Perturbative calculations for $K \to \pi \pi$ and $\varepsilon' / \varepsilon$

Based on work in collaboration with:

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Lattice Meets Continuum

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– ε'/ε interference of mixing and decay of $K \to \pi\,\pi$

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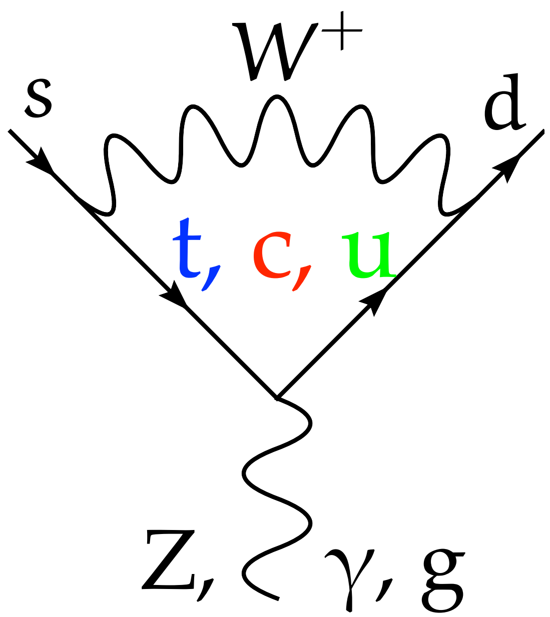
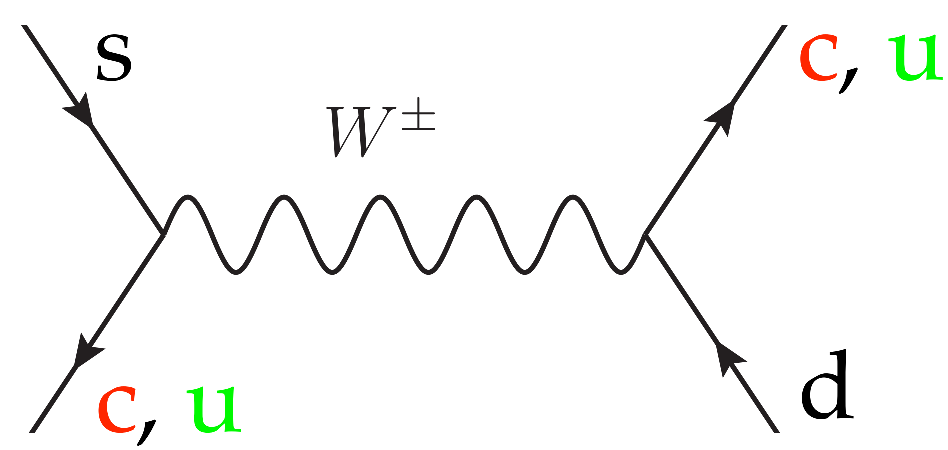
CKM Factors in Kaon physics



Loop K → π π



Tree K → π π

$$\mathrm{Im}V_{ts}^{*}V_{td} = -\mathrm{Im}V_{cs}^{*}V_{cd} = \mathcal{O}(\lambda^{5})$$

$$\mathrm{Re}V_{us}^{*}V_{ud} = -\mathrm{Re}V_{cs}^{*}V_{cd} = \mathcal{O}(\lambda^{1})$$

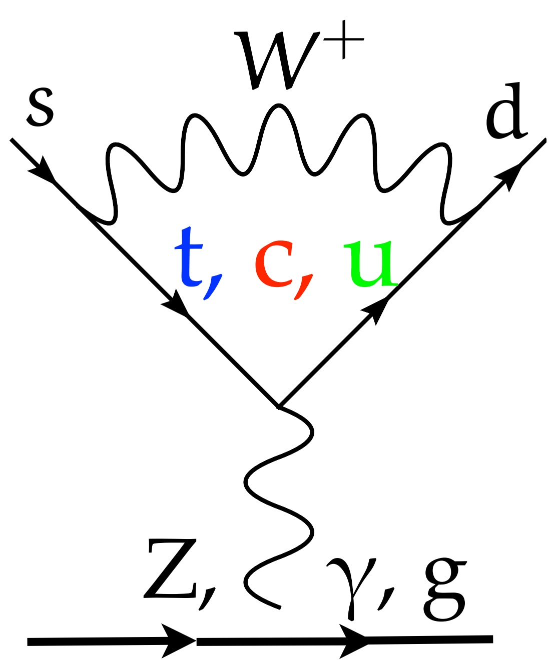
$$\mathrm{Im}V_{us}^{*}V_{ud} = 0$$

$$\mathrm{Re}V_{ts}^{*}V_{td} = \mathcal{O}(\lambda^{5})$$

$$\frac{\mathrm{Im}A_{K}}{\mathrm{Re}A_{K}} = \mathcal{O}(\lambda^{4})\text{x loop}$$

CP violation in decays is highly suppressed (λ^4) for Kaons

Contributions to $\varepsilon' / \varepsilon$



Using the GIM mechanism, we can eliminate: $V_{us}^* V_{ud} \rightarrow - V_{cs}^* V_{cd} - V_{ts}^* V_{td}$

Z-Penguin and Boxes (high virtuality): power expansion in: $A_c - A_u \propto 0 + O(m_c^2 / M_W^2)$

γ/g-Penguin (momentum expansion + e.o.m.): power expansion in: $A_c - A_u \propto O(\mathrm{Log}(m_c^2 / m_u^2))$

In the $K \rightarrow \pi\,\pi$ decay for $\varepsilon' / \varepsilon$ both contribute with opposite signs

Need good theory control that relies on both perturbative and non-perturbative (Lattice QCD) methods

New Physics Sensitivity

While the cancellation requires good theory control:

Additional QCD suppression in SM leads to
increased NP sensitivity
(better Z-Penguin if compared to non-oblique LEP and potentially $B_s \to \mu^+ \mu^-$ for Minimal Flavour Violation)

Additional CP violation suppression (λ^4) increases sensitivity to new sources of Flavour Violation.

$\varepsilon' / \varepsilon$ severely restrict the parameter space of new physics.

But we have to work hard to interpret deviations from the Standard Model.

K Meson Mixing

$\varepsilon' / \varepsilon$: Interference of mixing and decay

Schrödinger type equation for meson mixing

$$i\frac{d}{dt}\begin{pmatrix}|K^0(t)\rangle \\ |\overline{K}^0(t)\rangle\end{pmatrix} = \left[\begin{pmatrix}M_{11} & M_{12} \\ M_{12}^* & M_{11}\end{pmatrix} - \frac{i}{2}\begin{pmatrix}\Gamma_{11} & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_{11}\end{pmatrix}\right]\begin{pmatrix}|K^0(t)\rangle \\ |\overline{K}^0(t)\rangle\end{pmatrix}$$

Diagonalise

$$|K_S\rangle = p|K^0\rangle + q|\overline{K}^0\rangle$$
$$|K_L\rangle = p|K^0\rangle - q|\overline{K}^0\rangle$$

M_{12} from $\Delta_s = 2$ Box $\longleftrightarrow$ Electroweak process

$\Gamma_{12} \longleftrightarrow \Delta\Gamma$ maximal and $\Delta I = 1/2$ saturates $\Gamma_{12} = A_0 \overline{A}_0$

CP violation in Kaons

CP violation in mixing, interference & decay → non-zero

$$\eta_{+-} = \frac{\langle \pi^+\pi^- | K_L^0 \rangle}{\langle \pi^+\pi^- | K_S^0 \rangle} \qquad \eta_{00} = \frac{\langle \pi^0\pi^0 | K_L^0 \rangle}{\langle \pi^0\pi^0 | K_S^0 \rangle}$$

Only CP violation in mixing (Re ε), interference of mixing and decay (Im ε, Im ε') and direct CP violation (Re ε')

$$\epsilon_K = (\eta_{00} + 2\eta_{+-})/3 \qquad \epsilon' = (\eta_{+-} - \eta_{00})/3$$

Using: $\lambda_{ij} = \frac{q}{p}\frac{\langle \pi^i\pi^j | \bar{K}^0 \rangle}{\langle \pi^i\pi^j | K^0 \rangle}$ and $|1 - \lambda_{ij}| \ll 1$

$$\boxed{\epsilon' \approx \frac{1}{6}(\lambda_{00} - \lambda_{+-})} + \frac{1}{12}(\lambda_{00} - \lambda_{+-})(2 - \lambda_{00} - \lambda_{+-}) + \ldots$$

Formula for ε' / ε

a_0, a_2 & a_2^+ from experiment

[Cirigliano, et.al. `11]

a_0 & a_2: isospin amplitudes for isospin conservation

$$\langle \pi^0 \pi^0 | K^0 \rangle = a_0 e^{i\chi_0} + a_2 e^{i\chi_2} / \sqrt{2}$$

$$\langle \pi^+ \pi^- | K^0 \rangle = a_0 e^{i\chi_0} - a_2 e^{i\chi_2} \sqrt{2}$$

$$\langle \pi^+ \pi^0 | K^+ \rangle = 3 a_2^+ e^{i\chi_2^+} / 2$$

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Current theory gives us only: $A_I = \langle (\pi\pi)_I | \mathcal{H}_{\rm eff} | K \rangle$

Normalise to K^+ decay (ω_+, a) and ε_K ,
expand in A_2/A_0 and CP violation:

Formula for ε' / ε

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Current theory gives us only: $A_I = \langle (\pi\pi)_I | \mathcal{H}_{\text{eff}} | K \rangle$

Normalise to K^+ decay (ω_+, a) and ε_K ,
expand in A_2 / A_0 and CP violation:

$$\text{Re}\left(\frac{\epsilon'}{\epsilon}\right) \simeq \frac{\epsilon'}{\epsilon} = -\frac{\omega_+}{\sqrt{2}\,|\epsilon_K|}\left[\frac{\text{Im}A_0}{\text{Re}A_0}\,(1 - \hat{\Omega}_{\text{eff}}) - \frac{1}{a}\,\frac{\text{Im}A_2}{\text{Re}A_2}\right]$$

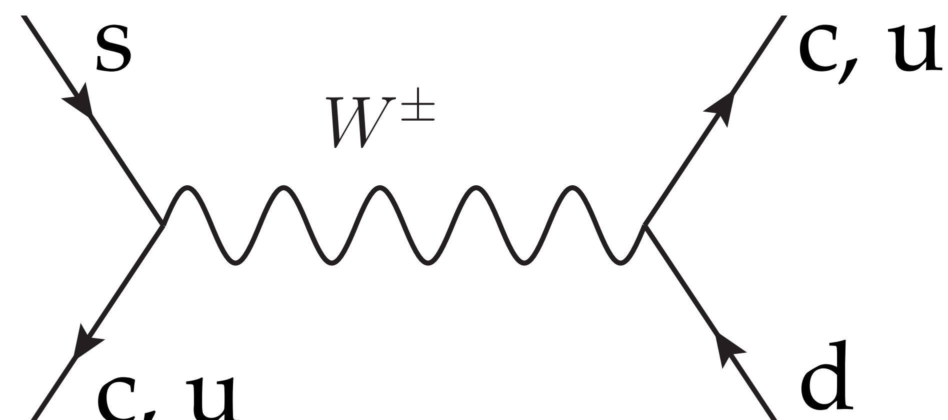
[Buras, MG, Jäger, Jamin `15]

Adjusted to keep electroweak penguins in Im A_0 [Cirigliano, et.al. `11]

Current-Current & CKM

Study Unitarity & CKM Elements to get Im A_I & Re A_I

We use unitarity to eliminate $V_{cs}^* V_{cd} = -V_{us}^* V_{ud} - V_{ts}^* V_{td} Q_2^c$



Current-current interactions:
Two contributions if $\mu > m_c$.

($\propto V_{ts}^* V_{td}$ and $\propto V_{us}^* V_{ud}$)

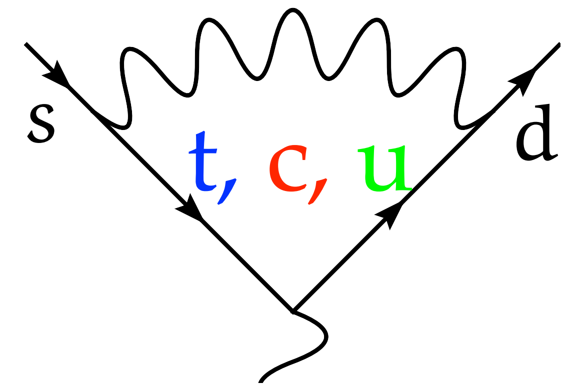
$$V_{us}^* V_{ud} Q_{1/2}^u + V_{cs}^* V_{cd} Q_{1/2}^c \to$$

$$V_{us}^* V_{ud} (Q_{1/2}^u - Q_{1/2}^c) - V_{ts}^* V_{td} Q_{1/2}^c$$

For $\mu < m_c$: $V_{ts}^* V_{td}$ is absent: $V_{us}^* V_{ud} Q_{1/2}^u$

Penguin & CKM

Penguins: $f(m_u) - f(m_c) = 0$:
Only $V_{ts}^* V_{td}$ contribution



$$\left\{V_{us}^* V_{ud} f(m_u) + V_{cs}^* V_{cd} f(m_c) + V_{ts}^* V_{td} f(m_t)\right\} Q_{\text{Penguin}} \to$$

$$\left\{V_{us}^* V_{ud}\left[f(m_u) - f(m_c)\right] + V_{ts}^* V_{td}\left[f(m_t) - f(m_c)\right]\right\} Q_{\text{Penguin}}$$

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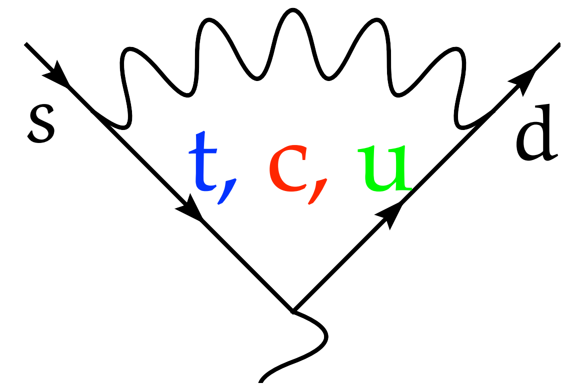

$$\{V_{us}^* V_{ud} f(m_u) + V_{cs}^* V_{cd} f(m_c) + V_{ts}^* V_{td} f(m_t)\} Q_{\text{Penguin}} \to$$
$$\{V_{us}^* V_{ud} [f(m_u) - f(m_c)] + V_{ts}^* V_{td} [f(m_t) - f(m_c)]\} Q_{\text{Penguin}}$$

$\mu > m_c$: $V_{ts}^* V_{td} Q^c_{1/2}$ mixes into $V_{ts}^* V_{td} Q_{\text{Penguin}}$ (like usual).

$\mu > m_c$: $V_{us}^* V_{ud} (Q^u_{1/2} - Q^c_{1/2})$ does not mix into Q_{Penguin} .

$\mu < m_c$: Match $V_{ts}^* V_{td} Q^c_{1/2}$ onto $V_{ts}^* V_{td} Q_{\text{Penguin}}$

→ CP violation from Q_{Penguin}

→ CP conserving from $Q^u_{1/2}$ (plus small Q_{Penguin})

Effective Hamiltonian

Currently we use the effective Hamiltonian below the charm:

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}}\, V_{ud}V^*_{us} \sum_{i=1}^{10} \big(z_i(\mu) + \tau\; y_i(\mu)\big)\, Q_i(\mu)\,, \qquad \tau \equiv -\,\frac{V_{td}V^*_{ts}}{V_{ud}V^*_{us}}$$

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current-current

QCD &

electroweak

penguins

$$Q_{1,2/\pm} = (\bar{s}_i u_j)_{V-A} \, (\bar{u}_k d_l)_{V-A}$$

$$Q_{3,\ldots,6} = (\bar{s}_i d_j)_{V-A} \sum_{q=u,d,s} (\bar{q}_k q_l)_{V\pm A}$$

$$Q_{7,\ldots,10} = (\bar{s}_i d_j)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}_k q_l)_{V\pm A}$$

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current-current QCD & electroweak penguins

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We have z_i & y_i at NLO [Buras et.al., Ciuchini et. al. `92 `93]

And now also a Lattice QCD calculation of: $\langle(\pi\pi)_I\,|\,Q_i\,|\,K\rangle = \langle Q_i\rangle_I$

by RBC-UKQCD [Blum et. al., Bai et. al. `15]

Operator Relations

$$Q_{3,\ldots,6} = (\bar{s}_i d_j)_{V-A} \sum_{q=u,d,s} (\bar{q}_k q_l)_{V\pm A}$$

$$Q_{7,\ldots,10} = (\bar{s}_i d_j)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}_k q_l)_{V\pm A}$$

3-Flavour Fierz identities:

$Q_4 = Q_3 + Q_2 - Q_1$

$Q_9 = 3/2\, Q_1 - Q_3$

$Q_{10} = Q_2 + Q_1 - Q_3$

Isospin Symmetry:

$\langle Q_3 \rangle_2 = \langle Q_4 \rangle_2 = 0$

All matrix elements $\langle Q_1 \rangle_2$, $\langle Q_2 \rangle_2$, $\langle Q_9 \rangle_2$, $\langle Q_{10} \rangle_2$ are proportional.

Im A_2 / Re A_2 – (V-A)x(V-A)

A_2 only contributes in the ratio Im A_2/Re A_2

Let us first consider only (V-A)x(V-A) operators:

$$Q_1 = (\bar{s}_\alpha u_\beta)_{V-A}\,(\bar{u}_\beta d_\alpha)_{V-A} \qquad Q_2 = (\bar{s}u)_{V-A}\,(\bar{u}d)_{V-A}$$

$$Q_9 = \frac{3}{2}\,(\bar{s}d)_{V-A}\sum_{q=u,d,s,c,b} e_q\,(\bar{q}q)_{V-A} \qquad Q_{10} = \frac{3}{2}\,(\bar{s}_\alpha d_\beta)_{V-A}\sum_{q=u,d,s,c,b} e_q\,(\bar{q}_\beta q_\alpha)_{V-A}$$

Isospin limit: $2 \langle Q_9 \rangle_2 = 2 \langle Q_{10} \rangle_2 = 3 \langle Q_1 \rangle_2 = 3 \langle Q_2 \rangle_2$

Re A_2: $(z_1+z_2)\langle Q_1+Q_2\rangle_2 = z_+\langle Q_+\rangle_2$ Im A_2: $y_9\langle Q_9\rangle_2 + y_{10}\langle Q_{10}\rangle_2$

$$\left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2}\right)_{V-A} = \mathrm{Im}\tau\,\frac{3(y_9+y_{10})}{2z_+}\,, \qquad \tau = \frac{V_{ts}^* V_{td}}{V_{us}^* V_{ud}}$$

Im A_0 / Re A_0 – (V-A)x(V-A)

More operators contribute to Im A_0 / Re A_0

$$\mathrm{Re}A_0 = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \left(z_+ \langle Q_+ \rangle_0 + z_- \langle Q_- \rangle_0 \right)$$

Fierz relations for (V-A)x(V-A) give, e.g.: $\langle Q_4 \rangle_0 = \langle Q_3 \rangle_0 + 2 \langle Q_- \rangle_0$

$$\left(\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} \right)_{V-A} = \mathrm{Im}\tau \, \frac{2 y_4}{(1+q) z_-} + \mathcal{O}(p_3)$$

Is only a function of Wilson coefficients and of the ratio

$$q = (z_+(\mu) \langle Q_+(\mu) \rangle_0) / (z_-(\mu) \langle Q_-(\mu) \rangle_0)$$

Expression with $p_3 = \langle Q_3 \rangle_0 / \langle Q_4 \rangle_0$ and EW penguins given in [Buras, MG, Jäger & Jamin `15]

(V-A)x(V+A) Contributions

Q_6 & Q_8 give the leading contribution to
$\mathrm{Im}A_0$ & $\mathrm{Im}A_2$ respectively

$$\left(\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right)_6 = -\frac{G_F}{\sqrt{2}}\,\mathrm{Im}\lambda_t\, y_6\, \frac{\langle Q_6\rangle_0}{\mathrm{Re}A_0}$$

$$\left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2}\right)_8 = -\frac{G_F}{\sqrt{2}}\,\mathrm{Im}\lambda_t\, y_8^{\mathrm{eff}}\, \frac{\langle Q_8\rangle_2}{\mathrm{Re}A_2}$$

Here: Take $\mathrm{Re}\,A_0$ from data

One can re-express $\langle Q_6\rangle_0$ & $\langle Q_8\rangle_2$ in terms of B_6 & B_8

Prediction for ε‘ / ε

I=2 Similarly for (V-A)x(V-A):

I=0 (V-A)x(V-A) I=2 (V-A)x(V-A)

$$\frac{\varepsilon'}{\varepsilon} = 10^{-4}\left[\frac{\mathrm{Im}\lambda_\mathrm{t}}{1.4\cdot 10^{-4}}\right]\left[a\left(1-\hat{\Omega}_\mathrm{eff}\right)\left(-4.1(8)+24.7\,B_6^{(1/2)}\right)+1.2(1)-10.4\,B_8^{(3/2)}\right]$$

(V-A)x(V+A) Matrix elements $B_6=0.57(19)$ and $B_8=0.76(5)$

from Lattice QCD [Blum et. al., Bai et. al. `15]

$$\left(\frac{\epsilon'}{\epsilon}\right)_\mathrm{SM} = 1.9(4.5)\times 10^{-4}$$

2.9 σ difference

$$\left(\frac{\epsilon'}{\epsilon}\right)_\mathrm{exp} = 16.6(2.3)\times 10^{-4}$$

Similar findings

[Kitahara, Nierste, Tremper 1607.06727]

quantity	error on ε'/ε
$B_6^{(1/2)}$	4.1
NNLO	1.6
$\hat{\Omega}_\mathrm{eff}$	0.7
p_3	0.6
$B_8^{(3/2)}$	0.5
p_5	0.4
$m_s(m_c)$	0.3
$m_t(m_t)$	0.3

NLO vs NNLO

Theory prediction only at NLO at the moment

Convergence at m_c is not clear – should calculate next order

Long term use Lattice QCD

Also the error estimate does not include $O(p^2/m_c^2)$ corrections which for $K \to \pi\,\pi$ are expected to be small

Status of ε'/ε NNLO

Energy	Fields	Order
μ_W	g,γ,W,Z,h, u,d,s,c,b,t	NNLO Q_1-Q_6 & Q_{8g} i) NNLO EW Penguins (traditional Basis) ii)
RGE	γ,g,u,d,s,c,b	NNLO Q_1-Q_6 & Q_{8g} iii)
μ_b	γ,g,u,d,s,c,b	NNLO Q_1-Q_6 iv)
RGE	γ,g,u,d,s,c	NNLO Q_1-Q_6 & Q_{8g} iii)
μ_c	γ,g,u,d,s,c	NLO Q_1-Q_{10} v)
RGE	γ,g,u,d,s	NNLO Q_1-Q_6 & Q8g iii)
$M_{Lattice}$	g,u,d,s	NLO Q_1-Q_{10} (traditional Basis) vi)

i) [Misiak, Bobeth, Urban]
ii) [Gambino,Buras, Haisch]
iii)[Gorbahn, Haisch]
iv)[Gorbahn, Brod]
v) [Buras, Jamin, Lautenbacher]
vi)[Blum et. al., Bai et. al. ‘15]

Factorisation

Traditional the contribution of running ($U(\mu, \mu_0)$) and matching ($M(\mu)$) are combined as:

$$\langle \vec{Q} \rangle^{(3)}(\mu_L) \vec{C}^{(3)}(\mu_L) = \langle \vec{Q} \rangle(\mu_L) U^{(3)}(\mu_L, \mu_c) M^{(34)}(\mu_c) U^{(4)}(\mu_c, \mu_b) M^{(45)}(\mu_b) U^{(5)}(\mu_b, \mu_W) \vec{C}^{(5)}(\mu_W)$$

Alternatively we can also factorise as

$$\begin{aligned} \langle \vec{Q} \rangle^{(3)}(\mu_L) \vec{C}^{(3)}(\mu) = & \langle \vec{Q} \rangle(\mu_L)^{(3)} u^{(3)}(\mu_L) \\ & u^{(3)^{-1}}(\mu_c) M^{(34)}(\mu_c) u^{(4)}(\mu_c) \\ & u^{(4)^{-1}}(\mu_b) M^{(45)}(\mu_b) u^{(5)}(\mu_b) \\ & u^{(5)^{-1}}(\mu_W) \vec{C}^{(5)}(\mu_W) \end{aligned}$$

or write in terms of scheme and scale independent quantities:

$$\langle \vec{Q} \rangle^{(3)}(\mu_L) \vec{C}^{(3)}(\mu) = \langle \hat{\vec{Q}} \rangle^{(3)} \hat{M}^{(34)} \hat{M}^{(45)} \hat{\vec{C}}^{(5)}$$

Schemes

The Matrix elements $<Q_i>_{0/2}^{\text{RI-SMOM}}$ are evaluated on the Lattice and renormalised in the RI-SMOM scheme.

The RI-SMOM renormalisation conditions (off-shell four point functions) make loop calculations very difficult.

Scheme change to MSbar known only at NLO
[Sturm, Lehner `11]

$$<Q_i>(\mu_L) = [T^{(0)} + \alpha_s(\mu_L)\ T^{(1)}(\mu_L)]_{ij} <Q_j>^{\text{RI-SMOM}}$$

At least expect good convergence (at least in the case of the three-point function used for mass renormalisation at NNLO [Gorbahn, Jäger `10] [Alemeda, Sturm `10])

RGI Scheme

Using the $\overline{\text{MS}}$ matrix elements $\langle\vec{Q}\rangle(\mu_L)$ and evolution $u(\mu_L)$ we have, e.g. in the three-flavour theory

$$\langle\vec{Q}\rangle^{(3)}(\mu_L)u^{(3)}(\mu_L) = \langle\hat{\vec{Q}}\rangle^{(3)}$$

or alternatively in terms of RI-SMOM parameters

$$\langle\vec{Q}\rangle^{(3)}_{\text{RI-SMOM}}(\mu_L)u^{(3)}_{\text{RI-SMOM}}(\mu_L) = \langle\hat{\vec{Q}}\rangle^{(3)}\,,$$

which would still be difficult.
But when $\mu \to \infty$ we are less sensitive to the loop correction

$$u^{(3)}_{\text{RI-SMOM}}(\mu) = \left(1 + \frac{\alpha_s(\mu)}{4\pi}J^{(1)} + \dots\right)u_0^{(3)}(\mu) \to \left(\frac{\alpha_s(\mu)}{4\pi}\right)^{\frac{-\gamma_0^T}{2\beta_0}}$$

Still the RGI objects might also be useful for the numerical evaluation:

RGI Numerics

All hatted quantities $\langle \hat{\vec{Q}} \rangle^{(3)}, \hat{M}^{(34)}, \hat{M}^{(45)}$ and $\hat{\vec{C}}^{(5)}$ and also their products

$$\hat{\vec{C}}^{(3)} = \hat{M}^{(34)} \hat{M}^{(45)} \hat{\vec{C}}^{(5)}$$

are formally scheme and scale independent.

The matrix elements $\langle \hat{\vec{Q}} \rangle$ satisfy $d = 4$ Fierz identities.

$\hat{\vec{C}}^{(3)}$ is μ *independent*, but shows *residual* μ *dependence*.

Plot this for the $\hat{y}(\mu_c)$ (the ones $\propto \mathrm{Im}(V_{ts}^{*} V_{td})$):

and for $\hat{z}(\mu_c)$ (relevant for Re A_0 and Re A_2)

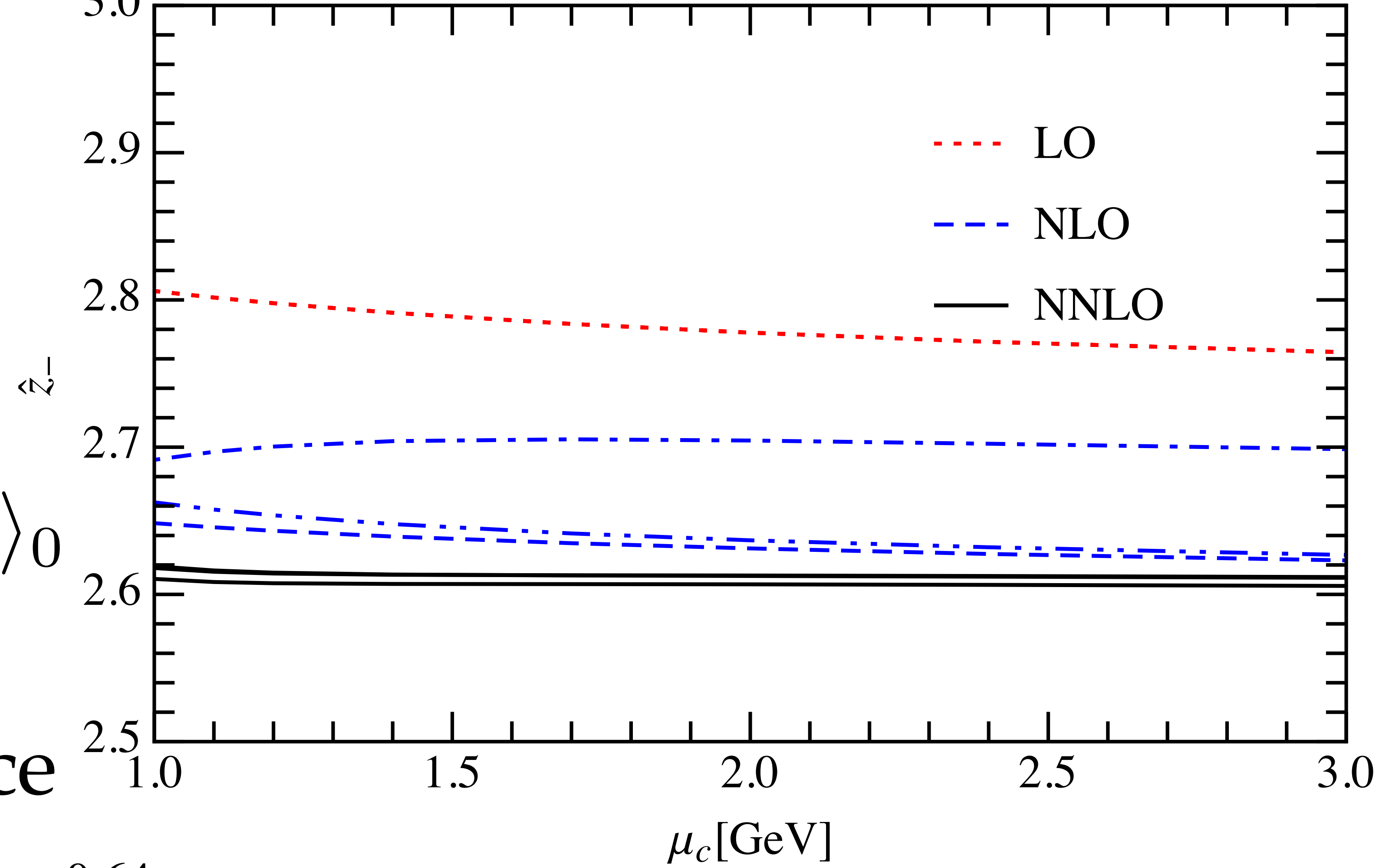
Use different RGE running (numerical or via Λ_{MS})

from $\alpha_s(M_Z)$ at LO, NLO & NNLO

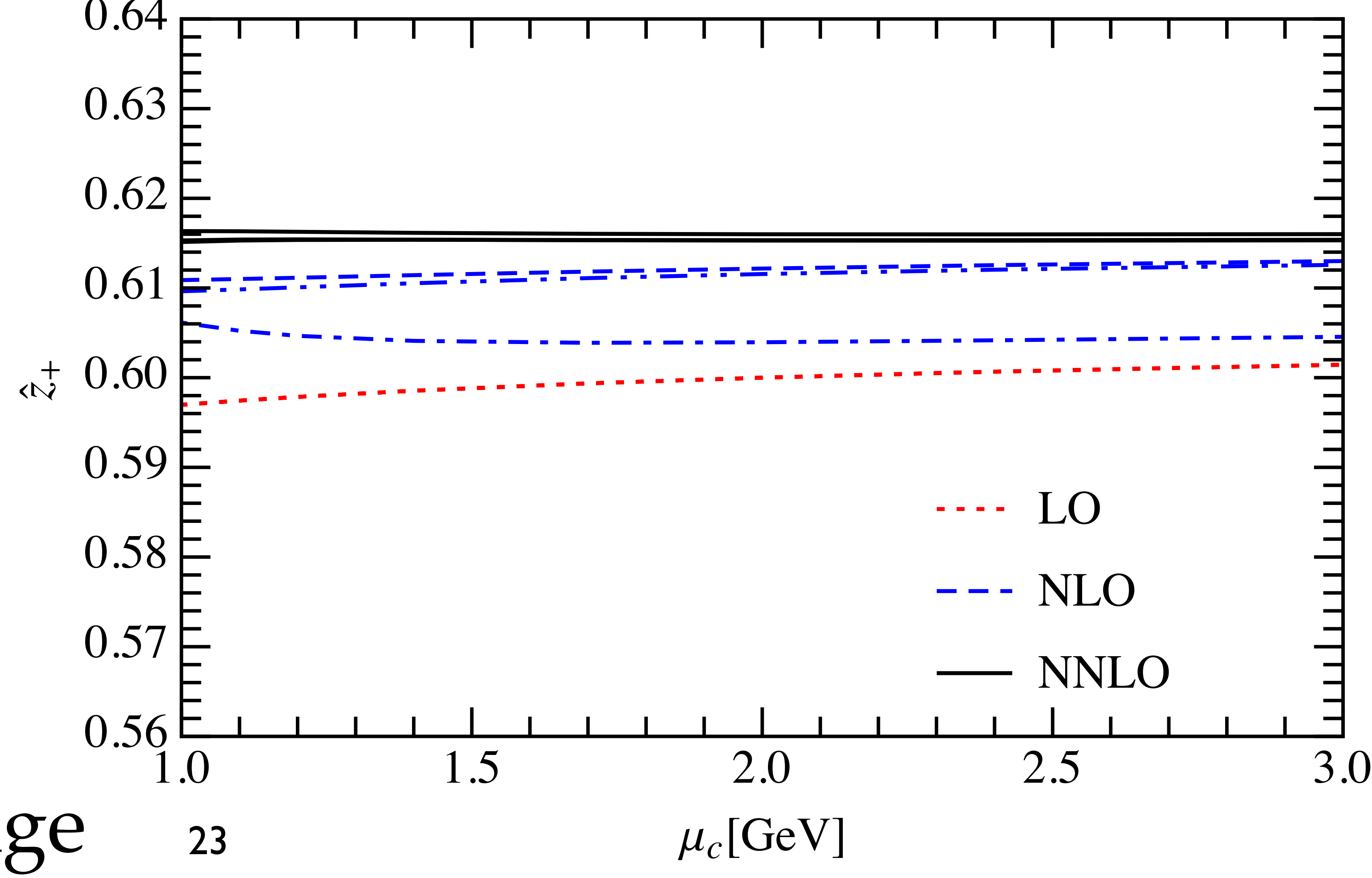
The Real Part of A_0 & A_2 is dominated by z_+ & z_-

$$\mathrm{Re}A_2 = \hat{z}_+ \langle \hat{Q}_+ \rangle_2$$

$$\mathrm{Re}A_0 = \hat{z}_+ \langle \hat{Q}_+ \rangle_0 + \hat{z}_- \langle \hat{Q}_- \rangle_0$$



The residual μ_c dependence reduces order by order



At NLO there is still a dependence on the implementation of α_s Running.

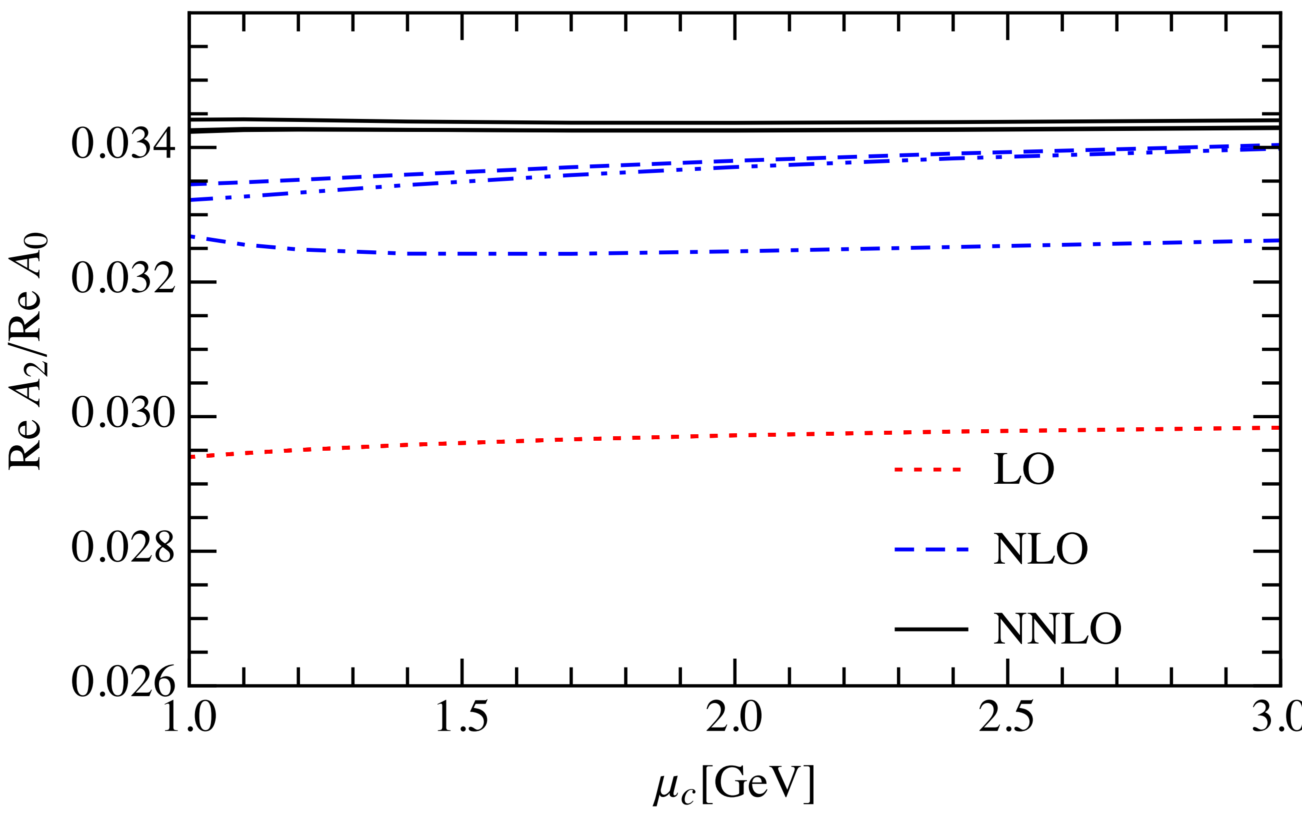
α_s dependence in the RI-SMOM → MSbar change

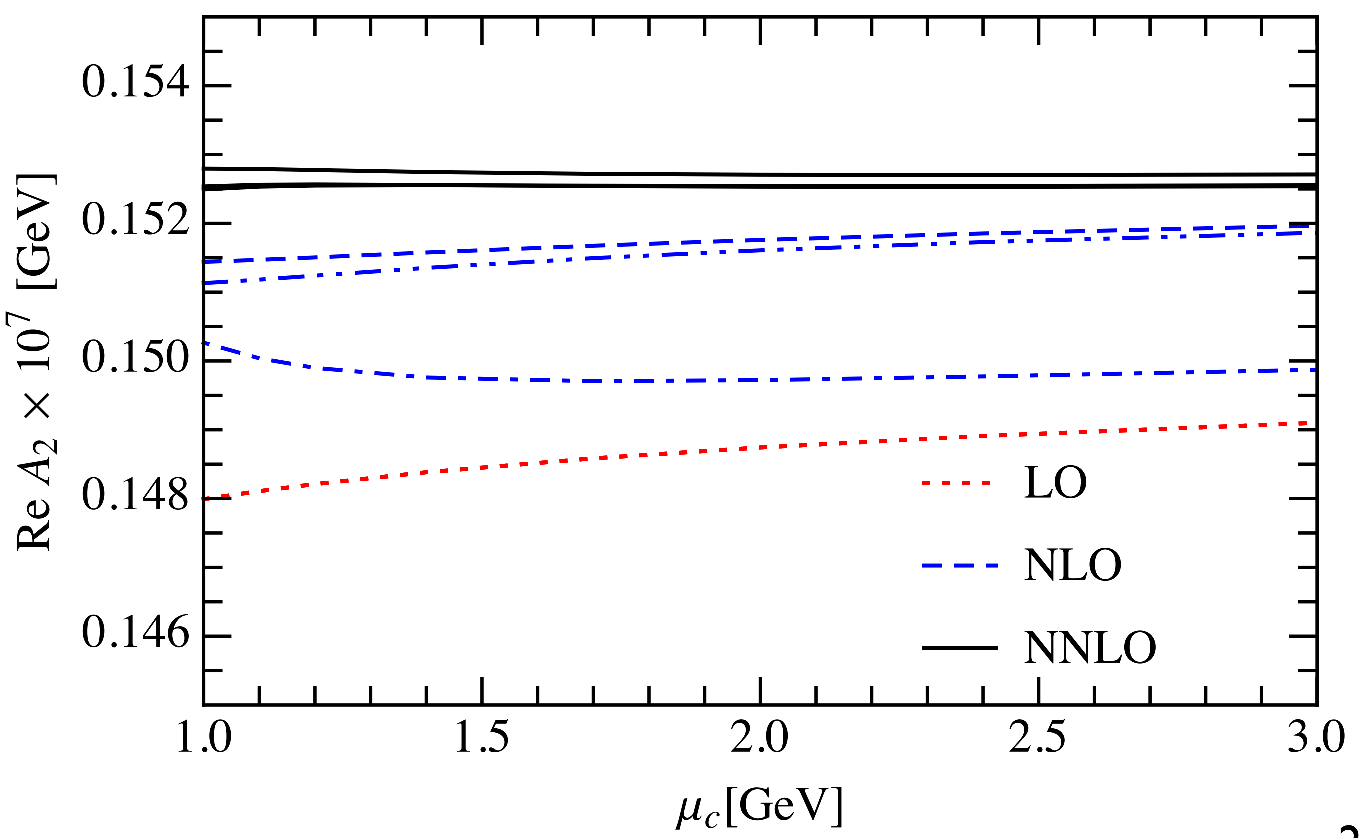
Transform Lattice RISMOM matrix elements to $\hat{q}$ scheme

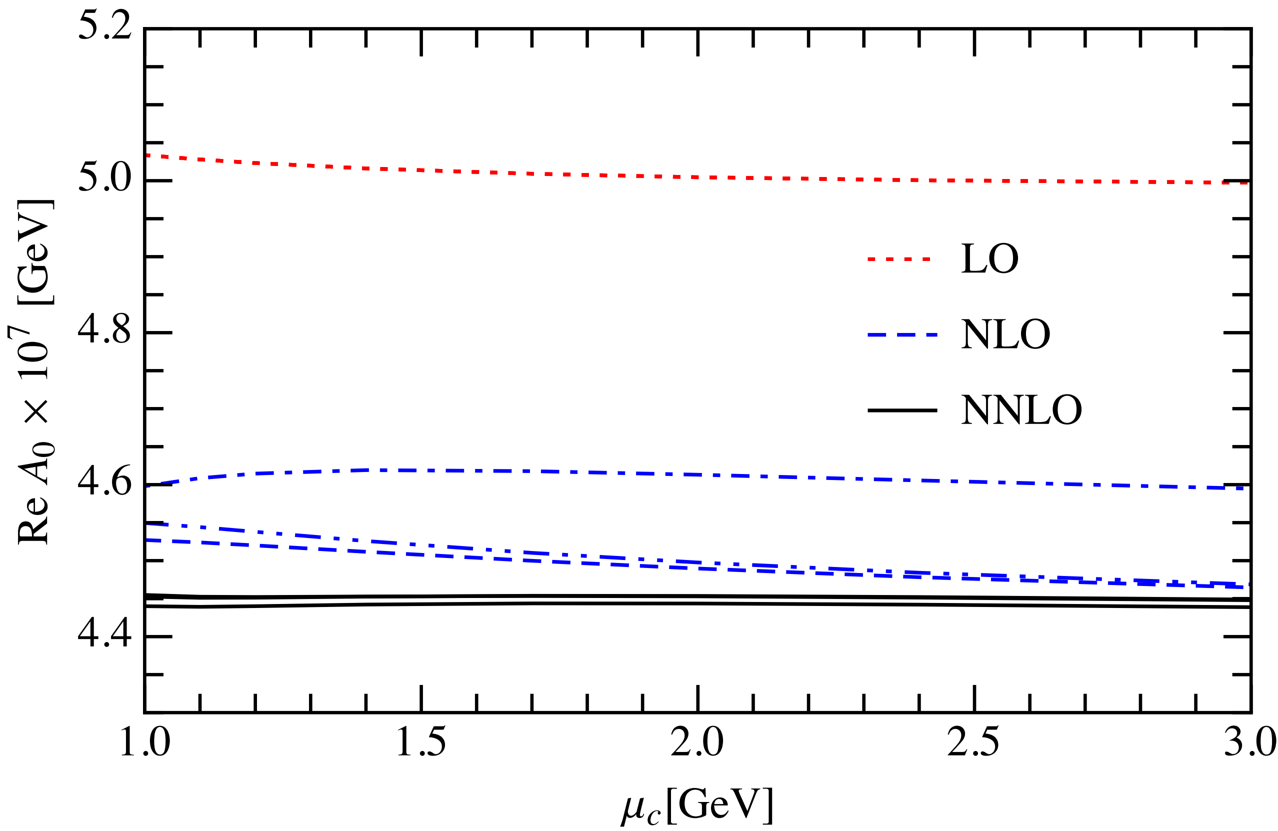
Re $A_0 = 33.2 \times 10^{-8}$ GeV

Re $A_2 = 1.48 \times 10^{-8}$ GeV

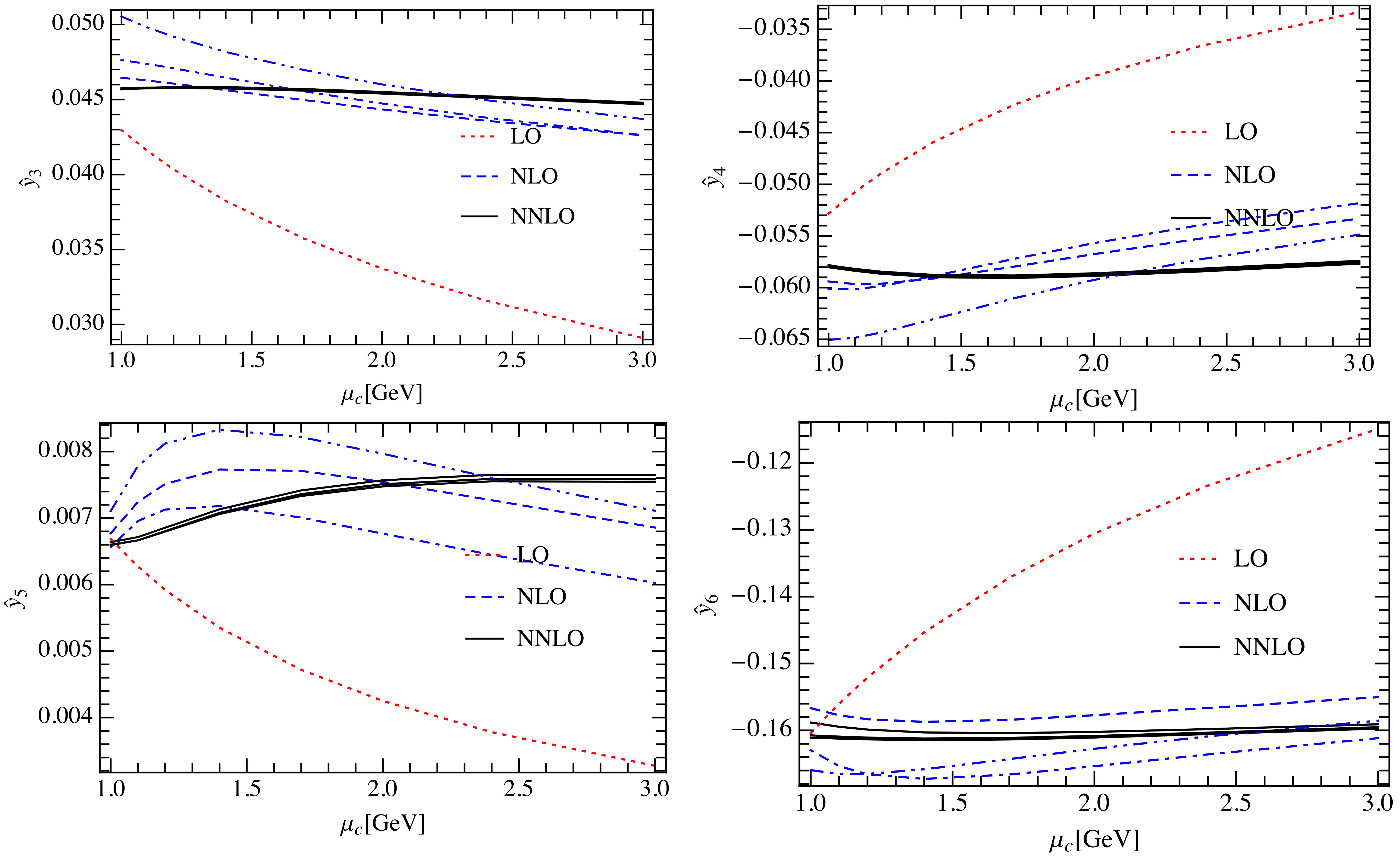
Lattice input to Re A_0 has still 20% / 25% stat / sys. uncertainty

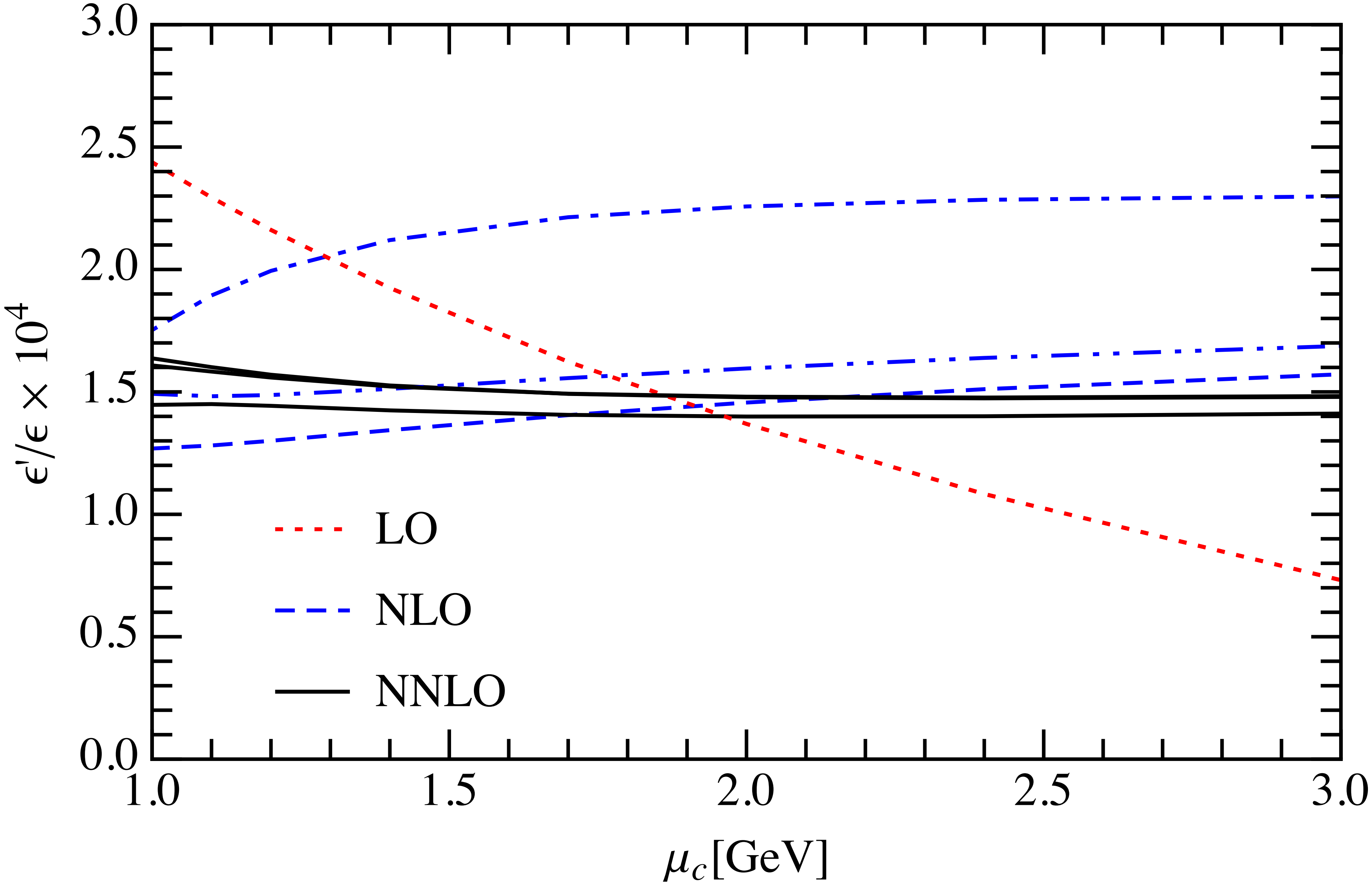






QCD Penguin scale uncertainty is reduced from NLO to NNLO





Plot residual μ_c dependence of the QCD contribution to $\varepsilon' / \varepsilon$

Uncertainty is significantly reduced by going to NNLO

There are still improvements:

e.g. better α_s implementation & better incorporation of subleading corrections – will not change the overall picture

Conclusions

Using Recent Lattice results and proper combination of results → tension in ε'/ε

Previous determination relied only on NLO calculation

NNLO analysis show that theory prediction for QCD Penguins in ε'/ε is under very good control.

Small shift in the Re A_0 & Re A_2.

Extend to EW penguins & Need further input from Lattice QCD.

NNLO Operator Basis

The traditional basis requires the calculation of traces with **γ5** .

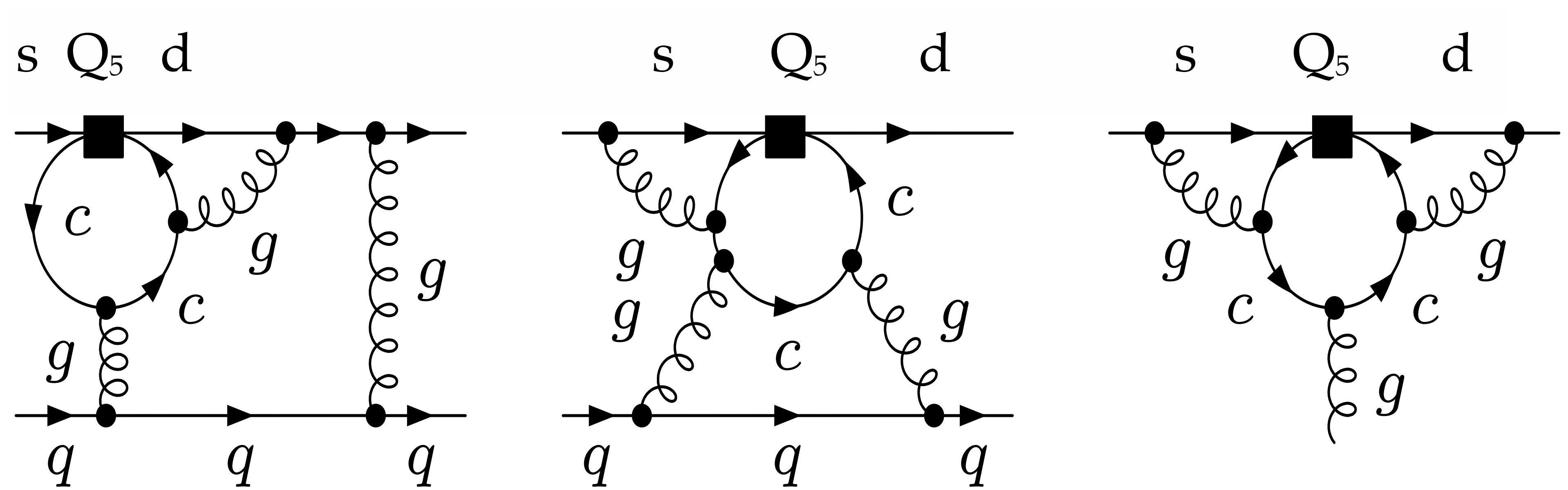
$$\mathcal{O}_{5,6} = (\bar{s}_i d_j)_{\text{V-A}} \sum_{u,d,s} (\bar{q}_k q_l)_{\text{V+A}}$$

Issues with the treatment of the γ_5 in D dimensions

Higher order calculations can be significantly simplified if we use a different operator basis.

$$\mathcal{O}^m_{5,6} = (\bar{s}_i \gamma_\mu \gamma_\nu \gamma_\rho P_L d_j)_{\text{V-A}} \sum_{u,d,s} (\bar{q}_k \gamma^\mu \gamma^\nu \gamma^\rho q_l)$$

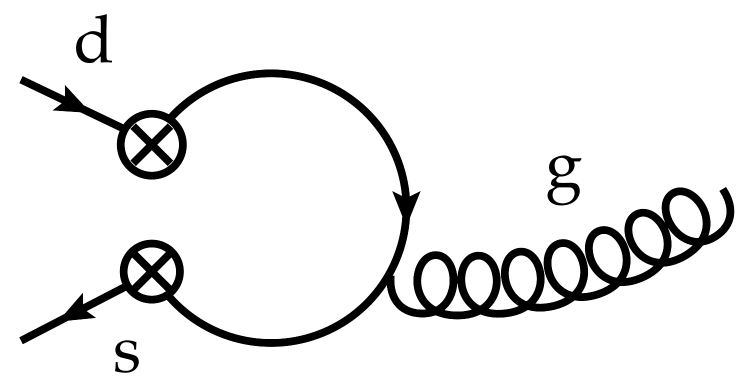
No trace of γ_5



Charm Matching NLO

O_1 & O_2 have the largest Wilson Coefficients.

Only one type of $s \to d$ gluon diagram for O_1 & O_2



We perform an off-shell matching:

expanding in external momentum $O(k^2)$

$$\mathcal{O}_{31} = \tfrac{1}{g}\bar{s}_L\gamma^\mu T^a b_L D^\nu G^a_{\mu\nu} + \mathcal{O}_4$$

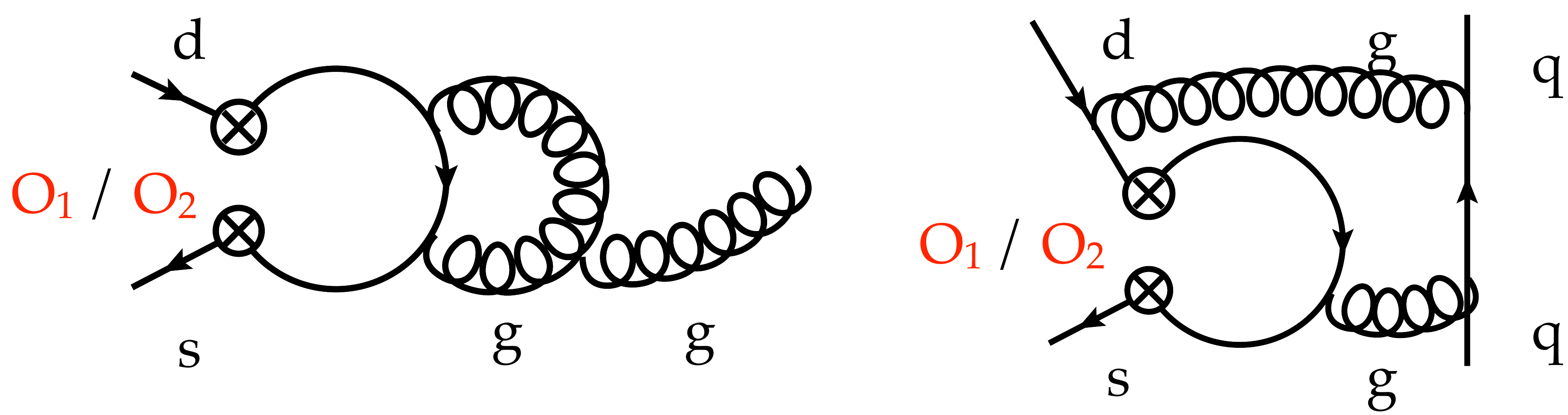
$$\mathcal{O}_4 = (\bar{s}_L\gamma^\mu T^a b_L)\sum_q(\bar{q}\gamma^\mu T^a q)$$

There are no one-light-particle-irreducible diagrams for $s \to d\,\bar{u}\,u$.

No evanescent operators are generated at one-loop.

NNLO Matching

There are Q_1 & Q_2 have the largest Wilson Coefficients.

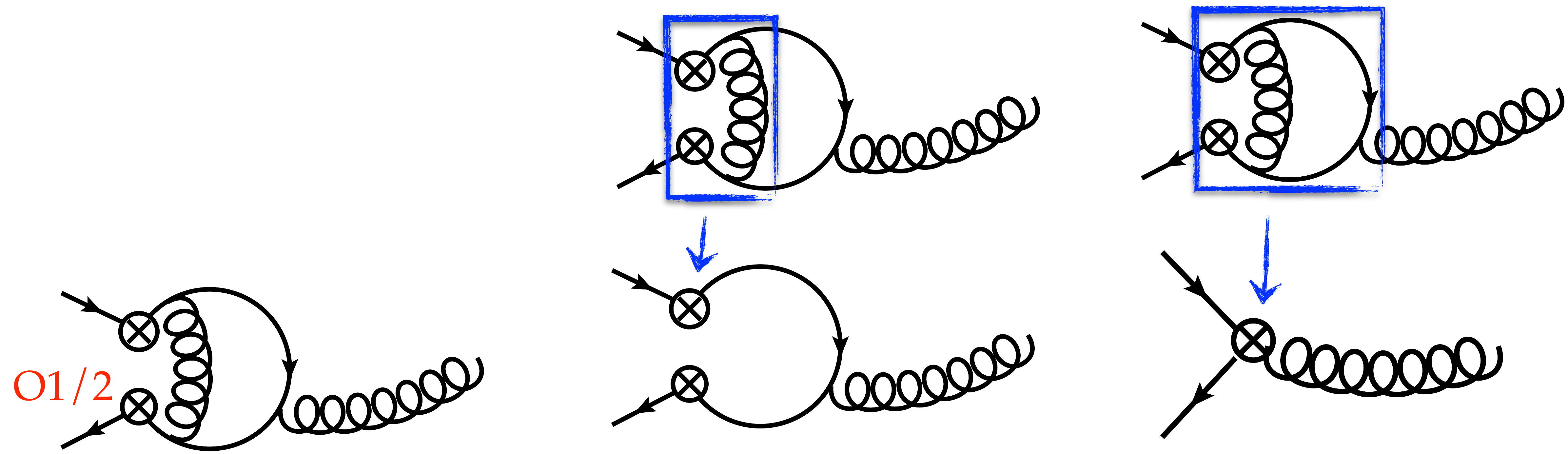


The calculation produces several types of structures,

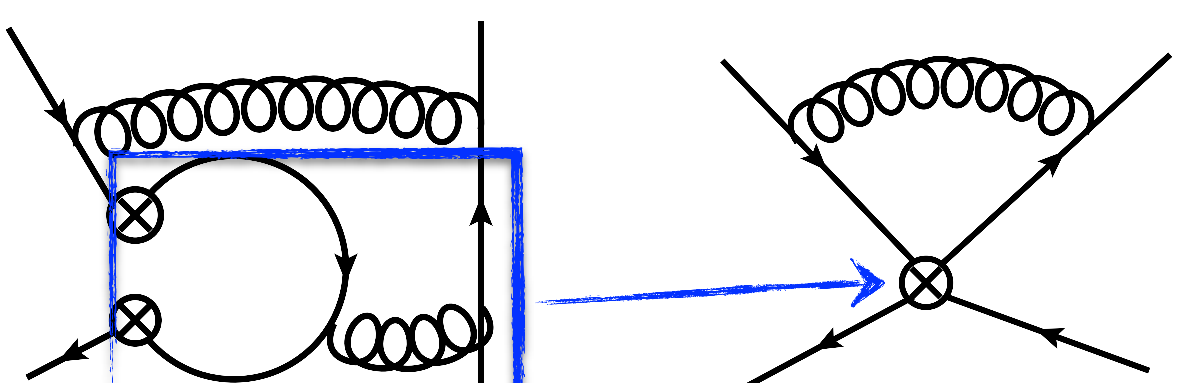
$$(\bar{s}_i \gamma^\mu P_L T^a_{ij} d_j) G^a_\mu k_1^2 \quad (\bar{s}_i \gamma_\nu T^a_{ij} P_L d_j) G^a_\mu k_1^\mu k_2^\nu \quad \ldots$$

– more than operators.

Renormalisation f=4



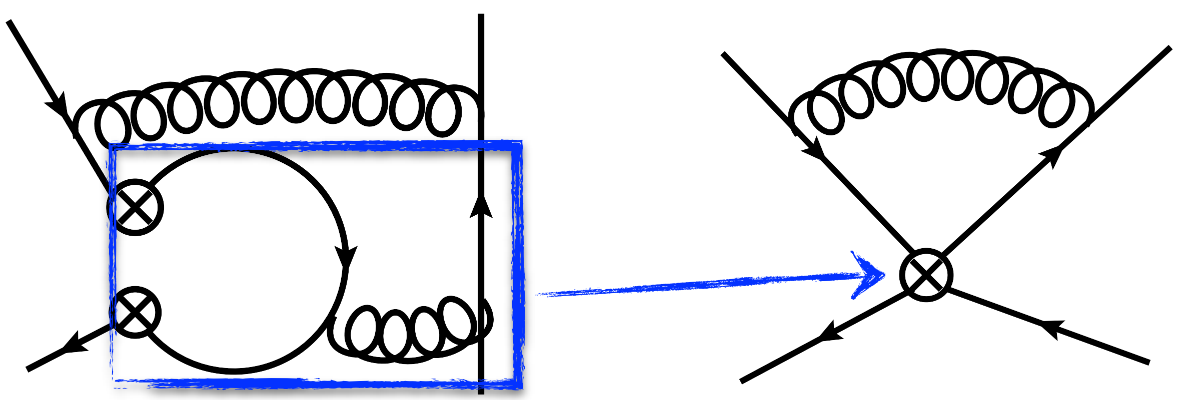
Our procedure: Full (f=4) theory is still divergent after renormalisation.



Counterterm matrix element vanishing for $m_s = m_d = m_u = 0$

Renormalisation f=3

Vanishing f=4 matrix element



Counterterm matrix element

vanishing for $m_s = m_d = m_u = 0$

Will be canceled in f=3 theory by
One-loop matching coefficient × one-loop operator mixing

Above sub-diagram → effective O_4 Wilson Coefficient (f=3)
→ The renormalisation $C_4 Z_{4,i} O_i$ cancels divergence.

$A_{full} = A_{eff}$ results then in finite threshold corrections

Results f=3

$A_{full} = A_{eff}$ results in finite matching.

Additional Check: All results can be projected onto the Physical and EOM vanishing Operator Basis.

The log(μ) dependence cancels analytically.

Note: Evanescent Operators only contribute in f=4 theory at NNLO

How to determine the residual uncertainty?

New Physics Operators

An SU(2)xU(1) invariant operator – written in terms of 2nd and 1st generation doublets S_L & D_L –

$$i(\bar{S}_L \gamma^\mu D_L)(\phi^\dagger \overleftrightarrow{D}_\mu \phi)$$

generates a s-d-Z penguin

$$\rightarrow -v M_Z Z_\mu (\bar{s}_L \gamma^\mu d_L)$$
$$+\text{up-type quarks}$$

Correlated effects in $K_L \rightarrow \pi^0 \bar{\upsilon}\, \upsilon$

Correlation broken if there are contributions from magnetic operators and (from Z' ...)

$$\phi \bar{S}_L \sigma^{\mu\nu} T^a d_R G^a_{\mu\nu} \rightarrow$$
$$v/\sqrt{2}\bar{s}_L \sigma^{\mu\nu} T^a d_R G^a_{\mu\nu}$$

Also Re A_0 could be modified?

New Physics in Re A_0

Suppose there is New Physics in Re A_0: $\quad H = \dfrac{(\mathrm{Re}A_0)_{\mathrm{SM}}}{(\mathrm{Re}A_0)_{\mathrm{EXP}}}$