scheme matching!

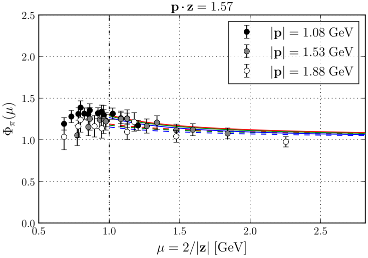
Large $|\vec{p}\cdot\vec{z}|/2$ values are **desirable** as this is conjugate to $\xi = 2u - 1$.

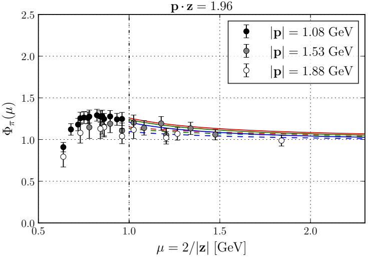
$|z|$ small $\Rightarrow$ large $|\vec{p}|$.

Results: factorization scale dependence of the X-space DA

$N_f = 2$ NP improved Wilson-clover quarks (old QCDSF ensemble)

$a^{-1} \approx 2.76\,\text{GeV},\ M_\pi \approx 290\,\text{MeV},\ L = 32a \approx 3.4/M_\pi.$





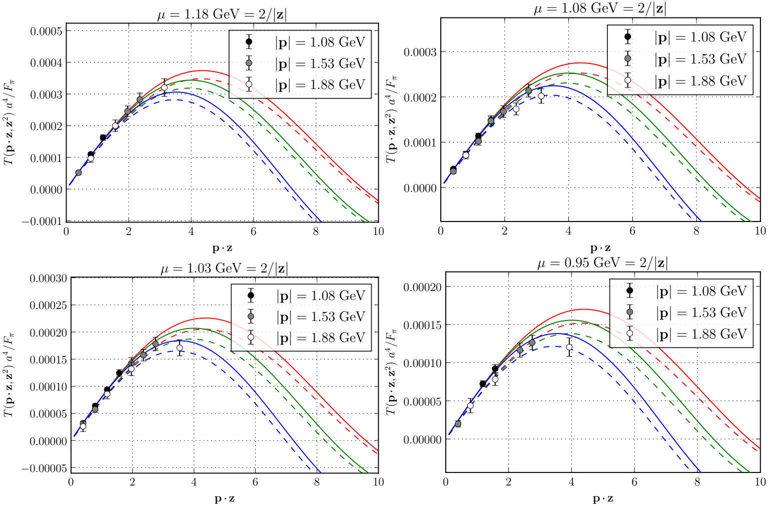
QCD factorization:

$$\frac{T(p\cdot z, z^2)}{F_\pi} = \frac{p\cdot z}{2\pi^2 z^4} \int_0^1 du\, e^{i(u-1/2)p\cdot z}\, H(u,\mu,z^2)\phi_\pi(u,\mu) + T^{\mathrm{HT}}\,,$$

We have computed $H(u,\mu,z^2)$ to order α_s. $T^{\mathrm{HT}} = \mathcal{O}(M_\pi^2 z^2, \Lambda^2 z^2)/z^4$ can be estimated from LCSR. [VA Novikov et al, NPB237(84)525; VM Braun, A Khodjamirian, M Maul, hep-ph/9907495]

Results II: $|\vec{p}\cdot\vec{z}|$ dependence ($\mu = \mu_R = 2/|z|$)

Solid curves include higher twist and mass correction estimates.



Note that $\vec{p}\cdot\vec{z}$ is conjugate to $u - 1/2$. The future: smaller a, larger $|\vec{p}|$.

Outlook

- The future of Gegenbauer/Mellin moments:
  - Continuum limit.
  - So far NP matching to RI'-MOM/SMOM schemes and from there perturbatively to $\overline{\mathrm{MS}}$ at NLO. We explore NNLO matching.
  - Also other meson and baryon DAs.
- "Direct" method:
  - We presented a proof of concept.
  - For $2/|z| \gtrsim 1\,\mathrm{GeV}$ we need $|\vec{p}| \gtrsim 4\,\mathrm{GeV}$ to reach "Ioffe times" $|p \cdot z|$ large enough to discriminate between DA parametrizations $\Rightarrow$ small lattice spacings are necessary.
  - Ideal: smaller $|z|$ to suppress higher twist effects $\Rightarrow$ even higher $|\vec{p}|$.
  - The future: a new algorithm enables smaller statistical errors $\Rightarrow$ small a, higher $|\vec{p}|$ for pion and kaon X-space DA (modulo computer time).
  - Other current-current combinations will provide cross-checks.
  - Already at the limit of the state-of-the art for the pion DA: what about nucleon PDFs?
  - Good news: matching to the $\overline{\mathrm{MS}}$ scheme is easier for currents without derivatives. The matching function requires "only" a continuum calculation $\Rightarrow$ NNLO is possible.